Joint CFO and Channel Estimation for CP-OFDM Modulated Two-Way Relay Networks

Gongpu Wang[†], Feifei Gao[‡], Yik-Chung Wu[*], and Chintha Tellambura[†]

[†]University of Alberta, Edmonton, Canada,
[‡]Jacobs University, Bremen, Germany
[*]The University of Hong Kong, Hong Kong

Email: gongpu@ece.ualberta.ca

ICC'10

Outline

- Introduction
- Previous Results
- Problem Formulation
- Proposed Solution
- Simulation Results
- Conclusion

Introduction

- Two-way relay networks (TWRN) can enhance the overall communication rate [Boris Rankov, 2006], [J.Ponniah, 2008].

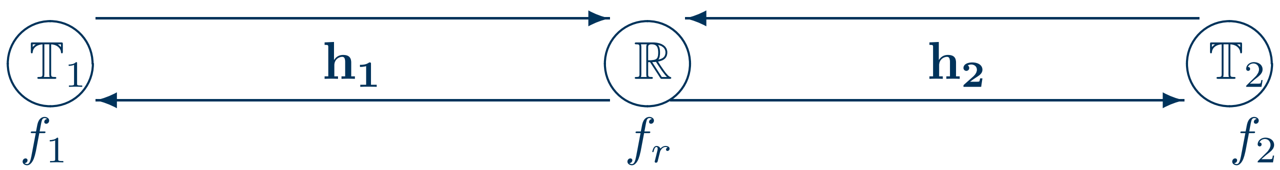

Figure 1: System configuration for two-way relay network.

CORE WIRELESS
UNIVERSITY OF ALBERTA

Previous Results

- Most existing works in TWRN assumed perfect synchronization and channel state information (CSI).
- Channel estimation problems in amplify-and-forward (AF) TWRN are different from those in traditional communication systems.
- Flat-fading and frequency-selective channel estimation and training design for AF TWRN has been done in [Feifei Gao, 2009].
- Our paper will focus on joint frequency offset (CFO) and channel estimation for AF-based cyclic-prefix (CP) OFDM-Modulated TWRN.

Joint CFO and Channel Estimation Problems

- With CFOs, the orthogonality between subcarriers will be destroyed in TWRN.
- Even with completed estimation, data detection is not simple as circular convolution no longer exists.
- How to estimate the mixed CFOs and channels and how to faciliate data detection?
- We introduce some redundancy and modify the cyclic-prefix (CP)-based OFDM TWRN system to facilitate both the joint estimation and detection.

CP-based OFDM protocol

- Traditionally, OFDM scheme at $\mathbb{T}_i$ will add L cyclic-prefix (CP), where L is the channel length. Relay $\mathbb{R}$ just sample, amplify and forward. But with CFO, this will not work in TWRN.
- Proposed CP-based OFDM protocol:
  1. $\mathbb{T}_i$ adds a CP of length $2L$ in the front of $\mathbf{s}_i$;
  2. $\mathbb{R}$ removes only the first L symbols in the CP.
- By doing so, $\mathbf{y}_{cp}$, received signal at Terminal $\mathbb{T}_1$, can then be expressed as

$$\mathbf{y}_{cp} = \alpha_{cp}\mathbf{H}_{cp}^{(N)}[(\underbrace{\boldsymbol{\Omega}^{(L+1)}[f_1 - f_r]\mathbf{h}_1) \circledast \mathbf{h}_1}_{\mathbf{a}}]\mathbf{s}_1 + \mathbf{n}_e$$
$$+ \alpha_{cp}e^{j2\pi vLT_s}\boldsymbol{\Gamma}^{(N)}[v]\mathbf{H}_{cp}^{(N)}[(\underbrace{\boldsymbol{\Omega}^{(L+1)}[f_2 - f_r]\mathbf{h}_1) \circledast \mathbf{h}_2}_{\mathbf{b}}]\mathbf{s}_2, \quad (1)$$

where $\boldsymbol{\Omega}^{(K)}[f] = \text{diag}\{e^{j2\pi f(K-1)T_s}, \ldots, e^{j2\pi fT_s}, 1\}$, $\circledast$ means the N-point circular convolution, and $\mathbf{H}_{cp}^{(N)}[\mathbf{x}]$ is the $N \times N$ circulant matrix with the first column $[\mathbf{x}^T, \mathbf{0}_{1\times(N-L-1)}^T]^T$.

