Functions of a Digital Repeater (or "Regenerator")

"3R" Repeater
- Retime
- Reshape
- Retransmit

Weak, distorted, noisy

Transmission medium
- Fiber
- Air
- Coax
- Twisted pair

Detector
- PIN diode
- Antenna
- Down conversion if not baseband

Low noise
Fixed gain

Nyquist shaping filter (may be adaptive)

Main Amp

Signal here has (ideally) now:
- Minimum possible
- Noise/bandwidth
- Zero ISI-shaping

Decision circuit

Retransmit
(Claro, or
Upconverter
& Antenna)

"Timing Recovery" path

Rocks onto discrete tone at symbol rate

Coarse BPF limits noise into PLL (Clock tone SNR)

"Timing Recovery" path

Nonlinear operation creates a spectral line at symbol rate
"Eye Diagram"

- Easy to display on oscilloscope: trigger sweep at $R_b$ or $R_b/2 \rightarrow$ portrays superposition of all signaling traces folded back as they relate to the issue of Retiming & Decision on the symbol values.
- Can be very diagnostic of problems due to incorrect equalizer filter shaping, excess noise, transmit pulse shape errors, timing jitter, accumulation, inadequate low frequency coupling, "dc wander" etc.

Nyquist filtering:

Imperfect overall channel shaping:

"perceptibly open eye"
Monitoring Transmission Quality by Eye Diagram

Required minimum bandwidth is

\[ B_T \geq r/2 \]

Nyqvist's sampling theorem:

Given an ideal LPF with the bandwidth B it is possible to transmit independent symbols at the rate:

\[ B_T \geq r/2 = 1/(2T_b) \]
How to interpretation an eye diagram

![Eye Diagram Diagram](image)

**Fig 1 – Eye opening, what it means**

The eye diagram is created by taking the time domain signal and overlapping the traces for a certain number of symbols. If we are sampling a signal at a rate of 10 samples per second and we want to take a look at two symbols, then we would cut the signal every 20 samples and overlap these. The overlapped signals show us a lot of useful information and this is called the eye diagram.

The open part of the signal represents the time that we can safely sample the signal with fidelity. So obviously we want an open looking eye and larger the opening (the white space in the middle) the better. For raised cosine signals, the larger the α, the wider the opening (See Fig. 2). The opening is smallest for $\alpha = .2$. So a smaller $\alpha$ will lead to larger errors if not sampled at the best sampling time which occurs at the center of the eye.

The horizontal band represents the amount of signal variation at the time it is sampled. This variation is seen in the scatter diagram and is directly related to the SNR of the signal. A small band means a large SNR.

The slope of the eye determines how sensitive the signal is to timing errors. A small slope allows eye to be opened more and hence less sensitivity to timing errors. The width of the crossover represents the amount of jitter present in the signal. Small is obviously better.
Mathematically perfect eye diagrams:

- Obtained through "Raised Cosine" spectral shapings.

\[ \chi = 0 \quad \text{Brickwall filter at the Nyquist rate} \]

\[ \chi = 0.3 \]

\[ \chi = 0.7 \]

\[ \chi = 1.0 \]

- As \( \chi \to 1 \), the eye diagram becomes more "open", hence less prone to errors in timing, and more resistant to noise.
Calculation of Bit (or Symbol) Error Rates

N.B. This is for "hard decision" decoding. More advanced methods exist for maximum-likelihood sequence decoding involving "soft" decisions on each received symbol. (Viterbi decoding)

Basic concept: Errors are made if a sample of the noise exceeds the decision distance between the signal voltage and the respective decision threshold.

Eq. ON-OFF (Unipolar) binary:

(1 threshold) 0 1 0 \[ \sqrt{\text{decision distance} = 0.5V} \]

- Signal: \[ \text{signal} \]
- Noise: \[ \text{noise} \]

\[ \text{noise: } \frac{\text{rms}}{\sqrt{2}} \text{ in } 1 = \sigma^2 \text{ is the noise power} \]

Thus, what is the probability that the (correct) value of 1 is mistakenly taken as a "0"?

\[ \text{prob. error} = 0.5V \]

\[ \text{decision distance} \]

\[ \text{Signal Value of Sample Time} \]

\[ \text{noise p.d.f.} \]

\[ \text{noise value} \leq 0.5 \]
Because problems involving the tail area of a normal distribution are so common, the form just arrived at is a well known standard function.

\[ Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} \, dx \]

\( Q \) is no closed form solution known. Widely tabulated in tables & books.

\[ Q(x) = \frac{1}{2} \operatorname{erfc}(\frac{x}{\sqrt{2}}) \]

for our problem

\[ p(\text{error}) = \frac{x}{2} \cdot p(0/1) + \frac{x}{2} \cdot p(1/0) = Q \left( \frac{\Delta}{\sigma} \right) \quad \text{(related to \text{"erfc\text{\textsc{m}}"})} \]

Complementary error function.

