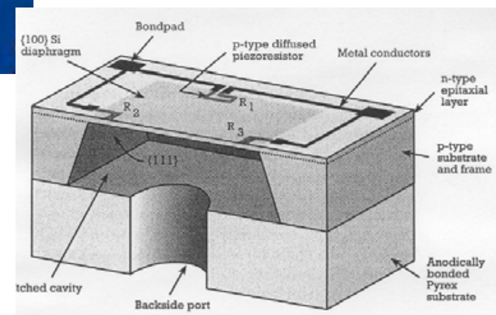
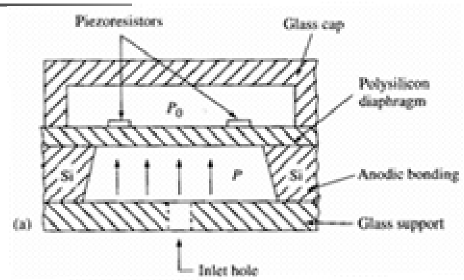
18. Piezoresistive Pressure Sensors : Case Study

Introduction
Piezoresistance
Motorola MAP sensor

Example: membrane pressure sensor



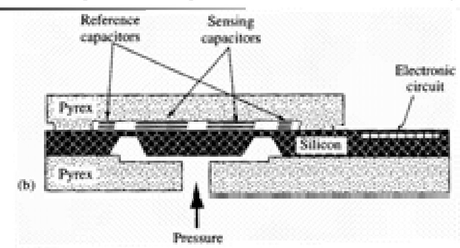
Piezoresistive pressure sensor design



- The piezoresistive straing gauge are usually made of doped poly-Si and are designed in pairs with a readout cicuitry (usually a Wheatstone bridge).
- While strain-pressure reponses of the membrane have been modelled in previous chapter, practical devices are usually rather calibrated, and their response stored on-chip in a look-up table.
- The response of the device to applied pressure is related through the mechanical response of membrane, piezoresistive response of transducer:

$$V_{out} \propto \Delta R \propto \Pi(P - P_0)$$

Capacitive pressure sensor design



- In this design, a capacitive bridge can be formed wit two reference capacitors, and the output voltage is related to the deflection of the membrane Δx and hence the differential pressure ($P - P_0$) through:

$$V_{out} \propto \Delta C \propto \Delta x \propto (P - P_0)$$

- By controlling the background pressure P_0 it is possible to fabricate the following types of pressure sensors:
  - An absolute pressure sensor that is referenced to vacuum ($P_0 = 0$)
  - A gauge-type pressure sensor that is referenced to atmoshperic pressure (P_0 = 1 atm)
  - A differential sensor where P_0 is maintained at a known value

Piezoresistive vs capacitive approaches

	Advantages	Disadvantages
Capacitive	More sensitive (polysilicon)	Large piece of silicon for bulk micromachining
	Less temperature-sensitive	Electronically more complicated
	More robust	Needs integrated electronics
Piezoresistive	Smaller structure than bulk capacitance	Strong temperature-dependence
	Simple transducer circuit	Piezocoefficient depends on the doping level
	No need for integration	

Piezoresistive designs are the most employed because of its low cost, robustness, and ease of circuit integration

Piezoresistive pressure sensors

- **This chapter introduces piezoresistive devices through the specific case study of membrane pressure sensors**
- **While other approaches such as capacitive effects can be used for such applications, silicons also possess the property of piezoresistance whose implementations as transduction mechanism in membrane is somewhat more straightforward**

