MATCHED NODES AND DUAL FEEDING: OPTIONS FOR HIGH AVAILABILITY PATH PROVISIONING IN SONET RING - BASED NETWORKS

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Abstract ---- We consider dual feeding as an alternative which can be more economic than matched nodes in some circumstances where redundant inter-ring transfers are warranted. We first develop an economic comparison of dual feeding (df) and matched nodes (mn) in terms of the resource consumption of each scheme and show that the choice can be made on an individual ring-by-ring basis with a simple decision criterion. An implication of this is that we need to be able to calculate the end-to-end unavailability of pure df, mn and mixed df-mn path constructions. The second contribution of the paper is therefore to provide mathematical models for the end to end unavailability of pure df, pure mn and mixed df-mn high availability path implementations. These models may eventually be used for on-line provisioning or path planning systems that can minimize service provisioning cost subject to an assured target level of service availability.

1. INTRODUCTION

Bidirectional line-switched rings (BLSR) are widely used for implementing broadband SONET transport networks. When a transport signal, an STS-1 for example, is in the body of a BLSR it is protected against any single node or link failure by the reverse-direction routing mechanism of the BLSR [1,2]. Therefore, two or more simultaneous failures are required to cause intra-ring outage. But if no special measures are taken there will be a single point of failure at each site where inter-ring transfers occur. This may be acceptable for most traffic given that the transfer usually occurs in an inside-plant C.O. environment and that the cross-office wiring (or fibering) may itself be 1:1 protection switched. Some customer leased signals, however, and / or special services signals established by the network operators may warrant routing that avoids any single points of failure. For handling these signals the “matched node” arrangement in Figure 1 has been used by SONET ring network planners to date [3].

![Figure 1: Matched node drop & continue inter-ring transfer arrangement](image-url)
With a matched node (mn) arrangement, the service path transfers redundantly via two matched (also called gateway) nodes using the drop and continue feature of SONET ADMs. In Figure 1 labels C1 through C4 designate the ADM cores involved in the redundant signal transfer and labels 1A to 4A designate the add/drop interfaces involved. The signal flow is drawn in a unidirectional manner but the complementary treatment for the opposite signal direction is implied. The transferring signal enters the primary gateway node C1 where it is dropped to C3 and continued on ring1 to the secondary node C2. The drop signal at C1 is supplied via a cross-office link to the primary destination gateway C3 for a standard tributary insertion (add). The continue signal in ring 1 is terminated at C2, dropped, and then added to node C4. The primary gateway C3 on the destination ring thereby receives two copies of the transferring signal and performs 1:1 receive selection. This transfer arrangement is resilient to failure of any one of the four ADM cores or add/drop interfaces. Less obviously, it protects against total loss of either gateway site (e.g.; loss of both nodes C1 and C3 or C2 and C4) and resists some other dual-failure combinations as well.

To appreciate the latter one must consider the ordinary line-level reverse direction routing reactions of rings 1 and 2 simultaneously with the tributary-level signal selection behaviour. For instance if the building housing C1 and C3 fails entirely from a fire or power loss, say, the tributary signal in ring 1 will be reverse routed onto protection as part of the OC-n line level reaction in ring 1 to C2 where C2 will substitute the protection signal for the normal signal from C1. The secondary transfer path then conveys the tributary to C4. But C4 is also in loopback mode in response to loss of node C3. C5 in ring 2 is also in loopback mode. The tributary is therefore reverse routed on protection bandwidth to C5 in ring 2, from where it resumes its normal routing.

1.1. The Dual-Feeding (df) Alternative

We return to the matched node arrangement to study its 2-failure unavailability and its resource consumption costs. Presently, however, we wish to introduce the idea of direct dual feeding (df) of a signal as an alternative to matched nodes. Figure 2 shows how redundant inter-ring signal transfer can be achieved by establishing two physically identifiable copies of the signal in the two rings, with direct transfers at the same two crossing points as in Figure 1.

![Figure 2: Dual feeding direct transfer inter-ring arrangement](image-url)
bandwidth use, add-drop interface use and ‘continue’ signal bandwidth consumption are all factors in determining which scheme will actually cost less in a given situation.

1.2. Functional Compatibility of mn and df Treatments

We now aim to establish that df and mn treatment choices can be determined and applied on a ring by ring basis. This is based on the following two observations: (i) mn on a “West” ring is functionally compatible and can be directly interfaced to df in an adjoining “East” ring, and vice-versa. (ii) the mn or df decision can be made independently within each ring based on the parameters of each ring alone. That is to say it does not require consideration of two (or more) rings at a time to decide which treatment is most economic.

