19. Capacitive Accelerometers : A Case Study

- Introduction
- Fundamentals of Quasi-Static Accelerometers
- Position Measurement with Capacitance
- Capacitive Accelerometer Case Study
- Position Measurements with Tunneling Tips

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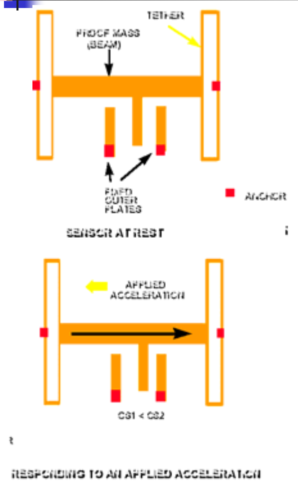
Introduction

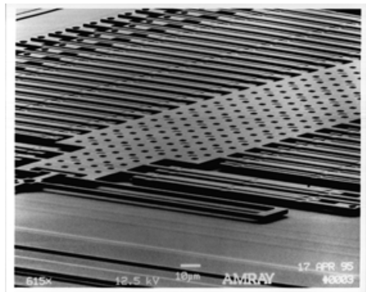
- The measurement of acceleration, in addition to being a central element of intertial guidance systems, has application in a wide variety of indutrial and sonsumer applications such as airbag deployment sensors in automobiles, vibration monitoring, and movement-based human/computer interfacing.
- Most acceleration sensors are of the "open-loop" kind, inasmuch as no feedback system is employed to offset the effect of the external acceleration being monitored.
- A "closed-loop" system, however, would also include a feedback mechanism that would monitor any displacement generated by the external acceleration, and would negate it by applying an opposite internal force applied through capacitive actuators.
- The offset force employed then becomes the actual output signal being read out.

Introduction (ctnd.)

- Most accelerometers are of the open-loop kind, but there are examples of closed-loop devices as well.
- In either case, a proof mass is held by some kind of elastic support attached to the rigid frame.
- Detection of acceleration is accomplished either by direct observation of the changed position of the proof mass (mostly accomplished through capacitive electrodes), or by detection of the deformation of the support (accomplished by piezoresistive or piezoelectric sensors.
- The case study presented here focuses on direct position measurements

Example of a position-monitoring device





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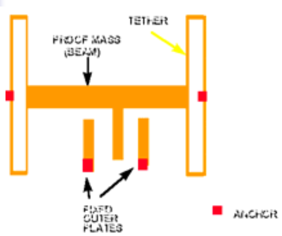
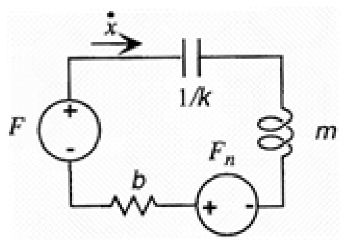
Lumped model of acceleration sensor



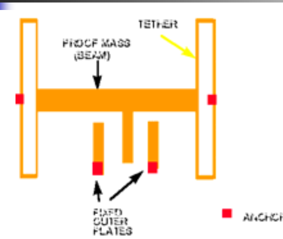


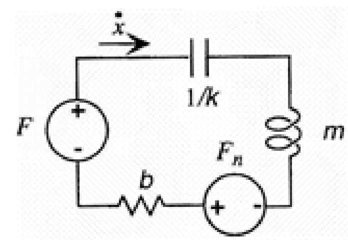
- Righthand figure shows a lumped electrical model for the spring-mass-dapshot system depicted to the left.
- F represents the external force and F_n a source of force noise
- The velocity response of this system (using Laplace transforms and related s-plane notation is:

$$\dot{x} = sx = \frac{F + F_n}{ms + b + k/s}$$

where m is the proof mass, k is the spring constant of the support, and the damping b usually comes from squeezed air damping.

Lumped model of acceleration sensor (ctnd.)





- The resulting force-displacement characteristic is:

$$x = \frac{F + F_n}{ms^2 + bs + k}$$

- This system has an undamped resonant frequency ω_0 = sqrt(k/m), and a quality factor equal to Q = $m\omega_0/b$.

Frequency considerations

- A "quasi-static" accelerometer is one in which the motion of the proof mass follows the time-evolution of the applied intertial force without significant retardation or attenuation.
- Therefore, one designs the accelerometer to have a frequency much larger than the expected maximum frequency component of the acceleration signal.
- In all of the following discussion, we shall assume that the frequencies of interest are well below ω_0. In that case, we can use the quasi-static response:

$$x = \frac{F + F_n}{k}$$

- The displacement and acceleration are scaled by the quare of the natural frequency:

$$x = \frac{a}{\omega_0^2}$$

- Thus, the scale factor depends only on the resonant frequency.
- For example, the detecion of 50 g using a 24.7 kHz device will result in displacement of the proof mass by only 20 nm.

