Channel inversion in MIMO systems over Rician fading

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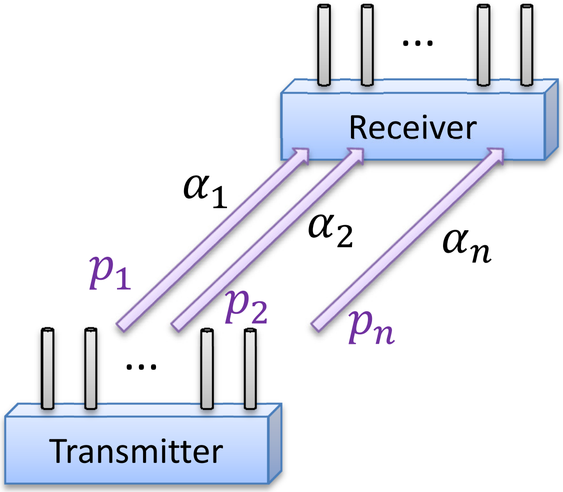
Overview

- Extends **[Senaratne2009]** on Rayleigh fading

- Scope:
  - MIMO, eigenmode transmission
  - Ricean fading
  - channel inversion (CI) power allocation scheme

allocated power → $p_i \propto \dfrac{1}{|\alpha_i|^2}$ ← α_i *- channel coefficient*



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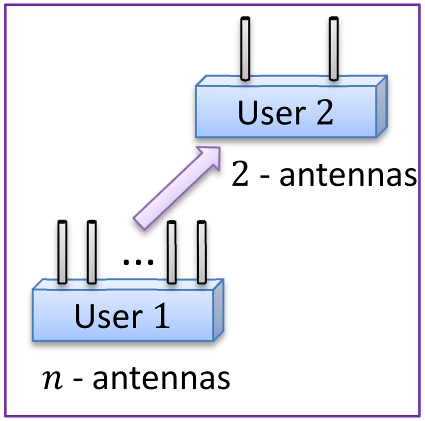
- Why CI?
  - CI (at tx.) ≈ ZF (at tx.) + instantaneous total power constraint
    - e.g. multi-user MIMO downlink
  - eigenmodes/ VCs with identical rx. SNR
    - multi-user fairness
    - optimal if same coding & modulation schemes
  - non-iterative
    - unlike water-filling

Overview...



- System model (for performance analysis):
    - Rank 2 channel matrix: $\boldsymbol{H}$ (restricted)
    - User 1 → User 2 (or User 2 → User 1) communication
    - $\boldsymbol{H}$ assumed to be $\boxed{2 \times n}$, $n \geq 2$

- MIMO channel:
    - $\boxed{\boldsymbol{H} = a\boldsymbol{H}_{sp} + b\boldsymbol{H}_{sc}}$, where $a^2 + b^2 = 1$

scatter component

specular (line-of-sight) component

- Rice factor: $K = \dfrac{a^2\|\boldsymbol{H}_{sp}\|_F^2}{b^2\|\boldsymbol{H}_{sc}\|_F^2}$
- non-centrality matrix: $\boldsymbol{\Omega} = \dfrac{a^2}{b^2}\boldsymbol{H}_{sp}\boldsymbol{H}_{sp}{}^H$
- $\boldsymbol{W} = \boldsymbol{H}\boldsymbol{H}^H$

fading characterized by:
eigenvalues of $\boldsymbol{W}$ *and* $\boldsymbol{\Omega}$

Overview...

- Let
  - $\{\lambda_1, \lambda_2\} = eig(\boldsymbol{W})$ ← *unordered eigenvalues*
  - $(\omega_1, \omega_2 | \omega_1 > \omega_2) = eig(\boldsymbol{\Omega})$ ← *ordered eigenvalues,*

    let $\boldsymbol{\omega} = [\omega_1, \omega_2]$

- SNR under eigenmode transmission: $\{\lambda_1, \lambda_2\}$

- SNR under eigenmode transmission + channel inversion $\propto \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \Lambda$ (say)
  - note: $\lambda_1 p_1 = \lambda_2 p_2, p_1 + p_2 = const$

