BEM-Based Estimation for Time-Varying Channels and Training Design in Two-Way Relay Networks

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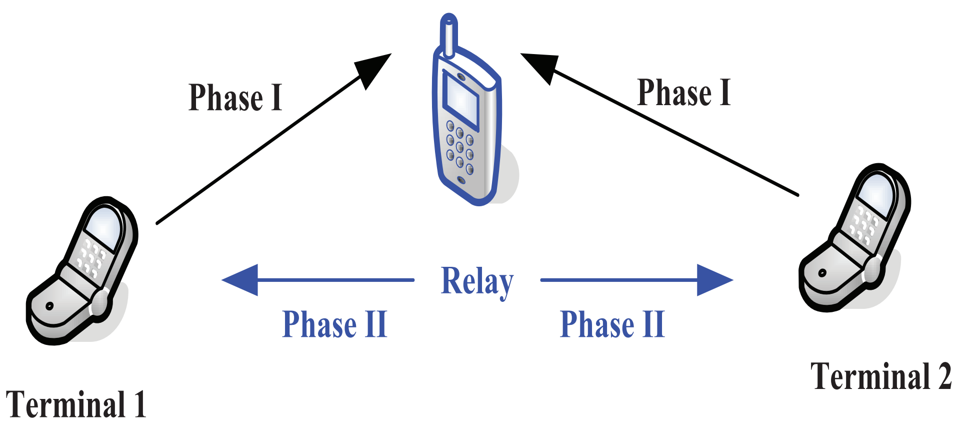
Outline

- Introduction
- Problem Formulation
- Proposed Solution
- Simulation Results
- Conclusion

Introduction

- Two-way transmission was firstly exploited by Shannon [Shannon, 1961].
- Two-way relay networks (TWRN) now has drawn much attention due to its improved spectral efficiency over one-way relay networks (OWRN).
- The overall communication rate between two source terminals in TWRN is approximately twice that achieved in OWRN [Rankov, 2006].



Problem Formulation

- For TWRNs, the relay and the two sources can be mobile, and the relative motion between any two nodes doubles the Doppler spread.
- This fact places additional demands on the estimation of time-varying channels.
- How to estimate time-varying channels $f[n]$ and $g[n]$? How to design training sequences ?