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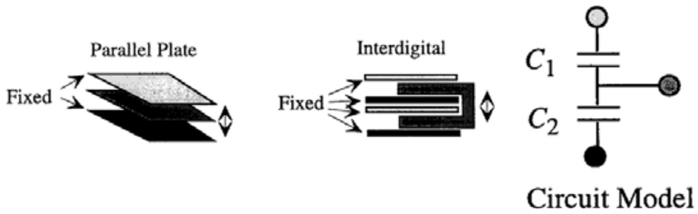
Transduction approaches

- Several approaches for electromechanical transduction have been discussed in previous section.
- These included capacitive, piezoresistive, piezoelectric, and quantum mechanical approaches.
- Optical position sensing in microstructures has also recently emerged as a promising approach in future devices.
- This section concentrates on capacitive approaches for transduction in accelerometers, and examine quantum mechanical approaches in some detail at end of section.

Standard capacitive transduction

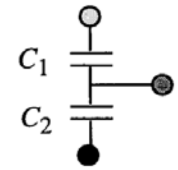
- Figure above shows three different design for the sensing of displacement using capacitively coupled surfaces.
- The parallel plate capacitor will vary through vertical motion of one plate with respect to the other
- The interdigitated capacitor will vary with the engagement of the fingers
- the fringing capacitance deploys an interdigital set of electrodes on one substrate, and detects the change of capacitance as the electrodes are brought into proximity with a third electrode.

Differential capacitive transduction



- Figure above shows an alternate approach that rather monitors the *difference* between two capacitors as one surface is moved with respect to two others.
- In all three examples, there are three electrodes used for the measurement, with two capacitors that are nominally of equal size when the moveable surface is centered.
- Such approach allows a linearization of the output signal about the balance point of the system

Differential capacitive transduction (ctnd.)



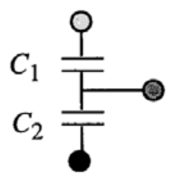
- Consider the parallel plate example. We define G_1 and G_2 as the gaps of the upper and lower electrodes, respectively. We assume an equal area for both capacitors. Voltages of $+V_s$ and $-V_s$ are applied to the upper and lower plates, respectively.
- The voltage appearing at the output is given by:

$$V_0 = -V_s + \frac{C_1}{C_1 + C_2}(2V_s) = \frac{C_1 - C_2}{C_1 + C_2} V_s$$

- This is rearranged to yield:

$$V_0 = \frac{G_1 - G_2}{G_1 + G_2} V_s$$

Differential capacitive transduction

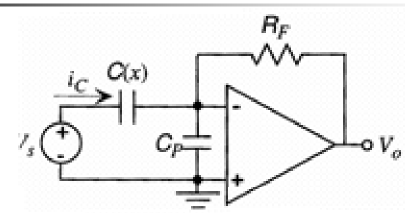


- ....this is rearranged to yield:

$$V_0 = \frac{G_1 - G_2}{G_1 + G_2} V_s$$

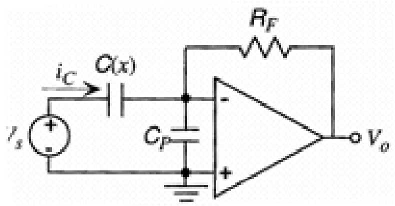
- If the two gaps are equal, then the output voltage is zero. However, if the middle plate moves so that one gap becomes larger than the other, then the output signal will linearly depend on that displacement.

Circuits for capacitance measurements



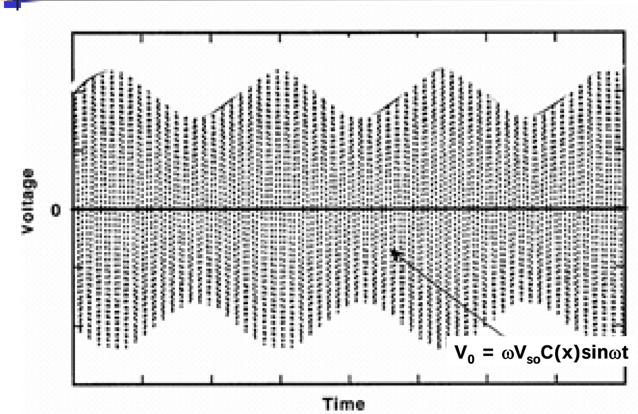
- Figure above shows a sensing circuit in which a transimpedance amplifier is used to capture the current flowing through the sensing capacitor C(x). A parasitic capacitance C_p is also included for modelling purposes.
- Because of the virtual ground at the op-amp input, there is negligible charge on the parasitic capacitance, and it does not affect the measurement.
- The output of this circuit is $V_0 = - R_f i_c$.
- If V_s is a DC source, then i_c (and thus V_0) will be proportional to the velocity dx/dt. To obtain *position,* one must therefore used a properly initialized integrator, or use a time-varying waveform.

Circuits for capacitance measurements (ctnd.)



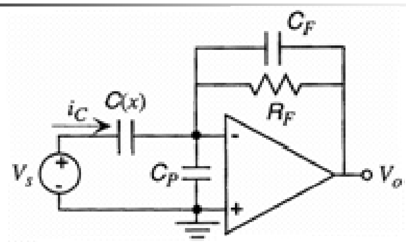
- If we were to use a sinusoidal waveform as the source $V_s = V_{so} \cos \omega t$, then the output of the amplifier will rather be $-\omega V_{so} C(x) \sin\omega t$.
- Thus the value of C(x) can now be determined from the amplitude of the output sinusoidal wave.
- However, if C(x) is also time varying (in vibration-monitoring applications, for instance), then the output signal will also have a component depending on dC/dt.
- This approach therefore requires to make the frequency ω of V_s sufficiently large to insure to have output signal be dominated by the value of C(x) rather than its time derivative.

