Network-Coded Cooperative MIMO with Outdated CSI and CCI

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***Abstract*—We study the effect of outdated channel state information (CSI) and co-channel interference (CCI) on the performance of relay selection (RS) network-coded cooperative (NCC) multiple-input multiple-output (MIMO) systems. Specifically, we consider a RS MIMO-NCC system where N single-antenna sources communicate with one multiple-antenna destination using M decode-and-forward (DF) multiple-antenna relays. The destination selects K best relays according to the quality of relay-destination channels. The selected relays apply network coding (NC) on the received sources' symbols using network code coefficients based on maximum distance separable (MDS) codes. The exact closed-form outage probability (OP) of the system is derived. The asymptotic high signal-to-noise ratio (SNR) OP is also obtained, through which the diversity order and the coding gain are found. Numerical results are further presented to illustrate the adverse effect of outdated CSI and CCI on the system performance and to validate the accuracy of our analysis.**

***Keywords*—Network-coded cooperation, outdated CSI, co-channel interference, outage probability, diversity order.**

I. Introduction

A. Background and Related Works

Cooperative communication (CC) is a well established technique to combat the multi-path induced fading inherent to the wireless channel [1]. CC systems, however, suffer from spectral inefficiency, since each relay requires multiple resource blocks when transmitting different sources' packets. Network-coded cooperation (NCC), a new family of CC, has been proposed to reduce the number of relay transmissions in multi-source cooperative networks [2]. Unlike conventional CC where the relays simply decode and forward sources' packets one by one, in NCC, each relay performs network coding (NC) [3] on the decoded data packets received from multiple sources and then forwards the coded version to the destination. Thus, the number of orthogonal resource blocks for the relaying does not increase with the number of sources; thereby NCC improves the energy efficiency and increases the system throughput. Relay selection (RS) based NCC can further improve the spectral efficiency of CC systems [4]–[8].

B. Contributions of This Paper

The main contributions of this paper compared to current NCC literature are threefold. First, many prior works on NCC build upon the assumption that the direct links between the sources and the destination are available [4]–[8]. This might not be a realistic assumption, in particular, when the sources are far from the destination and the direct links experience heavy path-loss and shadowing. This paper studies the outage performance and the diversity order of RS NCC in the absence of direct source-destination links. Second, so far, only one paper investigated the impact of outdated channel state information (CSI) on the performance of single-antenna RS NCC [7]. But this work has not been extended to RS multiple-input multiple-output (MIMO) NCC. Third, the performance of NCC subjected to co-channel interference (CCI) is not available. However, because of the aggressive frequency reuse, CCI is an important constraint for fifth generation (5G) and future wireless networks. Therefore, it is of both theoretical and practical interest to study the impact of outdated CSI and CCI on the performance of RS MIMO-NCC. The objective of this work is to remedy this gap.

In particular, we consider a dual-hop cooperative network that consists of $N > 1$ single-antenna sources, $M \geq 1$ decode-and-forward (DF) multiple-antenna relays and a single multiple-antenna destination. The destination selects K best relays that maximize the signal-to-noise ratio (SNR) of relay-destination channels. The selected relays apply NC on the received sources' symbols using network code coefficients based on maximum distance separable (MDS) codes. In our system setup, the relays use one transmit antenna to forward encoded signals to the destination. On the other hand, both the destination and relays employ selection combining (SC) for signal reception. For this system, we derive the exact outage probability (OP) in closed-form. To obtain further insights into the system-design parameters, the asymptotic high-SNR OP is also derived, through which the diversity order and the coding gain are quantified. Valuable insights and guidelines are provided to help the design of practical RS MIMO-NCC.

The rest of this paper is as follows: Section II explains the system and channel models. The exact OP and asymptotic analyses are presented in Section III. Numerical results are given in Section IV. Finally, we conclude in Section V.