9. Piezoresistive Pressure Sensors : Case Study

Introduction
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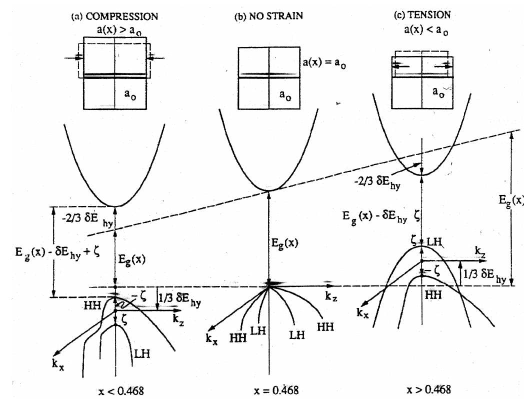
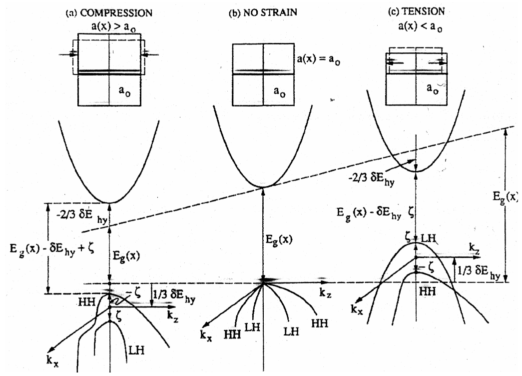
Piezoresistivity



- **Piezoresistivity is the dependence of electrical resistivity on strain**
- **Such an effect is related to the rearrangement of energy bands of a solid under applied strain (above)**

Piezoresistivity



- Such strain will modify both the bandgap as well as the effective masses (and thus mobilities) ascribed to the bands
- The effect is isotropic in as much as a given strain may increase resistivity along one direction while decreasing it along others

Analytic formulation in cubic materials

- Assuming effect is linear, the relationships between electric field and current density is given by:

$$\boldsymbol{\varepsilon} = [\boldsymbol{\rho}_e + \boldsymbol{\Pi} \cdot \boldsymbol{\sigma}] \cdot \mathbf{J}$$

  where ρ_e is the resistivity tensor, Π is the piezoresistive tensor, σ is the stress tensor, and J is the current density
- Note: while ε and J are vectors, ρ_e and σ are second rank tensors, while Π is a fourth rank tensor
- However, in a cubic crystal the resistivity tensor is diagonal and characterized by a unique diagonal value ρ_e
- In addition, as previously described, the stress tensor can be reduced to six independent elements and re-annoted as such:

$$\sigma_{11} = \sigma_1 \qquad \sigma_{23} = \sigma_{32} = \tau_{23}$$
$$\sigma_{22} = \sigma_2 \qquad \sigma_{31} = \sigma_{13} = \tau_{31}$$
$$\sigma_{33} = \sigma_3 \qquad \sigma_{12} = \sigma_{21} = \tau_{12}$$

Analytic formulation in cubic materials (ctnd.)

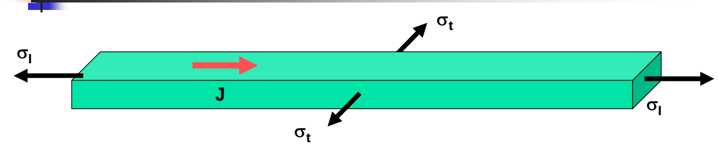
- Thus, the above equation can be written along the three principal directions of the cubic lattice

$$\frac{\varepsilon_1}{\rho_e} = [1 + \pi_{11}\sigma_1 + \pi_{12}(\sigma_2 + \sigma_3)]J_1 + \pi_{44}(\tau_{12}J_2 + \tau_{13}J_3)$$
$$\frac{\varepsilon_2}{\rho_e} = [1 + \pi_{11}\sigma_2 + \pi_{12}(\sigma_1 + \sigma_3)]J_2 + \pi_{44}(\tau_{12}J_1 + \tau_{23}J_3)$$
$$\frac{\varepsilon_3}{\rho_e} = [1 + \pi_{11}\sigma_3 + \pi_{12}(\sigma_1 + \sigma_2)]J_3 + \pi_{44}(\tau_{13}J_1 + \tau_{23}J_2)$$

where the three independent piezoresistive coefficients are:

$$\rho_e \pi_{11} = \Pi_{1111}$$
$$\rho_e \pi_{12} = \Pi_{1122}$$
$$\rho_e \pi_{44} = 2\Pi_{2323}$$

Longitudinal and transverse piezoresistance



- If a relatively long and narrow resistor is defined in a planar structure, then the primary current density and electric field are both along the long axis of the resistor.
- Structures are usually designed so that one of the axes of the principle in-plane stress is also along the resistor axis.
- This simplifies the set of equations to following simplified formulation:

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} = \pi_l \sigma_l + \pi_t \sigma_t$$

  where *R* is the resistance of the resistor and the subscripts ***l & t*** refer to transverse and longitudinal stresses along the resistor axis.

