Joint CFO and Channel Estimation for ZP-OFDM Modulated Two-Way Relay Networks

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- Two-way relay networks (TWRN) can enhance the overall communication rate [Boris Rankov, 2006], [J.Ponniah, 2008].

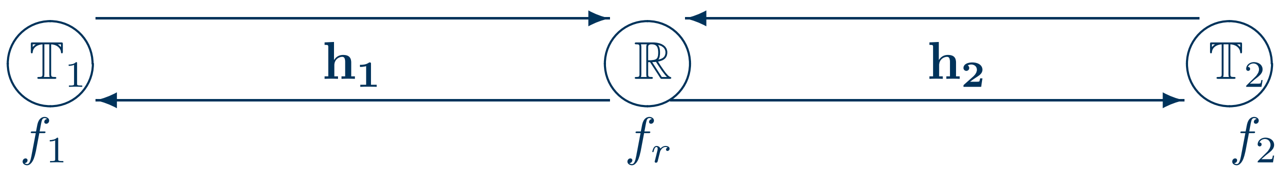

Figure 1: System configuration for two-way relay network.

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Previous Results

- Most existing works in TWRN assumed perfect synchronization and channel state information (CSI).
- Channel estimation problems in amplify-and-forward (AF) TWRN are different from those in traditional communication systems.
- Flat-fading and frequency-selective channel estimation and training design for AF TWRN has been done in [Feifei Gao, 2009].
- Our paper will focus on joint frequency offset (CFO) and channel estimation for AF-based OFDM-Modulated TWRN.

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Joint CFO and Channel Estimation Problems in TWRN

- With CFOs, the orthogonality between subcarriers will be destroyed in TWRN.
- Even with completed estimation, data detection is not simple as circular convolution no longer exists.
- How to estimate the mixed CFOs and channels and how to faciliate data detection?
- We introduce some redundancy and modify the OFDM TWRN system to facilitate both the joint estimation and detection.

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Signals at Relay

- The relay $\mathbb{R}$ will down-convert the passband signal by $e^{-j2\pi f_r t}$ and obtain

$$\mathbf{r}_{zp} = \sum_{i=1}^{2} \mathbf{\Gamma}^{(N+L)}[f_i - f_r]\mathbf{H}_{zp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i + \mathbf{n}_r, \qquad (1)$$

where $\mathbf{\Gamma}^{(K)}[f] = \text{diag}\{1, e^{j2\pi f T_s}, \ldots, e^{j2\pi f(K-1)T_s}\}$ and

$$\mathbf{s}_i = \mathbf{F}^H \tilde{\mathbf{s}}_i = \begin{bmatrix} s_{i,0} \\ s_{i,1} \\ \vdots \\ s_{i,N-1} \end{bmatrix} \qquad \mathbf{H}_{zp}^{(K)}[\mathbf{x}] \triangleq \underbrace{\begin{bmatrix} x_0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ x_P & \ddots & x_0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & x_P \end{bmatrix}}_{K \text{ columns}}$$

- Next, $\mathbb{R}$ adds L zeros to the end of $\mathbf{r}$ and scales it by the factor of α_{zp} to keep the average power constraint.

Signals at Terminal $\mathbb{T}_1$

- $\mathbb{T}_1$ will down-convert the passband signal by $e^{-\jmath 2\pi f_1 t}$ and get

$$\begin{aligned}\mathbf{y}_{zp} =& \alpha_{zp}\mathbf{\Gamma}^{(N+2L)}[f_r - f_1]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1]\mathbf{r}_{zp} + \mathbf{n}_1 \\ =& \alpha_{zp}\mathbf{\Gamma}^{(N+2L)}[f_r - f_1]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1] \\ & \times \left(\sum_{j=1}^{2}\mathbf{\Gamma}^{(N+L)}[f_i - f_r]\mathbf{H}_{zp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i\right) \\ & + \underbrace{\alpha_{zp}\mathbf{\Gamma}^{(N+2L)}[f_r - f_1]\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1]\mathbf{n}_r + \mathbf{n}_1}_{\mathbf{n}_e} \end{aligned} \tag{2}$$

- Next, using the following equalities

$$\mathbf{H}_{zp}^{(K)}[\mathbf{x}]\,\mathbf{\Gamma}^{(K)}[f] = \mathbf{\Gamma}^{(K+P)}[f]\mathbf{H}_{zp}^{(K)}\left[\mathbf{\Gamma}^{(K)}[-f]\mathbf{x}\right], \tag{3}$$

and

$$\mathbf{\Gamma}^{(K+P)}[f]\mathbf{H}_{zp}^{(K)}[\mathbf{x}] = \mathbf{H}_{zp}^{(K)}\left[\mathbf{\Gamma}^{(P+1)}[f]\mathbf{x}\right]\mathbf{\Gamma}^{(K)}[f]. \tag{4}$$