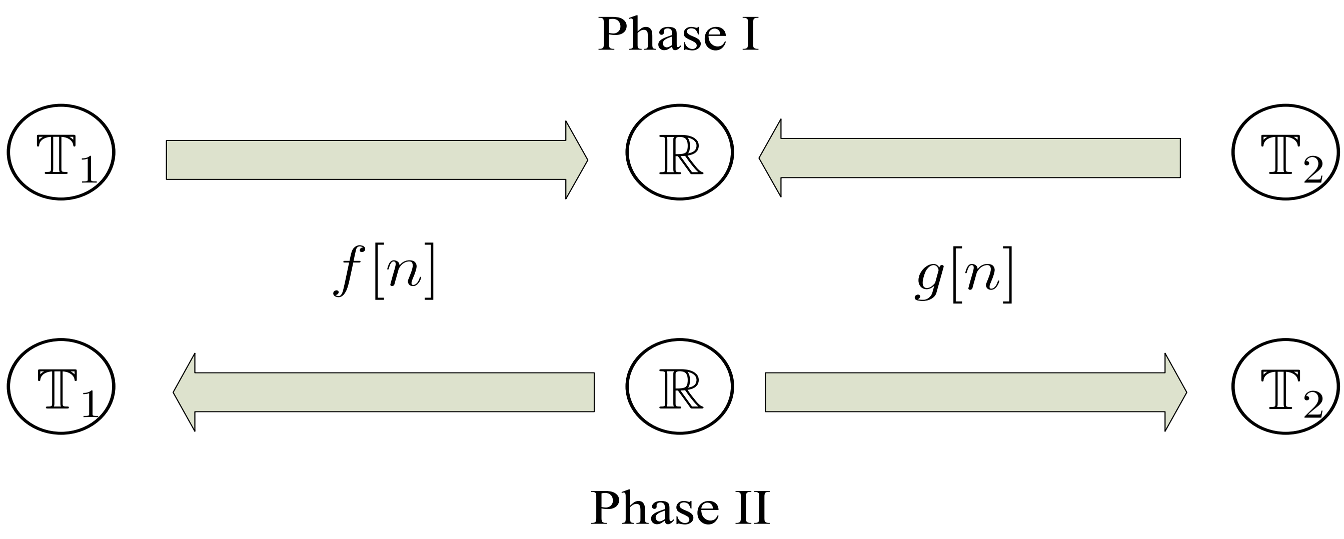


Figure 2: System model for TWRN with time varying channel.

BEM + TLS + Optimal Training Sequence

- The BEM of $f[n]$ and $g[n]$ can be expressed as

$$f[n] = \sum_{q=0}^{Q} f_q e^{jw_q n}, \quad 0 \le n \le N-1 \tag{1}$$

$$g[n] = \sum_{q=0}^{Q} g_q e^{jw_q n}, \quad 0 \le n \le N-1. \tag{2}$$

- The received signal at $\mathbb{T}_1$ can be written as

$$\begin{aligned} y[n+M+\Delta] =& \alpha\left(\sum_{q=0}^{Q} f_q e^{jw_q(n+M+\Delta)}\right)\left(\sum_{q=0}^{Q} f_q e^{jw_q n}\right) s_1[n] \\ &+ \alpha\left(\sum_{q=0}^{Q} f_q e^{jw_q(n+M+\Delta)}\right)\left(\sum_{q=0}^{Q} g_q e^{jw_q n}\right) s_2[n] \\ &+ \alpha f[n+M+\Delta] w_r[n] + w_1[n+M+\Delta]. \end{aligned} \tag{3}$$

BEM

- Define $\mathbf{A}$ as a $M \times (Q+1)^2$ Vandermonde matrix with the $(i(Q+1)+k)$th column

$$\mathbf{A}[:, i(Q+1)+k] = e^{jw_i(M+\Delta)} \begin{bmatrix} 1 \\ e^{j(w_i+w_k)} \\ \vdots \\ e^{j(M-1)(w_i+w_k)} \end{bmatrix}, \qquad (4)$$

$$i, k = 0, \ldots, Q,$$

- Then we can obtain

$$\mathbf{y} = \alpha \mathbf{S}_1 \mathbf{A} \mathbf{h}_1 + \alpha \mathbf{S}_2 \mathbf{A} \mathbf{h}_2 + \underbrace{\alpha \mathbf{f} \odot \mathbf{w}_r + \mathbf{w}_1}_{\mathbf{w}}, \qquad (5)$$

where $\odot$ denotes the Hadamard product, $\mathbf{h}_1$ is the $(Q+1)^2 \times 1$ vector with the $(i(Q+1)+k)$th entry $f_i f_k$, $\mathbf{h}_2$ is the $(Q+1)^2 \times 1$ vector with the $(i(Q+1)+k)$th entry $f_i g_k$, and $\mathbf{w}$ denotes the equivalent combined noise.

BEM (continued)

- Lemma: Matrix $\mathbf{A}$ defined in (4) has rank $2Q+1$.
- Combining those linearly dependent columns in $\mathbf{A}$, we can rewrite (5) as

$$\mathbf{y} = \alpha \mathbf{S}_1 \mathbf{A}_0 \mathbf{x}_1 + \alpha \mathbf{S}_2 \mathbf{A}_0 \mathbf{x}_2 + \mathbf{w}, \tag{6}$$

where $\mathbf{x}_i$ is a $(2Q+1) \times 1$ vector with entries

$$\mathbf{x}_1(m) = \sum_{i+k=m} e^{jw_i(M+\Delta)} f_i f_k, \quad m = 0, \ldots, 2Q \tag{7}$$

$$\mathbf{x}_2(m) = \sum_{i+k=m} e^{jw_i(M+\Delta)} f_i g_k, \tag{8}$$

$\mathbf{A}_0$ is an $M \times (2Q+1)$ Vandermonde matrix defined as

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ e^{j2w_0} & e^{j(w_0+w_1)} & \cdots & e^{j(w_{Q-1}+w_Q)} & e^{j2w_Q} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ e^{j2w_0(M-1)} & e^{j(w_0+w_1)(M-1)} & \cdots & e^{j(w_{Q-1}+w_Q)(M-1)} & e^{j2w_Q(M-1)} \end{bmatrix}$$