- Our approach:
  - characterization of Λ (i.e. PDF)
  - exact error and outage performance
  - high SNR regime

Analysis

- Joint PDF of λ_1, λ_2 **[Smith2004]**

$$f_{\lambda_1,\lambda_2}(\lambda_1, \lambda_2) = \frac{exp[-(\omega_1+\omega_2)]}{2} \frac{(\lambda_1-\lambda_2)(\lambda_1\lambda_2)^{\frac{n-2}{2}}}{(\omega_1-\omega_2)(\omega_1\omega_2)^{\frac{n-2}{2}}} exp[-(\lambda_1 + \lambda_2)]$$

$$\left(I_{n-2}\left(2\sqrt{\omega_1\lambda_1}\right)I_{n-2}\left(2\sqrt{\omega_2\lambda_2}\right) - I_{n-2}\left(2\sqrt{\omega_1\lambda_1}\right)I_{n-2}\left(2\sqrt{\omega_2\lambda_2}\right)\right)$$

$I_n(.)$ - Bessel I f^n

- PDF of Λ derived using

$$f_\Lambda(x) = \int_0^\infty \frac{(t+x)^2}{t^2} f_{\lambda_1,\lambda_2}\left(t + x, \frac{x(t+x)}{t}\right) dt$$

and infinite series expansion of $I_n(.)$

- case: $\omega_1 \geq \omega_2 > 0$ ← *Theorem 1*
- case: $\omega_1 > 0, \omega_2 = 0$ ← *rank-1 non-centrality matrix (e.g. directional antenna)*
- case: $\omega_1 = \omega_2 = 0$ ← *reduces to Rayleigh fading* **[Senaratne2009]**

Analysis - PDF of Λ

case: $\omega_1 \geq \omega_2 > 0$

cascaded infinite series

$$f_\Lambda(x) = \sum_{i=0}^{\infty}\sum_{j=0}^{i} \frac{\mathfrak{g}_{j,(i-j)}(\omega_1, \omega_2)}{(j+n-2)!\,(i-j+n-2)!\,j!\,(i-j)!} \sum_{p=0}^{i+2n-1} \binom{i+2n-1}{p} \exp(-2x)\, x^{i+2n-2} \left(K_{n+j-p}(2x) - K_{n+j-p-1}(2x)\right)$$

$K_n(.)$ - Bessel K f^n

where

$$\mathfrak{g}_{i,j}\,(\alpha, \beta) = \begin{cases} \dfrac{\exp\left(-(\alpha+\beta)\right)}{(\alpha-\beta)}\left(\alpha^i\beta^j - \alpha^j\beta^i\right), & \alpha \neq \beta \\ (i-j)\alpha^{i+j-1}\exp(-2\alpha)\,, & \alpha = \beta \end{cases}$$

Analysis - PDF of Λ

case: $\omega_1 \geq \omega_2 > 0$

$$f_\Lambda(x) \cong \sum_{i=0}^{I_{max}} \sum_{j=0}^{i} \frac{\mathfrak{g}_{j,(i-j)}(\omega_1,\omega_2)}{(j+n-2)!\,(i-j+n-2)!\,j!\,(i-j)!}$$

$$\sum_{p=0}^{i+2n-1} \binom{i+2n-1}{p}$$

$$\big(K_{n+j-p}(2x) -$$

where

$$\mathfrak{g}_{i,j}(\alpha,\beta) = \begin{cases} \dfrac{\exp(-(\alpha}{(\alpha - \beta} \\ (i-j) \end{cases}$$

for $\boldsymbol{\omega} = [1,4]$

ϵ	x	point of truncation (I_{max})	
		for $n = 2$	for $n = 4$
1×10^{-5}	0.1	20	9
	1.0	21	11
	5.0	1	12
1×10^{-13}	0.1	30	17
	1.0	31	19
	5.0	33	22

$I_{max} = i$ beyond which |terms| $< \ \epsilon \times \exp\big(-(\omega_1 + \omega_2)\big)$

Analysis - PDF of Λ...