II. System and Channel Models

Let us consider a dual-hop multi-source multi-relay cooperative network where N single-antenna sources $\mathcal{S} = \{S_n\}_{n=1}^{N}$ communicate with the destination D, equipped with $N_d \geq 1$ antennas, with the help of M DF relays $\mathcal{R} = \{R_m\}_{m=1}^{M}$. This setup may apply to the uplink cellular system where a group of single-antenna mobile users communicate with a multiple-antenna base station using multiple-antenna relays. Each relay has $N_r \geq 1$ receive antennas and uses only one antenna for transmission. We assume the direct links from the sources to the destination are not reliable and the sources' packets are transmitted only through the relays. This can happen due to propagation impairments such as shadowing and path-loss. The channels are assumed to follow a flat Rayleigh fading model. Let $\mathbf{h}_{S_n R_m} \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{h}_{R_m D} \in \mathbb{C}^{N_d \times 1}$, respectively, denote the single-input multiple-output (SIMO) channel vectors for the source-relay and relay-destination links whose elements are modeled as $\sim \mathcal{CN}(0,1)$. The j-th and the ℓ-th elements of $\mathbf{h}_{S_nR_m}$ and $\mathbf{h}_{R_mD}$ are denoted by $h^{(j,1)}_{S_nR_m}$ and $h^{(\ell,1)}_{R_mD}$. Further, the channels include independent additive white Gaussian noise (AWGN) terms with mean zero and variance one. We assume that the number of CCI signals impairing the relays and the destination are I_1 and I_2, respectively, and that the received interference signals at the relays and destination have identical average energy.

The transmission of the sources and the relays occurs in non-overlapping time-slots and a complete round of cooperation takes place in two phases.

A. First Phase: Source-Relay Transmission

In the first phase, the sources transmit their messages to the relays in N orthogonal time-slots. The relays employ SC to exploit receiver diversity. In particular, the best receiver antenna providing the maximum SNR between source S_n and relay R_m is selected for data reception. At the same time, relay R_m receives I_1 CCI signals. The instantaneous signal-to-interference plus noise ratio (SINR) for $S_n \to R_m$ $(\forall n, m)$ link can then be written as

$$\gamma_{nm} = \frac{\gamma^*_{nm}}{1+\gamma_{I_1}}. \tag{1}$$

In (1) $\gamma_{I_1} = \sum_{i=1}^{I_1} \mu_1 |g_{m,i}|^2$ where μ_1 is the interference transmit SNR and $g_{m,i}$ is the channel coefficient of the i^{th} interference at R_m. Also, $\gamma^*_{nm} = \bar{\gamma}|h^*_{nm}|^2$ where $\bar{\gamma}$ is the transmit SNR and $|h^*_{nm}|$ is given by

$$|h^*_{nm}| = \max_{1\leqslant j \leqslant N_r} \{|h^{(j,1)}_{S_nR_m}|\}. \tag{2}$$

At the end of the first phase, the relays which successfully decode all N sources' packets, send a flag packet to the destination, indicating that they are ready for cooperation. Let $\mathcal{A}$ denote the set of decodable relays with the cardinality of l. Mathematically speaking, this can be written as

$$\mathcal{A} \triangleq \{R_m \in \mathcal{R} : \gamma_{nm} > \gamma_{th},\ \forall n\}, \tag{3}$$

where γ_{th} is the predefined SINR threshold.

It is clear that the number of relays in $\mathcal{A}$, l, is upper bounded by the total number of available relays M, i.e., $l \leq M$. Note that $\mathcal{A}$ is a random set and l is thus a random variable.

B. Second Phase: Relay-Destination Transmission

The second phase lasts for K time-slots. At the beginning of each time-slot a relay in $\mathcal{A}$ which maximizes the SNR of relay-destination channels is selected for transmission. This procedure continues until K relays transmit. In particular, the selected relay R_{m^*} linearly combines the received sources' packets using a non-binary q-ary Galois field NC based on MDS codes [2]. The resulting network-coded packet is then forwarded to the destination by a single antenna at R_{m^*}. At the same time, the received signal at the destination is impaired by I_2 CCI signals. Since the destination employs SC, the SNR of relay-destination channel at selection instant t is given by

$$\hat{\gamma}_{m^*} = \bar{\gamma}|\hat{h}_{m^*}| = \bar{\gamma} \max_{R_m \in \mathcal{A}} \left\{ \max_{1\leqslant \ell \leqslant N_d} \{|h^{(\ell,1)}_{R_mD}|\} \right\}, \tag{4}$$

which may differ from the actual SNR $\gamma_{m^*} = \bar{\gamma}|h_{m^*}|^2$ during transmission time $t+\tau$ due to feedback delay. $\hat{h}_{m^*}$ and h_{m^*} are joint complex Gaussian distributions with correlation coefficients $0 \leqslant \rho \leqslant 1$. When $\rho = 1$, then the channels are perfectly correlated and RS is based on the perfect CSI. On the other hand, when $\rho = 0$, the channels are perfectly uncorrelated and RS is equivalent to random selection of relays from decoding set $\mathcal{A}$.