Total Least Square (TLS) Method

- Equation (6) can be rewritten as

$$\mathbf{y} = \underbrace{[\alpha \mathbf{S}_1 \mathbf{A}_0, \alpha \mathbf{S}_2 \mathbf{A}_0]}_{\mathbf{C}} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}}_{\mathbf{x}} + \mathbf{w}, \tag{9}$$

- By assuming that $\mathbf{y}$ and $\mathbf{C}$ contain disturbance $\mathbf{e}$ and $\mathbf{E}$, respectively, we obtain:

$$(\mathbf{C} + \mathbf{E})\mathbf{x} = \mathbf{y} + \mathbf{e}, \Leftrightarrow \underbrace{[-\mathbf{y}, \mathbf{C}]}_{\mathbf{J}} \mathbf{z} + \underbrace{[-\mathbf{e}, \mathbf{E}]}_{\mathbf{D}} \mathbf{z} = \mathbf{0}_{M \times 1}, \tag{10}$$

where $\mathbf{z} = [1, \mathbf{x}^T]^T$.

- The TLS finds $\mathbf{x}$, $\mathbf{E}$, and $\mathbf{e}$ from the following optimization:

$$\min_{\mathbf{D},\mathbf{x}} ||\mathbf{D}\mathbf{z}||_F^2, \tag{11}$$

$$s.t. \quad (\mathbf{y} + \mathbf{e}) \in \text{Range}(\mathbf{C} + \mathbf{E}), \tag{12}$$

where $|| \cdot ||_F^2$ denotes the Frobenius norm of a matrix.

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Total Least Square (TLS) Method

- We solve the following optimization:

$$\min_{\mathbf{z}} ||\mathbf{J}\mathbf{z}||_F^2, \tag{13}$$

- The solution to (13) is easily proved to be eigenvector of $\mathbf{J}^H\mathbf{J}$ that corresponds to the smallest eigenvalue.
- The estimation of $\mathbf{x}$ can be expressed as

$$\hat{\mathbf{x}} = \frac{1}{\mathbf{V}(1, 4Q+3)} \begin{bmatrix} \mathbf{V}(2, 4Q+3) \\ \vdots \\ \mathbf{V}(4Q+3, 4Q+3) \end{bmatrix}, \tag{14}$$

where $\mathbf{V}(m_1, m_2)$ is the (m_1, m_2)th entry of $\mathbf{V}$.

$$\mathbf{J} = \mathbf{U} \begin{bmatrix} \sigma_1(\mathbf{J}) & & & 0 \\ & \sigma_2(\mathbf{J}) & & \\ & & \ddots & \\ 0 & & & \sigma_{4Q+3}(\mathbf{J}) \end{bmatrix} \mathbf{V}^H, \tag{15}$$

Training Sequence Design

- Consider the optimization problem

$$\arg\min_{\mathbf{s}_1,\mathbf{s}_2} \qquad \kappa\big(\mathbf{C}(\mathbf{s}_1,\mathbf{s}_2)\big) = \frac{\sigma_1(\mathbf{C})}{\sigma_{4Q+2}(\mathbf{C})}, \qquad (16)$$

  where $\kappa(\cdot)$ denotes the condition number.

- ***Type-1 Training:*** The training symbol from $\mathbb{T}_1$ and $\mathbb{T}_2$ appear alternatively with maximum power P_s and arbitrary phase.

- ***Type-2 Training:*** We choose $|s_1[n]| = P_s$ with arbitrary phase and set $s_2[n] = (-1)^n s_1[n]$.

- **Theorem**: Denote $\mathbf{C}$ from type-1 training as $\mathbf{C}_1$, and that from type-2 training as $\mathbf{C}_2$. The condition numbers of $\mathbf{C}_1$ and $\mathbf{C}_2$ are the same.