Circuits for capacitance measurements (ctnd.)



Under these condictions, in the case of cyclic variations of C(x), the output signal is therefore an amplitude modulated signal that uses V_s as carrying frequency.

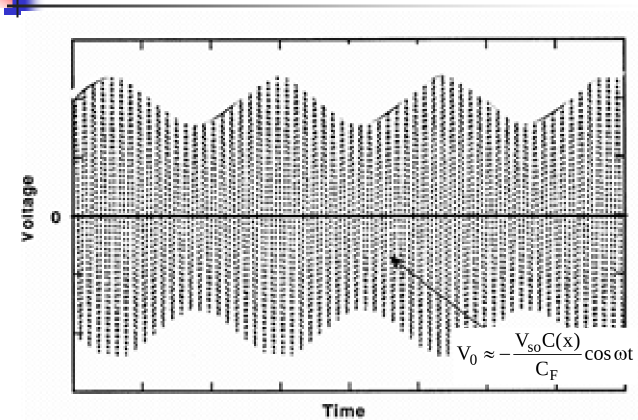
Adding a feedback capacitor



- When using a high-frequency AC source, so that the velocity-dependent component of the current can be ignored, then an addition of a capacitor in the feedback arm can also be employed.
- The value of R_f is chosen so that at the measurement frequency, the magniatude of $Z_c = 1/j\omega C$ is small compared to R_f.
- The output is then:

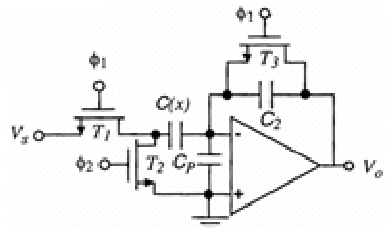
$$V_0 \approx -\frac{i_C}{sC_F} V_s \approx -\frac{C(x)}{C_F} V_s$$

Adding a feedback capacitor (ctnd.)



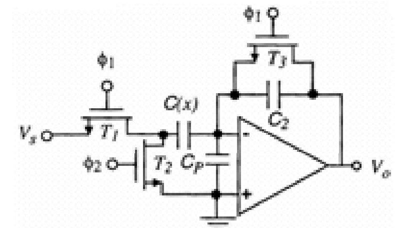
Addition of C_f removes the carrying frequency-dependence on the output modulation

Using a switch-capacitor inverter



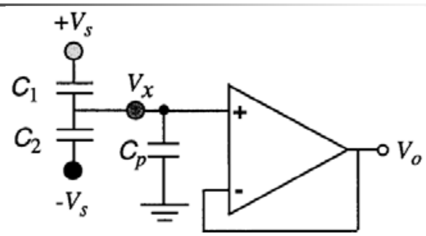
- Such designs uses two non-overlapping clock pulses ϕ_1 and ϕ_2 to switch the transistors from open circuits to closed connections.
- When ϕ_1 turns on transistors T_1 and T_3, the amplifier operates as a unity-gain buffer, and capacitor C(x) acquires a charge $C(x)V_s$.
- When ϕ_1 is turned off, it isolates C(x) and turns the op-amp into an integrator, ready to collect the charge from C(x).
- A short time later, ϕ_2 is turned on, grounding the left terminal of C(x) that creates a negative-going signal at the inverting input which drives V_0 positive, pulling the inverting node back towards zero.
- The circuit then settles to $V_0 = [C(x)/C_2]\ V_s$

Using a switch-capacitor inverter (ctnd.)



- The clock cycle is then repeated, charging C(x) to V_s and discharging C_2 back to zero.
- The output therefore alternates between zero and $[C(x)/C_2]\ V_s$.
- Putting this signal through a low-pass filter therefore provides an averaged output equal to f $[C(x)/C_2]\ V_s$, where f is the fractional duty cycle (equal to 0.5 for symmetric clock timings).

Readout of differential capacitors

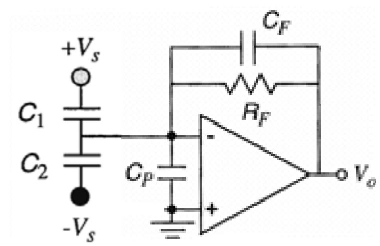


- When a differential capacitor is used, the voltage on the shared terminal is directly measured.
- Circuit above employs a unity-gain buffer to directly sense the output voltage labelled V_x.
- Assuming symmetric sinusoidal or pulse signals applied to the outer terminal, V(x) is given by:

$$V_x = \frac{C_1 - C_2}{C_1 + C_2 + C_P} V_s$$

- The parasitic capacitance C_p therefore affects the signal, as well as the calibration of the system