The SINR of $R_{m^*} \to D$ link can be expressed as

$$\gamma_m = \frac{\gamma_{m^*}}{1+\gamma_{I_2}}, \tag{5}$$

where $\gamma_{I_2} = \sum_{i=1}^{I_2} \mu_2 |g_i|^2$, μ_2 is the interference transmit SNR, and g_i is the channel coefficient of the i^{th} CCI at D.

III. Performance Analysis

A. CDF of Intermediate Links

To evaluate overall RS MIMO-NCC outage, we must compute the cumulative distribution function (CDF) of the SINR in the first hop ($S \to R$ links) and in the second hop ($R \to D$ links).

Lemma 1. The CDF of γ_{nm} (1) and γ_m (5) are, respectively, given by

$$F_{\gamma_{nm}}(\gamma) = \sum_{k=0}^{N_r} \frac{\binom{N_r}{k}(-1)^k}{\mu_1^{I_1}} \left(\frac{\bar{\gamma}\mu_1}{\bar{\gamma}+\gamma k \mu_1}\right)^{I_1} e^{-\frac{k\gamma}{\bar{\gamma}}}, \tag{6}$$

$$F_{\gamma_m}(\gamma) = 1 - \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{1+k} \left(\frac{\bar{\gamma}\big(1+(1-\rho)k\big)}{(1+k)\mu_2\gamma + \bar{\gamma}\big(1+(1-\rho)k\big)}\right)^{I_2} e^{-\frac{(1+k)\gamma}{(1+(1-\rho)k)\bar{\gamma}}}. \tag{7}$$

Proof. We first proceed to determine the CDF of (1). The CDF of γ^*_{nm} in (1) is given by $F_{\gamma^*_{nm}}(\gamma) = (1-e^{-\frac{\gamma}{\bar{\gamma}}})^{N_r}$. Applying binomial expansion, we have

$$F_{\gamma^*_{nm}}(\gamma) = \sum_{k=0}^{N_r} \binom{N_r}{k} (-1)^k e^{-\frac{k\gamma}{\bar{\gamma}}}. \tag{8}$$

On the other hand, the probability density function (PDF) of γ_{I_1} in (1) is given by

$$f_{\gamma_{I_1}}(y) = \frac{y^{I_1-1}}{\Gamma(I_1)\mu_1^{I_1}} e^{-\frac{y}{\mu_1}}, \tag{9}$$

where $\Gamma(\cdot)$ is the Gamma function.
The CDF of γ_{nm} can then be obtained as

$$F_{\gamma_{nm}}(\gamma) = \int_0^\infty F_{\gamma^*_{nm}}\big((1+y)\gamma\big) f_{\gamma_{I_1}}(y) dy. \tag{10}$$

Inserting (8) and (9) into (10), we have

$$F_{\gamma_{nm}}(\gamma) = \sum_{k=0}^{N_r} \frac{\binom{N_r}{k}(-1)^k}{\Gamma(I_1)\mu_1^{I_1}} e^{-\frac{\gamma k}{\bar{\gamma}}} \int_0^\infty e^{-\left(\frac{\gamma k}{\bar{\gamma}}+\frac{1}{\mu_1}\right)y} y^{I_1-1} dy. \tag{11}$$

Using $\int_0^\infty y^{\upsilon-1} e^{-\varphi y} dy = \varphi^{-\upsilon}\Gamma(\upsilon)$ [9], $F_{\gamma_{nm}}(\gamma)$ can be derived as (6).

