Superimposed Pilot Based
Joint CFO and Channel Estimation for
CP-OFDM Modulated Two-Way Relay Networks

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Outline

- Introduction
- Problem Formulation
- Proposed Solution
- Simulation Results
- Conclusion
Two-way transmission was firstly exploited by Shannon [Shannon, 1961].

Two-way relay networks (TWRN) now has drawn much attention due to its improved spectral efficiency over one-way relay networks (OWRN).

The overall communication rate between two source terminals in TWRN is approximately twice that achieved in OWRN [Rankov, 2006].

Figure 1: System configuration for two-way relay network.
Introduction

- Most existing works in TWRN assumed perfect synchronization and channel state information (CSI).

- Channel estimation problems in amplify-and-forward (AF) TWRN are different from those in traditional communication systems.


Problem Formulation

- However, only the convoluted channel parameters $a$ and $b$, and the mixed CFO value $v$ can be found in the previous works.

$$w = f_r - f_1, \quad a = (\Omega^{(L+1)}[-w]h_1) \otimes h_1,$$
$$v = f_2 - f_1, \quad b = (\Omega^{(L+1)}[v - w]h_1) \otimes h_2.$$

- The individual frequency and channel parameters remain unknown to the source nodes.

- How to estimate individual frequency $f_r, f_1, f_2$ and channel parameters $h_1$ and $h_2$?
Proposed Solution

- Problem: How to obtain $h_1$, $h_2$ and $w = f_r - f_1$?
- Our solution: superimposed pilots + iterative algorithms.

Figure 2: Two-way relay network with superimposed pilots at the relay node.
Superimposed Pilot Aided Estimation

- For CP-OFDM, the terminal node \( T_1 \) will receive

\[
y = \alpha \mathbf{H}^{(N)}_{cp} [\mathbf{a}] \mathbf{s}_1 + \alpha \mathbf{\Gamma}^{(N)}_L [\mathbf{v}] \mathbf{H}^{(N)}_{cp} [\mathbf{b}] \mathbf{s}_2 + \mathbf{\Gamma}^{(N)}_L [\mathbf{w}] \mathbf{H}^{(N)}_{cp} [\mathbf{h}_1] \mathbf{p}_0 + \mathbf{n}_e \\
= \alpha \mathbf{S}_1 \mathbf{a} + \alpha \mathbf{\Gamma}^{(N)}_L [\mathbf{v}] \mathbf{S}_2 \mathbf{b} + \mathbf{\Gamma}^{(N)}_L [\mathbf{w}] \mathbf{P} \mathbf{h}_1 + \mathbf{n}_e. \tag{1}
\]

where \( \mathbf{S}_j \) is the \( N \times (2L + 1) \) circulant matrix with the first column \( \mathbf{s}_i \), and \( \mathbf{P} \) is the \( N \times (L + 1) \) circulant matrix with the first column \( \mathbf{p}_0 \).

- \( \mathbf{h}_1, \mathbf{h}_2, \mathbf{v} \) and \( \mathbf{w} \) to be estimated.

- Iteration to further refine our estimates

\[
[v^{(1)}, w^{(1)}] = \arg \min_{v, w} (y - \alpha \mathbf{S}_1 \mathbf{a}^{(0)} - \alpha \mathbf{\Gamma}[v] \mathbf{S}_2 \mathbf{b}^{(0)} - \mathbf{\Gamma}[w] \mathbf{P} \mathbf{h}_1^{(0)})^H \\
\times \mathbf{R}^{-1} (y - \alpha \mathbf{S}_1 \mathbf{a}^{(0)} - \alpha \mathbf{\Gamma}[v] \mathbf{S}_2 \mathbf{b}^{(0)} - \mathbf{\Gamma}[w] \mathbf{P} \mathbf{h}_1^{(0)}),
\]

where \( \mathbf{R} \) is the covariance matrix of \( \mathbf{n}_e \).
Maximum Pilot Length

- Define $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_r$ as the frequency domain pilot index sets from $\mathbb{T}_1$, $\mathbb{T}_2$, and $\mathbb{R}$, with cardinality $K_1$, $K_2$, and $K_r$ respectively.

- We require $K_1 \geq L + 1$, $K_2 \geq L + 1$, $K_r \geq L + 1$ and $\mathcal{K}_1 \cup \mathcal{K}_2 \cup \mathcal{K}_r = \{1, \ldots, N\}$.