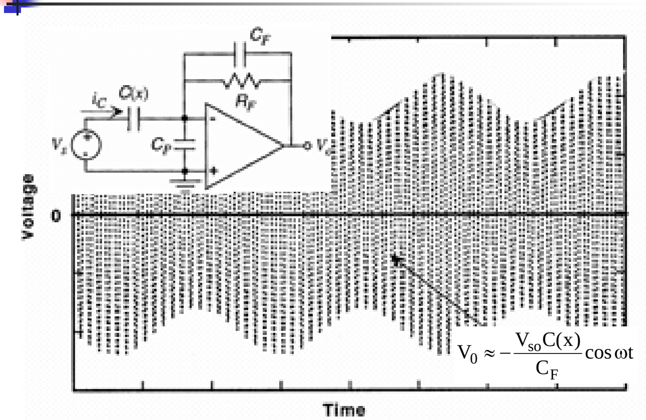
Readout of differential capacitors (ctnd.)



- Alternatively, one could use the inverting configuration with differential capacitor described previously.
- Using oppositely phased sinusoidal sourced for $+V_s$ and $-V_s$, the output of the circuit becomes :

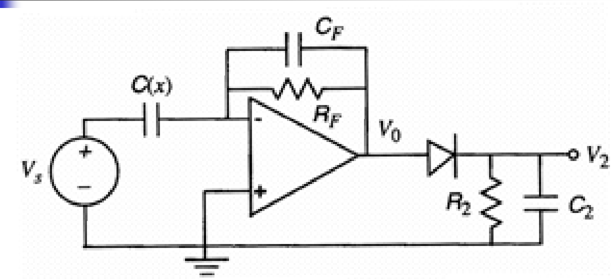
$$V_0 = -\frac{C_1 - C_2}{C_F} V_s$$

Demodulating circuitries



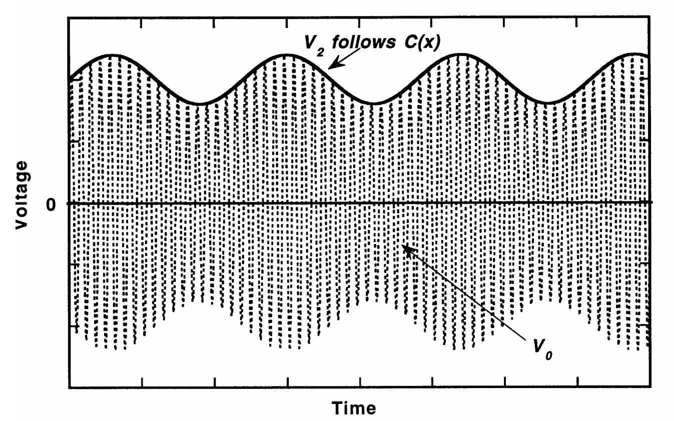
As mentioned previously, the op-amp output V_0 is a relatively high-frequency sinusoid proportional to the quantity C(x) of interest.

Demodulation using a peak detection circuitry



- If the R_2C_2 time constant is selected to be long enough compared to the period of the sinusoid, yet short compared to characteristic time for changes in C(x), the output V_2 is a slowly varying signal which follows the amplitude of the sinusoid (next slide).

Demodulation using peak detection circuitry (cntd.)



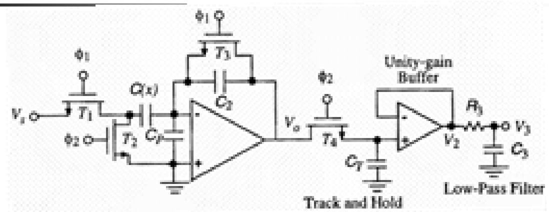
Synchronous demodulator circuit

- The synchronous demodulator is a circuit for demodulating a periodic wave function, wheter sinusoidal or pulsed.
- As example, assume that the wavefunction of interest is A(t) cosωt, where A(t) is a slowly varying amplitude corresponding to changes in C(x).
- If we multiply this by a reference sinusoid at the same frequency, B cos (ωt + θ), where θ is a phase shift, the result is:

$$[A(t)\cos\omega t][B\cos(\omega t+\theta)] = \frac{A(t)B}{2}[\cos\theta + \cos(2\omega t+\theta)]$$

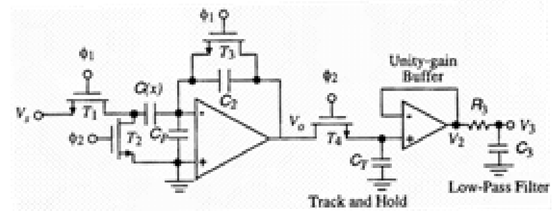
- If this is signal is put through a low-pass filter with a corner frequency that rejects the component at 2ω, the result of that filtering will be A(t)Bcosθ, ie. an amplified version of the original signal.
- Due to the phase-sensitivity of the output, it is essential for the reference signal to possess the correct phase.

Synchronous demodulator circuits (ctnd.)