Longitudinal and transverse piezo… (ctnd.)

- The orientations of resistor is not necessarily aligned with crystalline orientations of device
- The general expressions π_l and π_t are related to the original tensor through:

$$\pi_l = \pi_{11} - 2(\pi_{11} - \pi_{12} - \pi_{44})(l_1^2 m_1^2 + l_1^2 n_1^2 + m_1^2 n_1^2)$$

and

$$\pi_t = \pi_{12} + (\pi_{11} - \pi_{12} - \pi_{44})(l_1^2 l_2^2 + m_1^2 m_2^2 + n_1^2 n_2^2)$$

where (l_1, m_1, n_1) are the directional cosines between the longitudinal resistor direction and the crystal axis and (l_2, m_2, n_2) is the direction cosines between the transverse direction and the crystal axes

- Note: by this definition $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ given that these direction cosines are orthogonal to each other.

Longitudinal and transverse piezo… (ctnd.)

- Piezoresistors are often oriented in the [110] directions.
- The directional cosines are:

$$(l_1, m_1, n_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \quad (l_2, m_2, n_2) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

Thus:

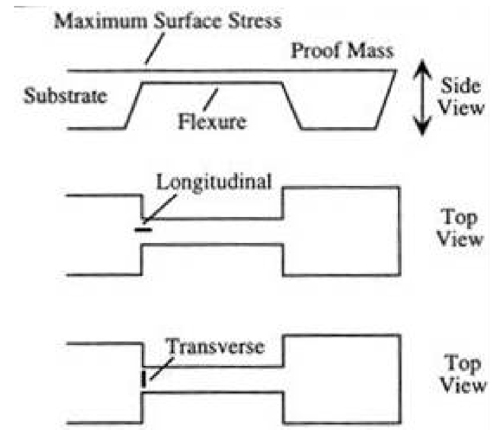
$$\pi_{l,110} = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$$

$$\pi_{t,110} = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})$$

Piezoresistive coefficients in Si

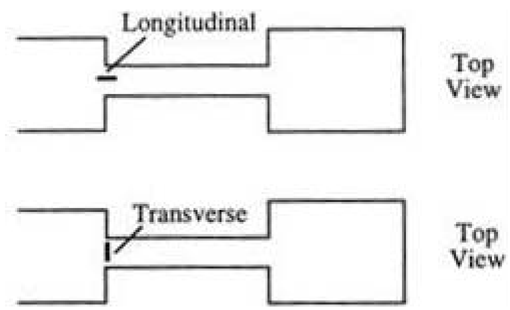
Type	Resistivity	π_{11}	π_{12}	π_{44}
Units	Ω-cm	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}
n-type	11.7	-102.2	53.4	-13.6
p-type	7.8	6.6	-1.1	138.1

Design example



- The resistors are fabrictated along the [110] directions

Design example (ctnd.)

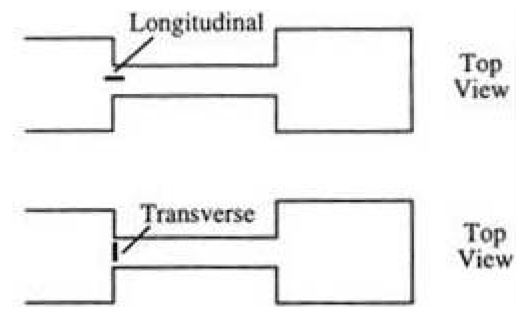


- **For n-type resistors:**

$$\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44}) = \frac{1}{2}(-102.2 + 53.4 - 13.6) = -31.2 \times 10^{11}\,\mathrm{Pa}^{-1}$$

$$\pi_t = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44}) = \frac{1}{2}(-102.2 + 53.4 - 13.6) = -17.6 \times 10^{11}\,\mathrm{Pa}^{-1}$$

Design example (ctnd.)



