Joint Frequency Offset and Channel Estimation Methods for Two-Way Relay Networks

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Introduction

- Two-way relay networks (TWRN) can enhance the overall communication rate [Boris Rankov, 2006], [J.Ponniah, 2008].

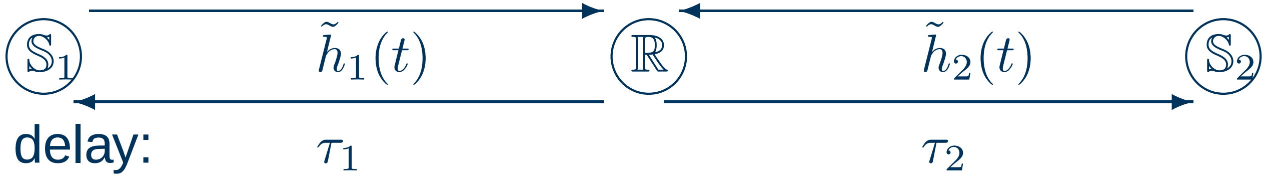

Figure 1: System configuration for two-way relay network.

Previous Results

- Most existing works in TWRN assumed perfect synchronization and channel state information (CSI).
- Channel estimation problems in amplify-and-forward (AF) TWRN are different from those in traditional communication systems.
- Flat-fading channel estimation and training design for AF TWRN has been done in [Feifei Gao, 2009].
- Our paper will focus on joint frequency offset (CFO) and channel estimation for AF TWRN.