Joint Estimation

- We can rewrite $\mathbf{y}_{cp}$ as

$$\mathbf{y}_{cp} = \mathbf{S}_1^{(N)}\mathbf{a} + \mathbf{\Gamma}^{(N)}[v]\mathbf{S}_2^{(N)}\mathbf{b} + \mathbf{n}_e, \tag{2}$$

  where $\mathbf{S}_i^{(N)}$ is the $N \times (2L+1)$ circulant matrix with first column $\mathbf{s}_i$.

- The LS estimation can be directly expressed as

$$\{\widehat{\mathbf{a}}, \widehat{\mathbf{b}}, \widehat{v}\} = \arg\min_{\mathbf{a},\mathbf{b},v} \|\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b}\|^2, \tag{3}$$

- Denote $\mathbf{C} = [\mathbf{S}_1, \mathbf{\Gamma}\mathbf{S}_2]$ and $\mathbf{d} = [\mathbf{a}^T, \mathbf{b}^T]^T$. Joint CFO and channel estimation:

$$\widehat{\mathbf{d}} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}, \tag{4}$$

$$\begin{aligned} \widehat{v} &= \arg\min_{v} \ ||\mathbf{y} - \mathbf{C}\widehat{\mathbf{d}}||^2 \\ &= \arg\max_{v} \ \mathbf{y}^H\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}. \end{aligned} \tag{5}$$

Approximated Cramér-Rao Bound (ACRB)

Let $\boldsymbol{\mu} = \mathbf{S}_1\mathbf{a} + \boldsymbol{\Gamma}\mathbf{S}_2\mathbf{b}$, and define

$$\boldsymbol{\eta} \triangleq [v, \Re\{\mathbf{a}\}^T, \Im\{\mathbf{a}\}^T, \Re\{\mathbf{b}\}^T, \Im\{\mathbf{b}\}^T]^T. \quad (6)$$

The Fisher Information Matrix (FIM) is obtained as:

$$\mathbf{F} = \frac{2}{\sigma_{ne}^2}\Re\left[\frac{\partial\boldsymbol{\mu}^H}{\partial\boldsymbol{\eta}}\frac{\partial\boldsymbol{\mu}}{\partial\boldsymbol{\eta}^T}\right] = \frac{2}{\sigma_{ne}^2}\begin{bmatrix} F_{11} & \mathbf{r}^T & \mathbf{s}^T \\ \mathbf{r} & \mathbf{K} & \mathbf{V}^T \\ \mathbf{s} & \mathbf{V} & \mathbf{N} \end{bmatrix}, \quad (7)$$

where

$$F_{11} = \mathbf{b}^H\mathbf{S}_2^H\mathbf{D}^2\mathbf{S}_2\mathbf{b}, \quad \mathbf{D} = 2\pi T_s \mathrm{diag}\{0, 1, \ldots, (N-1)\},$$