case: $\omega_1 > 0, \omega_2 = 0$

$$f_\Lambda(x) = \sum_{i=0}^{\infty} \frac{\omega_1^{i-1}\exp(-\omega_1)}{(n-2)!\,(i+n-2)!\,i!} \sum_{p=0}^{i+2n-1} \binom{i+2n-1}{p} \exp(-2x)\, x^{i+2n-2}$$
$$\left(K_{n+i-p}(2x) - K_{n-p}(2x) - K_{n+i-p-1}(2x) + K_{n-p-1}(2x)\right)$$

case: $\omega_1 = \omega_2 = 0$

$$f_\Lambda(x) = \frac{1}{(n-2)!\,(n-1)!} \sum_{p=0}^{2n} \binom{2n}{p} \exp(-2x)\, x^{i+2n-1}$$
$$(K_{n+i-p}(2x) - 2K_{n-p}(2x) + K_{n-p-1}(2x))$$

Applications

- Outage probability $F_\Lambda(x) = \int_0^x f_\Lambda(t)dt$

$$\int_0^x t^\mu \exp(-2t)\, K_\nu(2t)dt = \frac{\sqrt{\pi}}{4^{\mu+1}} \mathcal{G}_{2,3}^{2,1}\left(4x \middle| \begin{matrix} 1, \mu+1.5 \\ \mu+\nu+1, \mu-\nu+1, 0 \end{matrix}\right)$$

$\mathcal{G}_{p,q}^{m,n}(.\,|\substack{\cdots\\ \cdots})$ - Meijer G f^{n}

- Average symbol error rate $\mathcal{E}_\Lambda\left[\alpha\, \mathcal{Q}\left(\sqrt{2\beta\Lambda}\right)\right] = \frac{\alpha}{2}\sqrt{\frac{\beta}{\pi}}\int_0^\infty \frac{\exp(-\beta t)}{\sqrt{t}} F_\Lambda(t)dt$

$$\int_0^\infty t^{-0.5}\exp(-\beta t)\,\mathcal{G}_{2,3}^{2,1}\left(4t\middle|\begin{matrix}1,\rho\\ \mu,\nu,0\end{matrix}\right)dt = \beta^{-0.5}\mathcal{G}_{3,3}^{2,2}\left(\frac{4}{\beta}\middle|\begin{matrix}0.5,1,\rho\\ \mu,\nu,0\end{matrix}\right)$$

- Moment generating function

$$\mathcal{M}_\Lambda(s) = \mathcal{E}_\Lambda[exp(-s\Lambda)]$$

- High SNR behavior (e.g. diversity order = $n-1$ as in ZF)
  - note: $f_\Lambda(x)$ is a weighted sum of $\exp(-2x)\, x^\mu K_\nu(2x)$ terms

