Superimposed Pilot Based Joint CFO and Channel Estimation for CP-OFDM Modulated Two-Way Relay Networks

Gongpu Wang[†], Feifei Gao[‡], and Chintha Tellambura[†]

[†]University of Alberta, Edmonton, Canada,
[‡]Jacobs University, Bremen, Germany
Email: gongpu@ece.ualberta.ca

Globecom 2010

Outline

- Introduction
- Problem Formulation
- Proposed Solution
- Simulation Results
- Conclusion

CORE WIRELESS
UNIVERSITY OF ALBERTA

Introduction

- Two-way transmission was firstly exploited by Shannon [Shannon, 1961].
- Two-way relay networks (TWRN) now has drawn much attention due to its improved spectral efficiency over one-way relay networks (OWRN).
- The overall communication rate between two source terminals in TWRN is approximately twice that achieved in OWRN [Rankov, 2006].

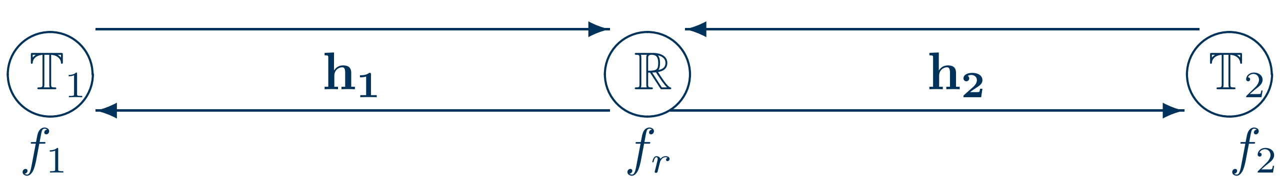

Figure 1: System configuration for two-way relay network.

CORE WIRELESS
UNIVERSITY OF ALBERTA

Introduction

- Most existing works in TWRN assumed perfect synchronization and channel state information (CSI).
- Channel estimation problems in amplify-and-forward (AF) TWRN are different from those in traditional communication systems.
- Flat-fading [Gao, 2009, TCOM] and frequency-selective [Gao, 2009, TSP] channel estimation and training design for AF TWRN.
- Joint frequency offset (CFO) and channel estimation: modeling [Wang, 2009, Globecom]; CP-OFDM [Wang, 2010, ICC]; ZP-OFDM [Wang, 2010, WCNC].

Problem Formulation

- However, only the convoluted channel parameters a and b, and the mixed CFO value v can be found in the previous works.

$$w = f_r - f_1, \quad \mathbf{a} = (\mathbf{\Omega}^{(L+1)}[-w]\mathbf{h}_1) \otimes \mathbf{h}_1,$$
$$v = f_2 - f_1, \quad \mathbf{b} = (\mathbf{\Omega}^{(L+1)}[v - w]\mathbf{h}_1) \otimes \mathbf{h}_2.$$

- The individual frequency and channel parameters remain unknown to the source nodes.
- How to estimate individual frequency f_r, f_1, f_2 and channel parameters $\mathbf{h}_1$ and $\mathbf{h}_2$?

Proposed Solution

- Problem: How to obtain $\mathbf{h}_1$, $\mathbf{h}_2$ and $w = f_r - f_1$?
- Our solution: superimposed pilots + iterative algorithms.