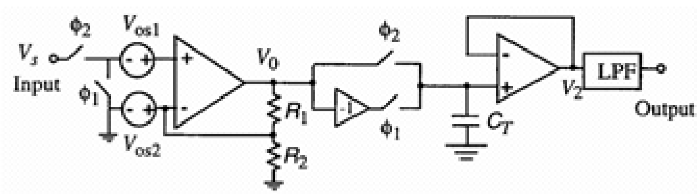
- One can alternatively use a track-and-hold circuit in combination with a switch, as illustrated above.
- When ϕ_2 is on, and the V_0 output equals $C(x)V_s/C_F$ the track and hold capacitor C_T is charged to that value through transistor T_4, so the output V_2 equals $V_{O.}$
- When ϕ_1 is on, C_T is disconnected, and hence holds the previous value of V_o. Meanwhile, C(x) charges up to its next measurement value
- Once ϕ_2 turns on again, the V_0 output signal is updated to this new value of $C(x)V_s/C_F$
- Thus, the output is a fair-step waveform that follows samples of C(x), one sample per clock cycle.

Synchronous demodulator circuits (ctnd.)



- The R_3C_3 section of the circuit constitutes a low-pass filter with a time constant long compared to the switching period, but short compared to expected time variations.

Chopper stabilized amplifiers

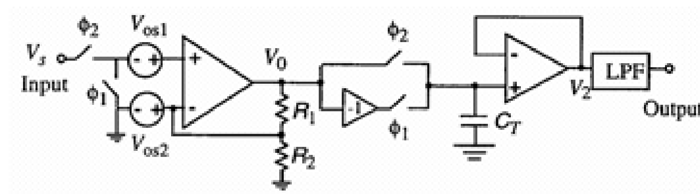


- Op-amp devices can have input offset voltages which can induce errors in the inferred capacitance values.
- A chopper-stabilized op-amp circuit (shown above) uses transistor switches to alternate the input of the non-inverting amplifier between the input signal and ground. The circuit also includes the internal offset voltages for purpose of modelling.
- The output voltage of the circuit is given by:

$$V_0 = \frac{A(R_1+R_2)}{AR_1+R_1+R_2}(v_+ - V_{os2})$$

- where A is the open-loop gain of the op-amp, and v^+ is the voltage at the non-inverting input of the op-amp.

Chopper stabilized amplifiers (ctnd.)



- In the limit of large A, this reduces to the familiar non-inverting gain $(R_1+R_2)/R_1$.
- During the ϕ_1 phase, v_+ is V_{os1} while during phase ϕ_2, v_+ is $V_s + V_{os1}$.
- Thus the V_0 signal is a square wave that alternates between the amplified difference in offset voltages, and the amplified plus the difference in offset voltages.
- Because the amplified offset voltages appear in both phases, they contribute to the DC average value of the V_0 signal, but not to the heigth of the square wave.

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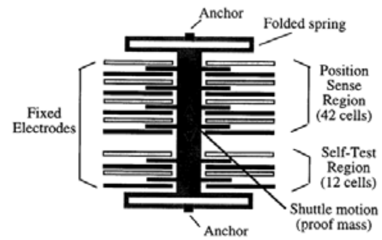
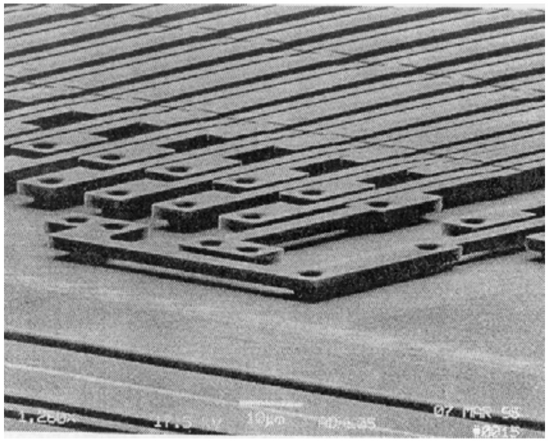
ADXL 150 accelerometer



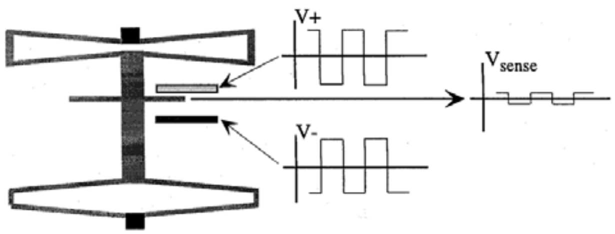
- **Monolithic device integrating poly-Si proof-mass, spring suport, and capacitive sensors together with electronic devices required to provide analog output proportional to acceleration.**
- **A moveable shuttle provides a proof mass, and is suspended on folded springs attached though anchor points.**
- **A number of cantilevered electrodes are positioned between two fixed electrodes, forming lateral differential capacitors.**
- **There is also a self-test region with similar electrode arrangement, but these electrodes are connected to a external drive circuit for purposes of testing operation of device.**

ADXL 150 accelerometer (ctnd.)



