Survivability considerations are increasingly important as the capacities of optical networks continue to grow, enabling ever greater demand quantities to be carried on a single fiber. Protection and restoration mechanisms are a key aspect in the design of such high-capacity optical networks and can significantly impact not only restoration speeds and switching complexities, but the overall cost and efficiency of a network. In this Special Issue on Protection, Restoration, and Disaster Recovery we need hardly belabor this motivation further. This article is in part based on excerpts and extension of very recent arguments so far only touched on as part of the book [1]. Our specific focus in this article is on network protection against span failures, which are undoubtedly the most frequent form of major outside plant damage to occur in practice. Many readers will already be familiar with synchronous optical network (SONET) rings for span protection. Rings are simple and fast, but it has been found that once deployed they are quite inflexible to changes in demand patterns and very inefficient in terms of total capacity relative to usable working capacity. In contrast, mesh-based survivable networks are increasing in popularity due to their greater flexibility in the face of dynamic demand and much greater efficiency and amenability to automated provisioning processes. The key idea in mesh-based networking, with either span restoration or end-to-end path restoration, is to use the entire network topology to support generalized rerouting of restoration flows. The main benefit of this is much higher levels of protection capacity sharing over nonsimultaneous failure scenarios.

While much attention continues to dwell on the “ring vs. mesh” debate of the last 10 years, the p-cycle concept has sort of “snuck in from left field” and made both prior alternatives somewhat obsolete for transport-level protection (as opposed to service layer protection, where shared mesh path protection remains attractive). We say this because the p-cycle paradigm simultaneously provides both the speed and the structural and switching simplicity of rings, and the much greater capacity efficiency of a mesh network as well as the freedom of shortest path routing of working paths. Indeed, in what follows we show that a Hamiltonian p-cycle defined on what we call a semi-homogeneous network literally realizes $1/(d - 1)$ redundancy (where $d$ is the network average nodal degree). This is most significant because previously $1/(d - 1)$ redundancy has only ever been an unattainable lower bound for span-restorable mesh networks.

1 By span we mean a link in the physical layer graph. A measure of physical capacity such as number of fibers or wavelengths is associated with a span (see [2]).
So what is a p-cycle? A p-cycle is a preconfigured cycle formed in the protection (or spare) capacity of a network. Complete network designs can have many p-cycles, overlapping and complementing each other to provide a complete and efficient solution for protection against span failures. The key to how a p-cycle differs from a conventional ring is the protection of straddling spans that have their endpoints on the p-cycle but are not on the cycle themselves. Straddling spans enjoy two units of capacity protection from each unit capacity p-cycle because restoration paths can be routed in both directions around the cycle in the event of a straddling span failure. This apparently small technical difference actually enables a complete jump from ring-like redundancies (over 100 percent) to mesh-like efficiency (well under 100 percent redundancy). But we still retain the structural simplicity and speed of a ring because only two nodes do any real-time switching and are fully preplanned for such actions [3, 4]. A less obvious but additional separate source of efficiency is that when straddling spans are admitted, demands can take shortest path routes over the graph, as opposed to ring-constrained routing. Figure 1 shows how the protection mechanism works. For a so-called on-cycle failure the action is essentially just that of a bidirectional line switched ring (BLSR); routing demand flows on a failed working span in the opposite direction around the cycle. The same access-to-protection type of switching occurs for a straddling span failure, but as Fig. 1c shows, the p-cycle can restore two units of working capacity.

It has previously been shown that p-cycles are the most efficient preconfigured pattern possible for network protection [5]. If hosted on optical crossconnects (OXCs) they have the added advantage that they can be created and updated on a unit capacity basis without the inherent modularity present in rings. These combined attributes have made p-cycles an option of considerable recent interest and study [3–11].