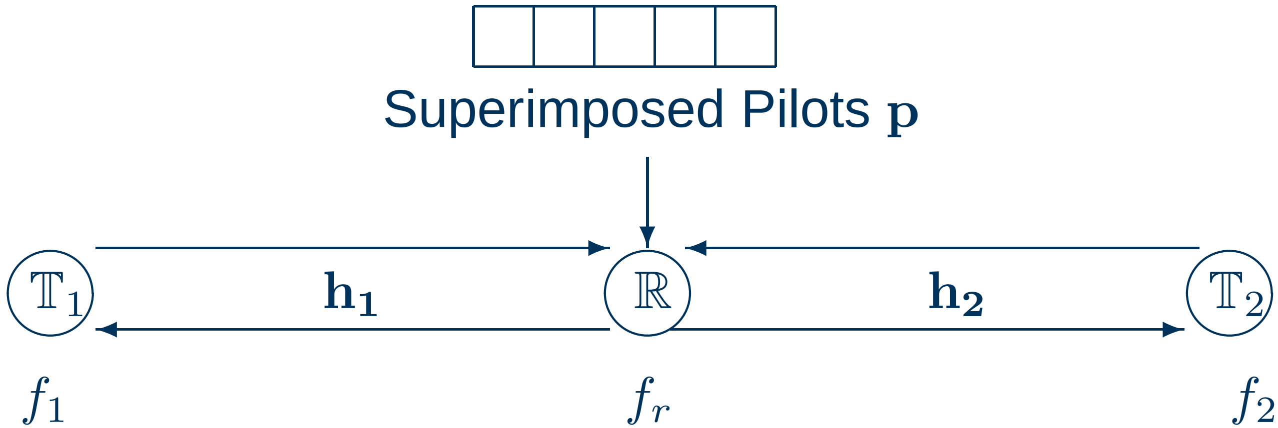

Figure 2: Two-way relay network with superimposed pilots at the relay node.

Superimposed Pilot Aided Estimation

- For CP-OFDM, the ternimal node $\mathbb{T}_1$ will receive

$$\mathbf{y} = \alpha \mathbf{H}_{cp}^{(N)}[\mathbf{a}]\mathbf{s}_1 + \alpha \mathbf{\Gamma}_L^{(N)}[v]\mathbf{H}_{cp}^{(N)}[\mathbf{b}]\mathbf{s}_2 + \mathbf{\Gamma}_L^{(N)}[w]\mathbf{H}_{cp}^{(N)}[\mathbf{h}_1]\mathbf{p}_0 + \mathbf{n}_e$$
$$= \alpha \mathbf{S}_1\mathbf{a} + \alpha \mathbf{\Gamma}_L^{(N)}[v]\mathbf{S}_2\mathbf{b} + \mathbf{\Gamma}_L^{(N)}[w]\mathbf{P}\mathbf{h}_1 + \mathbf{n}_e. \quad (1)$$

  where $\mathbf{S}_j$ is the $N \times (2L+1)$ circulant matrix with the first column $\mathbf{s}_i$, and $\mathbf{P}$ is the $N \times (L+1)$ circulant matrix with the first column $\mathbf{p}_0$.

- $\mathbf{h}_1$, $\mathbf{h}_2$, v and w to be estimated.

- Iteration to further refine our estimates

$$[v^{(1)}, w^{(1)}] = \arg\min_{v,w} (\mathbf{y} - \alpha \mathbf{S}_1\mathbf{a}^{(0)} - \alpha \mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)})^H$$
$$\times \mathbf{R}^{-1}(\mathbf{y} - \alpha \mathbf{S}_1\mathbf{a}^{(0)} - \alpha \mathbf{\Gamma}[v]\mathbf{S}_2\mathbf{b}^{(0)} - \mathbf{\Gamma}[w]\mathbf{P}\mathbf{h}_1^{(0)}),$$

  where $\mathbf{R}$ is the covariance matrix of $\mathbf{n}_e$.

Minimum Pilot Length

- Define $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_r$ as the frequency domain pilot index sets from $\mathbb{T}_1$, $\mathbb{T}_2$, and $\mathbb{R}$, with cardinality K_1, K_2, and K_r respectively.
- We require $K_1 \geq L+1$, $K_2 \geq L+1$, $K_r \geq L+1$ and $\mathcal{K}_1 \bigcup \mathcal{K}_2 \bigcup \mathcal{K}_r = \{1, \ldots, N\}$.
- Since $\mathbf{S}_j$ and $\mathbf{P}$ are columnwise circulant matrices, they can be represented as

$$\mathbf{S}_j = \mathbf{F}^H \text{diag}\{\tilde{\mathbf{s}}_j\} \mathbf{F}_{[:,1:2L+1]} = \mathbf{F}^H_{[:,\mathcal{K}_j]} \text{diag}\{\breve{\mathbf{s}}_j\} \mathbf{F}_{[\mathcal{K}_j,1:2L+1]} \quad (2)$$

$$\mathbf{P} = \mathbf{F}^H \text{diag}\{\tilde{\mathbf{p}}_0\} \mathbf{F}_{[:,1:L+1]} = \mathbf{F}^H_{[:,\mathcal{K}_r]} \text{diag}\{\breve{\mathbf{p}}_0\} \mathbf{F}_{[\mathcal{K}_r,1:L+1]}. \quad (3)$$