Simulation Results

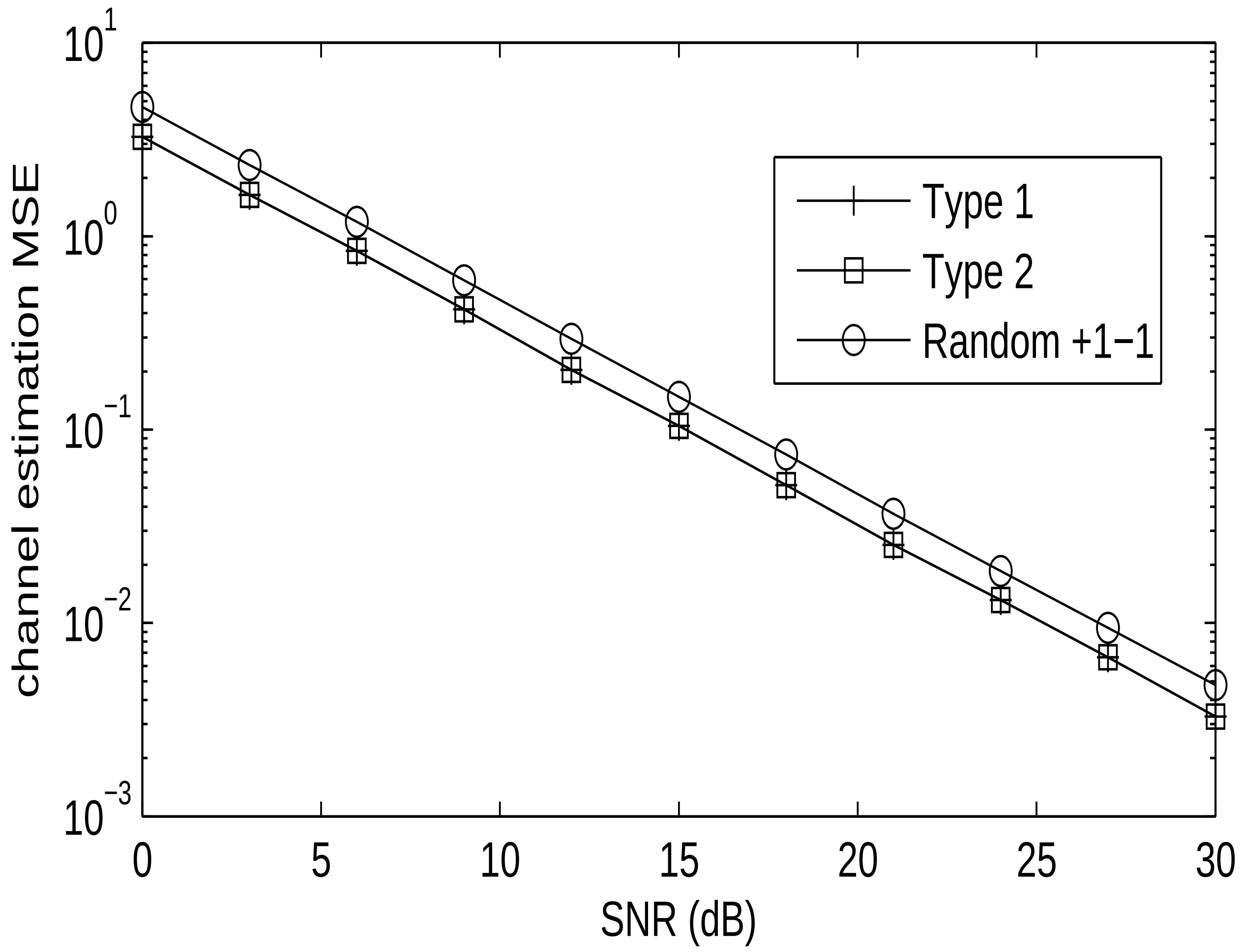

Figure 3: Channel estimation MSE of $\mathbf{x}$

Simulation Results

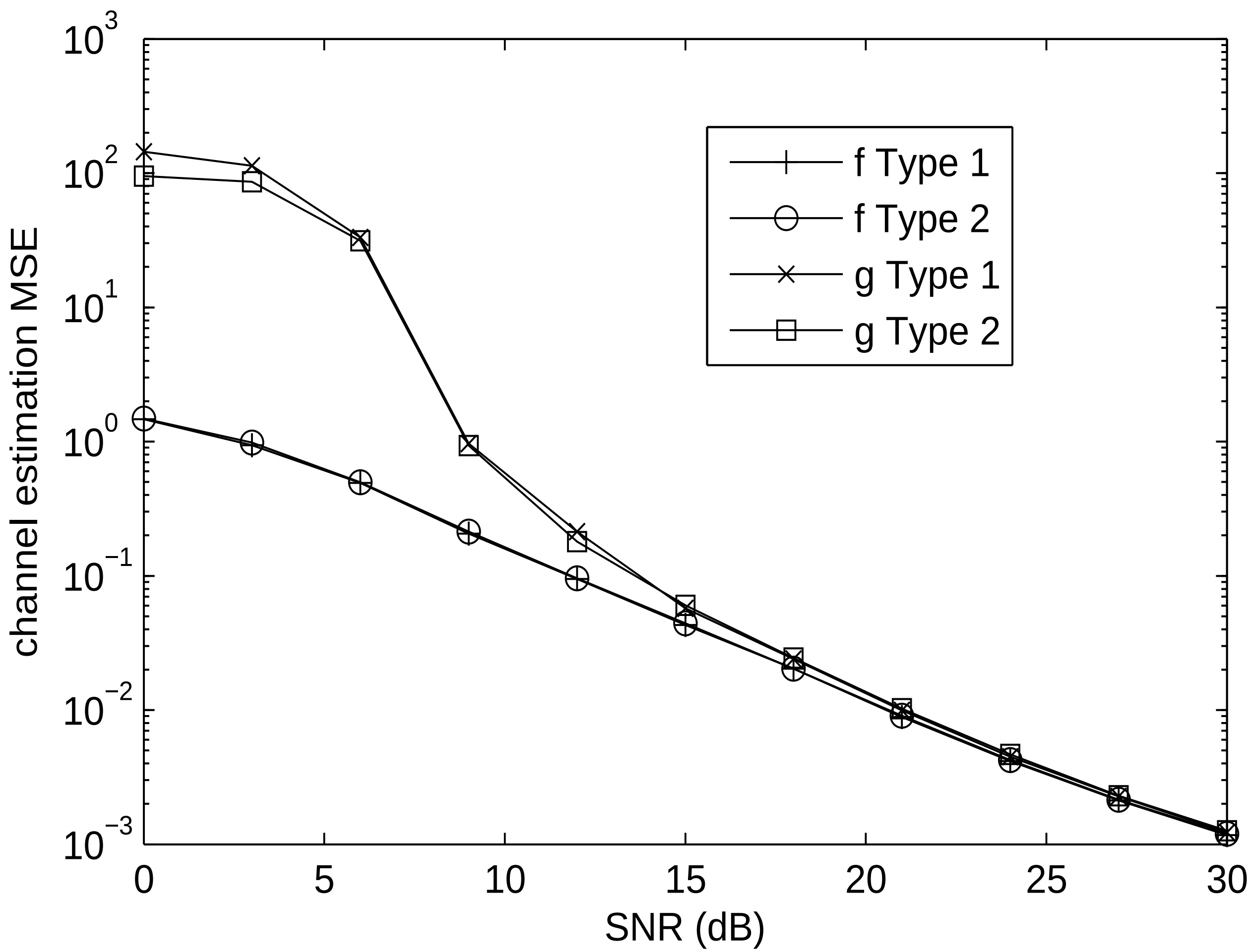

Figure 4: Channel estimation MSE of $\mathbf{f}$ and $\mathbf{g}$