Two particular points about p-cycles are especially relevant to this work. One is the recognition, first given in [5], that Hamiltonian cycles are of particular interest as p-cycles. A network is said to be Hamiltonian if a single cycle exists that visits all nodes exactly once. If a network is Hamiltonian (and contains suitable working capacity still requiring protection), it permits formation of p-cycles that are especially efficient in terms of the ratio of working capacity protected to the amount of protection capacity used to form the corresponding p-cycle. The high efficiency arises because, for an investment of N unit hops of protection capacity, in a network of S spans, \(N = 2(S - N)\) units of working capacity can be protected, where \(N\) is the number of nodes (and hence, \(S - N\) is the number of spans that have a straddling relationship to the Hamiltonian p-cycle). For example, a 25-node 50-span network could contain a p-cycle that is only 33 percent redundant. In contrast, a ring protects \(N\) hops of working capacity with \(N\) hops of protection.

A concept that applies to any p-cycle is its efficiency score. In [9] the a priori efficiency (AE) of a cycle as a candidate to be a p-cycle is defined as

\[
AE(p) = \sum_{i \in S} x_{p,i} / \sum_{j \in S} \delta_{p,j} \cdot c_j, \tag{1}
\]

where \(x_{p,i}\) is the number of protection paths that p-cycle \(p\) can provide to span \(i\). If the candidate cycle does not either include span \(i\) or have it as a straddler, \(x_{p,i} = 0\). If the cycle includes the span in an on-cycle relationship, \(x_{p,i} = 1\), and \(x_{p,i} = 2\) if a straddling relationship exists. \(\delta_{p,j}\) is 1 if p-cycle \(p\) crosses span \(j\), and 0 otherwise. The cost of constructing a cycle on span \(j\) is \(c_j\), which has a value of 1 when considering only hop counts, but can more generally be distance-weighted or based on any other cost measurement. The measure is called the a priori efficiency because in the context of [9] the relative merit of candidate cycles is assessed in advance of knowing whether working capacity will be present to exploit the potential protection relationships a cycle has from its relation to the topology alone. Here, unless stated otherwise, our context is such that \(AE\) will reflect actual efficiency, but to avoid confusion we stick with the terminology \(AE\). A Hamiltonian cycle has, by definition, the minimum number of on-cycle spans and at the same time a maximum number of straddling spans, and is frequently (but not always) the most efficient candidate to create a p-cycle.

Homogeneous Networks

A separate point of background, not initially related to p-cycles, is the recent appearance of papers on routing, wavelength assignment, and protection design that assume only two working fibers per span and an identical number of wavelength channels on each span. This is the paradigm in which generalized loopback [12] and Ellinas’ concept of orientable cycle double covers [13] are both proposed. It is also the paradigm under which p-cycles on Hamiltonian graphs were recently considered by Huang and Copeland [7]. It is not altogether clear if such “flat capacity” is just a simplifying assumption or meant to reflect a view that assumes that with DWDM one can support almost any arbitrarily high number of channels per fiber. One can then imagine that in small or moderate-sized networks, it may be that all required capacity can be supported on a single pair of fibers per span, one for each direction. This gives rise to the paradigm of homogenous networks, or flat capacity networks, where every span of the net-

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2 While terms such as hops and capacity units permit a general logical discussion of networks, these can be either fibers or lightwave channels in the context of a dense wavelength-division multiplexing (DWDM)-based optical network.
work is assumed to have equal (and essentially unlimited or fully adequate) capacity for any anticipated demand pattern. Under the homogenous network assumption, network planning problems become uncapacitated versions of the more general capacitated design problems that have been common in research to date. They collapse the view of the network to that of a simple graph, not a capacitated multigraph. It is hard to judge, at present, how practical or economically realistic this might be because the implication is that a homogeneous network must implement a DWDM transmission system on every fiber that is set by the requirements of the single highest channel requirement of any span under the actual demand pattern. Even if the demand matrix is fully uniform, the load on each span is highly nonuniform. So the homogeneous view inherently assumes that, for example, two-fiber systems capable of supporting 512 channels per fiber would not cost significantly more than two pairs of fibers at 256 channels each for the most highly loaded span and mixtures of systems of 256, 64, 32, or 16 channels (as needed) on multiple fiber pairs on other spans. To prove-in economically a homogenous network thus seems to assume the most advanced and cost-reduced future DWDM technology. This being said, it may well come to pass, and in any case the supposition of homogeneity reduces future DWDM technology. From [4] serves in the general capacitated case (and of course also applies in the homogeneous capacity case).