Minimum Pilot Length (continued)

- Define $\bar{\mathcal{K}}_1$ as the complement set of $\mathcal{K}_1$. Multiplying both sides with $\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}$ yields

$$\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y} = \underbrace{\left[\alpha \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[v]\mathbf{F}^H_{[:,\mathcal{K}_2]}\text{diag}\{\breve{\mathbf{s}}_2\} \;\; \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[w]\mathbf{P}\right]}_{\mathbf{C}_1}\underbrace{\begin{bmatrix}\breve{\mathbf{b}}\\ \mathbf{h}_1\end{bmatrix}}_{\mathbf{d}_1} + \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{n}_e, \quad (4)$$

where $\breve{\mathbf{b}} = \mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{b}$ is the DFT response of $\mathbf{b}$ on the subcarrier set $\mathcal{K}_2$ and $\mathbf{C}_1$ is an $(N-K_1)\times(K_2+L+1)$ matrix.

- Two-dimensional seach estimator:

$$\{\hat{v},\hat{w}\} = \arg\max_{v,w}\; \mathbf{y}^H\mathbf{F}^H_{[\bar{\mathcal{K}}_1,:]}\mathbf{C}_1(\mathbf{C}_1^H\mathbf{C}_1)^{-1}\mathbf{C}_1^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}. \quad (5)$$

$$\hat{\mathbf{d}}_1 = (\mathbf{C}_1^H\mathbf{C}_1)^{-1}\mathbf{C}_1^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}. \quad (6)$$

Minimum Pilot Length (continued)

- Note that

$$\breve{\mathbf{b}} = \mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1]\mathbf{h}_2. \quad (7)$$

- Then, $\mathbf{h}_2$ can be estimated as

$$\hat{\mathbf{h}}_2 = (\mathbf{F}_{[\mathcal{K}_2,1:2L+1]}\mathbf{H}_{zp}^{(L+1)}[\mathbf{\Omega}^{(L+1)}[v-w]\mathbf{h}_1])^{\dagger}\breve{\mathbf{b}}. \quad (8)$$

- Iteration can be applied to further improve the estimation performance.
- The minimum number of N is $3L+5$, when sets are disjoint and $K_1 = K_2 = L+1$, $K_r = L+3$.

CORE WIRELESS
UNIVERSITY OF ALBERTA

A Special Case

- In practical applications, the relay terminal is often a simple device while the two source terminals may employ high-precision synchronization circuits.
- Thus, it is reasonable to expect the CFO between the two source terminals to be negligible.
- Then we can obtain

$$\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y} = \underbrace{\alpha\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[v]\mathbf{F}^H_{[:,\mathcal{K}_2]}}_{\approx\mathbf{0}}\text{diag}\{\breve{\mathbf{s}}_2\}\mathbf{F}_{[\mathcal{K}_i,1:2L+1]}\mathbf{b} + \underbrace{\mathbf{G}_{[\bar{\mathcal{K}}_1,:]}\mathbf{\Gamma}[w]\mathbf{P}}_{\mathbf{C}_2}\mathbf{h}_1 + \mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{n}_e, \quad (9)$$

- CFO w can be estimated as

$$\hat{w} = \arg\max_{w}\ \mathbf{y}^H\mathbf{F}^H_{[\bar{\mathcal{K}}_1,:]}\mathbf{C}_2(\mathbf{C}_2^H\mathbf{C}_2)^{-1}\mathbf{C}_2^H\mathbf{F}_{[\bar{\mathcal{K}}_1,:]}\mathbf{y}, \quad (10)$$