Numerical Results

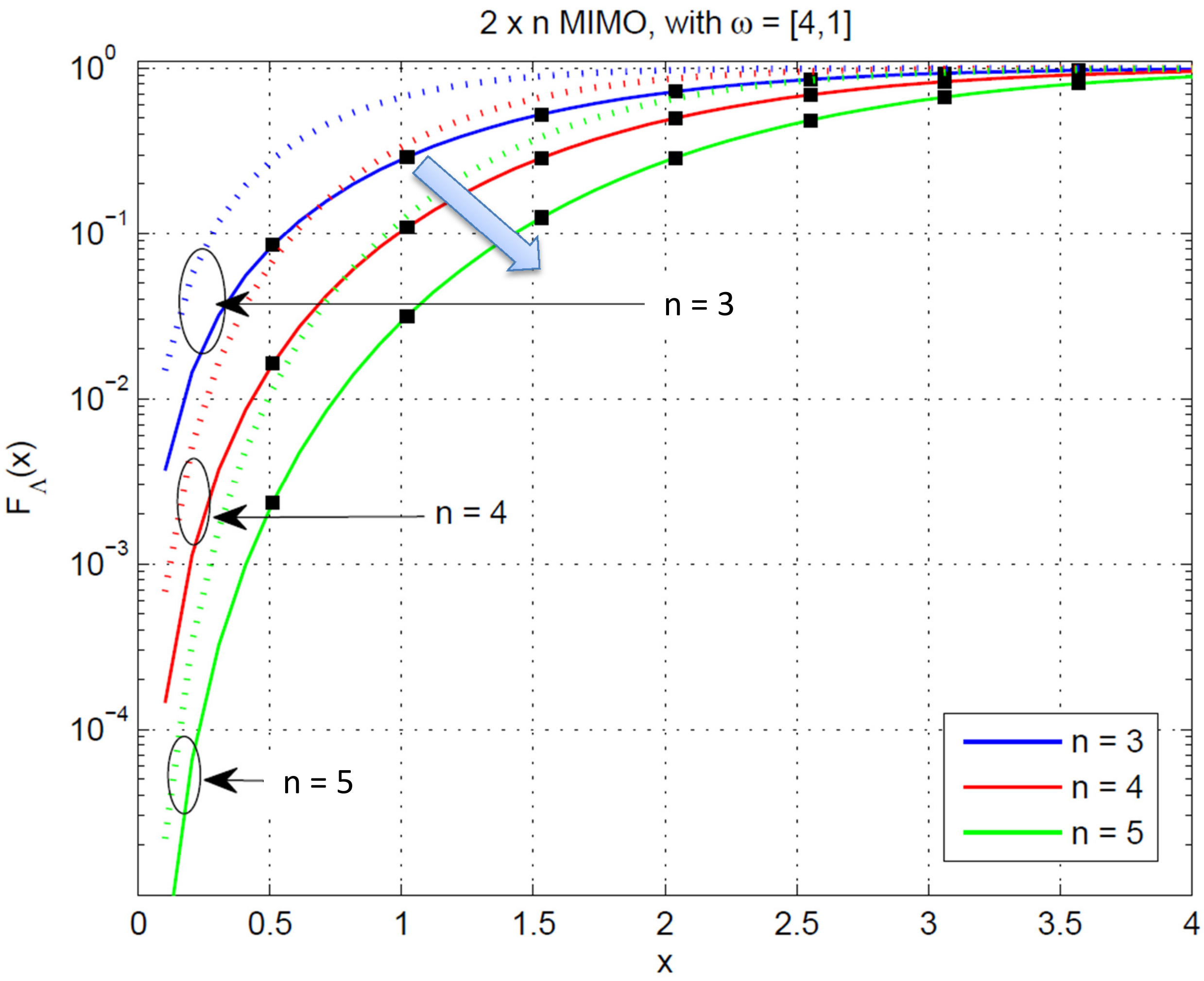


Fig. 3. The cdf, analytic (solid lines) vs. simulated (■), of Λ in (3) over a $2 \times n$ MIMO system, in Rician fading modeled by a rank-2 non-centrality matrix having eigenvalues $[4, 1]$. The cdf for Rayleigh fading is shown in dotted lines.

Numerical Results...

