18. Piezoresistive Pressure Sensors: Case Study

Introduction
Piezoresistance
Motorola MAP sensor

Piezoresistive pressure sensor design

- The piezoresistive strain gauge are usually made of doped poly-Si and are designed in pairs with a readout circuitry (usually a Wheatstone bridge).
- While strain-pressure responses of the membrane have been modelled in previous chapter, practical devices are usually rather calibrated, and their response stored on-chip in a look-up table.
- The response of the device to applied pressure is related through the mechanical response of membrane, piezoresistive response of transducer:
  \[ V_{\text{out}} \propto \Delta R \propto \Pi(P - P_0) \]

Example: membrane pressure sensor

Capacitive pressure sensor design

- In this design, a capacitive bridge can be formed with two reference capacitors, and the output voltage is related to the deflection of the membrane \( \Delta x \) and hence the differential pressure \( (P - P_0) \) through:
  \[ V_{\text{out}} \propto \Delta C \propto \Delta x \propto (P - P_0) \]
- By controlling the background pressure \( P_0 \) it is possible to fabricate the following types of pressure sensors:
  - An absolute pressure sensor that is referenced to vacuum \( (P_0 = 0) \)
  - A gauge-type pressure sensor that is referenced to atmospheric pressure \( (P_0 = 1 \text{ atm}) \)
  - A differential sensor where \( P_0 \) is maintained at a known value
Piezoresistive vs capacitive approaches

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitive</td>
<td>More sensitive (polysilicon)</td>
</tr>
<tr>
<td></td>
<td>Less temperature-sensitive</td>
</tr>
<tr>
<td></td>
<td>More robust</td>
</tr>
<tr>
<td>Piezoresistive</td>
<td>Smaller structure than bulk capacitance</td>
</tr>
<tr>
<td></td>
<td>Simple transducer circuit</td>
</tr>
<tr>
<td></td>
<td>No need for integration</td>
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</tbody>
</table>

Piezoresistive designs are the most employed because of its low cost, robustness, and ease of circuit integration.

9. Piezoresistive Pressure Sensors : Case Study

Introduction
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Piezoresistive pressure sensors

- This chapter introduces piezoresistive devices through the specific case study of membrane pressure sensors.
- While other approaches such as capacitive effects can be used for such applications, silicons also possess the property of piezoresistance whose implementations as transduction mechanism in membrane is somewhat more straightforward.

Piezoresistivity

- Piezoresistivity is the dependence of electrical resistivity on strain.
- Such an effect is related to the rearrangement of energy bands of a solid under applied strain (above).
Piezoresistivity

- Such strain will modify both the bandgap as well as the effective masses (and thus mobilities) ascribed to the bands.
- The effect is isotropic in as much as a given strain may increase resistivity along one direction while decreasing it along others.

Analytic formulation in cubic materials

- Assuming effect is linear, the relationships between electric field and current density is given by:

\[ \varepsilon = [\rho + \Pi \cdot \sigma] \cdot J \]

where \( \rho \) is the resistivity tensor, \( \Pi \) is the piezoresistive tensor, \( \sigma \) is the stress tensor, and \( J \) is the current density.

- Note: while \( \varepsilon \) and \( J \) are vectors, \( \rho \), \( \Pi \), and \( \sigma \) are second rank tensors, while \( \Pi \) is a fourth rank tensor.

- In addition, as previously described, the stress tensor can be reduced to six independent elements and re-annoted as such:

\[ \sigma_{11} = \sigma_{22} = \sigma_{33}, \quad \sigma_{23} = \sigma_{32} = \tau_{23}, \quad \sigma_{13} = \sigma_{31} = \tau_{31}, \quad \sigma_{12} = \sigma_{21} = \tau_{12} \]

Analytic formulation in cubic materials (ctnd.)