Simulation Results

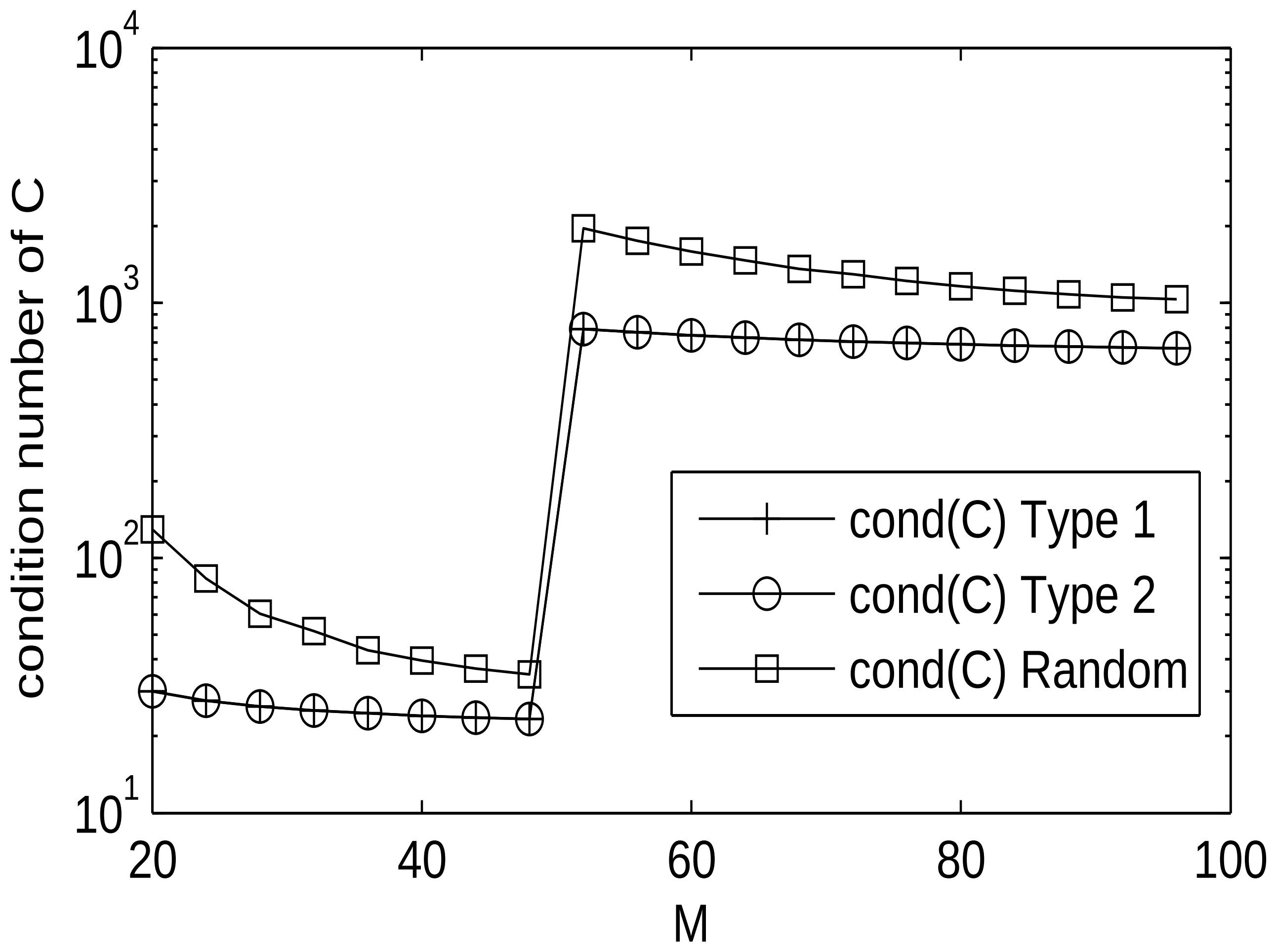

Figure 5: Condition number of A_0

Conclusion

- **Our contribution:**
  1. Time-varying channel estimation in AF TWRN
  2. BEM + TLS
  3. Training sequence design.
- **Problem:** How to improve channel estimation performance ?

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Conclusion

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- **Problem:** How to improve channel estimation performance ?
- **ICC 2011**