Point (i) is a simple observation that is explained in Figure 3 by illustrating an example of an mn-to-df conversion connecting successive rings in a signal path.

Point (ii) is a less obvious claim. It says that from an economic viewpoint we can choose the treatment independently in each ring and the resulting complete path will be cost minimal; i.e., that the sum of the economically best treatments in each ring is globally the best treatment for this problem. The path unavailability implications will not have a similar independence property and will be separately assessed. To explain this point regarding economic optimization of the path consider Figure 4.

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**Figure 3:** Matched nodes (mn) in ring 1 with dual feeding (df) in ring 2

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**Figure 4:** Considerations for ring-by-ring choice between mn or df.
In Figure 4 the $mn$ and $df$ approaches are illustrated at a whole ring level showing the signal routing that each implies within the ring. In a ring configured for $mn$ style redundant interfaces the working signal is physically present only on the $\{E1, Wa, E2\}$ path shown. $Wa$ is the number of spans along the working path between the primary matched nodes and $E1$, $E2$ are the number of spans between redundant entry and exit node locations, respectively. For generality we consider general values of $E1, E2$ but often $E=1$ in practice. Figure 4(b) shows the dual fed alternative at the same whole ring level. $\{Wa\}$ is the same as in $mn$. $\{Wb\}$ is the path of the explicit additional signal feed that is added in the $df$ approach.

The two approaches are functionally equivalent at interfaces X and Y. There is no observation at interface Y (other than possibly a long term difference in unavailability) that would allow one to tell the difference between (a) and (b) in Figure 4. Therefore (a) and (b) must be functionally interchangeable with respect to interface Y. But the same arguments apply to the prior ring at the interface X. Therefore we should be technically free to choose between $mn$ or $df$ independently within each ring as the signal arrangement in one ring has no functional influence or dependence on the configuration in an adjacent ring. While the unavailability of the complete path construction will indeed depend on the ring by ring choices, it seems clear from an economic decision viewpoint that we can optimize the treatment in each ring individually.

### 1.3. Outline of Remaining Developments

We now proceed as follows: Section 2 develops an economic criterion for the choice between $df$ or $mn$ treatment in any given ring. Section 3 is then concerned with calculating the unavailability of path constructions of pure $df$ rings, pure $mn$ rings and of path constructions with mixed $df-mn$ treatments along the end to end route of a path. The unavailability analysis complements the economic criterion for ring by ring choice by ensuring that the availability implications of such economic path constructions can be assessed. Section 4 summarizes and discusses continuing work.

### 2. ECONOMIC DECISION CRITERION

#### 2.1. Framework for Resource Cost Assessment

For generality we avoid dependence on specific cost data. We take a general approach towards the weighted conversion of equipment items and other resources consumed into a resource consumption function that is a surrogate for cost. Three cost characteristics are considered: distance-dependent transmission costs, ADM core costs, and tributary-related ADM interface costs. Models with access costs are included in [4] but excluded here because the access architecture is independent of the intra-ring routing choices. Reference [4] considered only pure dual-fed or pure matched-node path types, not the mixed $mn-df$ constructions now also considered. Figure 5 helps explain the variables in the resource cost assessment of alternatives.

The pro-rated cost of passing one tributary signal unit through one ADM chassis is defined as the fundamental “unit cost” (1.0). An ADM core or ‘chassis’ refers to all the common equipment infrastructure (card cage, backplane, power, control processors, etc. plus East and West dual optical line interfaces) of the ADM before any tributary cards are added. Relative to this unit cost $\beta$ is defined as the cost of an add/drop interface for one signal unit, and $\alpha$ is the relative cost of a unit distance of transmission for intra-ring optical line transport. $\alpha$ and $\beta$ are both pro-rated to the ADM core transit cost on a unit bandwidth basis. This may not always be apparent for the optical line cost factor ($\alpha$) because the line OC-n rate is normally the same as the ADM backplane bandwidth so $\alpha$

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1. re: notation: $/Wa/$ is the set of all elements (OC-n line km, ADM nodes, optical interfaces etc.) in the specified path segment. $Wa$ is the number of spans in $/Wa/$.
is also the ratio of the whole optical line cost (per km) to the whole ADM chassis cost.