Problem Formulation

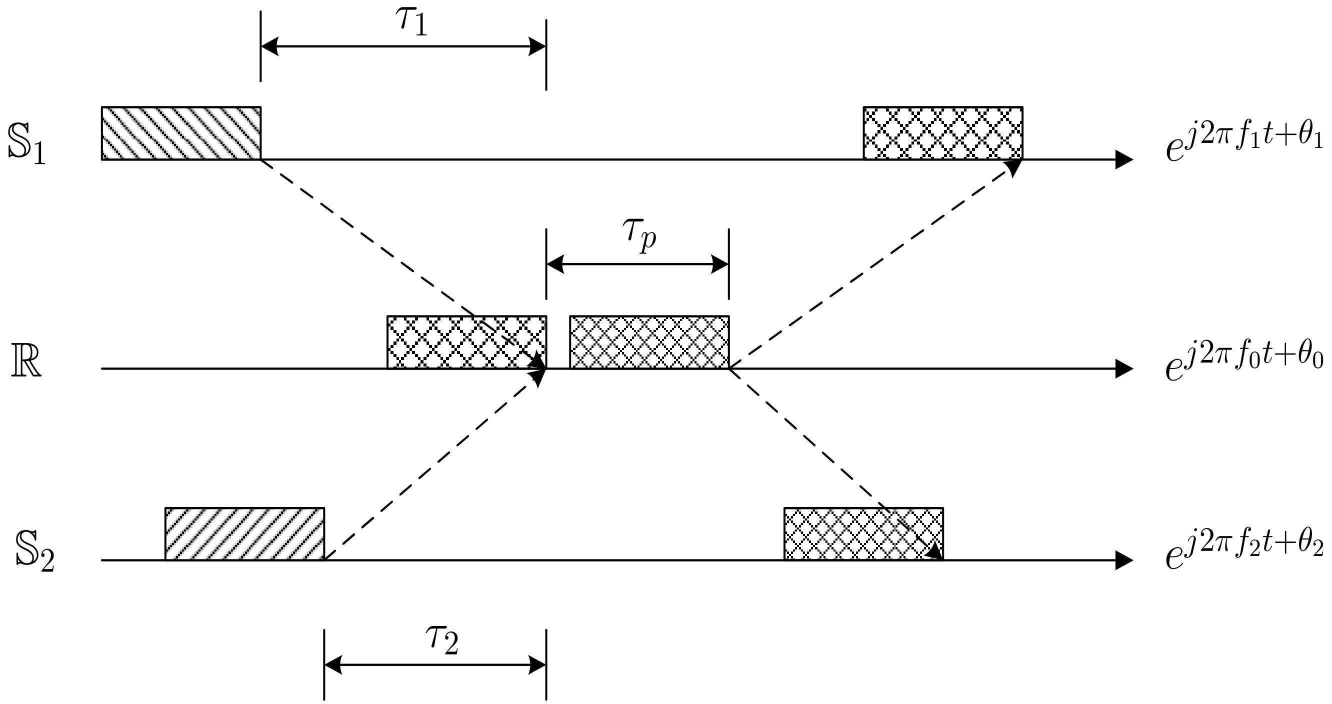

Figure 2: Illustration of channel delay and signal processing delay in two-way relay transmission.

Problem Formulation

- The passband signal sent by $\mathbb{S}_i$ is

$$\tilde{s}_i(t) = \sum_{m=-\infty}^{+\infty} s_i[m]p(t - mT_s)e^{j2\pi f_i t}. \tag{1}$$

- The received signal in $\mathbb{R}$ is

$$\tilde{r}_r(t) = \sum_{i=1}^{2} \tilde{h}_i(t) * \tilde{s}_i(t) + \tilde{n}_r(t) \tag{2}$$

- The passband signal sent out by relay is then

$$\tilde{r}_s(t - \tau_p) = \alpha \sum_{i=1}^{2} \tilde{h}_i(t) * \tilde{s}_i(t - \tau_p) + \alpha \tilde{n}_r(t - \tau_p) \tag{3}$$

Problem Formulation

- The data received at $\mathbb{S}_1$ is

$$
\begin{aligned}
\tilde{y}(t) &= \tilde{h}_1(t) * \tilde{r}_s(t-\tau_p) + \tilde{n}_1(t), \\
&= \alpha \sum_{i=1}^{2} \Big(\big((\tilde{h}_1(t) * \tilde{h}_i(t)) e^{-j2\pi f_i t} \big) * s_i(t-\tau_p) \Big) \\
&\qquad \times e^{j2\pi f_i (t-\tau_p)} + \alpha \tilde{h}_1(t) * \tilde{n}_r(t-\tau_p) + \tilde{n}_1(t). \qquad (4)
\end{aligned}
$$

- Then $\mathbb{S}_1$ down-convert $\tilde{y}(t)$ to baseband by $e^{-j2\pi f_1 t}$

$$
\begin{aligned}
y(t) &= \tilde{y}(t) e^{-j2\pi f_1 t} \\
&= \alpha \Big(\Big((\tilde{h}_1(t) * \tilde{h}_1(t)) e^{-j2\pi f_1 t} \Big) * s_1(t-\tau_p) \Big) e^{j\phi_1} \\
&+ \alpha \Big(\Big((\tilde{h}_1(t) * \tilde{h}_2(t)) e^{-j2\pi f_2 t} \Big) * s_2(t-\tau_p) \Big) e^{j2\pi v t + j\phi_2} \\
&+ \alpha \Big(\tilde{h}_1(t) * \tilde{n}_r(t-\tau_p) \Big) e^{-j2\pi f_1 t} + \tilde{n}_1(t) e^{-j2\pi f_1 t}. \qquad (5)
\end{aligned}
$$

Problem Formulation

- The baseband signal is rewritten as

$$\begin{aligned} y(t) =& a(t) * s_1(t-\tau_p) + b(t) * s_2(t-\tau_p)e^{j2\pi v t} \\ &+ h_1(t) * (\tilde{n}_r(t-\tau_p)e^{-j2\pi f_1 t}) + \tilde{n}_1(t)e^{-j2\pi f_1 t}. \end{aligned} \tag{6}$$