$$\mathbf{r} = \begin{bmatrix} -\Im(\mathbf{S}_1^H\mathbf{D}\boldsymbol{\Gamma}\mathbf{S}_2\mathbf{b}) \\ \Re(\mathbf{S}_1^H\mathbf{D}\boldsymbol{\Gamma}\mathbf{S}_2\mathbf{b}) \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} -\Im(\mathbf{S}_2^H\mathbf{D}\mathbf{S}_2\mathbf{b}) \\ \Re(\mathbf{S}_2^H\mathbf{D}\mathbf{S}_2\mathbf{b}) \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} \Re(\mathbf{S}_1^H\mathbf{S}_1) & -\Im(\mathbf{S}_1^H\mathbf{S}_1) \\ \Im(\mathbf{S}_1^H\mathbf{S}_1) & \Re(\mathbf{S}_1^H\mathbf{S}_1) \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} \Re(\mathbf{S}_2^H\mathbf{S}_2) & -\Im(\mathbf{S}_2^H\mathbf{S}_2) \\ \Im(\mathbf{S}_2^H\mathbf{S}_2) & \Re(\mathbf{S}_2^H\mathbf{S}_2) \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} \Re(\mathbf{S}_2^H\boldsymbol{\Gamma}^H\mathbf{S}_1) & -\Im(\mathbf{S}_2^H\boldsymbol{\Gamma}^H\mathbf{S}_1) \\ \Im(\mathbf{S}_2^H\boldsymbol{\Gamma}^H\mathbf{S}_1) & \Re(\mathbf{S}_2^H\boldsymbol{\Gamma}^H\mathbf{S}_1) \end{bmatrix}.$$

The CRB of CFO is the upper-left block in $\mathbf{F}^{-1}$, which can be explicitly calculated as:

$$\mathrm{ACRB}(v) = \frac{\sigma_{ne}^2}{2}[F_{11} - \mathbf{t}_1^T \mathbf{Q}_1^{-1} \mathbf{t}_1]^{-1}, \tag{8}$$

where

$$\mathbf{t}_1 = \begin{bmatrix} \mathbf{r} \\ \mathbf{s} \end{bmatrix}, \quad \mathbf{Q}_1 = \begin{bmatrix} \mathbf{K}, \mathbf{V}^T \\ \mathbf{V}, \mathbf{N} \end{bmatrix}.$$

The ACRBs of the channel estimates $\mathbf{a}$, $\mathbf{b}$ are then given by

$$\mathrm{ACRB}(\mathbf{a}) = \mathbf{A}\mathbf{F}^{-1}\mathbf{A}^H, \tag{9}$$

$$\mathrm{ACRB}(\mathbf{b}) = \mathbf{B}\mathbf{F}^{-1}\mathbf{B}^H, \tag{10}$$

were

$$\mathbf{A} = [\mathbf{0}_{(2L+1)\times 1}, \mathbf{I}, j\mathbf{I}, 0 \times \mathbf{I}, 0 \times \mathbf{I}]$$
$$\mathbf{B} = [\mathbf{0}_{(2L+1)\times 1}, 0 \times \mathbf{I}, 0 \times \mathbf{I}, \mathbf{I}, j\mathbf{I}],$$

where $\mathbf{I}$ means $(2L+1) \times (2L+1)$ identity matrix.