- Since $S_j$ and $P$ are columnwise circulant matrices, they can be represented as

$$S_j = F^H \text{diag}\{\tilde{s}_j\} F[:,1:2L+1] = F_{[:,K_j]}^H \text{diag}\{\tilde{s}_j\} F_{[K_j,1:2L+1]}$$

(2)

$$P = F^H \text{diag}\{\tilde{p}_0\} F[:,1:L+1] = F_{[:,K_r]}^H \text{diag}\{\tilde{p}_0\} F_{[K_r,1:L+1]}.$$  

(3)
Define $\bar{\mathcal{K}}_1$ as the complement set of $\mathcal{K}_1$. Multiplying both sides with $F[\bar{\mathcal{K}}_1,:]$ yields

$$F[\bar{\mathcal{K}}_1,:] y = \underbrace{\alpha F[\bar{\mathcal{K}}_1,:] \Gamma[v] F^H[:,\mathcal{K}_2] \text{diag}\{\bar{s}_2\} F[\bar{\mathcal{K}}_1,:] \Gamma[w] P}_{C_1} \begin{bmatrix} \bar{b} \\ h_1 \end{bmatrix} + F[\bar{\mathcal{K}}_1,:] n_e,$$

where $\bar{b} = F[\mathcal{K}_2,1:2L+1] b$ is the DFT response of $b$ on the subcarrier set $\mathcal{K}_2$ and $C_1$ is an $(N - K_1) \times (K_2 + L + 1)$ matrix.

Two-dimensional search estimator:

$$\{\hat{v}, \hat{w}\} = \arg \max_{v,w} y^H F^H[\bar{\mathcal{K}}_1,:] C_1 (C_1^H C_1)^{-1} C_1^H F[\bar{\mathcal{K}}_1,:] y.$$  

$$\hat{d}_1 = (C_1^H C_1)^{-1} C_1^H F[\bar{\mathcal{K}}_1,:] y.$$
Note that

\[
\hat{\mathbf{b}} = \mathbf{F}_{[K_2, 1:2L+1]} \mathbf{H}_{zp}^{(L+1)} [\Omega^{(L+1)} [v - w]h_1] h_2.
\]  

Then, \( h_2 \) can be estimated as

\[
\hat{h}_2 = (\mathbf{F}_{[K_2, 1:2L+1]} \mathbf{H}_{zp}^{(L+1)} [\Omega^{(L+1)} [v - w]h_1])^\dagger \hat{\mathbf{b}}.
\]

Iteration can be applied to further improve the estimation performance.

The minimum number of \( N \) is \( 3L + 5 \), when sets are disjoint and \( K_1 = K_2 = L + 1, K_r = L + 3 \).
A Special Case

- In practical applications, the relay terminal is often a simple device while the two source terminals may employ high-precision synchronization circuits.

- Thus, it is reasonable to expect the CFO between the two source terminals to be negligible.

- Then we can obtain

\[
\mathbf{F}_{[\bar{K}_1,:]} \mathbf{y} = \alpha \mathbf{F}_{[\bar{K}_1,:]} \Gamma [v] \mathbf{F}^H_{[,:\bar{K}_2]} \text{diag}\{\tilde{s}_2\} \mathbf{F}_{[\bar{K}_1,1:2L+1]} \mathbf{b} \\
\approx 0 \\
+ \mathbf{G}_{[\bar{K}_1,:]} \Gamma [w] \mathbf{P} \mathbf{h}_1 + \mathbf{F}_{[\bar{K}_1,:]} \mathbf{n}_e, \tag{9}
\]

- CFO \( \omega \) can be estimated as

\[
\hat{\omega} = \arg \max_\omega \mathbf{y}^H \mathbf{F}^H_{[\bar{K}_1,:]} \mathbf{C}_2 (\mathbf{C}_2^H \mathbf{C}_2)^{-1} \mathbf{C}_2^H \mathbf{F}_{[\bar{K}_1,:]} \mathbf{y}, \tag{10}
\]
Simulation Results: Minimum Pilot Length Case

Figure 3: CFO estimation MSE versus SNR: $N = 14$
Simulation Results: Minimum Pilot Length Case

Figure 4: Channel estimation MSE versus SNR: $N = 14$
Simulation Results: A Special Case

Figure 5: CFO estimation MSE versus SNR: $N = 9$
Simulation Results: A Special Case

Figure 6: Channel estimation MSE versus SNR: $N = 9$
Conclusion

- Estimate individual channel parameters by introducing superimposed pilots at the relay node and using iterative algorithms.
- Less training is needed.
- Special case: small $v$.

Table 1: Comparison between adapted CP-OFDM and superimposed pilot aided CP-OFDM.

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<thead>
<tr>
<th></th>
<th>Minimum Pilot Length</th>
<th>Estimated Parameters</th>
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</thead>
<tbody>
<tr>
<td>adapted CP-OFDM</td>
<td>$4L + 3$</td>
<td>a, b and $v$</td>
</tr>
<tr>
<td>superimposed pilot</td>
<td>$3L + 5$</td>
<td>$h_1$, $h_2$, a, b, $v$ and $w$</td>
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Questions and discussion?

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