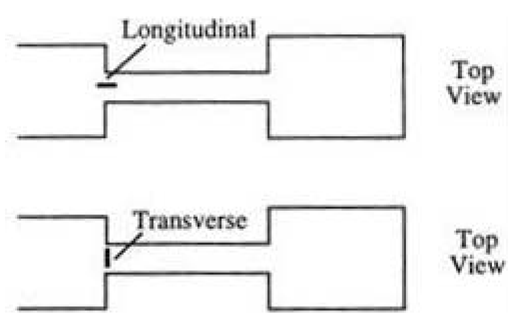
- **For p-type resistors:**

$$\pi_l = 71.8 \times 10^{11}\,\mathrm{Pa}^{-1}$$

$$\pi_t = -66.3 \times 10^{11}\,\mathrm{Pa}^{-1}$$

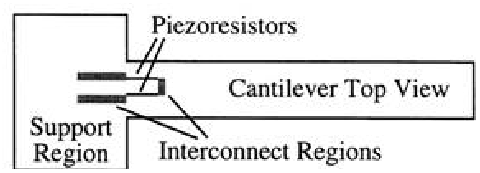
thus, p-type is better suited to perform piezoresistive readout in this direction

Design example (ctnd.)



- **Which is better, a longitudinal or a transverse resistor?**
- **The transverse resistor is fully plunged in region of maximum strain, but will also be greatly affected by placement error**
- **The longitudinal resistor spans over a wider region of stresses. It will be less sensitive, but will be less prone to alignment errors from device to device**

Numerical example



- **The n-type cantilever is 200 μm long 20 μm wide and 5 μm thick. It is bent by a point load on its end. The p=type piezoresistors are 20 μm long and 2μm wide. A force of 10 μN is applied at extremity of device. Calculate change of resistance**
- **From Senturia, section 9.3:**

$$w_{max} = \left(\frac{4L^3}{EWH^3}\right)F = \left(\frac{4(200\times10^{-6})^3}{(160\times10^{9})(20\times10^{-6})(5\times10^{-6})^3}\right)10\times10^{-6} = 0.8\mu m$$

- **The cantilever deflection at any point *x* is given by:**

$$w(x) = \frac{FL}{2EI}x^2\left(1 - \frac{x}{3L}\right)$$

Numerical example (ctnd.)

- **The radius of curvature is then given by:**

$$\frac{1}{\rho(x)} = \left|\frac{\partial^2 w}{\partial x^2}\right| = \frac{F}{EI}(L-x)$$

- **Since $\sigma = -zE/\rho$, the stress at surface (z =-H/2) is given by:**

$$\sigma(x)\frac{EH}{2\rho} = \frac{FH}{2I}(L-x)$$

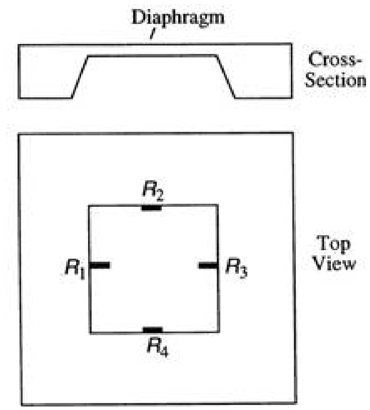
- **Given that $I = WH^3/12$:**

$$\sigma(x) = \frac{6F}{H^2 w}(L-x) = 1.2\times10^{11}(L-x)$$

- **It therefore spans from σ = 24 MPa at x = 0 to 21.6 MPa at x = 20 μm**
- **An average stress of σ = 22.8 MPa is therefore used to calculate change of resistance. We finally get:**

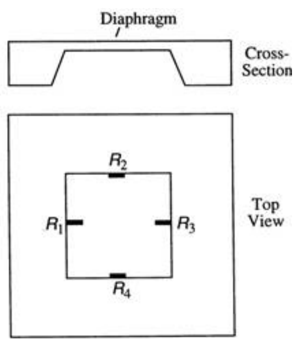
$$\frac{\Delta R}{R_0} = (67.6\times10^{-11})(22.8\times10^{6}) = 1.54\,\%$$

Alternate design



- **All four resistors are aligned along one of the [110] directions, and aligned with the principle axes of stresses**

Alternate design (cntd)