- Following the traditional approach [M.Morelli 2000], we can obtain

$$\mathbf{y} = \mathbf{S}_1\mathbf{a} + \boldsymbol{\Gamma}\mathbf{S}_2\mathbf{b} + \mathbf{H}_1\mathbf{n}_r + \mathbf{n} \tag{7}$$

where

$$\boldsymbol{\Gamma} = \mathrm{diag}\left(1, e^{j2\pi v T_s}, \ldots, e^{j2\pi v(N-1)T_s}\right),$$

$$\mathbf{H}_1 = \alpha \begin{bmatrix} h_1[L] & \ldots & h_1[0] & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & h_1[L] & \ldots & h_1[0] \end{bmatrix}.$$

Estimation Methods

- Approximated ML Estimation
- Nulling Based LS Method

Approximated ML Estimation

- Maximizing the probability density function (pdf) of $\mathbf{y}$:

$$p(\mathbf{y}|\mathbf{\Theta}) = \frac{1}{\pi^N \det(\mathbf{H}_1\mathbf{H}_1^H + \mathbf{I})} \times$$
$$\exp\{(\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b})^H(\mathbf{H}_1\mathbf{H}_1^H + \mathbf{I})^{-1}(\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b})\}$$

- When the number of the channel taps is large, $\mathbf{H}_1\mathbf{H}_1^H$ can be approximated by

$$\mathbf{H}_1\mathbf{H}_1^H \approx \sum_{l=0}^{L} \sigma_{h,l}^2\mathbf{I} \qquad (8)$$

- Then the ML estimation is

$$\{\widehat{\mathbf{a}}, \widehat{\mathbf{b}}, \widehat{v}\} = \arg\min_{\mathbf{a},\mathbf{b},v} \|\mathbf{y} - \mathbf{S}_1\mathbf{a} - \mathbf{\Gamma}\mathbf{S}_2\mathbf{b}\|^2. \qquad (9)$$

Approximated ML Estimation

- Denote $\mathbf{C} = [\mathbf{S}_1, \mathbf{\Gamma}\mathbf{S}_2]$ and $\mathbf{d} = [\mathbf{a}^T, \mathbf{b}^T]^T$. As long as $N > 4L + 2$, $\mathbf{d}$ can be estimated as

$$\widehat{\mathbf{d}} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y}. \tag{10}$$

- CFO estimation is

$$\begin{aligned} \widehat{v} &= \arg\min_{v} \ ||\mathbf{y} - \mathbf{C}\widehat{\mathbf{d}}||^2 \\ &= \arg\max_{v} \ \mathbf{y}^H\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{y} \\ &= \arg\max_{v} \ g(v). \end{aligned} \tag{11}$$

Cramér-Rao Bound of CFO Estimation of AML

$$\mathbf{F} = \frac{2}{\sum_l \sigma_{h,l}^2 + 1} \begin{bmatrix} F_{11} & \mathbf{r}^T & \mathbf{s}^T \\ \mathbf{r} & \mathbf{K} & \mathbf{V}^T \\ \mathbf{s} & \mathbf{V} & \mathbf{N} \end{bmatrix}, \qquad (12)$$

where

$$F_{11} = \mathbf{b}^H \mathbf{S}_2^H \mathbf{D}^2 \mathbf{S}_2 \mathbf{b}, \quad \mathbf{D} = 2\pi T_s \mathrm{diag}\{0, 1, \ldots, (N-1)\},$$