Using $\alpha$, $\beta$ as study parameters for comparative assessment of alternatives we can obtain useful general conclusions without loss of scope from dependence on specific cost data. These parameters also define a parameter space which can systematically represent variations in geographic scale, and the effects of a variety of technology assumptions. For instance, in metropolitan networks spans often do not require regenerators and previously placed fibre is available in existing ducts. Then distance-related transmission costs are often treated as negligible, and the whole planning problem is about minimizing the terminal equipment. This is the $\alpha = zero$ situation in our framework. On the other hand, in a long-haul network the cost of the transmission systems on a span may be greater than the nodal terminal costs, in which case $\alpha$ is a significant value reflecting the ratio of a km of OC-n line system relative to the OC-n ADM chassis cost. The role of $\alpha$ in determining a cost-effective routing scheme is therefore importantly related to geographical network scale. On the other hand $\beta$ is more likely to vary in dependence on the technology epoch assumed. For instance the relative cost of an STS3c add/drop interface to the same bandwidth proportional cost will differ or an OC-48 and OC-192 ADM core. More extensive use of this framework is made in [4] for comparative study of unavailability and cost in various end-to-end path models including access costs and access effects on path availability.

2.2. Resource Cost for Matched Node Treatment

We consider the resource cost of the $mn$ configuration in two steps: First we consider the ADM core consumption, add/drop interface consumption and distance-bandwidth span consumption costs of the $W_a$ path through the ring between the primary matched nodes. Then we add considerations of the drop and continue setups at each side of the ring. Inside the ring on the $W_a$ signal path, there are $W_w$ working spans and $W_a + 1$ nodes. However, the cores of the entry and egress ADM’s on each ring are only “half-transited” by the tributary signal of interest. At the other ADMs in $W_a$ the core is fully traversed. The total ADM core consumption is therefore $W_a + 1 - 2(1/2) = W_a$ unit costs per ring. One add/drop interface is used on each side of the ring in $W_a$. $W_a$ spans of line-level transport are used in each ring where $L_s(i)$ will be the length of the $i^{th}$ span.

To add the matched node setup to the $W_a$ path the ADM core consumption goes up by a 1/2 core transit at the primary and secondary gateway node. The continue signal completes a transit of the primary node cores and a half-transit is added at each secondary node. Additional whole core transits occur if either entry or egress separations are more than one span apart. In general $E-1$ additional core transits and $E$ spans for the continue signal are added to the $W_a$ path for each egress or exit separation of $E$. Finally the drop and continue setup adds one more add / drop interface at each side

Figure 5: Elemental unavailability and resource cost models for ring network elements
of the ring. The resource cost of \( mn \) (in order of cores, spans, interfaces) is therefore:

\[
C_{mn} = (W_a + E_1 + E_2) + \alpha \cdot \left( \sum_{i \in \{W_a, E_1, E_2\}} L_3[i] \right) + 4\beta
\]  

(1)

2.3. Resource Cost of Dual-Fed Treatment

The corresponding resource cost for \( df \) is obtained from Figure 4 (b) as follows: The \( df \) path pair almost circumnavigates each ring. In total there are \( W_a + W_b = (S - E_1 - E_2) \) spans traversed. \( S \) is the number of spans in a ring. All ADM cores on the \( \{A\} \) or \( \{B\} \) paths are fully traversed by either the A or B signal feed but at the ends of each feed the respective signal copies only “half transit” the ADM core for a total ADM core consumption of \( (S - E_1 - E_2) \) units of core ADM capacity. There are also 4 add/drop interfaces consumed. Therefore we can write:

\[
C_{df} = (S - E_1 - E_2) + \alpha \cdot \left( \sum_{i \in \{W_a, W_b\}} L_3[i] \right) + 4\beta
\]

(2)

2.4. Decision Criterion

By recognizing that \( W_b = S - W_a - E_1 - E_2 \) and that the number of add-drop interfaces as well as \( W_a \) is common to both expressions (1) and (2) we can form a decision variable \( \Delta C \):

\[
\Delta C = (C_{mn} - C_{df})
\]

(3)

\[
= W_a + 2 \cdot (E_1 + E_2) - S + \alpha \cdot \left( \sum_{i \in \{E_1, E_2\}} L_3[i] - \sum_{i \in \{W_a, W_b\}} L_3[i] \right)
\]

If \( \Delta C \) is positive the \( df \) treatment is more cost effective than using matched nodes. To understand the factors affecting the \( df-mn \) criterion further let us consider the special case where all spans are of equal length (i.e., cost) \( L_s \). Then the \( mn-df \) cost difference simplifies further to:

\[
\Delta C = [W_a + 2 \cdot (E_1 + E_2) - S] \cdot (1 + \alpha \cdot L_3)
\]

(4)

The criterion simplifies to this form because with the half-transit considerations for entry-egress ADM cores, the number of spans traversed is numerically equal to the number of core bandwidth units used, each of the latter costing 1.0 and each of the former costing \( \alpha L_s \).