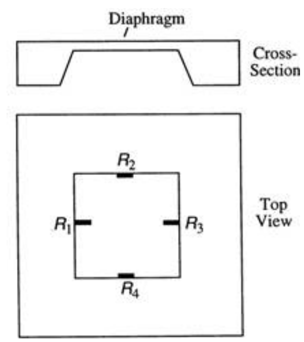


- **Resistors R_1 and R_3 experience stresses σ in their longitudinal direction and a stress $\upsilon\sigma$ in their transverse direction:**

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \pi_l\sigma + \pi_t\upsilon\sigma = (67.6\times10^{-11})\sigma$$

(note: the Poisson ratio in the [110] direction is ν = 0.064)

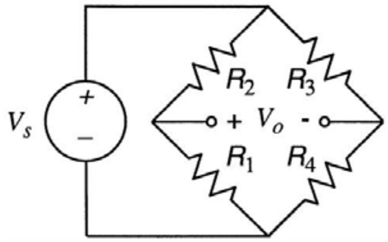
Alternate design (ctnd.)



- **Resistors R_2 and R_4 experience stresses σ in their transverse direction and a stress $\upsilon\sigma$ in their longitudinal direction:**

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = \upsilon\sigma\pi_l + \sigma\pi_t = -61.7\times10^{-11}\sigma_l$$

Alternate design (ctnd.)



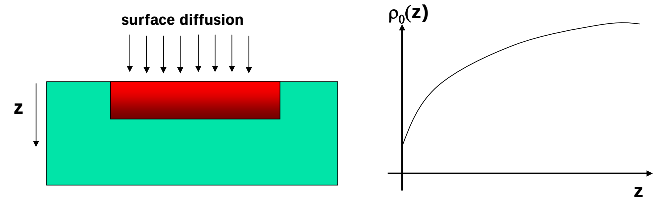
- Connecting the four resistors in a Wheatstone bridge configuration, we get:

$$\frac{V_0}{V_s} = \frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_3+R_4)} \approx \frac{\alpha_1+\alpha_2}{2(1+\alpha_1-\alpha_2)}$$

where:

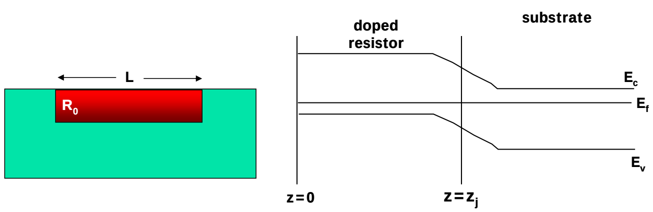
$$\alpha_1 = (\pi_l + \upsilon\pi_t)\sigma$$
$$\alpha_2 = (\upsilon\pi_l + \pi_t)\sigma$$

Averaging over doping variations



- Real life piezoresistors will present some degree of non-uniformity with respect to the doping levels and the stress distribution they are subjected to.
- For instance, creation of piezoresistor through surface diffusion doping will create a depth-dependent profile of the unstrained resistivity ρ_0, as seen above.

Averaging over doping variations (cntd)

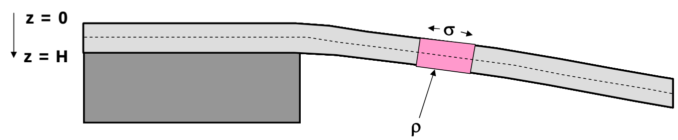


- To calculate the nominally unstrained resistance R_0 , we evaluate the following integral:

$$\frac{1}{R_0} = \int_0^{z_j} \frac{W}{L\rho_{e,0}(z)} \partial z$$

where z_j is the junction depth equal to the edge of the space-charge junction.