Furthermore, the PDF of γ_{m^*} in (5) can be obtained by taking the average of the conditional PDF $f_{\gamma_{m^*}|\hat{\gamma}_{m^*}}(\gamma|\hat{\gamma})$ over the PDF of $\hat{\gamma}_{m^*}$. This can be written as

$$f_{\gamma_{m^*}}(\gamma) = \int_0^\infty f_{\gamma_{m^*}|\hat{\gamma}_{m^*}}(\gamma|\hat{\gamma}) f_{\hat{\gamma}_{m^*}}(\hat{\gamma}) d\hat{\gamma}. \tag{12}$$

The conditional PDF is given by [10]

$$f_{\gamma_{m^*}|\hat{\gamma}_{m^*}}(\gamma|\hat{\gamma}) = \frac{1}{(1-\rho)\bar{\gamma}} e^{-\frac{\rho\hat{\gamma}+\gamma}{(1-\rho)\bar{\gamma}}} \mathcal{I}_0\left(\frac{2\sqrt{\rho\gamma\hat{\gamma}}}{(1-\rho)\bar{\gamma}}\right), \tag{13}$$

where $\mathcal{I}_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. Furthermore, the CDF of $\hat{\gamma}_{m^*}$ is given by $F_{\hat{\gamma}_{m^*}}(\hat{\gamma}) = (1 - e^{-\frac{\hat{\gamma}}{\bar{\gamma}}})^{lN_d}$. Thus, the PDF of $\hat{\gamma}_{m^*}$ can be written as

$$f_{\hat{\gamma}_{m^*}}(\hat{\gamma}) = \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{\bar{\gamma}} e^{-\frac{(1+k)\hat{\gamma}}{\bar{\gamma}}}. \tag{14}$$

Substituting (13) and (14) into (12), we have

$$\begin{aligned} f_{\gamma_{m^*}}(\gamma) = &\sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{(1-\rho)\bar{\gamma}^2} e^{-\frac{\gamma}{(1-\rho)\bar{\gamma}}} \\ &\times \int_0^\infty e^{-\upsilon\hat{\gamma}} \mathcal{I}_0(2\sqrt{\varphi\hat{\gamma}}) d\hat{\gamma}, \end{aligned} \tag{15}$$

where $\upsilon = \frac{1+(1-\rho)k}{(1-\rho)\bar{\gamma}}$ and $\varphi = \frac{\rho\gamma}{(1-\rho)^2\bar{\gamma}^2}$. Finally, solving the integral by using $\int_0^\infty e^{-\upsilon x}\mathcal{I}_0(2\sqrt{\varphi x})dx = \frac{1}{\upsilon}e^{\frac{\varphi}{\upsilon}}$ [9], $f_{\gamma_{m^*}}(\gamma)$ can be derived as

$$f_{\gamma_{m^*}}(\gamma) = \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{(1+(1-\rho)k)\bar{\gamma}} e^{-\frac{(1+k)\gamma}{(1+(1-\rho)k)\bar{\gamma}}}. \tag{16}$$

From (16), the CDF of γ_{m^*} can be obtained as

$$F_{\gamma_{m^*}}(\gamma) = 1 - \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{1+k} e^{-\frac{(1+k)\gamma}{(1+(1-\rho)k)\bar{\gamma}}}. \tag{17}$$

Therefore, $F_{\gamma_m}(\gamma)$ can be formulated as

$$\begin{aligned} F_{\gamma_m}(\gamma) = 1 &- \frac{lN_d}{\Gamma(I_2)\mu_2^{I_2}} \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k}{1+k} \\ \times e^{-\frac{(1+k)\gamma}{(1+(1-\rho)k)\bar{\gamma}}} &\int_0^\infty y^{I_2-1} e^{-\frac{((1+k)\mu_2\gamma+\bar{\gamma}(1+(1-\rho)k))y}{\bar{\gamma}(1+(1-\rho)k)\mu_2}} dy. \end{aligned} \tag{18}$$

Solving the integral in (18), one can obtain (7). This completes the proof. □

B. Overall Outage Probability

The probability that l relays succeed to recover all sources' messages can be written as

$$\Pr\{|\mathcal{A}| = l\} = \binom{M}{l} \mathsf{P}_s^l (1-\mathsf{P}_s)^{M-l}, \tag{19}$$

where $\mathsf{P}_s = \left(1 - F_{\gamma_{nm}}(\gamma_{th})\right)^N$.