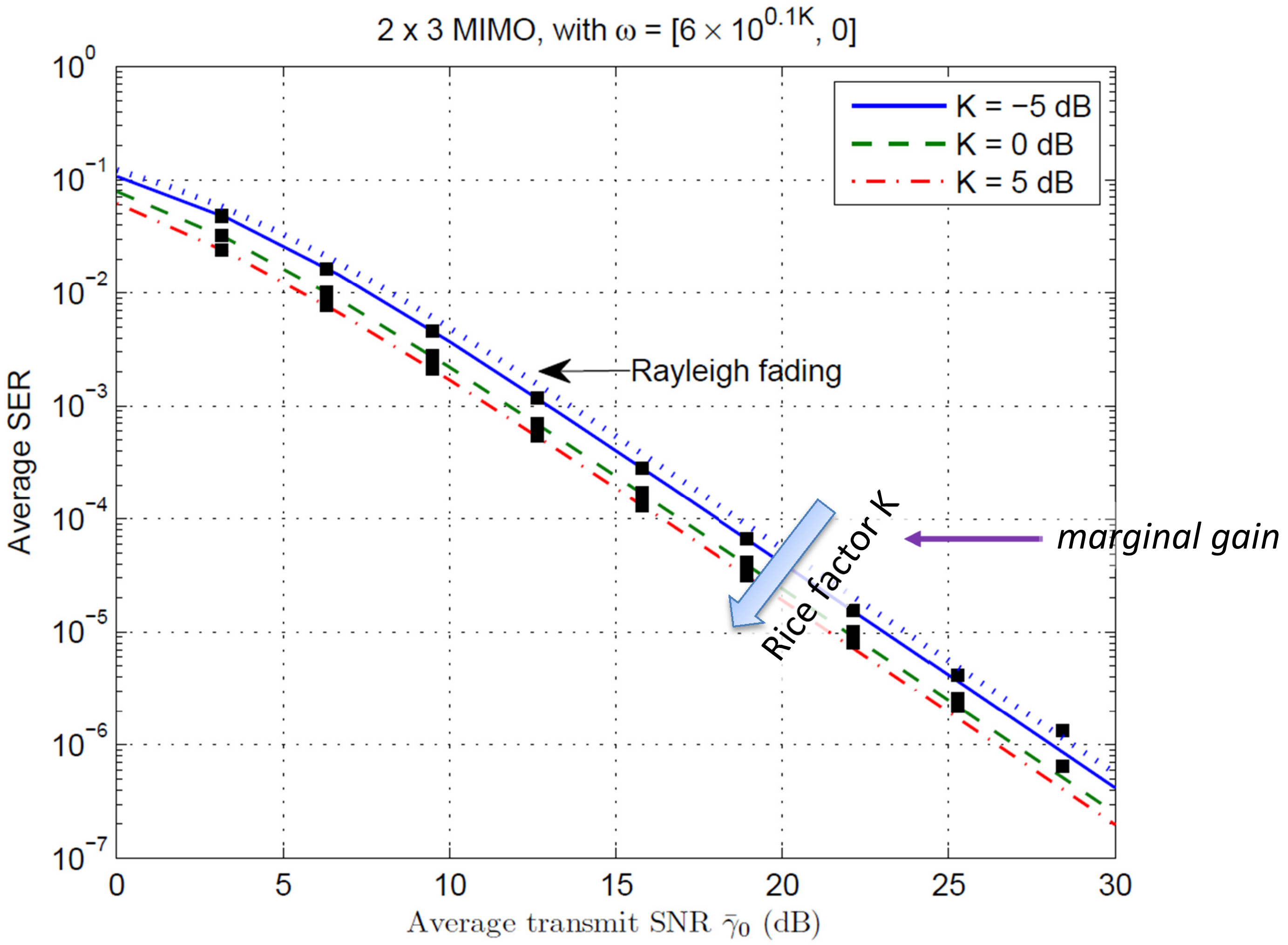


Fig. 4. The average SER for BPSK (i.e. $\alpha = 1, \beta = \frac{b^2}{2}\bar{\gamma}_0$), analytic vs. simulated (■), for a 2×3 MIMO system, in Rician fading modeled by a rank-1 non-centrality matrix parameterized [17] by $\{\theta_t = 20^o, \theta_r = 10^o, d = 1\}$.

↑

[Smith2004]

Numerical Results...