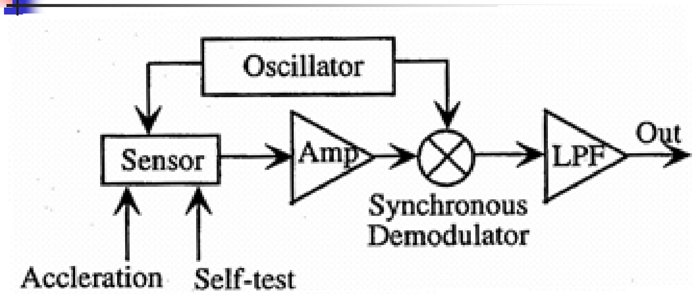
- **Enlarged view…**

Operation of device



- **Figure above illustrates the exagerated motion of the proof mass shuttle in response of inertial acceleration.**
- **A proof-mass displacement unbalances the differential capacitor.**
- **The two fixed electrodes are driven with oppositely polarized plane waves that measure the unbalance of the differential capacitors.**
- **This output is amplified, synchronously demodulated, and low-pass filtered to provide the output signal.**

Circuit block diagram

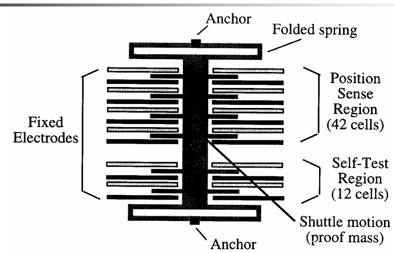


- **Original ADXL 50 employed a closed-loop system which strongly depended on the linearity and accuracy of the feedback force actuators.**
- **More recent devices such as the ADXL 150 rather amployed a open-loop system, as shown above, which will rather depend on the linearity and accuracy of the restoring springs and sensing capacitors**

Specifications

Property	Specification
Sensitivity	38mV/g
Full-scale range	$\pm$ 50 g
Transfer function form	see text
Package type	14-pin cerpak
Temperature range	-40 to +85°C
Supply voltage	4 - 6 V
Nonlinearity	0.2 %
Package alignment error	$\pm$ 1°
Transverse sensitivity	$\pm$ 2%
Zero-g output voltage (Bias)	$V_s/2 \pm 0.35$ V
Temperature drift (from 25°C to T_{min} or T_{max})	0.2 g
Noise from 10 Hz to nominal bandwidth	1 m$g/\sqrt{\text{Hz}}$
Clock noise	5 mV peak-to-peak
Bandwidth	400 or 1000 Hz, customer choice
Temperature drift of bandwidth	50 Hz
Sensor resonant frequency	24 kHz
Self test output change	400 mV
Absolute maximum acceleration	2000 g (unpowered)
	500 g (powered)
Drop test	1.2 meters
Min/max storage temperature	-65 to 150 °C
Max lead temperature (10 seconds)	245 °C

Sensor design and modeling

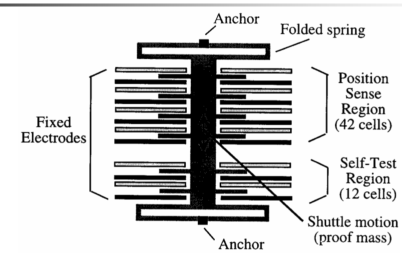


- **The shuttle electrodes and folded spring are fabricated out of H = 2 um thick poly-Si. The nominal gap between fingers is G_0 = 1.3 um, and the length of the overlap region is L_0 = 104 μm.**
- **Given there are 42 identical capacitors in parallel, the total value of the sense capacitor is given by:**

$$C_{sense} \approx 42 \frac{\varepsilon_0 H L_0}{G_0 \pm y}$$

where y is the displacement to be measured.

Sensor design and modeling (ctnd.)

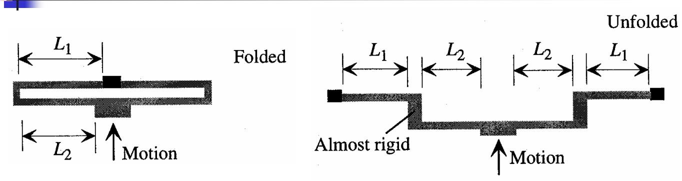


- **Assuming small displacements (y << G_0), we get:**

$$C_{sense} \approx 60\left[1 \pm \frac{y}{G_0}\right] \quad \text{(fF)}$$

- **The actual capacitance will be larger due to the presence of fringing fields on the edges of the plates.**

Sensor design and modeling (ctnd.)

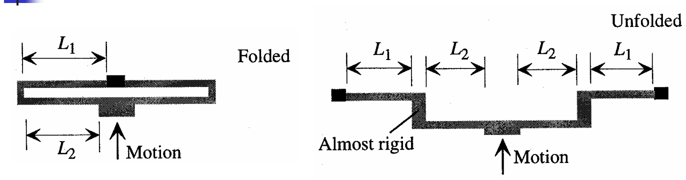


- Figure above illustrates how the folded springs with two segments L_1 and L_2 could be "unfolded" to create two connected doubly-clamped beams of length $2L_1$ and $2L_2$.
- The total displacement can be found from the sum of the compliances of the two beams, leading to a net spring constant:

$$k = \frac{F}{c} = \left(\frac{\pi^4}{6}\right)\left[\frac{EWH^3}{(2L_1)^3 + (2L_2)^3}\right]$$

- where F is the applied load, c is the mass displacement, E is Young's modulus, W is the width of the beam, and H is its thickness. Here W is the poly-Si thickess since the cantilevers are bending in plane.