Averaging over stress variations



- The neutral axis is at z = H/2 thus:

$$\sigma(z) = \frac{E(H/2 - z)}{\rho}$$

- Transverse stresses are neglected, thus, the stress-induced resistivity change is:

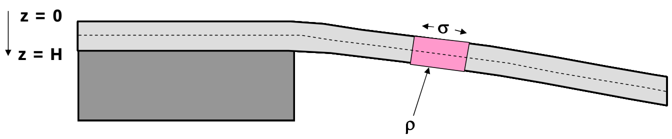
$$\rho_e(z) = \rho_{e,0}(z)[1 + \pi_l\sigma_l(z)]$$

- Thus:

$$\frac{1}{R} = \int_0^{z_j} \frac{W}{L\rho_{e,0}(z)}[1 - \pi_l\sigma_l(z)]\partial z$$

where we assumed that: $[1+\pi_l\sigma]^{-1} \approx [1+\pi_l\sigma]$

Averaging over stress variations (cntd)

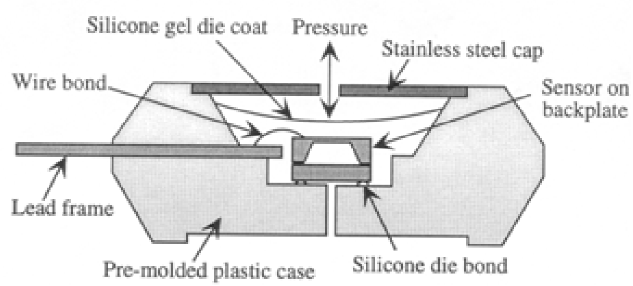


- **Rearranging:**

$$R = R_0 \left[1 + R_0 \int_0^{z_j} \frac{W}{L\rho_{e,0}(z)} \pi_l \sigma_l(z) \partial z \right]$$

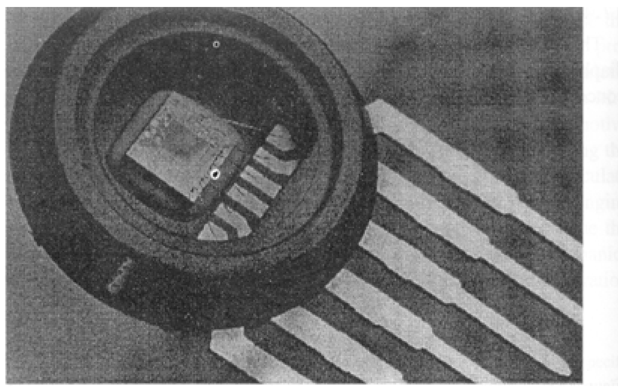
9. Piezoresistive Pressure Sensors : Case Study

Motorola MAP sensor



- **The Motorola MAP sensor has been developped to measure the absolute pressure in the intake of automobile engines**

Motorola MAP sensor



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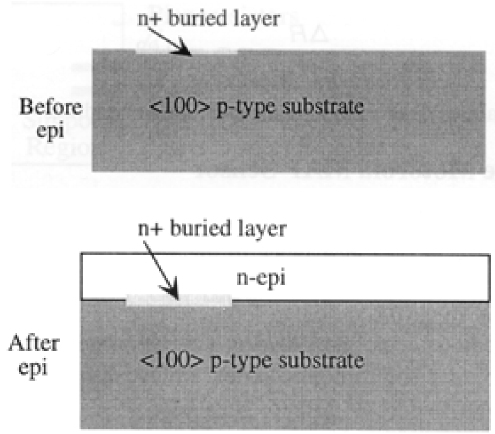
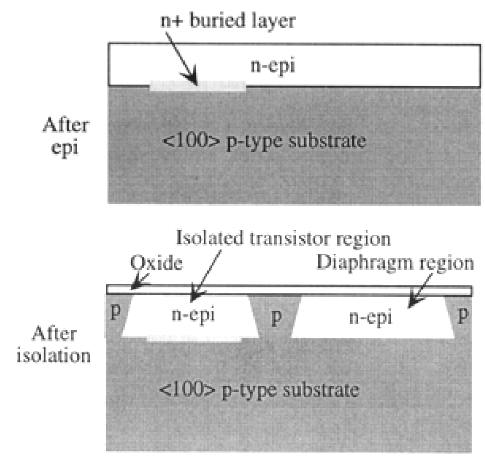
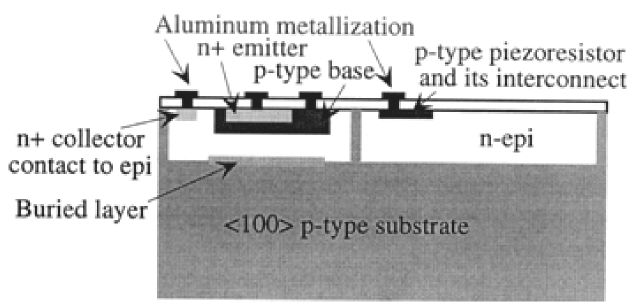
Motorola MAP sensor: transistor fabrication