$$\mathbf{r} = \begin{bmatrix} -\Im(\mathbf{S}_1^H \mathbf{D} \boldsymbol{\Gamma} \mathbf{S}_2 \mathbf{b}) \\ \Re(\mathbf{S}_1^H \mathbf{D} \boldsymbol{\Gamma} \mathbf{S}_2 \mathbf{b}) \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} -\Im(\mathbf{S}_2^H \mathbf{D} \mathbf{S}_2 \mathbf{b}) \\ \Re(\mathbf{S}_2^H \mathbf{D} \mathbf{S}_2 \mathbf{b}) \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} \Re(\mathbf{S}_1^H \mathbf{S}_1) & -\Im(\mathbf{S}_1^H \mathbf{S}_1) \\ \Im(\mathbf{S}_1^H \mathbf{S}_1) & \Re(\mathbf{S}_1^H \mathbf{S}_1) \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} \Re(\mathbf{S}_2^H \mathbf{S}_2) & -\Im(\mathbf{S}_2^H \mathbf{S}_2) \\ \Im(\mathbf{S}_2^H \mathbf{S}_2) & \Re(\mathbf{S}_2^H \mathbf{S}_2) \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} \Re(\mathbf{S}_2^H \boldsymbol{\Gamma}^H \mathbf{S}_1) & -\Im(\mathbf{S}_2^H \boldsymbol{\Gamma}^H \mathbf{S}_1) \\ \Im(\mathbf{S}_2^H \boldsymbol{\Gamma}^H \mathbf{S}_1) & \Re(\mathbf{S}_2^H \boldsymbol{\Gamma}^H \mathbf{S}_1) \end{bmatrix}.$$

$$\mathrm{CRB}_1(v) = \frac{\sum_l \sigma_{h,l}^2 + 1}{2} [F_{11} - \mathbf{t}_1^T \mathbf{Q}_1^{-1} \mathbf{t}_1]^{-1}, \qquad (13)$$

where

$$\mathbf{t}_1 = \begin{bmatrix} \mathbf{r} \\ \mathbf{s} \end{bmatrix}, \quad \mathbf{Q}_1 = \begin{bmatrix} \mathbf{K}, \mathbf{V}^T \\ \mathbf{V}, \mathbf{N} \end{bmatrix}.$$

Nulling Based Method

- Left-multiply both sides of (7) with $\mathbf{J}^H$, we obtain:

$$\mathbf{J}^H\mathbf{y} = \mathbf{0} + \underbrace{\mathbf{J}^H\boldsymbol{\Gamma}\mathbf{S}_2}_{\mathbf{G}}\mathbf{b} + \mathbf{J}^H(\mathbf{H}\mathbf{n_r} + \mathbf{n}), \quad (14)$$

- The estimation of $\mathbf{b}$ can be immediately found as

$$\hat{\mathbf{b}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{y}. \quad (15)$$

- CFO is estimated as

$$\hat{v} = \arg\max_{v}\ \mathbf{y}^H\mathbf{J}\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y} \quad (16)$$

- The least square (LS) estimation of channel $\mathbf{a}$ is obtained from

$$\hat{\mathbf{a}} = (\mathbf{S}_1^H\mathbf{S}_1)^{-1}\mathbf{S}_1^H(\mathbf{y} - \hat{\boldsymbol{\Gamma}}\mathbf{S}_2\hat{\mathbf{b}}), \quad (17)$$

where $\hat{\boldsymbol{\Gamma}} = \mathrm{diag}\{1, e^{j2\pi\hat{v}T_s}, ..., e^{j2\pi\hat{v}(N-1)T_s}\}$.

CRB of CFO Estimation of the nulling based method

The FIM is calculated as:

$$\mathbf{F} = \frac{2}{\sum_l \sigma_{h,l}^2 + 1} \begin{bmatrix} F'_{11} & \mathbf{t}_2^T \\ \mathbf{t}_2 & \mathbf{Q}_2 \end{bmatrix}, \qquad (18)$$

where

$$\begin{aligned} F'_{11} &= \mathbf{b}^H \mathbf{S}_2^H \mathbf{D} \mathbf{\Gamma}^H \mathbf{J} \mathbf{J}^H \mathbf{D} \mathbf{\Gamma} \mathbf{S}_2 \mathbf{b}, \\ \mathbf{t}_2 &= \begin{bmatrix} -\Im(\mathbf{S}_2^H \mathbf{\Gamma}^H \mathbf{J} \mathbf{J}^H \mathbf{D} \mathbf{\Gamma} \mathbf{S}_2 \mathbf{b}) \\ \Re(\mathbf{S}_2^H \mathbf{\Gamma}^H \mathbf{J} \mathbf{J}^H \mathbf{D} \mathbf{\Gamma} \mathbf{S}_2 \mathbf{b}) \end{bmatrix}, \\ \mathbf{Q}_2 &= \begin{bmatrix} \Re(\mathbf{G}^H \mathbf{G}) & -\Im(\mathbf{G}^H \mathbf{G}) \\ \Im(\mathbf{G}^H \mathbf{G}) & \Re(\mathbf{G}^H \mathbf{G}) \end{bmatrix}. \end{aligned}$$