Simulation Results

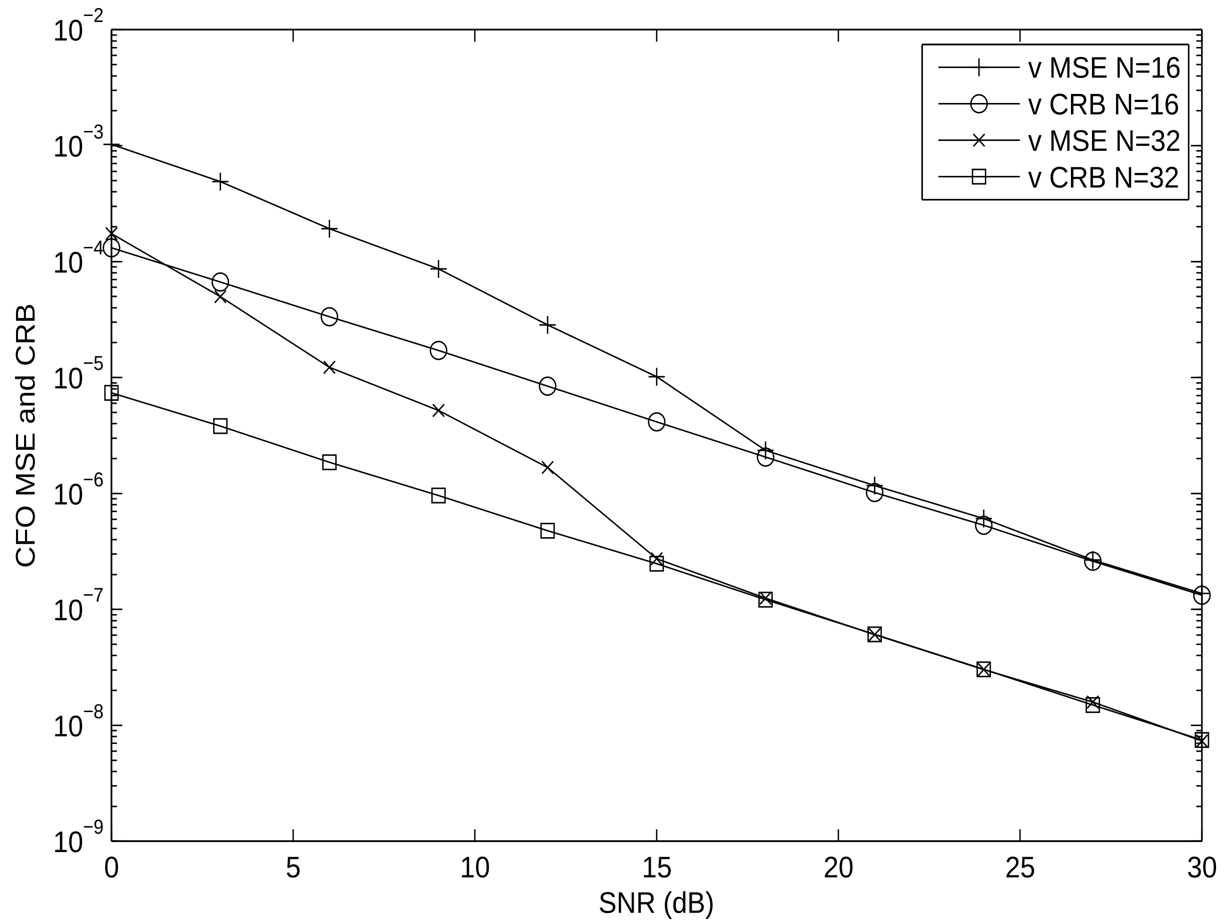

Figure 2: CFO estimation MSEs versus SNR for $N = 16,\ 32$, respectively.