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Signals at Terminal $\mathbb{T}_1$

- $\mathbf{y}_{zp}$ can be rewritten as

$$\mathbf{y}_{zp} = \alpha_{zp}\mathbf{H}_{zp}^{(N+L)}[\mathbf{\Gamma}^{(L+1)}[f_r - f_1]\mathbf{h}_1]\mathbf{H}_{zp}^{(N)}[\mathbf{h}_1]\,\mathbf{s}_1 + \mathbf{n}_e$$
$$+ \alpha_{zp}\mathbf{\Gamma}^{(N+2L)}[f_2 - f_1]\mathbf{H}_{zp}^{(N+L)}[\mathbf{\Gamma}^{(L+1)}[f_r - f_2]\mathbf{h}_1] \times \mathbf{H}_{zp}^{(N)}[\mathbf{h}_2 \qquad (5)$$

- We further note that $\mathbf{H}_{zp}^{(N+L)}[\mathbf{x}_1]\mathbf{H}_{zp}^{(N)}[\mathbf{x}_2] = \mathbf{H}_{zp}^{(N)}[\mathbf{x}_1 \otimes \mathbf{x}_2]$ where $\otimes$ denotes the linear convolution.
- Hence $\mathbf{y}_{zp}$ is finally written as

$$\mathbf{y}_{zp} = \alpha_{zp}\mathbf{H}_{zp}^{(N)}\Big[\underbrace{(\mathbf{\Gamma}^{(L+1)}[f_r - f_1]\mathbf{h}_1) \otimes \mathbf{h}_1}_{\mathbf{a}_{zp}}\Big]\mathbf{s}_1 + \mathbf{n}_e$$
$$+ \alpha_{zp}\mathbf{\Gamma}^{(N+2L)}[\underbrace{f_2 - f_1}_{v}]\mathbf{H}_{zp}^{(N)}\Big[\underbrace{(\mathbf{\Gamma}^{(L+1)}[f_r - f_2]\mathbf{h}_1) \otimes \mathbf{h}_2}_{\mathbf{b}_{zp}}\Big]\mathbf{s}_2, \qquad (6)$$

where $\mathbf{a}_{zp}$, $\mathbf{b}_{zp}$ are the $(2L+1) \times 1$ equivalent channel vectors and v is the equivalent CFO.

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Joint CFO and Channel Estimation

- We then obtain

$$\mathbf{y} = \mathbf{S}_1\mathbf{a} + \boldsymbol{\Gamma}\mathbf{S}_2\mathbf{b} + \mathbf{n}_e. \tag{7}$$

- Since $\mathbf{S}_1$ is a tall matrix, it is possible to find a matrix $\mathbf{J}$ such that $\mathbf{J}^H\mathbf{S}_1 = \mathbf{0}$.
- Left-multiplying $\mathbf{y}$ by $\mathbf{J}^H$ gives

$$\mathbf{J}^H\mathbf{y} = \mathbf{0} + \underbrace{\mathbf{J}^H\boldsymbol{\Gamma}\mathbf{S}_2}_{\mathbf{G}}\mathbf{b} + \underbrace{\mathbf{J}^H\mathbf{n}_e}_{\mathbf{n}}. \tag{8}$$

- Joint CFO estimation and channel estimation

$$\hat{v} = \arg\max_{v}\ \mathbf{y}^H\mathbf{J}\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}, \tag{9}$$

$$\hat{\mathbf{b}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}, \tag{10}$$

$$\hat{\mathbf{a}} = (\mathbf{S}_1^H\mathbf{S}_1)^{-1}\mathbf{S}_1^H(\mathbf{y} - \hat{\boldsymbol{\Gamma}}\mathbf{S}_2\hat{\mathbf{b}}). \tag{11}$$

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Performance Analysis

- At high SNR, the perturbation of the estimated CFO can be approximated by

$$\Delta v \triangleq \hat{v}_0 - v_0 \approx -\frac{\dot{g}(v_0)}{\mathsf{E}\{\ddot{g}(v_0)\}}, \qquad (12)$$

where $g(v) = \mathbf{y}^H \mathbf{J}\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}$.