- Thus, the above equation can be written along the three principal directions of the cubic lattice:

\[ \begin{align*}
\rho_{11} &= \left[1 + \Pi_{1111} + \Pi_{1212} (\sigma_2 + \sigma_3)\right] J_1 + \Pi_{1414} (\tau_1 J_2 + \tau_3 J_3) \\
\rho_{12} &= \left[1 + \Pi_{1122} + \Pi_{1212} (\sigma_1 + \sigma_3)\right] J_2 + \Pi_{1414} (\tau_1 J_1 + \tau_3 J_3) \\
\rho_{13} &= \left[1 + \Pi_{1133} + \Pi_{1212} (\sigma_1 + \sigma_2)\right] J_3 + \Pi_{1414} (\tau_1 J_1 + \tau_2 J_2)
\end{align*} \]

where the three independent piezoresistive coefficients are:

\[ \rho_{11} = \Pi_{1111}, \quad \rho_{12} = \Pi_{1122}, \quad \rho_{13} = 2\Pi_{1212} \]

Longitudinal and transverse piezoresistance

- If a relatively long and narrow resistor is defined in a planar structure, then the primary current density and electric field are both along the long axis of the resistor.

- Structures are usually designed so that one of the axes of the principle in-plane stress is also along the resistor axis.

- This simplifies the set of equations to following simplified formulation:

\[ \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} = \Pi_{1111} \]

where \( R \) is the resistance of the resistor and the subscripts \( l & t \) refer to transverse and longitudinal stresses along the resistor axis.
Longitudinal and transverse piezo... (ctnd.)

- The orientations of resistor is not necessarily aligned with crystalline orientations of device
- The general expressions \( \pi_l \) and \( \pi_t \) are related to the original tensor through:
  \[
  \pi_l = \pi_{11} - 2(\pi_{11} - \pi_{12} - \pi_{44})(l_1^2n_1^2 + l_1^2m_1^2 + m_1^2n_1^2)
  \]
  and
  \[
  \pi_t = \pi_{12} + (\pi_{11} - \pi_{12} - \pi_{44})(l_2^2l_2^2 + m_2^2m_2^2 + n_2^2n_2^2)
  \]
  where \((l_1, m_1, n_1)\) are the directional cosines between the longitudinal resistor direction and the crystal axis and \((l_2, m_2, n_2)\) is the direction cosines between the transverse direction and the crystal axes
- Note: by this definition \(l_1l_2 + m_1m_2 + n_1n_2 = 0\) given that these direction cosines are orthogonal to each other.

Piezoresistive coefficients in Si

<table>
<thead>
<tr>
<th>Type</th>
<th>Resistivity</th>
<th>( \pi_{11} )</th>
<th>( \pi_{12} )</th>
<th>( \pi_{44} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>( \Omega \cdot \text{cm} )</td>
<td>(10^{-11} \text{ Pa}^{-1} )</td>
<td>(10^{-11} \text{ Pa}^{-1} )</td>
<td>(10^{-11} \text{ Pa}^{-1} )</td>
</tr>
<tr>
<td>n-type</td>
<td>11.7</td>
<td>-102.2</td>
<td>53.4</td>
<td>-13.6</td>
</tr>
<tr>
<td>p-type</td>
<td>7.8</td>
<td>6.6</td>
<td>-1.1</td>
<td>138.1</td>
</tr>
</tbody>
</table>

Design example

- Piezoresistors are often oriented in the [110] directions.
- The directional cosines are:
  \[
  (l_1, m_1, n_1) = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right) \quad (l_2, m_2, n_2) = \left( -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right)
  \]
  Thus:
  \[
  \pi_{l,110} = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})
  \]
  \[
  \pi_{t,110} = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})
  \]

The resistors are fabricated along the [110] directions.
Design example (ctnd.)