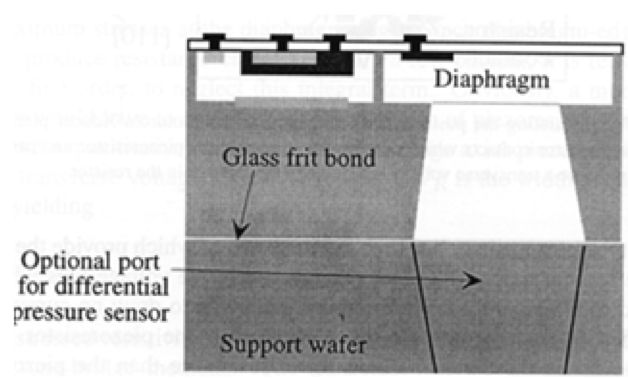
Motorola MAP sensor: transistor fabrication



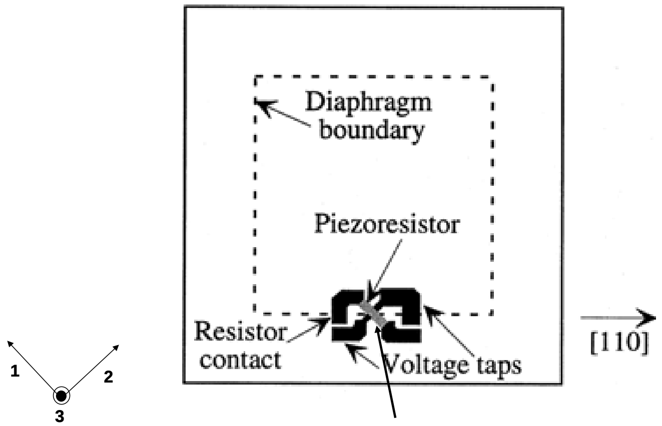
Motorola MAP sensor: diaphragm fabrication



Motorola MAP sensor: diaphragm fabrication (cntd.)

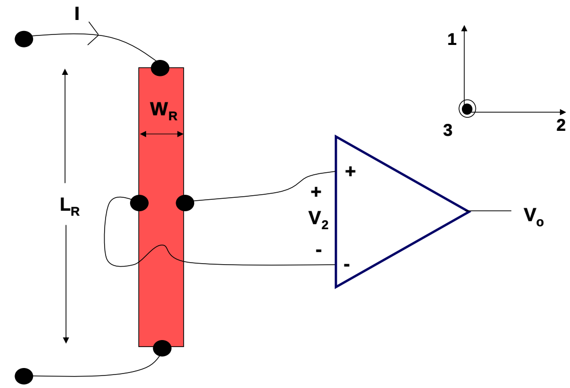


Design of piezoresistor



piezoresistor in the [100] direction

Design of piezoresistor (ctnd.)



- The resistor is probed in a four-point configuration as follows:

Design of piezoresistor (ctnd.)

- The current density J in resistor is $J = J_1$ along resistor axis but is $J = 0$ along the other direction.
- However the field in the transverse direction is not zero due to the piezoresistive properties of the materials
- Using the piezoresistive equations:

$$\frac{\varepsilon_1}{\rho_e} = [1+\pi_{11}\sigma_1+\pi_{12}(\sigma_2+\sigma_3)]J_1 + \pi_{44}(\tau_{12}J_2+\tau_{13}J_3)$$

$$\frac{\varepsilon_2}{\rho_e} = [1+\pi_{11}\sigma_2+\pi_{12}(\sigma_1+\sigma_3)]J_2 + \pi_{44}(\tau_{12}J_1+\tau_{23}J_3)$$

$$\frac{\varepsilon_3}{\rho_e} = [1+\pi_{11}\sigma_3+\pi_{12}(\sigma_1+\sigma_2)]J_3 + \pi_{44}(\tau_{13}J_1+\tau_{23}J_2)$$

since $J_2 = J_3 = 0$ and $\sigma_3 = \tau_{13} = \tau_{23} = 0$, we obtain:

$$\varepsilon_1 = \rho_e(1+\pi_{11}\sigma_1+\pi_{12}\sigma_2)J_1$$
$$\varepsilon_2 = \rho_e\pi_{44}\tau_{12}J_1$$
$$\varepsilon_3 = 0$$

Design of piezoresistor (ctnd.)