As seems reasonable from Figure 4, Eq. (4) tells us that large rings (large in terms of number of spans \( S \)) favour the matched node treatment. This is offset in favour of dual feeding, however, if the entry and egress separation sum \( E_{\text{tot}} = (E_1 + E_2) \) is large or if the \( \{W_a\} \) path is long.

While \( \Delta C \) is the actual cost difference, if we are interested in a binary decision criteria, then we can further reduce this to:

\[
W_a \geq S - 2 \cdot E_{\text{tot}} \quad \rightarrow \quad df, \quad \text{otherwise} \quad mn
\]

(5)

as a criterion for triggering the \( df \) approach rather than \( mn \). Note in applying Eq.(5) that the minimum meaningful value for \( E_{\text{tot}} \) is 2 and \( W_a > 0 \) is required to be meaningful. Similarly the minimum size ring for the \( df-mn \) treatments to be meaningfully considered is at \( S = 4 \). Although it is trivial to obtain Table I from Eq. (5) doing so brings out some useful guidelines emerging from this and demonstrates considerable scope for \( df \) applications. One implication for instance is that in rings of 5 spans or less dual feeding is always preferred. Table I shows that there is considerable scope for dual feeding in larger rings as well. In Table I \( W_{a\ast} \) denotes the length of the basic signal path through the given ring at or above which \( df \) proves in. \( \Delta C = 0 \) is strictly a ‘tie’ between \( mn \) and \( df \) but is considered here as...
a case where \( df \) proves in because -if all else is equal- \( df \) eliminates the matched node 1:1 select function and is thought to be generally easier to establish, operate, and manage. Table I shows that even in a 10 span ring, if the working path through the ring is 6 hops or more, dual feeding will be cheaper than matched nodes. Even at the limiting ring size \((S=16)\) dual feeding can still be advantageous if the baseline working path is relatively long and/or significantly separated entry/egress location pairs are possible.

Table I Some Illustrative Ring Conditions where \( df \) proves in

<table>
<thead>
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<th>( S )</th>
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<th>( S )</th>
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<td>16</td>
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<td>always ( df )</td>
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<tr>
<td>10</td>
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<td>5</td>
<td>x</td>
<td>always ( df )</td>
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<td>6</td>
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<td>4</td>
<td>x</td>
<td>always ( df )</td>
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Note that in developing the decision criterion we have assumed that the crossing locations used for dual feeding are the same as would be used for matched node pairs. This may not always be the case in practice: It is easy for a \( df \) path pair to have any \( E \) separation whereas it is operationally more complex to establish and operate multi-hop continue signal arrangements for the \( mn \) alternative. This is an additional unquantified factor in favour of the \( df \) option. Comparing the two with identical crossing points is generous towards \( mn \) because it does not penalize \( mn \) in any way for the practical difficulties of extended continue signals. To compare \( df \) and \( mn \) assuming that matched node pairs must always be adjacent nodes the above is then valid with \( E_{\text{tot}}=2 \) only.

3. PATH UNAVAILABILITY ANALYSES

Because we have proposed two different treatment options for each ring in a redundant path implementation and because the reason for such treatments in the first place is to achieve especially high availability, we must now give consideration to the calculation of end to end path availability of pure \( df \), pure \( mn \) and mixed \( df-mn \) path constructions. We proceed by first considering the 2-failure \textit{intra-ring} outage contributions from each ring in the path depending on its internal configuration. Then we consider the dual-failure combinations that can cause outage due to failures in elements involved in the \textit{inter-ring} transfer interfaces. Standard methods and assumptions for availability of repairable actively maintained systems are used [5]. The elemental unavailability variables are illustrated in Figure 5. \( U_{sl} \) is the unavailability per km of optical line length excluding ADM nodes and ADM optical interfaces. \( U_s = U_{sl} \times L_s \) is used directly in the following, where \( L_s \) is the length of a fiber span in a BLSR. \( U_{nl} \) is the unavailability of a tributary signal path through the ADM core. It includes optical interfaces but no add/drop interfaces. \( U_{na} \) is the unavailability of the add/drop interface and half of any cross-office connection path to effect a tributary transit from one ring to another.