The CRB of frequency offset estimation is:

$$\mathrm{CRB}_2(v) = \frac{\sum_l \sigma_{h,l}^2 + 1}{2} [F'_{11} - \mathbf{t}_2^T \mathbf{Q}_2^{-1} \mathbf{t}_2]^{-1}. \qquad (19)$$

Simulation Results

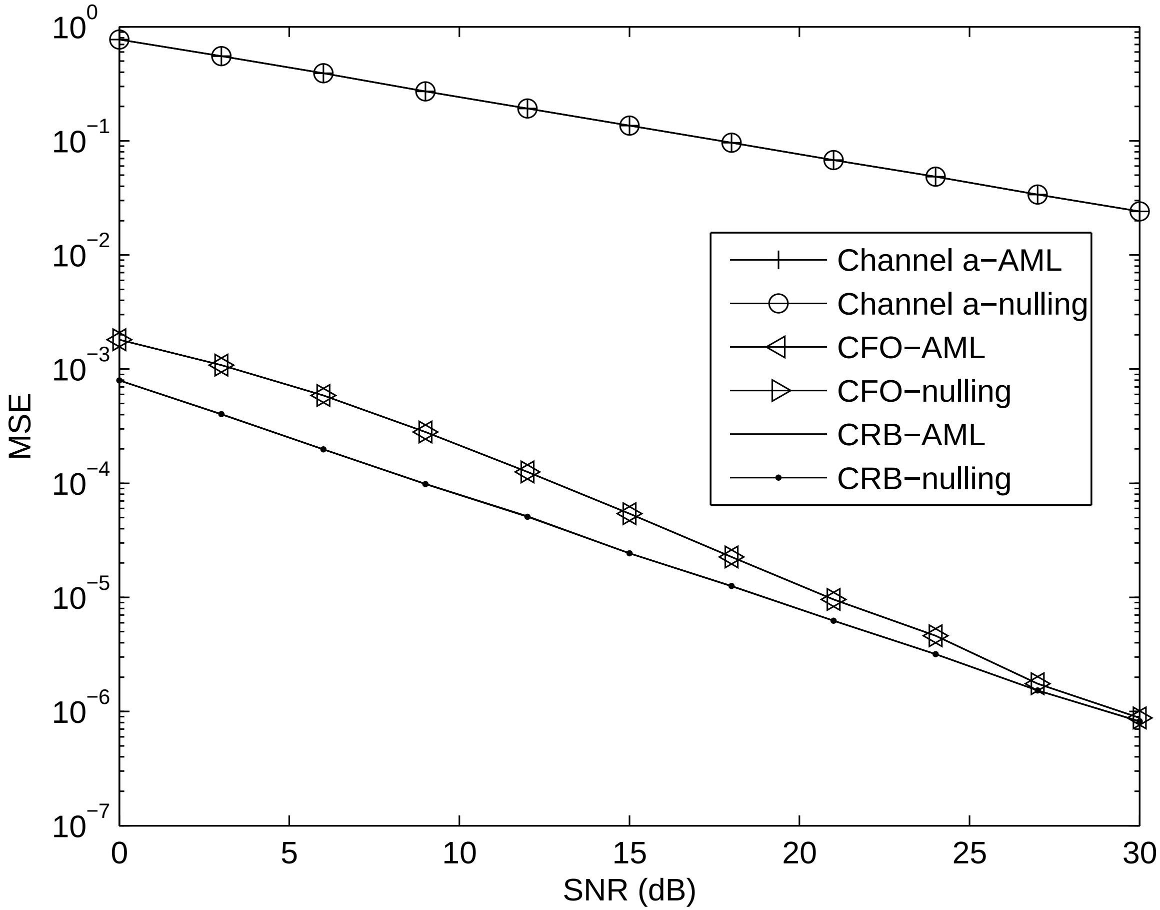

Figure 3: MSEs of CFO and channel estimation versus SNR for AML and nulling based method with orthonormal $\mathbf{J}$