We introduce it to illustrate certain properties of the capacitated design problem; later we rely on its solutions to also test hypotheses about the homogeneous case.

**The Role of Hamiltonians in Capacitated (Non-Homogeneous) p-Cycle Design**

As a final preliminary we need to touch on the optimal design of p-cycles in the general case where the capacity to be protected may be different on each span. The following model (from [4]) serves in the general capacitated case (and of course also still applies in the homogeneous capacity case). We introduce it to illustrate certain properties of the capacitated design problem; later we rely on its solutions to also test hypotheses about the homogeneous case.

**Capacitated p-Cycle Design (CPD):**

minimize: \[
\sum_{j \in S} c_j \cdot s_j
\]  
subject to: \[
s_j = \sum_{p \in P} \delta_{p,j} \cdot n_p \quad \forall j \in S
\]

\[
w_i \leq \sum_{p \in P} x_{p,i} \cdot n_p \quad \forall i \in S
\]

All \( n_p \) and \( s_j \) variables are nonnegative integers. The model minimizes the cost-weighted sum of all protection capacity placed. Equation 3 generates the spare capacity required in terms of the number of unit-capacity instances of the \( n_p \)th candidate cycle chosen for the design. \( \delta_{p,j} \) is an input parameter (not a variable), equal to 1 if the \( p \)th candidate cycle overlies span \( j \), 0 otherwise. Equation 4 makes sure that the set of chosen p-cycles (i.e., those with \( n_p \geq 1 \)) provide enough protection relationships, of either an on-cycle \( (x_{p,i} = 1) \) or a straddling span nature \( (x_{p,i} = 2) \) to protect all working capacity \( (w_i) \) on each span. All demands are first shortest path routed (or routed any other way desired) to generate the \( w_i \) capacities. The set of candidate p-cycles is found by a preprocessing program that also generates the \( \delta_{p,j} \) and \( x_{p,i} \) parameters which encode the layout of each cycle and the protection relationships it has to each network span. Although formally this is an integer linear programming (ILP) problem (which is in the class of NP-hard problems), it is in fact a relatively simple model to formulate and solves to optimality very quickly. Highly effective cycle preselection strategies (based on AE scores) are also available to reduce problem sizes on large network cases [9].

Let us now inspect some proper-

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**Figure 2.** In the optimal capacitated design problem many p-cycles are also Hamiltonians (adapted from [1]).
ties of strictly optimal p-cycle designs obtained with this model in a network that is differentially capacitated. Figure 2a shows a test network of 23 spans and 13 nodes which is Hamiltonian and has the working capacity counts (wi) that arise from least-hop routing of one demand unit per origin-destination (O-D) pair over the graph. This network has 410 distinct simple cycles to consider in its optimal design and was solved to optimality with CPLEX 7.1 in 0.63 s. The optimal p-cycle design is detailed in Fig. 2b–f for the case where cost is unit hop (i.e., all cij = 1). Seven distinct unit-capacity p-cycles exist in the solution, five of which are Hamiltonians. The sixth (Fig. 2e) is very nearly a Hamiltonian as well. The x1 and x2 notation in Fig. 2 denotes the number of unit-capacity p-cycles instantiated on each of the solution cycles shown. This sample result illustrates an important general point: the optimal solution to a capacitated problem involves a number of individual p-cycles, many of which may tend individually to be Hamiltonian, but in addition includes p-cycles that are not based on Hamiltonian cycles.

The optimal design in Fig. 2 has a logical redundancy of 53.8 percent. In the corresponding distance-weighted optimal design (where cij is proportional to the length of edge j as drawn), 13 unit-capacity p-cycles are employed on seven distinct cycles, but only two p-cycles are also Hamiltonians. The distance-weighted redundancy is 65.9 percent. To compare the seven-p-cycle solution in Fig. 2 to the best case if only one Hamiltonian p-cycle were employed, we can observe that with a single p-cycle the entire p-cycle would have to have a capacity of seven. This is because (in the best case) the span with wi = 14 (the maximum present) would have a straddling relationship to the single assumed Hamiltonian p-cycle. In this case the single p-cycle would require 13(7) = 91 units of protection capacity. There being a total of 158 units of working capacity, this represents a logical redundancy of 57.6 percent. In contrast the seven-cycle optimal solution is constructed with only 85 units of protection capacity (for 53.8 percent redundancy). Although this is not an extremely large difference, it does illustrate the point that even in a simple example network, a multiple-cycle solution is indeed more efficient.