3.1. Intra-Ring Dual Failure Transport Outage Contributions

In this section we develop the expressions \( Td_1(W,S) \) and \( Td_2(W,S,E) \) for the \textit{intra-ring} contributions to path unavailability with a single-fed signal and a dual-fed signal path through a BLSR. This is a step towards calculating the unavailability of multi-ring paths of \( mn \), \( df \) and \( mn-df \) mixed constructions. The “\( d \)” in \( Td_1 \), \( Td_2 \) is to designate that it is a dual-failure analysis. The 1,2, subscripts denote whether a single or dual signal instance physically traverses the ring in question. Thus \( Td_d(W,S) \)
will be applicable to rings in \( mn \) configuration and \( Td_2() \) to \( df \) rings. \( Td_f() \) and \( Td_2() \) functions include the optical line side signal path failures at the primary entry and egress nodes, and the ADM cores, but not the add/drop side (or tributary-level) failures at these nodes or failures involving the drop and continue setups for \( mn \). The latter are accounted for in the \textit{inter}-ring transfer terms of the path-level models that follow.

\( Td_f(W,S) \) is the line-level intra-ring transport unavailability of a single-fed signal path traversing \( W \) spans of a BLSR having \( S \) spans in total. It accounts for all double-failure event combinations with consideration of the line-switched protection mechanism. \( Td_f(W,S) \) is derived as follows: Let the set of \( W \) spans and \( (W+1) \) nodes in the normal path of a signal feed \( X \) be called the \textit{forward path} of \( X \), denoted \( \text{For} \{X\} \). Let the set of all other nodes and spans in the ring be \( \text{Rev} \{X\} \), \textit{i.e.}, \( \text{Rev} \{X\} = \{S \cdot \text{For} \{X\} \} \) where set \( \{S \} \) is the complete ring. \( \text{Rev} \{X\} \) has \( S-W \) spans. For outage of a single-fed path within a ring, it is necessary and sufficient that one failed element (node or span) belong to \( \text{For} \{X\} \) and the other to \( \text{Rev} \{X\} \). The relevant failure pairs consist of: a span in \( \text{For} \{X\} \) and a span in \( \text{Rev} \{X\} \), a span in \( \text{For} \{X\} \) and a node in \( \text{Rev} \{X\} \) or vice-versa, and a node in \( \text{For} \{X\} \) and a node in \( \text{Rev} \{X\} \). Triple failure combinations are ignored. Summing and simplifying all contributions we obtain:

\[
Td_1[W,S] = W(S-W)Us^2 + (W+1)(S-W-1)Unl^2 + (2W(S-W-1) + S) \cdot UsUnl
\]

(6)

This result is essentially the same as for the corresponding case in [6]. A more detailed explanation of its derivation and of \( Td_2() \) which follows is available in [4].

A slightly different function, \( Td_2() \), is needed for the \textit{intra}-ring transport unavailability of a BLSR where dual redundant signal feeds are explicitly provisioned through the BLSR. For a single-fed path the forward path plus reverse path for line switched protection circumnavigates the whole ring, \( \text{For} \{X\} \) + \( \text{Rev} \{X\} \) = \( S \). But when dual feeding the relevant element subsets do not circumnavigate the ring. When dual feeding \( A \) and \( B \) paths, outage requires one failure in \( \{\text{For} \{A\} \text{ AND } \text{Rev} \{B\}\} \) and the other in \( \{\text{Rev} \{A\} \text{ AND } \text{For} \{B\}\} \). This is the only way two failures can be positioned so as to simultaneously fail \( A \) and \( B \) forward feeds and both of their respective line-switched restoration paths. However, because the \( A \) and \( B \) signal feeds are disjoint \( \{\text{Rev} \{A\} \text{ AND } \text{For} \{B\}\} = \{\text{For} \{B\}\} \) because \( \text{For} \{B\} \) is necessarily a \textit{subset} of the path \( \text{Rev} \{A\} \). Likewise, \( \{\text{For} \{A\} \text{ AND } \text{Rev} \{B\}\} = \{\text{For} \{A\}\} \). Hence outage requires simultaneous failures in \( A \) and \( B \) feeds and there is no exposure to failures on spans or nodes not directly on one of the \( A/B \) feeding paths. For example, when a dual fed path pair enter and leave the ring one node apart \( \{\text{For} \{A\}\} + \{\text{For} \{B\}\} = S-2 \), so there are two fewer spans to contribute to the failure combinations than with \( Td_1() \) for the \( A \) signal feed alone as single feed in the same ring. From these considerations, the dual-failure combinations that bring down both \( A \) \& \( B \) feeds of a dual-fed path pair in a BLSR is obtained as:

\[
Td_2[S, W_a, W_b, W_{tot}, E_{tot}] = W_aW_bUs^2 + (W_a + 1)(W_b + 1)Unl^2 + (2W_aW_b + W_a + W_b)UsUnl
\]

(7)

where \( W_a \) is the length of the \( A \) path and \( S \) and \( E_{tot} \) are as above.