Simulation Results: Minimum Pilot Length Case

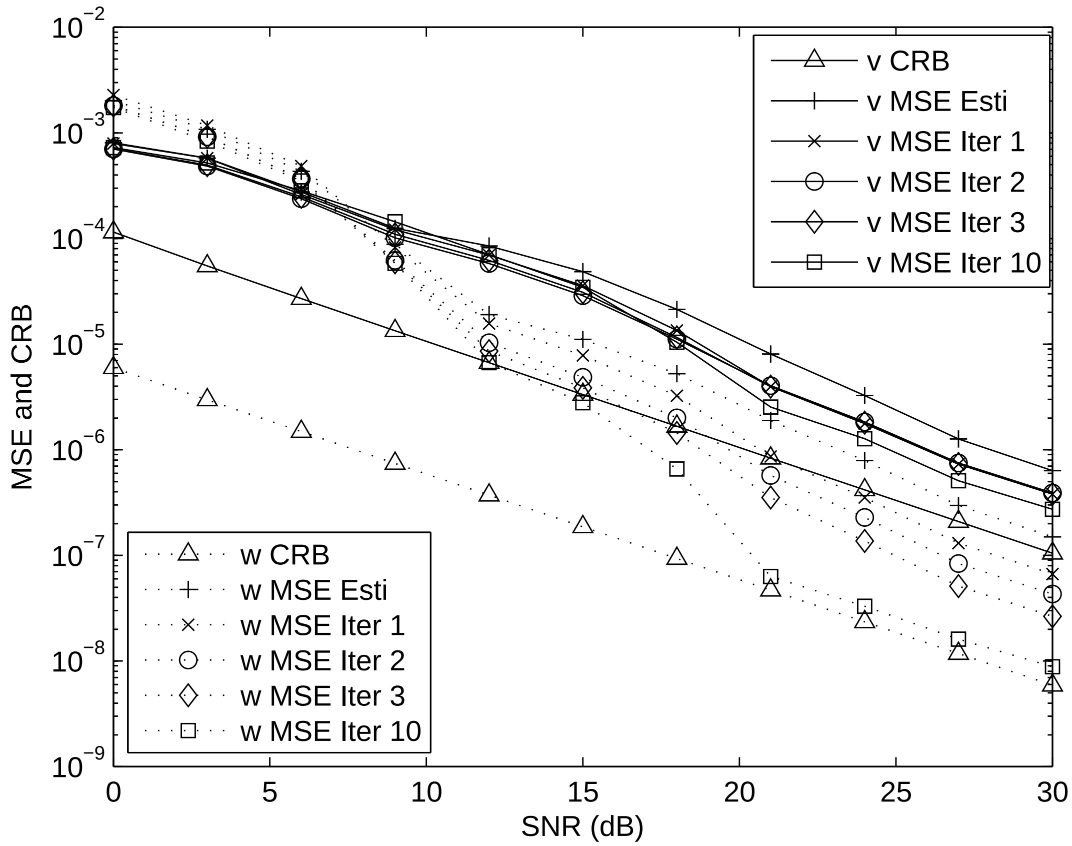

Figure 3: CFO estimation MSE versus SNR: $N = 14$

Simulation Results: Minimum Pilot Length Case

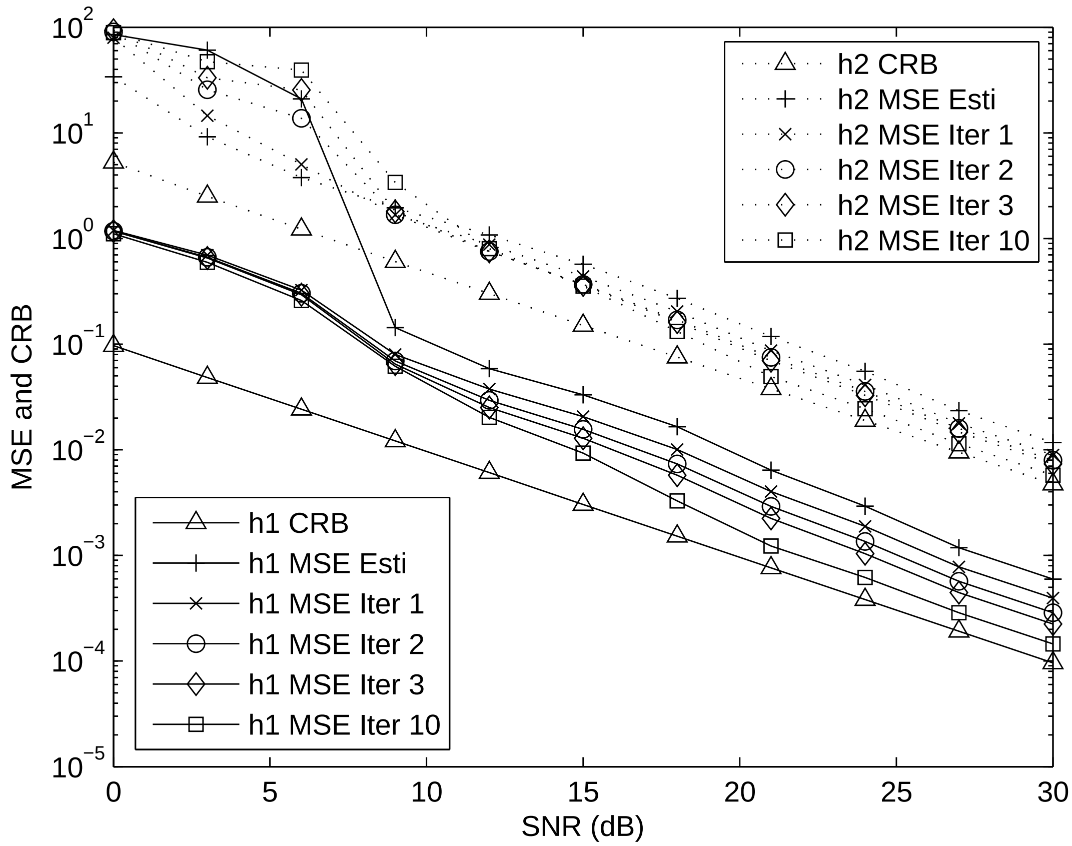

Figure 4: Channel estimation MSE versus SNR: $N = 14$

Simulation Results: A Special Case

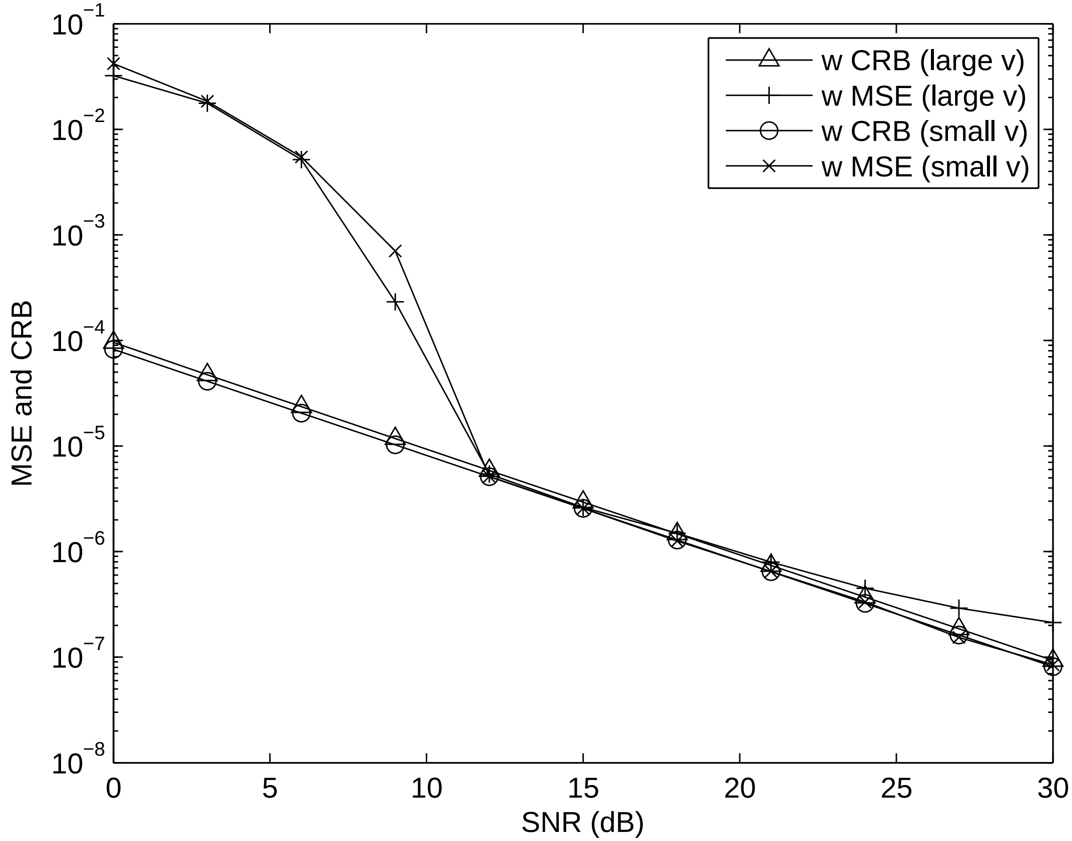