Sensor design and modeling (ctnd.)



- Using $L_1 = L_2 = 120\ \mu m$, we obtain k = 2.8 N/m for each anchor. Combining the effect of the two anchors, we would get k = 5.6 N/m.
- The mass of the proof mass is $m = 2.2 \times 10^{-10}$ kg, yielding a natural resonant frequency of f_0 = 24.7 kHz. The stated bandwidth of 1 kHz is therefore substantially less than this resonant frequency.
- Using a Couette flow model, a damping coefficient would be:

$$b = \frac{\eta A}{h} = 2.8 \times 10^{-7}\ (N \cdot s)/m$$

- corresponding to $Q = m\omega_0/b \sim 120$, which is much more than specified value. Other dissipation mechanisms may therefore be involved.

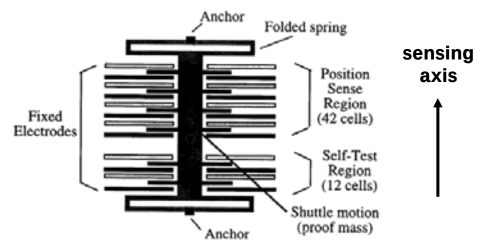
Noise and accuracy

- The sensitivity of the accelerometer is determined by the noise, which is specified as 1 mg/Hz$^{0.5}$ in a bandwidth from 10 Hz to 1000 Hz.
- This is about twice the noise estimate ascribed to Brownian motion.
- Applied to the maximum bandwidth of 1000 Hz, this correspond to an acceleration noise of 32 mg.
- Such sensitivity corresponds to a proof-mass positioning error of:

$$\delta x = \frac{\partial a}{\omega_0^2} = \frac{(32x10^{-3})(9.8)}{(1.55x10^5)^2} = 0.013\ nm$$

- Accuracy of fabricated electrodes is critical. A mismatch between the capacitor gaps of 1 % will yield to a net capacitive force of 0.01 μN when a voltage of 2.5 V is being applied.
- This is enough to move the shuttle by 2 nm, corresponding to an offset of the acceleration signal of almost 5 g.
- Cross-axis sensitivity is also another important characteristic. The device should be sensitive along one axis only, and that axis clearly marked on the chip package. Thus misalignment of the device relative to these markings could lead to apparent cross-axis sensitivities.

Intrinsic cross-axis sensitivity



- However, the device itself could also possess some of its own cross-axis sensitivity.
- The high stiffess of the folded beam design minimizes device response along the other in-plane other axis.
- In the out-of-plane direction, however, device stiffness is comparable to the in-plane direction being sensed.
- However, cyclic motion along that out-of-plane direction will be greatly attenuated due to the dominance of squeezed-air damping along that direction

8. Capacitive Accelerometer : A Case Study

Introduction
Fundamentals of Quasi-Static Accelerometers
Position Measurement with Capacitance
Capacitive Accelerometer Case Study
Position Measurements with Tunneling Tips

Tip-sample tunnel current under bias

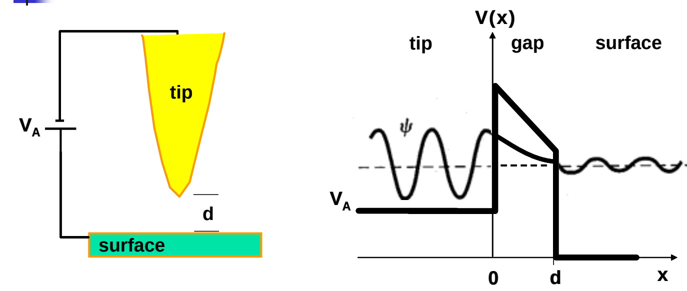


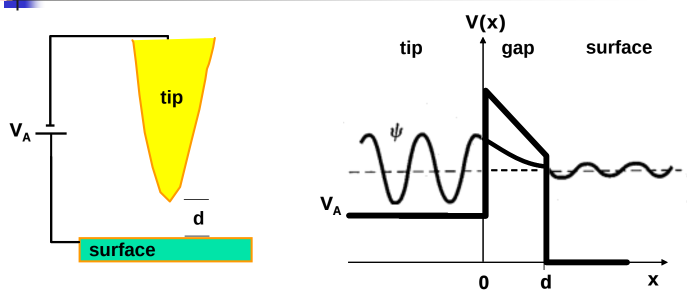
Diagram shows an electron of energy E incident from left of a potential barrier (eg. the gap between a metal tip and a metal surface).

Classically, electron would simply reflect off and remain on left-hand side.

Quantum mechanically, the wave function extends into the barrier (decaying exponentially) and through the other side.

There therefore is finite probability for the electron to travel through the barrier, thus inducing measurable "tunneling current".

Tip-sample tunnel current under bias (ctnd.)