An insight obtained from the process of developing these expressions is that dual-feeding through a BLSR does not radically improve the unavailability relative to single feeding because the state of the restoration path \( \text{Rev} \{A\} \) for signal feed \( A \) is highly correlated with events on the forward path of the (ideally independent) \( B \) feed and vice-versa. In other words, the explicitly duplicated signal feed plays a role that is almost but not wholly functionally equivalent to the ordinary line protection mechanism in the sense that most of the 2-failure combinations that bring down \( \text{For} \{X\} \) and \( \text{Rev} \{X\} \) for the single fed case also simultaneously fail \( \text{For} \{A\} \) and \( \text{For} \{B\} \) in dual feeding. This understanding could be helpful when faced with explicit customer demands for their services to be literally dual fed over fully redundant paths end-to-end: if the underlying transport is a BLSR based network then in general there is little extra \textit{intra}-ring protection from dual feeding through a BLSR.
On the other hand, dual feeding should not be ruled out but should be considered primarily for the economic benefit in suitable ring situations. Dual feeding will have some availability gain, however, to the extent that the overlap of $For{B}$ and $Rev{A}$ paths (and vice-versa) is not complete; i.e., the $\{E1\}+\{E2\}$ sets are removed from the 2-failure outage combinations when dual feeding. The greatest intra-ring unavailability benefit from dual feeding is therefore when the sum of entry and exit node pair separations is large.

### 3.2. Unavailability of a Pure Matched Node Path

It is clear from inspection of Figure 1 that the matched node arrangement protects the inter-ring transfer against failure of any one of the four ADM’s performing add/drop functions in the matched node arrangement. As previously explained, however, it also makes the inter-ring transfer immune to total loss of either gateway site (e.g; both C1 and C3 or C2 and C4) due to combined actions of each BLSR and the matched node select function. Through similar case-by-case functional reasoning all possible 2-element failures in the matched node system have been considered for their outage-causing effects. The result is summarized in Appendix A. The main finding is that only simultaneous pairs of same-ring (e.g., C1, C2) or diagonal cross-ring (e.g., C1, C4) ADM core or add/drop interface failures can defeat the matched node arrangement. Simultaneous failure of both ADM cores on one side of the gateway is also outage causing but is separately accounted for in $Td1()$ as a dual-failure intra-ring outage. Additionally, due to the line-level BLSR reaction, a span failure between the primary and secondary nodes on either ring and another single element failure in the inter-ring interface will not defeat the $mn$ scheme. Such failures are isolated within their respective rings and have no opportunity to combine into an outage-causing dual failure. The remaining failure combinations in the $mn$ interface are detailed in Appendix A. The result is that each inter-ring transfer in an $mn$ path contributes $2U_{NL}^2 + 4U_{NA}^2 + 8U_{NL}U_{NA}$. Therefore, the overall unavailability of a redundant path construction which begins at the inputs to the add/drop interfaces on the redundant entry and egress nodes in the first and last rings and contains K rings all interconnected via matched nodes is:

$$U_{mn}[K] = 2(U_{NA}^2 + 2U_{NA}U_{NL}) + 2(K-1)(U_{NL}^2 + 2U_{NA}^2 + 4U_{NA}U_{NL}) + K \sum_{i=1}^{K} T_{d1}[W_i, S_i]$$

### 3.3. Unavailability of a Pure Dual Fed Path

In dual feeding the separate A B feeds make their way individually across the inter-ring interfaces, as in Figure 2. Inside each ring the intra-ring dual failure outage is accounted for by $Td2()$ terms. Therefore, as in $mn$, single span or ADM node failures within a BLSR are isolated and do not accumulate in combinations end to end over the A/B paths. In a $df$ path construction, however, one failure at any inter-ring transfer site on feed “A” can combine with a single failure at any other (or the same) transfer point on the “B” path causing outage. There are therefore $2K$ add/drop interface points on each path A or B (K-1 full transfer interfaces at 2 each, plus 1 add/drop interface at entry and exit points on each feed on each outer ring) which may combine with any of the like numbered points on the other path to cause outage. If events on the crossing points of the “A” path are independent of those on the “B” path, the unavailability from end-to-end combinable inter-ring transfers is $(2KU_{NA})^2$.