Simulation Results

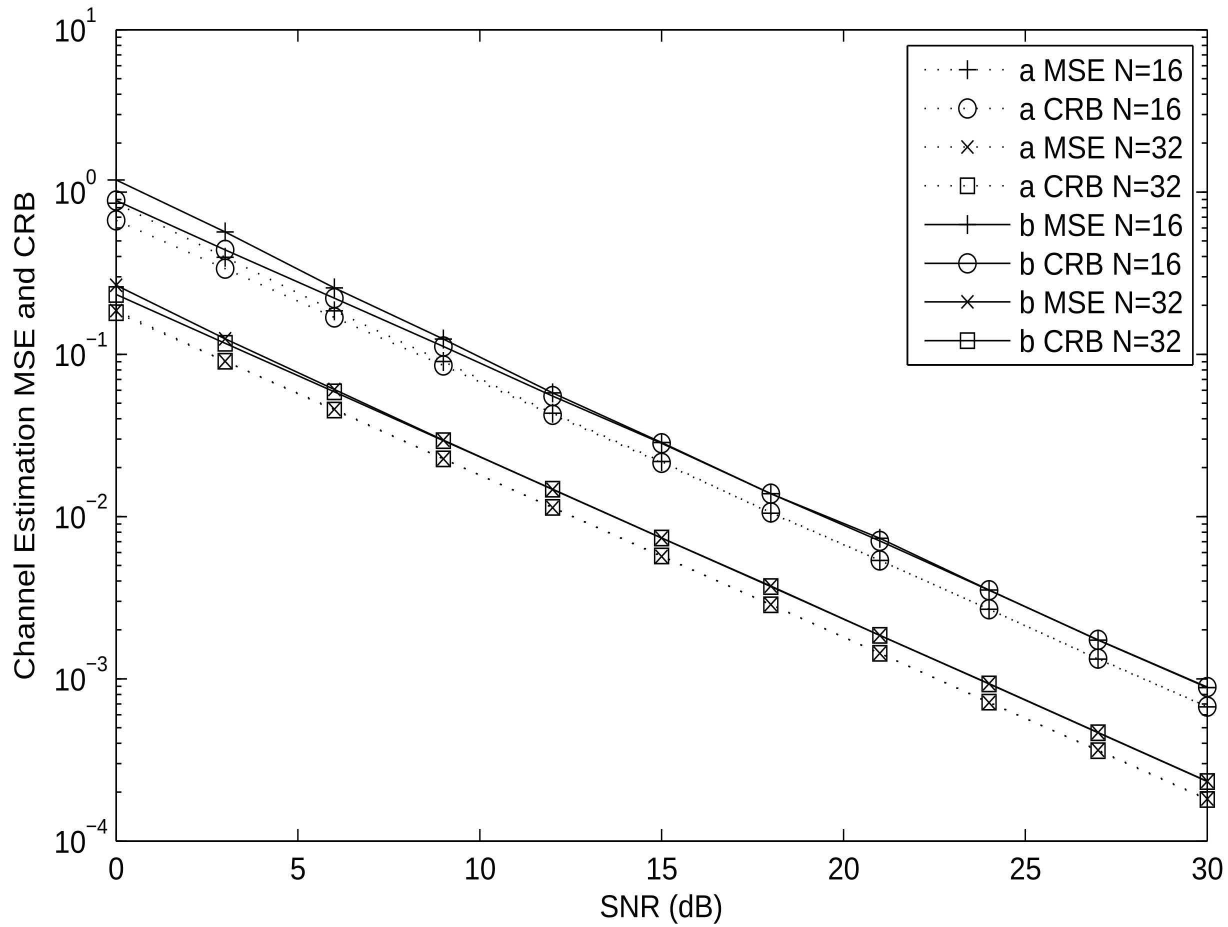

Figure 3: Channel estimation MSEs versus SNR for $N = 16,\ 32$, respectively.