Simulation Results

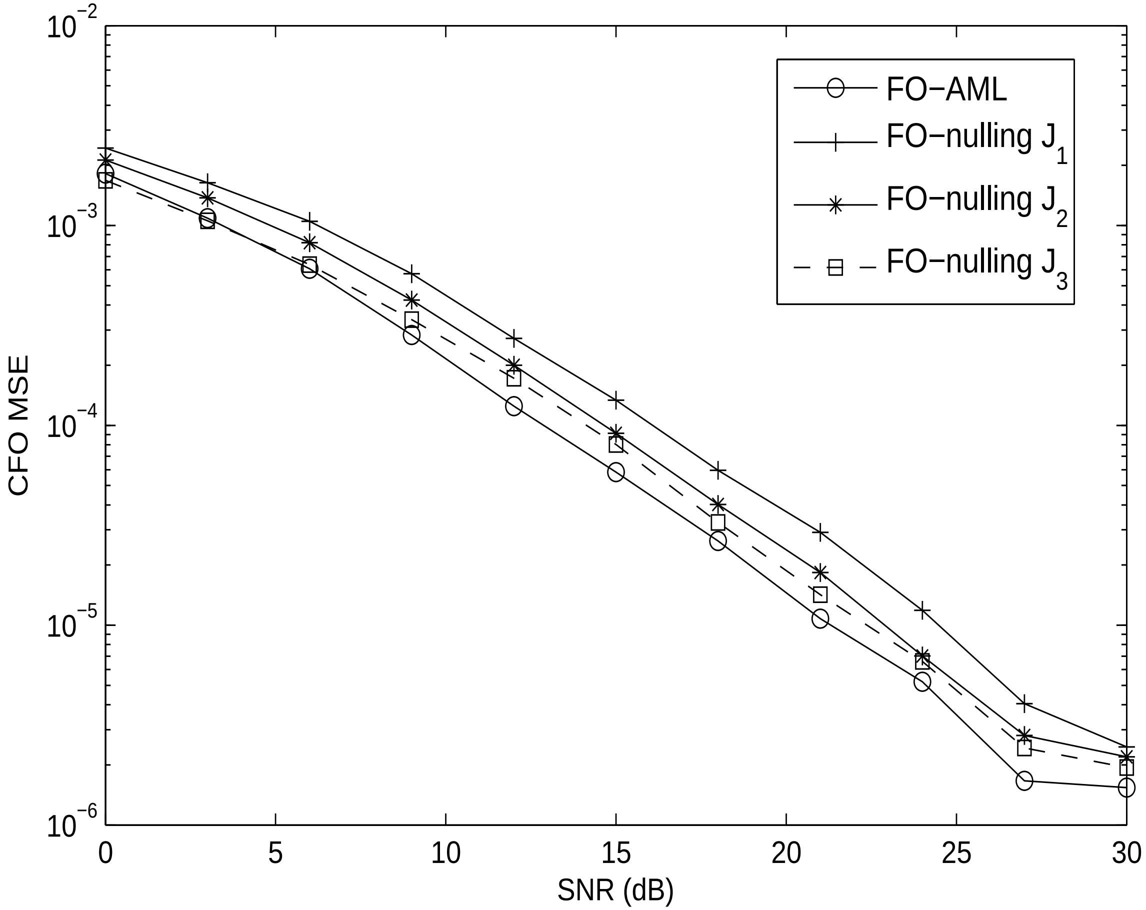

Figure 4: MSEs of CFO estimation versus SNR for AML and nulling based method with random $\mathbf{J}$