Some other two-element failure combinations exist which must happen together at the same interface to cause outage. At an inter-ring interface a diagonally opposite pair of core losses, each individually recoverable from an intra-ring line transport view, but occurring on both tributary feeds at the same transfer interface will fail the service. This is different from dual failures of both nodes on one ring at the transfer interface which is also outage-causing but is accounted for in $Td2()$. Similarly, a core loss in one ring combined with a diagonally opposite add/drop interface loss at the
same transfer interface is another localized dual failure combination at the pure $df$ transfer interface that causes outage and is not accounted for in the end-to-end non-isolated failure combinations. The latter add $2U_{NL}^2 + 4U_{NL}U_{NA}$ at each inter-ring transfer interface. Overall, the unavailability for a pure $df$ service path of $K$ rings between the inputs to add/drop interfaces at redundant entry and egress locations is:

$$U_{df}[K] = 4(U_{NL}U_{NA} + (K \cdot U_{NA})^2) + 2(K - 1)(U_{NL}^2 + 2U_{NL}U_{NA}) + \sum_{i=1}^{K} T_{di}[W_{Ai}, S_i, E_i]$$  \hspace{1cm} (9)

To assist in reading this expression: The first term is for diagonal access point failures plus non-isolated inter-ring cumulative failures. The second term is for isolated inter-ring interface failure combinations and the last term is the cumulative intra-ring outage contribution.

### 3.4. Unavailability of a Mixed df-mn Path Constructions

In a mixed path implementation, such as outlined in Figure 6, the total path unavailability may be obtained by breaking the path into segments of rings that share the same routing strategy [7]. Within these segments the unavailability can be calculated from the expressions (8) and (9). However, the embedding of a $df$ segment between $mn$ segments introduces a class of outage-causing failure combinations that are not accounted for in the pure segment models themselves. Consider $df$ segment 1 in Figure 6:

![Figure 6: Example of a mixed df-mn path construction: $N_{df} = N_{mn} = 2$; $K_{df[i]} = \{2, 1\}$; $K_{mn[j]} = \{1, 2\}$](image)

Dual failures of add/drop interfaces and/or node cores at nodes A, B or G, H cause outage but are accounted for in the $mn$ segment 1 and $mn$ segment 2 models. The same combinations at nodes C,D or E,F and the cumulative non-isolated C,F or D,E type failures in $df$ are similarly already accounted for in the $df$ segment 1 model. But consider a single failure in $mn$ ring 1 at node A and in $mn$ ring 4 at node H. Such failures, in separate $mn$ rings, would not previously have been outage-causing as single failures are isolated in a pure $mn$ segment. But when they occur in combination at the interfaces to the $df$ segment the failure at node A voids the $df$ “$A$” feed entirely through $df$ segment 1 and the failure at H prevents the surviving $df$ “$B$” feed from getting into the next $mn$ segment. This $mn$-$df$-$mn$ combination is not accounted for in any of the segment models themselves. It adds $2U_{NA}^2 + 4U_{NA}U_{NL}$ for each embedded $df$ segment but does not exist for a non-embedded $df$ segment, such as ring 6 above, that is at the outside of the path. Next, if $mn$ node A (core or add/drop interface) fails, then any single inter-ring failure on the $df$ “$B$” feed is also a new outage-causing inter-segment failure combination. This will add $4(U_{NL} + U_{NA})(K_{df[i]} - 1)(2(U_{NL} + U_{NA}))$ for an embedded $df$ segment because there are 4 $mn$ nodes that have this cross relationship to one of the $df$ “$A$” or “$B$” paths. The leading coefficient will be 2 for a non-embedded $df$ segment. Single intra-ring failures on feeds of the $df$ segment are isolated from this $mn$ node cross dependency by the BLSR line mechanism. Overall the unavailability of a mixed path construction becomes:

$$U_{dfmn}[N_{df}, N_{mn}] = \sum_{j=1}^{K_{mn}} U_{mn}[K_{mn[j]}] + \sum_{i=1}^{N_{df}} \{U_{df}[K_{df[i]}] + 2 \delta_{df[i]}(U_{NA}^2 + 2U_{NA}U_{NL})\}$$  \hspace{1cm} (10)
where \( U_{df} \) and \( U_{mn} \) are given by Eq’s (8) and (9) and \( N_{df} \) and \( N_{mn} \) are the number of dual fed and matched node contiguous segments in the overall path. \( K_i \) is the number of rings in the \( i \)th \( df \) segment and \( K_j \) is the number of rings in the \( j \)th \( mn \) segment and \( \delta_{df}[i] = 1 \) if the \( i \)th \( df \) segment is embedded, otherwise \( \delta_{df}[i] = 0 \).