Also, the probability that ζ relays (out of K selected relays) are not in outage given that $|\mathcal{A}| = l$ is computed as

$$\Pr\{|\mathcal{E}| = \zeta|l\} = \binom{K}{\zeta}\left(F_{\gamma_m}(\gamma_{th})\right)^{K-\zeta}\left(1 - F_{\gamma_m}(\gamma_{th})\right)^{\zeta}. \tag{20}$$

In NCC, the destination solves the linear equations transmitted by the selected K relays to recover the original N sources' packets. Thus, at least N successful transmissions are required. Since direct source-destination links are not available, the number of selected relays K must be at least equal to the number of sources N i.e., $N \leqslant K$. An outage occurs if fewer than N network-coded packets are received by the destination. The overall OP can then be obtained using the law of total probability and is given by (21).

$$\mathcal{P}_{\text{out}} = \sum_{l=0}^{M}\sum_{\zeta=0}^{N-1} \Pr\{|\mathcal{E}| = \zeta|l\}\Pr\{|\mathcal{A}| = l\}. \tag{21}$$

C. Asymptotic Analysis

Although the OP expression in (21) is exact, direct insights into the effect of the feedback delays and CCI on the system performance are desirable. In this subsection, we thus derive the asymptotic outage expression.

Theorem 1. The asymptotic high-SNR OP for perfect CSI ($\rho = 1$) is given by

$$\mathcal{P}_{\text{out}}^{\infty} \overset{\bar{\gamma}\to\infty}{\approx} (\mathcal{C}_1.\bar{\gamma})^{-G_{d_1}}, \tag{22}$$

where diversity order G_{d_1} and the coding gain $\mathcal{C}_1$ are, respectively, given by

$$G_{d_1} = M\min\{N_r, (K-N+1)N_d\}, \tag{23}$$

$$\mathcal{C}_1 = \begin{cases} \frac{\varpi_1^{(1)^{-\frac{1}{MN_r}}}}{\gamma_{th}} & K-N+1 > \frac{N_r}{N_d} \\ \frac{\varpi_1^{(2)^{-\frac{1}{M(K-N+1)N_d}}}}{\gamma_{th}} & K-N+1 < \frac{N_r}{N_d} \\ \frac{\varpi_1^{(3)^{-\frac{1}{MN_r}}}}{\gamma_{th}} & K-N+1 = \frac{N_r}{N_d} \end{cases} \tag{24}$$

in which $\varpi_1^{(1)} = (N\mathcal{H}_1(N_r))^M$, $\varpi_1^{(2)} = \binom{K}{N-1}(\mathcal{H}_2(MN_d))^{K-N+1}$, $\varpi_1^{(3)} = (N\mathcal{H}_1(N_r))^M + \sum_{l=1}^{M}\binom{M}{l}(N\mathcal{H}_1(N_r))^{M-l}\binom{K}{N-1}(\mathcal{H}_2(lN_d))^{K-N+1}$, and

$$\mathcal{H}_i(a) = \sum_{k=0}^{a} \frac{\binom{a}{k}\mu_i^k\Gamma(I_i+k)}{\Gamma(I_i)}. \tag{25}$$

Further, for outdated CSI ($\rho \neq 1$), $\mathcal{P}_{\text{out}}^{\infty}$ can be derived as

$$\mathcal{P}_{\text{out}}^{\infty} \overset{\bar{\gamma}\to\infty}{\approx} (\mathcal{C}_2.\bar{\gamma})^{-G_{d_2}}, \tag{26}$$

where

$$G_{d_2} = \min\{MN_r, K-N+1\}, \tag{27}$$

$$\mathcal{C}_2 = \begin{cases} \frac{\varpi_2^{(1)^{-\frac{1}{MN_r}}}}{\gamma_{th}} & K-N+1 > MN_r \\ \frac{\varpi_2^{(2)^{-\frac{1}{K-N+1}}}}{\gamma_{th}} & K-N+1 < MN_r \\ \frac{\left(\varpi_2^{(1)}+\varpi_2^{(2)}\right)^{-\frac{1}{K-N+1}}}{\gamma_{th}} & K-N+1 = MN_r \end{cases} \tag{28}$$

with $\varpi_2^{(1)} = (N\mathcal{H}_1(N_r))^M$, $\varpi_2^{(2)} = \binom{K}{N-1}\mathcal{T}^{K-N+1}$, and

$$\mathcal{T} = \sum_{k=0}^{MN_d-1} \frac{MN_d\left(\Gamma(I_2) + \mu_2\Gamma(I_2+1)\right)\binom{MN_d-1}{k}(-1)^k}{\left(1+(1-\rho)k\right)\Gamma(I_2)}. \tag{29}$$