- For n-type resistors:
  \[
  \pi_n = \frac{1}{2} (\pi_{11} + \pi_{12} + \pi_{44}) = \frac{1}{2} (-102.2 + 53.4 - 13.6) = -31.2 \times 10^{11} \text{Pa}^{-1}
  \]
  \[
  \pi_n = \frac{1}{2} (\pi_{11} + \pi_{12} - \pi_{44}) = \frac{1}{2} (-102.2 + 53.4 - 13.6) = -17.6 \times 10^{11} \text{Pa}^{-1}
  \]

Design example (ctnd.)

- For p-type resistors:
  \[
  \pi_p = 71.8 \times 10^{11} \text{Pa}^{-1}
  \]
  \[
  \pi_p = -66.3 \times 10^{11} \text{Pa}^{-1}
  \]
  thus, p-type is better suited to perform piezoresistive readout in this direction

Design example (ctnd.)

- Which is better, a longitudinal or a transverse resistor?
  - The transverse resistor is fully plunged in region of maximum strain, but will also be greatly affected by placement error
  - The longitudinal resistor spans over a wider region of stresses. It will be less sensitive, but will be less prone to alignment errors from device to device

Numerical example

- The n-type cantilever is 200 \( \mu \text{m} \) long 20 \( \mu \text{m} \) wide and 5 \( \mu \text{m} \) thick. It is bent by a point load on its end. The p-type piezoresistors are 20 \( \mu \text{m} \) long and 2\( \mu \text{m} \) wide. A force of 10 \( \mu \text{N} \) is applied at extremity of device. Calculate change of resistance
- From Senturia, section 9.3:
  \[
  w_{\text{max}} = \frac{4l^3}{EWH^3} F = \left( \frac{4(200 \times 10^{-6})^3}{(160 \times 10^{-6})(20 \times 5 \times 10^{-9})} \right) 10 \times 10^{-6} = 0.8 \mu \text{m}
  \]
  \[
  w(x) = \frac{FL}{2EL} x^2 \left( 1 - \frac{x}{3L} \right)
  \]
Numerical example (ctnd.)

- The radius of curvature is then given by:
  \[ \frac{1}{\rho(x)} = \frac{1}{E} \int_{-\infty}^{\infty} \frac{F(\pi - x)}{E} \, d\pi \]
- Since \( \sigma = -2E/\rho \), the stress at surface \( z = -H/2 \) is given by:
  \[ \sigma(x) = \frac{EH}{2p} \cdot (L-x) \]
- Given that \( I = WH^3/12 \) :
  \[ \sigma(x) = \frac{6F}{H^2w} (L-x) = 1.2 \times 10^9 (L-x) \]
- It therefore spans from \( \sigma = 24 \) MPa at \( x = 0 \) to 21.6 MPa at \( x = 20 \) \( \mu m \)
- An average stress of \( \sigma = 22.8 \) MPa is therefore used to calculate change of resistance. We finally get:
  \[ \frac{\Delta R}{R_0} = \left( 67.6 \times 10^{-11} \right) (22.8 \times 10^6) = 1.54 \%

Alternate design (ctnd.)

- Resistors \( R_1 \) and \( R_3 \) experience stresses \( \sigma \) in their longitudinal direction and a stress \( \sigma \) in their transverse direction:
  \[ \frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \pi_1 \sigma + \pi_1 \nu \sigma = (67.6 \times 10^{-11}) \sigma \]
  (note: the Poisson ratio in the [110] direction is \( \nu = 0.064 \))

- Resistors \( R_2 \) and \( R_4 \) experience stresses \( \sigma \) in their transverse direction and a stress \( \sigma \) in their longitudinal direction:
  \[ \frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = \nu \sigma \pi_1 + \sigma \pi_1 = -61.7 \times 10^{-11} \sigma_1 \]
Connecting the four resistors in a Wheatstone bridge configuration, we get:

\[
\frac{V_s}{V_i} = \frac{R_4 - R_3}{(R_1 + R_2)(R_3 + R_4)} \approx \frac{\alpha_1 + \alpha_2}{2(1 + \alpha_1 - \alpha_2)}
\]

where:

\[
\alpha_1 = (\tau_1 + \nu \sigma_1) \sigma \\
\alpha_2 = (\nu \sigma_1 + \pi_1) \sigma
\]

Averaging over doping variations

Real life piezoresistors will present some degree of non-uniformity with respect to the doping levels and the stress distribution they are subjected to.