4. CONCLUDING DISCUSSION

This work shows that an economic benefit may result from considering the option of dual feeding in some contexts where matched node arrangements would today be specified. The economic decision criterion is simple and applies on a ring by ring basis. The resulting end-to-end path constructions that result may conceivably be of a pure \( df \), pure \( mn \) or of a mixed \( df-mn \) construction. We have therefore developed analytical models that can be applied to any such path construction to compute its theoretical path availability. The ability to construct the min-cost path and compute its unavailability at provisioning time may be used on-line in future provisioning systems.

Continuing work in this area is extending the analytical models to include “extended transfer” sections between rings, where unprotected point to point transmission system of some length replace the cross-office inter-ring transfer connections to join distant rings. Segments of \( df \) path constructions where the rings traversed are physically and/or logically diverse i.e., the A and B feeds of the path pair are not always on the same logical rings as they are here, are also being considered. An eventual aim is to support a future on-line provisioning system which finds the globally minimum cost path implementation that meets a contracted customer level of design availability.

A final practical point for users of this work is that, as given here, the \( Td1() \) and \( Td2() \) functions are stated for the case of equal length spans. More generally every span has a distinct real-valued length. Taking this into account considerably complicates the form of the expressions that would otherwise have been presented in this paper. Programmed implementations of these equations that consider arbitrary individual span lengths may be made available upon request.

Acknowledgement

Lance Doherty, an undergraduate in the Engineering Physics program at the University of Alberta is thanked for his work as both Summer Student and Dean’s Award Student related to this project. This includes programming the generalized unavailability and cost expressions and studies of techniques for searching amongst alternative redundant path constructions in a network (work not reported here).

References


1. This must total 28 to confirm that all possible two-element failures amongst the set of 4 ADM cores and 4 add/drop interfaces for the sf-mn inter-ring transfer arrangement case have been considered because \( \binom{8}{2} = 28 \).

### Appendix A: Dual-Failure Analysis for mn Transfer Arrangement

<table>
<thead>
<tr>
<th>Dual-element failure (Description)</th>
<th>Example of failure class</th>
<th>Unavailability (per individual combination)</th>
<th>No. of combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 2 ADM cores (nodes) on the same ring</td>
<td>C1-C2</td>
<td>covered by ( Td/l() )</td>
<td>2</td>
</tr>
<tr>
<td>2) 2 ADM cores at opposite facing nodes</td>
<td>C1-C3</td>
<td>not outage causing</td>
<td>2</td>
</tr>
<tr>
<td>3) cores of diagonally opposite ADMs</td>
<td>C1-C4</td>
<td>( U_{nl}^2 )</td>
<td>2</td>
</tr>
<tr>
<td>4) two add/drop I/F’s on different nodes of the same ring</td>
<td>1A-2A</td>
<td>( U_{na}^2 )</td>
<td>2</td>
</tr>
<tr>
<td>5) two add/drop I/F’s on diagonally opposite nodes</td>
<td>1A-4A</td>
<td>( U_{na}^2 )</td>
<td>2</td>
</tr>
<tr>
<td>6) two add/drop I/F’s on facing opposite nodes</td>
<td>1A-3A</td>
<td>not outage causing</td>
<td>2</td>
</tr>
<tr>
<td>7) add/drop I/F at a node on ring1 and ADM core loss at diagonally opposite node</td>
<td>1A-4C</td>
<td>( U_{na}U_{nl} )</td>
<td>4</td>
</tr>
<tr>
<td>8) core loss on one ring with add/drop I/F loss at other node on same ring</td>
<td>C1-2A</td>
<td>( U_{na}U_{nl} )</td>
<td>4</td>
</tr>
<tr>
<td>9) core loss and same-node add/drop I/F</td>
<td>C1-1A</td>
<td>not outage causing</td>
<td>4</td>
</tr>
<tr>
<td>10) core loss and add/drop I/F at facing node</td>
<td>C1-3A</td>
<td>not outage causing</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Combinations(^1) =</td>
<td>28</td>
<td></td>
</tr>
</tbody>
</table>