It makes sense that fewer Hamiltonians also arise in the distance-weighted optimal solution. Under distance-weighted capacity, a Hamiltonian has no particular guarantee of efficiency that is higher than a non-Hamiltonian for which the ratio of total straddling span distances protected is high relative to the p-cycle circumference.

Comparing Hamiltonian and CPD Solutions

To further highlight the difference between a single Hamiltonian and the corresponding multiple-cycle solution, optimal designs were obtained using both the CPD model and a variant where only one unique cycle was allowed (although any number of unit-capacity p-cycle instances were possible). This modified model will thus always find a single Hamiltonian cycle as the solution, except where no Hamiltonian cycle exists, in which case the problem will be infeasible. In those cases the constraint was relaxed slightly to permit a two-cycle solution as the closest feasible approximation. Both of these models were applied to a family of networks, created from a master network with 20 nodes and 40 spans (d = 4.0). Individual spans were removed in a pseudo-random way to create a suite of progressively sparser networks, each with one less span than the last (the same method used in [14]). Span costs (cij values) were distance-weighted, and a single unit of demand between every O-D pair was used. The CPD designs were solved to within 0.5 percent of optimality, and the Hamiltonian solutions were all optimal. The two-cycle solutions where a Hamiltonian was not possible were strictly within 0.1 percent of optimal, and in many cases exact. These were all obtained with CPLEX 7.5.

Figure 3 depicts the ratio of the distance-weighted capacities with respect to the multiple-cycle CPD solutions (represented by the horizontal axis). The master 20-node 40-span network is shown in the inset. We can see that for single Hamiltonian solutions, there is a significant capacity premium over designs incorporating a variable number of p-cycles, chosen to complement each other in an optimal way. On average, the Hamiltonian cycle found was 52.9 percent more expensive than the CPD p-cycle design. Because CPLEX finds the best Hamiltonian possible to minimize the total distance-weighted capacity cost, other Hamiltonians may be even more costly. Two-cycle solutions in the sparser non-Hamiltonian networks were more reasonable, on average only 17.1 percent more expensive. As the network becomes more richly connected, the capacity ratio increases because the collection of smaller cycles CPD can use to create an optimal design is expanded.

p-Cycle Design in Homogeneous Hamiltonian Networks

Let us now consider homogeneous networks where the graph is also Hamiltonian. Intuition strongly suggests in this case that a single Hamiltonian p-cycle will be the optimal (i.e., minimum cost) solution, and this is borne out. In an N-node network, such a cycle has N hops and every span of the network will have either an on-cycle or a straddling relationship to the Hamiltonian cycle, so the one cycle protects all spans. It is a simple exercise to show that a single Hamiltonian p-cycle is the minimum-cost solution when allocating spare capacity to protect such a network against any single span failure. If cost is hop-based, then any Hamiltonian cycle is equally efficient (at a cost of N). If it is distance-based, and more than one Hamiltonian cycle exists, then the shortest such cycle is the most capacity-efficient choice for a single network-protecting p-cycle.

It is interesting, therefore, to look at some strictly optimal results in this regard. The prediction is that any time the working capacity to be protected is homogeneous on every span, the general-purpose capacitated p-cycle design model (CPD) will produce one p-cycle, and it will be Hamiltonian (if the network is Hamiltonian). We tested this using the network shown in Fig. 4a. As predicted, a single Hamiltonian p-cycle,
in this case the one shown in Fig. 4b, always constitutes the optimal solution. Span distances were hop-based in this result, and the resulting Hamiltonian p-cycle has a logical redundancy of $13/23 = 56.5$ percent, which is exactly $2/d$, a bound we can predict below. When span distances are changed from hop counts to the Euclidean distance between endpoints as they appear in the plane of the diagram, a slightly different Hamiltonian arises with distance-weighted redundancy of 47.1 percent.