Simulation Results

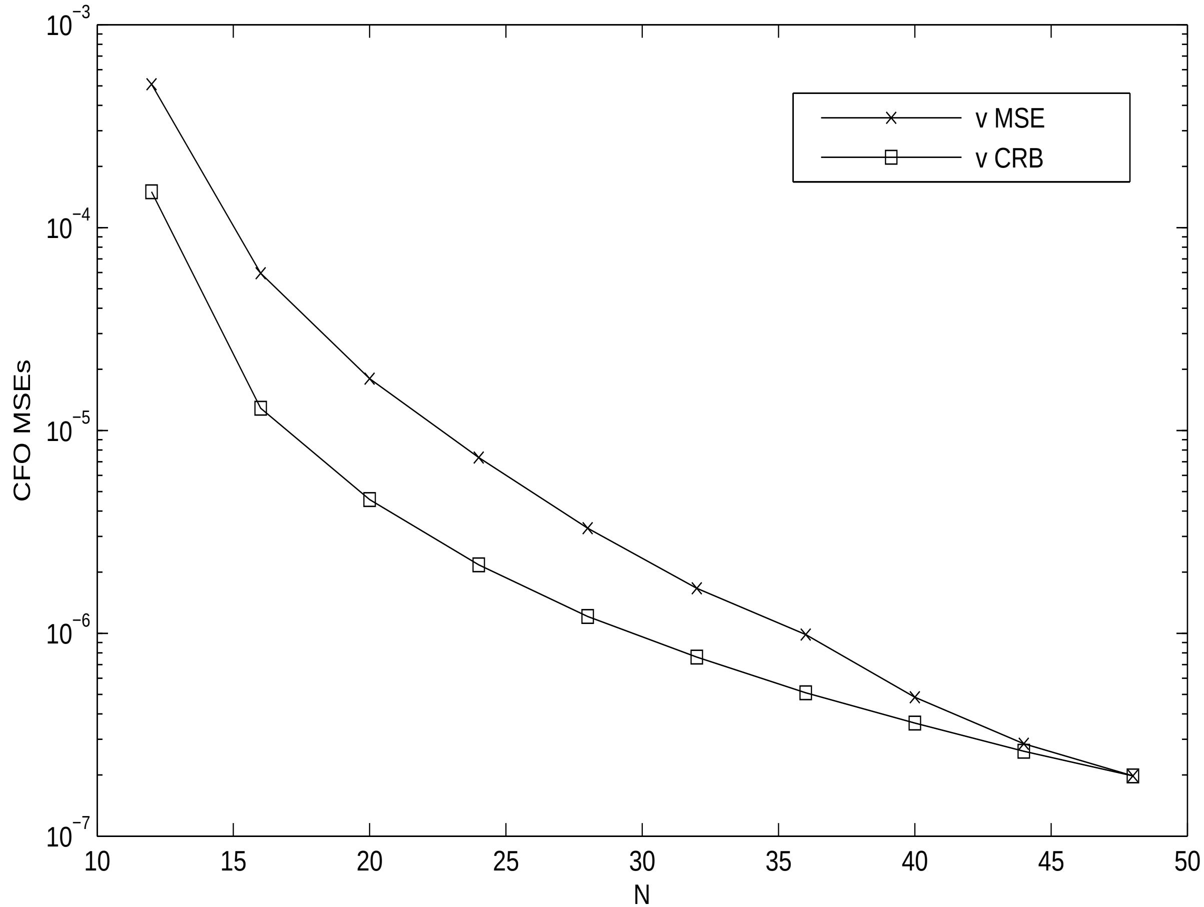

Figure 4: CFO estimation MSEs versus N for SNR$=10$ dB.