- The NLS estimation of CFO is unbiased and its MSE is

$$\mathsf{E}\{\Delta v^2\} = \frac{\sigma_{ne}^2}{2\mathbf{b}^H\dot{\mathbf{G}}^H[\mathbf{I} - \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H]\dot{\mathbf{G}}\mathbf{b}}. \qquad (13)$$

- The channel estimation $\hat{\mathbf{b}}$ is unbiased and its MSE is

$$\mathsf{MSE}\{\mathbf{b}\} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\dot{\mathbf{G}}\mathbf{b}\mathbf{b}^H\dot{\mathbf{G}}^H\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathsf{E}\{\Delta v^2\} + \sigma_{ne}^2(\mathbf{G}^H\mathbf{G})^{-1}. \qquad (14)$$

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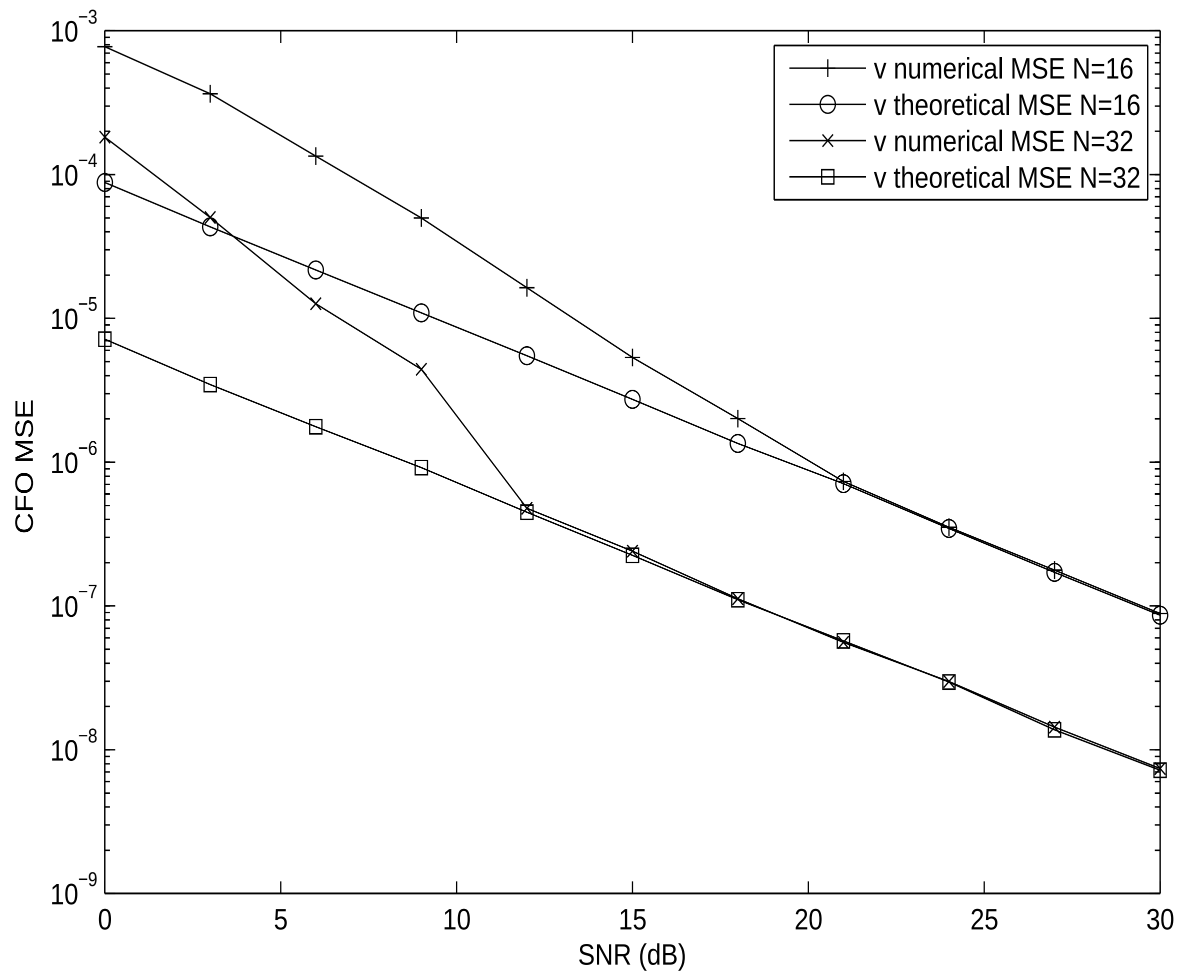