If we know that the optimal solution for a homogeneous Hamiltonian network is always a Hamiltonian p-cycle, we can also derive some general bounds on efficiency. Consider a network of degree $d$, with $N$ nodes and $S$ spans, and assume it is Hamiltonian. Taking note that $d = 2S/N$, a simple initial view of the logical (i.e., hop-count-based) redundancy in the homogeneous network case would be

$$
\sum_{j \in S} s_j / \sum_{j \in S} w_j = \frac{N}{S} \frac{N}{Nd / 2} = 2/d.
$$

Note an implication of Eq. 5, quite in line with intuition in this case, is that in order to reach the minimum redundancy of $2/d$, the solution must be based on a single p-cycle, which therefore must also be Hamiltonian. If any more than one cycle were used, the number of spare links in the redundancy calculation would be higher than $N$.

This result ($2/d$) would be the lower bound on redundancy for a strictly homogeneous network. However, under strict homogeneity we fail to reflect or otherwise exploit an important aspect of p-cycles: that a straddling span can actually have twice as much working capacity as the capacity of the p-cycle that protects it. This is a special sense in which p-cycles would support selected departures from strict homogeneity (discussed below). If a span is a straddler to the network Hamiltonian p-cycle, its particular fiber (or channel) count can in fact be doubled, without any change or addition required to retain full protection. We will return to look at how this could be implemented. But for now, if we take this into account in the lower bound, the sum of working channels changes from being just $S$ to

$$
\sum_{j \in S} w_j = N + 2(S - N) = N + 2\left(\frac{Nd}{2} - N\right) = N(d - 1)
$$

so the redundancy becomes

$$
\sum_{j \in S} s_j / \sum_{j \in S} w_j = \frac{N}{N(d - 1)} = \frac{1}{d - 1},
$$

which implies that the lower-bounding redundancy of a span-restorable mesh network is exactly realized! (Less directly the same result is reached as a lower bound in [5], but not associated with a constructible way to build a network that exactly realizes the bound.)

If we now revisit the capacitated p-cycle design above, we see that its logical redundancy of 53.8 percent is below the prediction of $2/d$ for this network if it was homogeneous, but above $1/(d - 1)$ (which is 39.4 percent) if it was semi-homogeneous in the way defined above. This is consistent with the set of p-cycles for the capacitated problem having been chosen in a way that exploits the two-times protection factor of straddling spans (which the homogenous model cannot reflect) but also not always being able to associate two working capacity units with every possible straddling relationship (which the semi-homogenous view assumes). Thus, there is clearly an advantage of the capacitated p-cycle design problem that is not exploited in a strictly homogeneous network.

It is tempting at this point to want to generalize that in a homogeneous Hamiltonian network, the p-cycle with the highest individual efficiency (in the $AE$ sense) must always be a Hamiltonian p-cycle. If true, this would lend itself nicely to a simple search procedure to find a single minimum cost p-cycle to protect an entire network (or designated subnetwork). Interestingly, however, it can be demonstrated that this will not always be the case. Compare a Hamiltonian cycle $C_1$ to the “best” non-Hamiltonian cycle $C_2$ possible in the network, where $AE$ is the figure of merit. We can let the latter cycle be just one hop shorter than $C_1$, so it visits just one less node than the Hamiltonian. The best case for $C_2$ is then also that the excluded node is a degree-2 node. We can then observe that $C_2$ has one less on-cycle span (from being one hop shorter) and one less straddling span (because a total of two spans lose protection). The $AE$ of the Hamiltonian (with $N$ on-cycle spans and $S - N$ straddlers) is thus $[N + 2(S - N)]/N = (2S - N)/N$. For the near-Hamiltonian possessing $N - 1$ spans on the cycle and $S - N - 1$ straddlers, the $AE$ is $[(N - 1) + 2(S - N - 1)]/(N - 1) = (2S - N - 3)/(N - 1)$. Thus, the Hamiltonian cycle will have a higher individual efficiency when

$$
\frac{(2S - N)}{N} \geq \frac{(2S - N - 3)}{(N - 1)}.
$$

Substituting $S = Nd/2$ and simplifying, we are left with $d < 4$. This indicates that for $d \geq 4$ there may exist a cycle with $AE$ score greater than or equal to that of a Hamiltonian. To