CORE WIRELESS
UNIVERSITY OF ALBERTA

Simulation Results

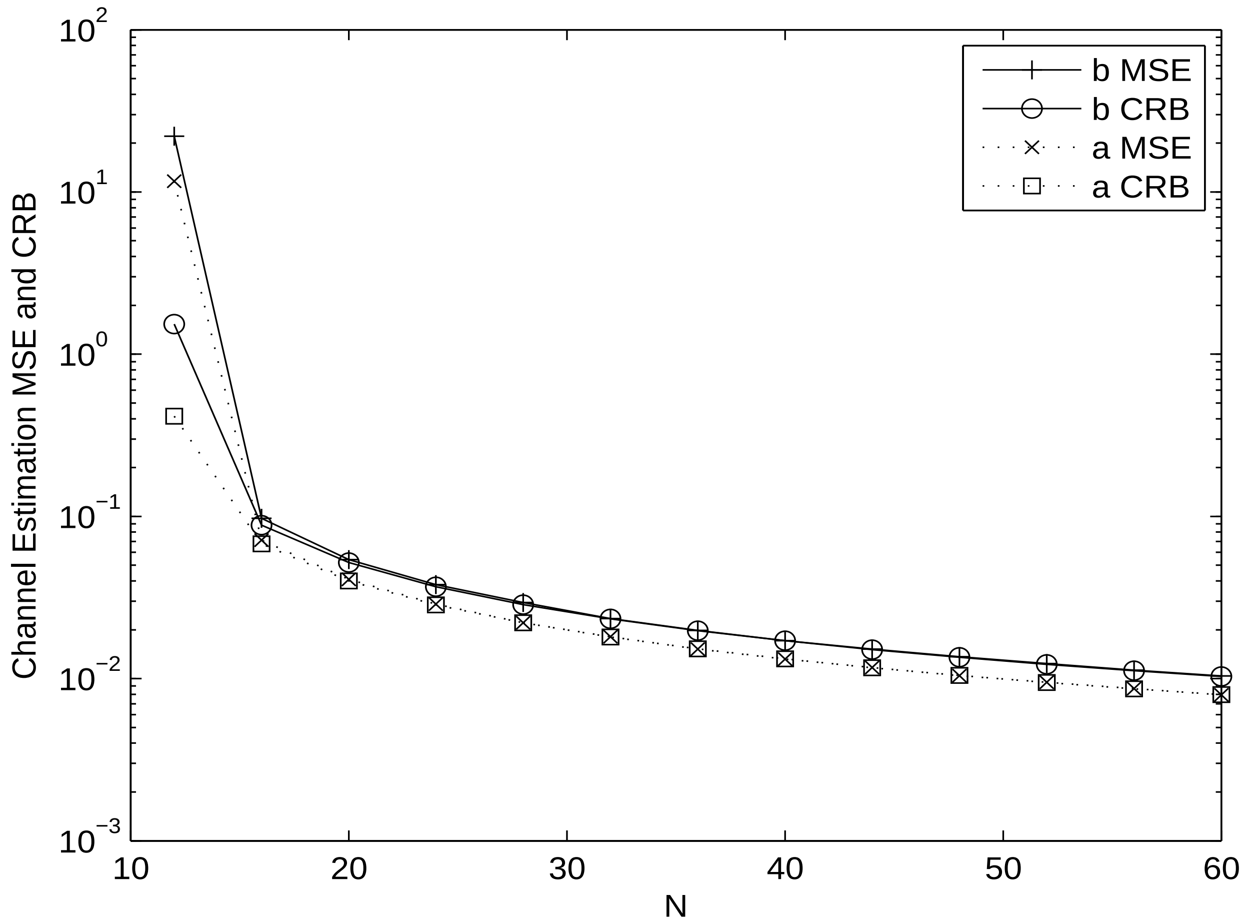

Figure 5: Channel estimation MSEs versus N for SNR$= 10$ dB.

Conclusion

Table 1: Comparison between ZP- and CP-OFDM modulated TWRN.

	ZP	CP
Transmitter activity	add L zeros suffix	add $2L$ cyclic prefix
Relay activity	add L zeros suffix	remove L prefix
Destination activity	None	remove L prefix
Received signal length	$N + 2L$	N
Spectrum efficiency	$N/(2N + 3L)$	$N/(2N + 3L)$
Required Pilot Length	$N \geq 2L + 3$	$N \geq 4L + 3$

Note: ZP-based OFDM protocl is proposed in our 2010 WCNC paper.

iCORE WIRELESS

UNIVERSITY OF ALBERTA

Conclusion

1. Adapt CP-based OFDM transmission scheme.
2. Suggest joint estimation method of CFO and channels.
3. Find ACRB.

Conclusion

1. Adapt CP-based OFDM transmission scheme.
2. Suggest joint estimation method of CFO and channels.
3. Find ACRB.

Prolbem: We only find combined channels $\mathbf{a}$ and $\mathbf{b}$. How to obtain individual frequency and channel parameters $\mathbf{h}_1$ and $\mathbf{h}_2$? (Globecom 2010)

ICORE WIRELESS

UNIVERSITY OF ALBERTA

Conclusion

Questions and discussion?

Email: gongpu@ece.ualberta.ca