For instance, creation of piezoresistor through surface diffusion doping will create a depth-dependent profile of the unstrained resistivity \( \rho_0 \) as seen above.

Averaging over doping variations (cntd)

To calculate the nominally unstrained resistance \( R_0 \), we evaluate the following integral:

\[
\frac{1}{R_0} = \int_0^{z_j} \frac{W}{L \rho_{0,e}(z)} \, dz
\]

where \( z_j \) is the junction depth equal to the edge of the space-charge junction.

Averaging over stress variations

The neutral axis is at \( z = H/2 \) thus:

\[
\sigma(z) = E(z/2 - z)
\]

Transverse stresses are neglected, thus, the stress-induced resistivity change is:

\[
\rho_e(z) = \rho_{e,0}(z)[1 + \tau_1 \sigma_1(z)]
\]

Thus:

\[
\frac{1}{R} = \int_0^{z_j} \frac{W}{L \rho_{e,0}(z)}[1 - \tau_1 \sigma_1(z)] \, dz
\]

where we assumed that:

\[
[1 + \tau_1 \sigma_1]^{-1} = [1 + \tau_1 \sigma_1]
\]
Averaging over stress variations (cntd)

\[ \rho \sigma_z = 0 \]

Rearranging:

\[ R = R_0 \left[ 1 + R_B \int_0^H \frac{\partial \sigma_0}{L \rho_0(z)} \pi_0 \sigma(z)dz \right] \]

9. Piezoresistive Pressure Sensors: Case Study

9.1 Introduction
9.2 Piezoresistance
9.3 Motorola MAP sensor

Motorola MAP sensor

The Motorola MAP sensor has been developed to measure the absolute pressure in the intake of automobile engines.
Motorola MAP sensor: transistor fabrication

Before epi

- n+ buried layer
- <100> p-type substrate

After epi

- n+ buried layer
- n-epi
- <100> p-type substrate

Motorola MAP sensor: diaphragm fabrication

- Aluminum metallization
- n+ emitter
- p-type base
- p-type piezoresistor and its interconnect
- n+ collector contact to epi
- Buried layer
- <100> p-type substrate

Motorola MAP sensor: diaphragm fabrication (cntd.)

- Glass frit bond
- Diaphragm
- Optional port for differential pressure sensor
- Support wafer
Design of piezoresistor

- The resistor is probed in a four-point configuration as follows:

\[
\begin{align*}
V_2 & = \frac{W_R}{L_R} V_1 \\
V_o & = \frac{W_R}{L_R} V_1
\end{align*}
\]

- However the field in the transverse direction is not zero due to the piezoresistive properties of the materials.

- Using the piezoresistive equations:

\[
\begin{align*}
\frac{\sigma_1}{\rho_e} &= \frac{1 + \pi_1 (\sigma_1 + \sigma_2) + \pi_3 (\sigma_1 + \sigma_3) + \pi_{44} (\sigma_1 \sigma_2)}{\rho_e} \\
\frac{\sigma_2}{\rho_e} &= \frac{1 + \pi_2 (\sigma_2 + \sigma_3) + \pi_3 (\sigma_2 + \sigma_3) + \pi_{44} (\sigma_2 \sigma_3)}{\rho_e} \\
\frac{\sigma_3}{\rho_e} &= \frac{1 + \pi_3 (\sigma_3 + \sigma_1) + \pi_3 (\sigma_3 + \sigma_1) + \pi_{44} (\sigma_3 \sigma_1)}{\rho_e}
\end{align*}
\]

since \( J_2 = J_3 = 0 \) and \( \sigma_1 = \tau_{13} = \tau_{23} = 0 \), we obtain:

\[
\begin{align*}
e_1 &= \frac{\sigma_1}{\rho_e} (1 + \pi_1 (\sigma_1 + \sigma_2) + \pi_{44} (\sigma_1 \sigma_2)) I_1 \\
e_2 &= \frac{\sigma_2}{\rho_e} (1 + \pi_2 (\sigma_2 + \sigma_3) + \pi_{44} (\sigma_2 \sigma_3)) I_2 \\
e_3 &= \frac{\sigma_3}{\rho_e} (1 + \pi_3 (\sigma_3 + \sigma_1) + \pi_{44} (\sigma_3 \sigma_1)) I_3
\end{align*}
\]

- The voltage across the piezoresistor is given by:

\[
\begin{align*}
V_1 &= L_R \frac{I_2}{\sigma_1} (1 + \frac{I_2}{\sigma_1}) \\
V_1 &= L_R \frac{I_2}{\sigma_1} (1 + \frac{I_2}{\sigma_1}) (\pi_1 (\sigma_1 + \sigma_2) + \pi_{44} (\sigma_1 \sigma_2)) I_1 \\
V_1 &= L_R \frac{I_2}{\sigma_1} (1 + \frac{I_2}{\sigma_1}) (\pi_1 (\sigma_1 + \sigma_2) + \pi_{44} (\sigma_1 \sigma_2)) I_1
\end{align*}
\]

- The transverse voltage is given by:

\[
\begin{align*}
V_2 &= \frac{W_R}{L_R} V_1 \\
V_2 &= \frac{W_R}{L_R} \frac{I_2}{\sigma_1} (1 + \frac{I_2}{\sigma_1}) (\pi_1 (\sigma_1 + \sigma_2) + \pi_{44} (\sigma_1 \sigma_2)) I_1 W_R
\end{align*}
\]

Thus the transverse voltage \( V_2 \) only depends on the shear stress present in regions between the two taps.
Stress analysis

- A uniform plate under uniform pressure possess a displacement function of:
  \[ w(x, y) = \frac{C_1}{4} \left[ 1 + \cos\left(\frac{2\pi x}{L}\right) \right] \left[ 1 + \cos\left(\frac{2\pi y}{L}\right) \right] \]
  with \( C_1 \) is the displacement in center of plate, and is given by:
  \[ P = -\pi^3 \frac{E H^3}{6(1-\nu^2)} C_1 \]
  where \( P \) is the applied pressure.
- The radius of curvature in the middle of the edge is given by
  \[ \frac{1}{\rho_x} = \frac{\partial^2 w}{\partial x^2} \bigg|_{x=L/2,y=0} = \left(\frac{2\pi}{L}\right)^2 \frac{C_1}{2} \]
- The related surface stress at that location is:
  \[ \sigma_x = \frac{E H}{2\rho_x} \]
  Thus:
  \[ \sigma_x = \frac{\pi^3}{2} \left(1-\nu^2\right) \left(\frac{L}{H}\right)^2 \frac{P}{2} \]

Stress analysis (ctnd.)

- Now, let's calculate the shear stress on the resistors arranged as above.
- The axial stresses in the \( x \) and \( y \) directions add up to a shear stress given by:
  \[ \tau_{12} = \frac{\sigma_x - \sigma_y}{2} = 0.144 \left(\frac{L}{H}\right)^2 \frac{P}{2} \]
  Thus:
  \[ \frac{V_2}{V_1} = 1.44 \left(\frac{W_R}{L_R}\right) \left(\frac{W_{ik}}{L_{ik}}\right) \frac{P}{2} \]
  With \( \kappa_{ik} = 1.38 \times 10^{-11} \text{Pa}^{-1}, L_i/H_i = 50, \) and \( W_R/L_R = 1/5 \) we get:
  \[ \frac{V_2}{V_1} = 0.098 \text{ mV/V} \times \text{ kPa} \]