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**Figure 4.** In a homogeneous (Hamiltonian) network a single Hamiltonian p-cycle is optimal (adapted from [1]): a) working capacity (bidirectional fiber pairs); b) a single network-protecting p-cycle.
illustrate this, Fig. 5 shows an example network and two modified versions. In Fig. 5a we can see that the degree is less than 4 and the Hamiltonian has the highest $AE$. In Fig. 5b where a span is added to make $d$ exactly 4, the $AE$ scores of the two $p$-cycles are identical. When a second new span is added in Fig. 5c to increase the average nodal degree even further, the shorter, near-Hamiltonian $p$-cycle has the highest $AE$ value.

Note, however, that in all cases the single Hamiltonian $p$-cycle remains a better global design choice as the near-Hamiltonian $p$-cycle does not cover the network entirely, implying that a second $p$-cycle is still required for full protection, which would indeed reduce the overall network $AE$ score below the score with a single Hamiltonian. Although the point may seem subtle, it is an insightful theoretical observation, and its practical implication is in regard to the search strategy to find the minimum cost network design: a strategy based solely on finding the cycle with maximum $AE$ will not always result in finding a Hamiltonian. This also resolves a small quandary in our research group in that the DCPC protocol [15] is a distributed procedure that can find the maximum $AE$ $p$-cycle in polynomial time. If this were to be equivalent to finding a Hamiltonian cycle, we would have solved an NP-hard problem in polynomial time!

To create a non-Hamiltonian network, we can start from one that is Hamiltonian and add certain arrangements of nodes and spans. In addition, at a degree-2 node we can observe that at least one cycle must cross the two spans at the node for a single $p$-cycle to cover it, or for the node to be included in a Hamiltonian cycle. Thus, any construction where three or more chain subnetworks are incident on some “anchor” node will force the network to be non-Hamiltonian. For instance, when three chains are incident to a common node, two cycles are required to visit all nodes. When five chains are incident, three cycles will be required. This inference was validated on the networks in Fig. 6, where A serves as a node that is common to two incident

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The larger Hamiltonian $p$-cycle has an $AE$ value (hop-count-based) that is higher than a), equal to b), or lower than c), its near-Hamiltonian counterpart as the average nodal degree is increased (adapted from [1]).}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Degree 2 chains attached to a common node can force a network to be non-Hamiltonian: a) three chains incident on a node — two $p$-cycles required; b) five chains incident on a node — three $p$-cycles required.}
\end{figure}
chains in one case (Fig. 6a) and three incident chains in Fig. 6b (amid other spans incident in both cases). Figures 6a and b show the strictly optimal p-cycle solutions for these cases.

To further test if three cycles were truly necessary in the network of Fig. 6b, a constraint was added to CPD to force a two-cycle solution, if possible. CPLEX then reported that the problem was infeasible. This verifies (at least in this case) the need for three p-cycles where five chains converge on one node. Thus, it is not possible theoretically to argue that a two-p-cycle solution is sufficient in a non-Hamiltonian homogeneous network case. In practice, however, nodes having five or more incident chains will be fairly rare and one can expect that two p-cycle solutions will very often be feasible for a homogeneous non-Hamiltonian network.

The solutions in Fig. 6 were obtained from optimal solution of the CPD model above. An interesting question is whether or not there is any rule or guideline for the cycles that will be formed. Results from two other test networks and the example in Fig. 6a give empirical indications that the minimum cost solution for the case with three incident chains will tend to involve one large “near-Hamiltonian” (i.e., a cycle of \( N - 1 \) hops), complemented by the smallest second p-cycle that then achieves full coverage. The case with five incident chains is more complex; in Fig. 6b none of the p-cycles chosen are either the longest or shortest in the network. Together the three cycles represent an optimal solution, but it does not seem possible to offer a general guideline as in the three-chain scenario.