Proof. In high SNR regime i.e., $\bar{\gamma} \to \infty$, we have $F^{\infty}_{\gamma^*_{nm}}(\gamma_{th}) = (\gamma_{th}/\bar{\gamma})^{N_r}$. Then by substituting this expression and (9) into (10), we have

$$F^{\infty}_{\gamma_{nm}}(\gamma_{th}) = \int_0^{\infty} \frac{\gamma_{th}^{N_r}(1+y)^{N_r} y^{I_1-1}}{\bar{\gamma}^{N_r}\Gamma(I_1)\mu_1^{I_1}} e^{\frac{-y}{\mu_1}} dy. \tag{30}$$

Finally, by performing binomial expansion and solving the integral, we obtain

$$F^{\infty}_{\gamma_{nm}}(\gamma_{th}) = \sum_{k=0}^{N_r} \frac{\binom{N_r}{k}\mu_1^k \Gamma(I_1+k)}{\Gamma(I_1)} \left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{N_r}. \tag{31}$$

Similarly, the asymptotic expression for $F_{\gamma_m}(\gamma_{th})$ when $\rho = 1$ can be derived as

$$F^{\infty}_{\gamma_m}(\gamma_{th}) = \sum_{k=0}^{lN_d} \frac{\binom{lN_d}{k}\mu_2^k \Gamma(I_2+k)}{\Gamma(I_2)} \left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^{lN_d}. \tag{32}$$

on the other hand, when $\rho \neq 1$, $F^{\infty}_{\gamma_{m^*}}(\gamma_{th})$ can be well approximated as

$$F^{\infty}_{\gamma_{m^*}}(\gamma_{th}) = \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k lN_d}{1+(1-\rho)k} \left(\frac{\gamma_{th}}{\bar{\gamma}}\right). \tag{33}$$

Using (33), we obtain

$$\begin{aligned} F^{\infty}_{\gamma_m}(\gamma_{th}) &= lN_d\left(\Gamma(I_2) + \mu_2\Gamma(I_2+1)\right) \\ &\times \sum_{k=0}^{lN_d-1} \frac{\binom{lN_d-1}{k}(-1)^k}{(1+(1-\rho)k)\Gamma(I_2)} \left(\frac{\gamma_{th}}{\bar{\gamma}}\right). \end{aligned} \tag{34}$$

Plugging these expressions in (21) and considering the dominant terms, one can obtain (22) and (26). □

D. Remarks and Guidelines

The following remarks can be drawn from (23) and (24):

Remark 1. The maximum achievable diversity is given by (23). It is either equal to MN_r or $M(K-N+1)N_d$. If $K-N+1 > N_r/N_d$, then the diversity is determined by $G_{d_1} = MN_r$ which is a function of M and N_r and is independent of other system parameters N, K, N_d. This implies that adding more antennas at the destination N_d, and selecting more relays K not only do not change the diversity but also increase the complexity and decrease the system throughput. On the other hand, if $K-N+1 < N_r/N_d$, then the diversity is given by $G_{d_1} = M(K-N+1)N_d$ which is a function of all system parameters except the number of antennas at relays N_r. Interesting but counter intuitively, here increasing the number of sources N decreases the diversity.

Remark 2. The optimal number of selected relays that maximizes the achievable diversity is a function of N_r, N_d, and N and is equal to (35). It can be seen that K_{opt_1} is inversely proportional to N_d. Therefore, by adding more antennas at the destination (i.e., increasing the complexity), the number of relays to be selected for achieving the maximum diversity can be decreased (i.e., reducing the relay transmissions). This clearly shows a trade-off between the system complexity and system throughput.

$$K_{\text{opt}_1} = \left\lceil \frac{N_r}{N_d} + N - 1 \right\rceil. \tag{35}$$

Remark 3. In our proposed system model, the relay uses only one transmit antenna for data transmission. Transmit antenna selection (TAS), which maximizes the relay-destination SNR, can also be employed at the relays. If TAS is used, then the diversity in (23) changes to

$$G_d^{\text{TAS}} = \min\{MN_r, M(K-N+1)N_rN_d\} = MN_r. \tag{36}$$

Thus, TAS improves the diversity only when $K-N+1 < N_r/N_d$. However, when $K-N+1 > N_r/N_d$, the system with TAS provides coding gain without diversity advantages.