Figure 5: CFO estimation MSE versus SNR: $N = 9$

Simulation Results: A Special Case

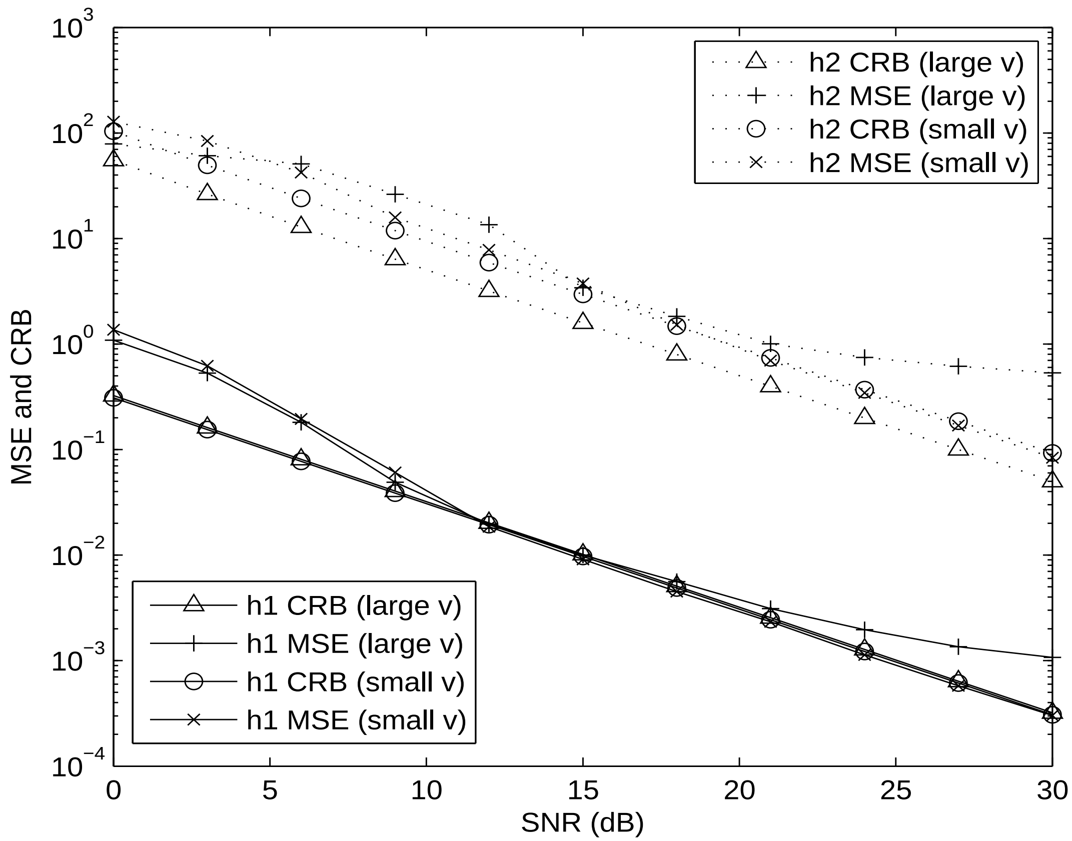

Figure 6: Channel estimation MSE versus SNR: $N = 9$

Conclusion

- Estimate individual channel parameters by introducing superimposed pilots at the relay node and using iterative algorithms.
- Less training is needed.
- Special case: small v.

Table 1: Comparison between adapted CP-OFDM and superimposed pilot aided CP-OFDM.

	Minimum Pilot Length	Estimated Parameters
adapted CP-OFDM	$4L+3$	$\mathbf{a}$, $\mathbf{b}$ and v
superimposed pilot	$3L+5$	$\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{a}$, $\mathbf{b}$, v and w

ICORE WIRELESS

UNIVERSITY OF ALBERTA

Questions and discussion?

Email: gongpu@ece.ualberta.ca