Semi-Homogeneous Networks Inspired by p-Cycles

Let us now return to the observation that the logical redundancy of 53.8 percent for the capacitated solution illustrated in Fig. 2 is below the lower bound prediction of \( 2d/3 \) for a homogeneous Hamiltonian network. The reason this happens is that the differentially capacitated situation allows some of the p-cycles to take advantage of the 2-to-1 ratio of working capacity on straddling spans, relative to the p-cycle capacity. As explained, this is an advantage the strictly homogeneous network model fails to exploit. Indeed, it makes us realize that a special class of p-cycle-based survivable networks is possible which are semi-homogeneous in that all spans have at least one working fiber pair, but spans that straddle the network’s Hamiltonian p-cycle can have two.

The point is illustrated in Fig. 7, which is based on the Hamiltonian p-cycle of Fig. 4 and shows how we would “revise” the strictly homogeneous case to recognize the working fiber counts that can actually be handled by the one Hamiltonian p-cycle involved. This semi-homogeneous case is where the Hamiltonian p-cycle actually realizes \( 1/(d - 1) \) redundancy, which for this case is the impressively low value of 39.4 percent! In other words, the Hamiltonian p-cycle has 13 spans but now protects a maximum of up to \( 13 + 2(10) = 33 \) working fibers, giving 39.4 percent redundancy. Figure 7b shows a blowup view of what is actually implied if the p-cycle is implemented to work at the dark fiber switching level. On a span of the p-cycle itself, one fiber pair is devoted to the p-cycle and the other is for working demand. But each straddling span can have up to two fiber pairs devoted solely to bearing working channels. This is called a semi-homogeneous network because spans are still not generally capacitated but have either one or two fiber pairs for working capacity. Note that as so far used homogeneity is defined in terms of the working capacity on each span, and it is in this sense that this arrangement is semi-homogeneous. Rather ironically, however, because of the \( 2x \) efficiency of p-cycles with respect to straddlers, this actually makes the total fiber counts constant everywhere at two fiber pairs per span in the semi-homogeneous network. Although this results in a total of four fibers per span, it is not the same as prior four-fiber networks (e.g., generalized loopback networks or cycle groups) that are 100 percent redundant on every span because here, two out of two fiber pairs are working on straddling spans. Only on-cycle spans are made up of matched working and protection fibers in a semi-homogeneous p-cycle network. An alternate implementation technique, in Fig. 7c, would use waveband or channel group definitions to keep the fiber counts at one pair per span but use straddling span fibers wholly for working demand and on-cycle fiber pairs half for protection and half for working.

Concluding Discussion

An important clarification of this work is to show that while a single Hamiltonian p-cycle will be optimal in a homogeneous Hamiltonian network, a single p-cycle is not optimal if the network has differential span capacities, even if it is Hamiltonian. The reason is that an assembly of individual smaller p-cycles can conform themselves to the local structure of the working capacity more efficiently than a single Hamiltonian p-cycle.
cycle can be capacitated to protect all spans in the capacitated case. Some of these may individually be Hamiltonian p-cycles, but a capacitated design will generally also include p-cycles that are not Hamiltonian. The CPD formulation can easily be used to find this optimal assembly of p-cycles. This is an important clarification in light of recent work [7] which may seem to suggest that use of a single Hamiltonian p-cycle would be a viable strategy in all cases. Indeed, a single Hamiltonian p-cycle is functionally sufficient, but in a capacitated network context its cost (and dual-failure unavailability) may be much higher than an optimized assembly of distinct unit-capacity p-cycles.

Building on the idea of homogeneous networks, we have pointed out that the p-cycle technique lends itself in a rather defining way to the concept of semi-homogeneous networks that can have either one or two units of working capacity on their straddlers but exactly one on each on-cycle span. This can be thought of as an enhanced or future-proofed form of homogeneous network. It is of both practical and theoretical significance that such a network has a capacity efficiency that realizes the $1/(d-1)$ lower bound of redundancy for any span-restorable mesh network. In practice the concept also provides for an initially homogeneous network protected by a single p-cycle that has the option to double its working capacity on any straddling span without any additional protection requirements. An area for further work is to consider the optimized design of such semi-homogeneous networks in which the choice of Hamiltonian p-cycle is cognizant of the demand flow over spans, tending to arrange high-load spans to be configured as straddlers on the network p-cycle.

References


Biographies

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