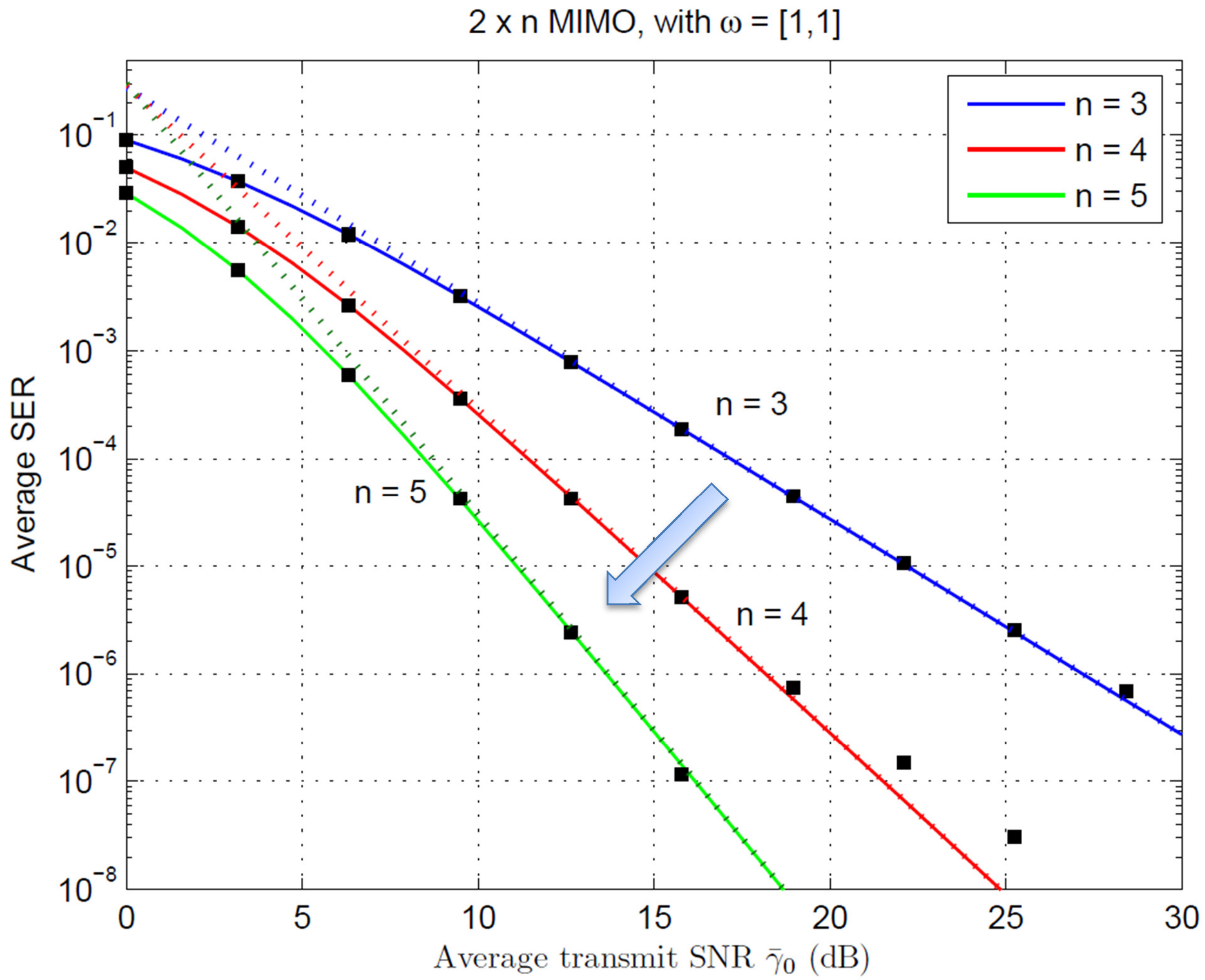


Fig. 5. The average SER for BPSK (i.e. $\alpha = 1, \beta = \frac{b^2}{2}\bar{\gamma}_0$), analytic (solid lines) vs. simulated (■), for a $2 \times n$ MIMO system, in Rician fading modeled by a rank-2 non-centrality matrix having equal eigenvalues of 1, for $b = \sqrt{2}$. Asymptotic SER curves are shown in dotted lines.

References

- **[Senaratne2009]** D. Senaratne and C. Tellambura, “Performance analysis of channel inversion over MIMO channels,” in Proc. IEEE Global Communication Conference, Honolulu, HI, Dec. 2009.
- **[Smith2004]** P. J. Smith and L. M. Garth, “Exact capacity distribution for dual MIMO systems in Ricean fading,” IEEE Commun. Lett., vol. 8, no. 1, pp. 18–20, Jan. 2004.

Conclusion

- Performance analysis of
  - MIMO eigenmode transmission
  - channel inversion power allocation
  - Ricean fading
- Exact outage, avg. symbol error rate, moment generating function results
- High SNR behavior
  - same diversity order as ZF
- Observations: SER improvement
  - marginal with Rice factor (K)
  - significant with # antennas (n)

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CI + eigenmode tx. - alternative to/ varient of ZF