- The voltage across the piezoresistor is given by:

$$V_1 = \int_0^{L_R}\varepsilon_1\partial x_1 = \int_0^{L_R}\rho_e(1+\pi_{11}\sigma_1+\pi_{12}\sigma_2)J_1$$

$$V_1 = \rho_e L_R J_1\left[1+\frac{1}{L_R}\int_0^{L_R}(\pi_{11}\sigma_{11}+\pi_{12}\sigma_{12})\partial x_1\right]$$

- The transverse voltage is given by:

$$V_2 = \int_0^{W_R}\varepsilon_2\partial x_2 = \int_0^{W_R}\rho_e\pi_{44}\tau_{12}J_1\partial x_2 = \rho_e\pi_{44}\tau_{12}J_1W_R$$

$$V_2 = \pi_{44}\tau_{12}\left(\frac{W_R}{L_R}\right)V_1$$

- Thus the transverse voltage V_2 only depends on the shear stress present in regions between the two taps.

Stress analysis

- A uniform plate under uniform pressure possess a displacement function of:

$$w(x,y)=\frac{C_1}{4}\left[1+\cos\left(\frac{2\pi x}{L}\right)\right]\left[1+\cos\left(\frac{2\pi y}{L}\right)\right]$$

with C_1 is the displacement in center of plate, and is given by:

$$P=\frac{\pi^4 EH^3}{6(1-\upsilon^2)L^4}C_1$$

where P is the applied pressure.

- The radius of curvature in the middle of the edge is given by

$$\frac{1}{\rho_x}=\frac{\partial^2 w}{\partial x^2}\bigg|_{x=L/2,y=0}=\left(\frac{2\pi}{L}\right)^2\frac{C_1}{2}$$

- The related surface stress at that location is:

$$\sigma_x=\frac{EH}{2\rho_x}$$

Thus:

$$\sigma_x=\frac{1}{\pi^2}(1-\upsilon^2)\left(\frac{L}{H}\right)^2 P$$

Stress analysis

- Since we are dealing with a plate, the y direction stress at center of the edge is:

$$\sigma_y=\upsilon\sigma_x$$

- Using υ = 0.06 for the [110] direction:

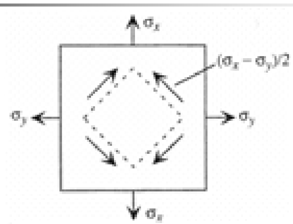
$$\sigma_x=0.606\left(\frac{L}{H}\right)^2 P$$

- A numerical analysis of that structure would actually yield:

$$\sigma_x=0.294\left(\frac{L}{H}\right)^2 P$$

which is a factor of two off from our analytical solution. This numerical result is used for the rest of the analysis.

Stress analysis (ctnd.)



- Now, lets calculate the shear stress on the resistors arranged as above.
- The axial stresses in the *x* and *y* directions add up to a shear stress given by:

$$\tau_{12}=\frac{\sigma_x-\sigma_y}{2}=0.141\left(\frac{L}{H}\right)^2 P$$

- Thus:

$$\frac{V_2}{V_1}=\pi_{44}\tau_{12}\left(\frac{W_R}{L_R}\right)=0.141\pi_{44}\left(\frac{L}{H}\right)^2\left(\frac{W_R}{L_R}\right)P$$

- With $\pi_{44}=138\times10^{-11}\,\mathrm{Pa}^{-1}, L/H=50, \text{and } W_R/L_R=1/5$ we get:

$$\frac{V_2}{V_1}=0.096\frac{\mathrm{mV}}{\mathrm{V\cdot kPa}}$$