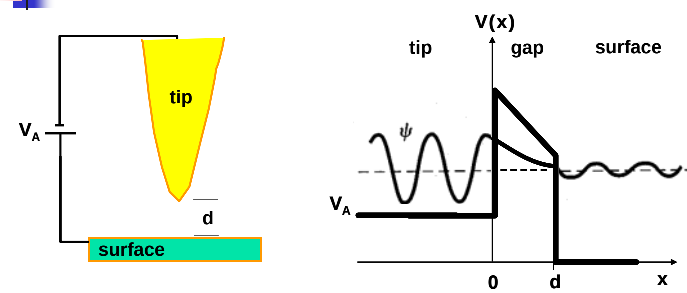
This tunneling current will be roughly exponentially proportional on gap:

$$I \propto V_A e^{-\alpha_I \sqrt{\Phi} d}$$

where α_I = 1.025 $A^{-1}eV^{-0.5}$, and Φ is the effective height of the tunnel barrier, which is about Φ = 0.2 eV.

Typical current levels are nanoamps when the distance d is of order of 1 nm and the bias V_A is ~0.2 V.

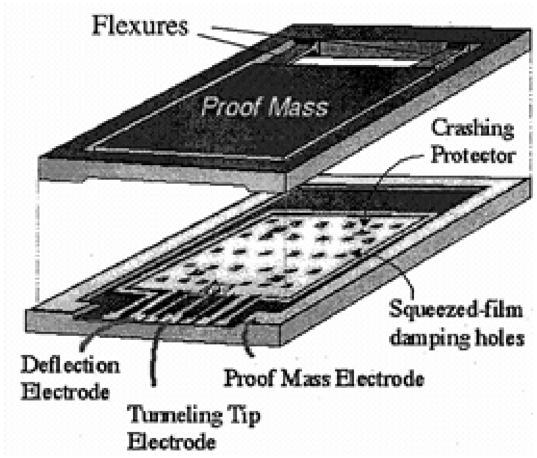
Tip-sample tunnel current under bias (ctnd.)



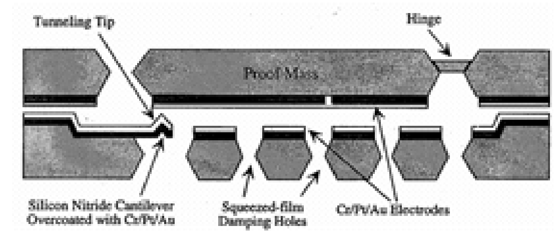
A displacement change of only 0.01 nm will result in a 4.5 % change of tunnel current, well within measuring abilities.

A most sensitive approach for measuring nanoscale displacements is to leverage this rapidly decaying exponential dependence of tunnel current between a tunneling tip and an electrode.

Tunneling accelerometer



Tunneling accelerometer (ctnd.)



As fabricated, the gap between tip and proof mass is relatively large, and a large DC voltage (V_{DC} ~25 V) applied between electrodes and mass is required to bring the two surfaces within tunneling range.

A small time-varying v_{ac} is added to this DC voltage whose amplitude is controlled by a feedback system.

Under closed-loop operation, v_{ac} is constantly adjusted to to keep the tunneling current constant, which in turn is equivalent to monitoring the inertial forces that are bending the cantilever.

Noise characteristics

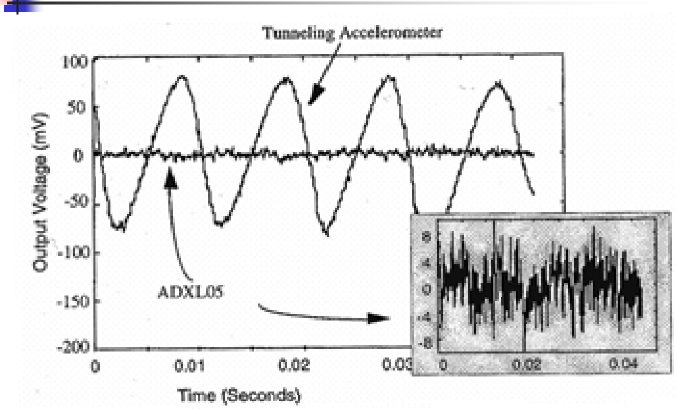
- **The noise characteristics of such device are quite interesting given the strong non-linear reponse of the device.**
- **A "gauge factor" is defined as the ratio of fractional change in tunnel current to the fractional change in gap:**

$$G.F. = \left|\frac{\delta I_t / I_t}{\delta d_t / d_t}\right| = \frac{d_t}{0.046\,\text{nm}}$$

where d_t ~ 1 nm. This gauge factor is ~22, about twenty times more sensitive to the gauge factor of capacitance sensors, which is ~1.

- **This means that the tunnel sensor itself acts as if it has a preamplifying stage built right into its operation. As a result, the noise performance of the device will be directly dominated by the intrinsic noise in the device itself.**
- **The noise equivalence of these devices approach 20 ng/(Hz)$^{0.5}$, about 50 times more sensitive than the commercial ADXL devices.**

Comparative performance



Comparison of the response of a tunneling accelerometer to that on Analog Devices ADXL05.
Both devices are subjected to a sinusiodal acceleration of 1.5 mg at 100 Hz