Simulation Results

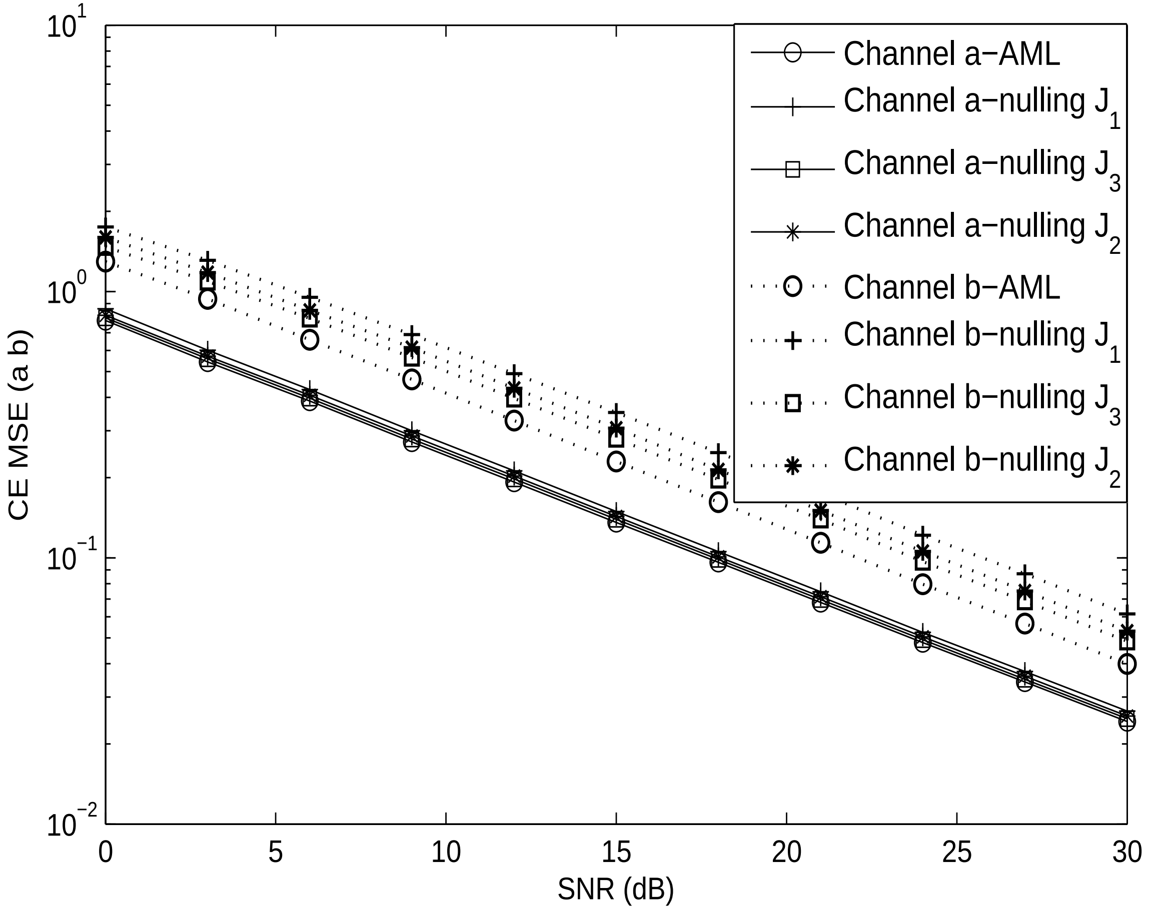

Figure 5: MSEs of channel estimation versus SNR for AML and nulling based method with random $\mathbf{J}$

Conclusions

- Formulate the signal model for two-way relay networks with frequency synchronization errors in a frequency selective environment.
- Develop two joint CFO and channel estimations methods, i.e., AML and nulling-based methods.
- Find CRB of CFO for each method and compare performance of the two methods.
- Here, relay only acts as a repeater. If relay down converts the received signal, the problem will become more complex and interesting.
- Our future work – WCNC and ICC 2010.

Questions and discussion