Remark 4. Although CCI does not impact diversity order (for fixed interference powers), it degrades the coding gain. When $K-N+1 > N_r/N_d$, the system parameters of the first hop (i.e., N_r, I_1, μ_1) impact the coding gain. When $K-N+1 < N_r/N_d$, the coding gain is determined by N_d, I_2 and μ_2. For $K-N+1 = N_r/N_d$, the coding gain is affected by the system parameters associated with both hops.

Remark 5. For single-antenna NCC i.e., when $N_d = N_r = 1$, the diversity in (23) is reduced to $G_{d_1} = M$. Thus, the proposed RS strategy increases the diversity from $M-N+1$ (earlier reported in [2]) to M. Based on (35), the optimal number of relays to be selected is $K_{\text{opt}_1} = N$.

On the other hand, from (27) and (28), we have the following remarks:

Remark 6. Outdated CSI degrades diversity order from G_{d_1} (23) to G_{d_2} (27). Thus, if ρ is not equal to one, the diversity is independent of number of antennas at the destination. Further, if TAS is used at the relays, the diversity does not change and is equal to (27), meaning that in the case of outdated CSI, TAS does not provide any diversity advantages. Also, the optimal number of selected relays that maximizes the diversity is

$$K_{\text{opt}_2} = MN_r + N - 1. \tag{37}$$

Remark 7. The coding gain is determined by the system parameters in the first hop, second hop, and both hops when $K-N+1 > MN_r$, $K-N+1 < MN_r$, and $K-N+1 = MN_r$, respectively.

IV. Numerical Results and Discussion

In this section, we present Monte-Carlo simulations to validate the derived analytical expressions. Unless otherwise stated, we assume $N = 5$, $I_1 = 2$, $I_2 = 3$, $\mu_1 = 0$ dB, $\mu_2 = 0$ dB and $\gamma_{th} = 0$ dB.

Fig. 1 depicts the outage (21) and asymptotic curves (22), (26) for different values of ρ and K when $M = 4$, $N_r = 3$, and $N_d = 2$. It can be readily checked that when $\rho = 1$ the optimal number of relays to be selected is $K_{\text{opt}_1} = 6$ (35). Therefore, as K increases from five to six the diversity increases from $G_{d_1} = M(K-N+1)N_d = 8$ to its maximum value $G_{d_1} = MN_r = 12$. However, the diversity for $K = 7$ is identical to that of $K_{\text{opt}_1} = 6$. This implies that selecting more than K_{opt_1} relays does not increase the diversity, confirming the statements in Remark 2. Also, the slope of the asymptotic curves reveals that for $\rho = 0.8$ the diversity significantly reduces to $G_{d_2} = K-N+1$ (27) and is equal to one, two, and three for $K = 5$, 6, and 7.

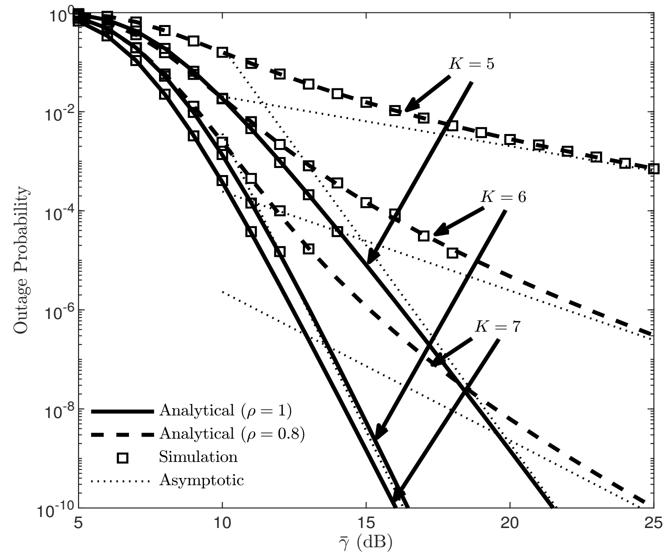

Figure 1. OP versus $\bar{\gamma}$ for different values of ρ and K when $M = 4$, $N_r = 3$, and $N_d = 2$.