Figure 2: Numerical and Theoretical MSEs of CFO versus SNR

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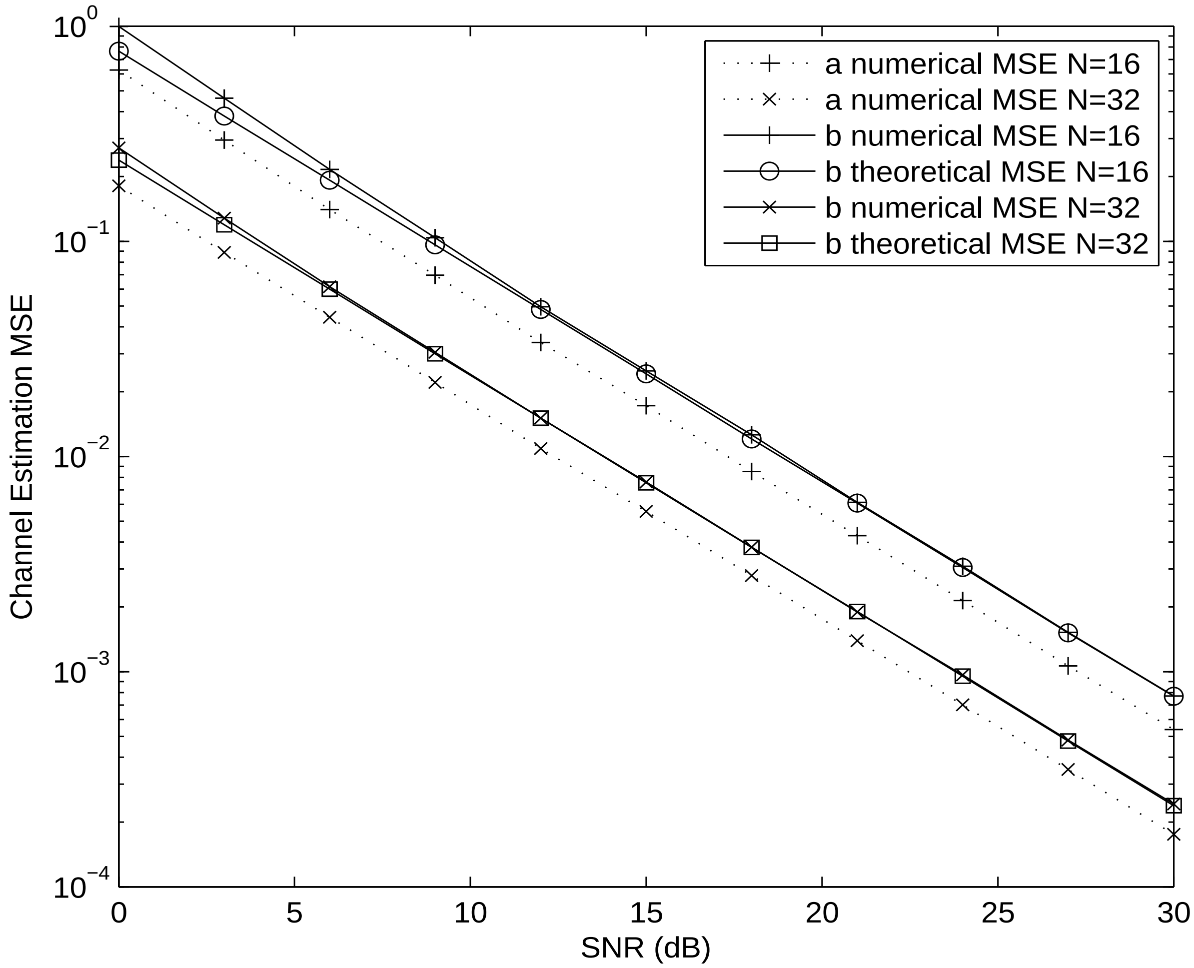

Figure 3: Numerical and Theoretical MSEs of Channel Estimation versus SNR

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1. Adapt ZP-based OFDM transmission scheme.

2. Suggest joint estimation method of CFO and channels.

3. Performance analysis: prove unbiasedness and give closed-form MSE expression.

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1. Adapt ZP-based OFDM transmission scheme.
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3. Performance analysis: prove unbiasedness and give closed-form MSE expression.

Problem: How to obtain individual frequency and channel parameters? (Globecom 2010)

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Questions and discussion?

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