Approximating \( Q(x) \) for arguments \( x > 2 \)

\[ Q(x) \approx \frac{1}{\sqrt{2\pi}} \left( 1 - \frac{0.7}{x^2} \right) e^{-x^2/2} \]

\( x \) is the "signal to noise ratio" (absolute)

\( 20 \log(x) \) or \( 20 \log(\frac{\Delta}{\sigma}) \) is the SNR in dB.

Note: \( x \) is always \( \frac{\text{decision distance}}{\text{rms noise}} \).

With multiple decision thresholds, several decision error possibilities may have to be added up.
More generally, let $A = \text{decision distance (volts)}$ and $\sigma = \text{rms noise (volts)}$

then $P(\text{error}) = \frac{1}{2} P(0\mid 1) + \frac{1}{2} P(1\mid 0)$

$\text{sent 1, decided zero, sent zero, decided 1}$

$= \text{prob (noise voltage} > A)$

If the noise is Gaussian, this is given just as the "tail area" of the normal distribution

for Gaussian noise

$$p(v_n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{v_n^2}{2\sigma_n^2}}$$

and the "tail area" of interest is:

$$p(\text{error}) = \int_{-\infty}^{\infty} p(v_n) \, dv \quad \text{or} \quad \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx$$

if $x$ is defined as $(\frac{A}{\sigma})$ **Digital Signal to Noise (post-detection) Ratio**
Noise Equivalent Bandwidth (NEBW)

Actual filter:

White noise "filling" the channel

\[ |H(f)| \]

\[ |H(0)| \]

\[ n_0 \text{ watts/H}_2 \]

\[ f \]

Q. How much (white) noise power does this filter admit in total?

Say noise power spectral density = \( n_0 \) watts/H\(_2\)

A. \( N_{\text{out}} = n_0 \int |H(f)|^2 df \) (watts) (or \( V^2 \) in)

Q. How wide would a brick-wall filter have to be to admit the same total power?

A. The "Noise Equivalent BW of \( |H(f)| \)."

i.e.) \( N_{\text{out}} = n_0 \frac{B}{|H(0)|^2} = n_0 \int |H(f)|^2 df \)

(brick wall) \( \text{NEBW} \)

\[ B(H_2) = \frac{\int |H(f)|^2 df}{|H(0)|^2} \]

Noise equivalent BW
Graphical view:

- \( |H(\omega)| \) is just any constant overall gain that the filter may apply. Often \(|H(\omega)| = 1\) is assumed if not stated.

Q. So, what is the Noise Equivalent BW of an ideal Nyquist filter (for impulse transmission pulses) or any "Raised Cosine" filter for zero ISI shaping?

A. \( B = \frac{1}{2} \) the symbol rate. \( \text{NEBW} = \text{Nyquist rate} \)

Q. Is this still true if for instance the noise was not white, but say \((1/f)\)-like, or \(\alpha f^2\) ?

A. No. In the presence of "colored" noise power spectral densities we have to calculate output noise power in general as: \( N_{out} = \int N(f) \cdot |H(f)|^2 df \) (watts)
Problem example:

- **ON-OFF** (Unipolar) signalling is used with 500mV "ON" voltage and 0.010V "OFF" voltage.
- The receiver **NEBW** is 50 MHz and the white noise power spectral density present at the receiver input is 96 pW/Hz.
- Estimate the BER (bit error ratio) = $p(error)$

**First determine the "decision distance":**

\[
\text{ON} = 500 \text{mV} \\
\text{OFF} = 10 \text{mV} \\
\text{Decision distance (logic zero)} = 245 \text{mV} \\
\text{Decision distance (logic one)} = 245 \text{mV}
\]

- **Next determine the rms noise:**

\[
\begin{align*}
\text{NEBW} & = 50 \text{ MHz} \\
\text{LPF} & \\
\text{Total noise} & = 96 \times 10^{-12} \frac{\text{W}}{\text{Hz}} \\
& = 2.3 \text{ mW} \\
& \text{implies } \sigma_{\text{noise}} = \sqrt{2.3} \text{ mW} \\
& = 48 \text{ mV}
\end{align*}
\]
So we have the situation

\[ p(\text{noise}) \]

\[ \text{error (zero to one)} \]

\[ 245 \text{ mV} \]

\[ \text{dec distance} = 245 \text{ mV} \]

\[ 48 \text{ mV} \]

or in other words

\[ x = \left( \frac{245}{48} \right) = 5.1 \]

or

\[ 20 \log (5.1) = 14 \text{ dB} \]

and (lookup or approx. equation) \( \text{BER} \approx 1.7 \times 10^{-4} \)

Note: In other places for unipolar signalling you may see \( \text{BER} = Q\left( \frac{Ap}{2\Delta V} \right) \) where \( Ap = \text{peak on voltage} \)

This is the same as we did because we had \( Ap = 500 \text{ mV} \) and \( Ap/2 \) was the decision margin or decision distance, aside from the 10 mV of incomplete "extinction" in the OFF state.