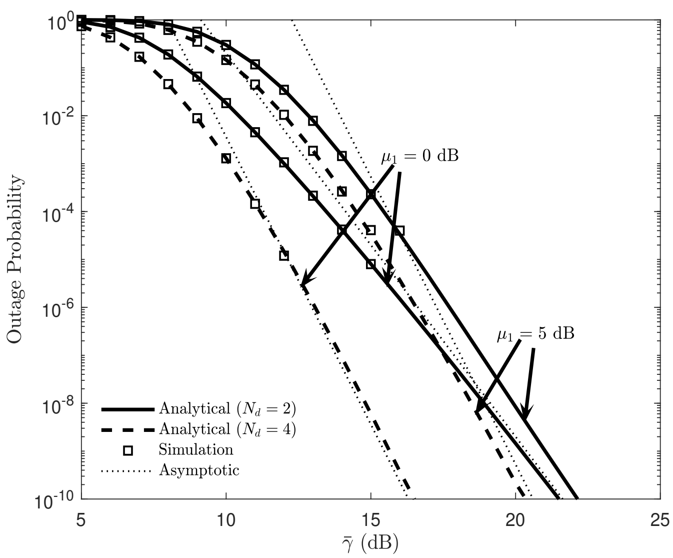

Figure 2. OP versus $\bar{\gamma}$ for different values of N_d and μ_1 when $M = 4$, $K = 5$, $N_r = 3$, and $\rho = 1$.

Fig. 2 illustrates the effect of CCI on the OP of RS MIMO-NCC, assuming different values of N_d and μ_1 when $M = 4$, $K = 5$, $N_r = 3$, and $\rho = 1$. As can be seen, CCI degrades the coding gain, rather than the diversity. Also, when $N_d = 2$ (which satisfies $K - N + 1 < N_r/N_d$), the value of μ_1 does not change the system performance as $\bar{\gamma} \to \infty$ and the curve corresponding to $\mu_1 = 5$ dB converges to that of $\mu_1 = 0$ dB, confirming Remark 4.

Fig. 3 compares the outage performance of RS MIMO-NCC with TAS and without TAS (SIMO) at the relays when $M = 3$, $K = 6$, $N_d = 2$, $N_r = 3, 5$, and $\rho = 1$. It is observed that TAS improves the diversity from $G_{d_1} = M(K - N + 1)N_d = 12$ (23) to $G_d^{\mathrm{TAS}} = MN_r = 15$ (36) when $N_r = 5$ ($K - N + 1 < N_r/N_d$). When $N_r = 3$ ($K - N + 1 > N_r/N_d$), however, TAS provides the diversity of nine which is identical to that

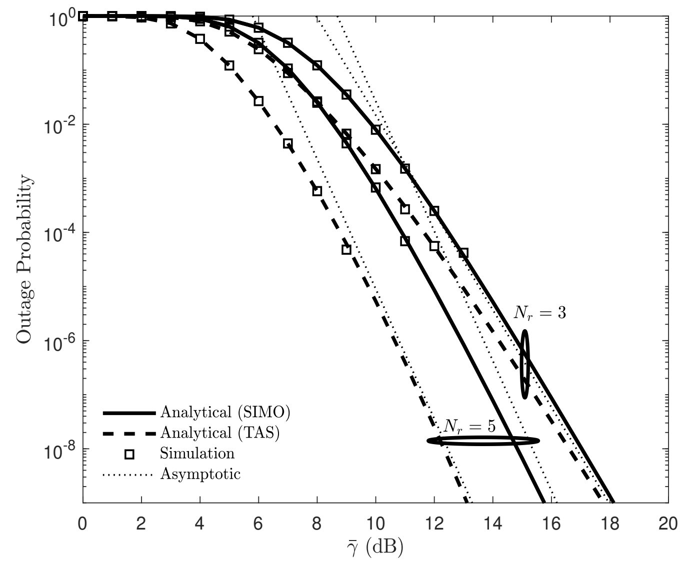

Figure 3. Comparison between SIMO/SC and TAS/SC when $M = 3$, $K = 6$, $N_d = 2$, $N_r = 3$, 5, and $\rho = 1$.

of without TAS. This confirms the statements in Remark 3.

V. Conclusions

We proposed a RS MIMO-NCC which provides most of the MIMO benefits while employing only one transmit/receive chain at the relays and destination. Our analysis revealed that RS MIMO-NCC incurs substantial performance losses with outdated CSI and CCI. These had not been analyzed before. Several design guidelines for practical RS MIMO-NCC systems were provided. A future research area is to study the performance of MIMO-NCC with other antenna strategies.

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