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Opportunistic Scheduling in Network-Coded Cooperative Systems

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***Abstract*—In this paper, we study the performance of opportunistic source selection (OSS) in multisource multirelay network-coded cooperative (NCC) systems. For this system, we derive the exact outage probability and asymptotic measures such as the diversity and coding gains. The derived analytical results provide an efficient means to evaluate the impact of different system parameters on the system performance. Our results reveal that the proposed NCC system greatly exploits the diversity gains in multisource multirelay NCC systems and thus provides a considerable performance improvement. From the derived closed-form diversity gains, we also evaluate the optimal number of selected relays that maximizes the achievable diversity gain. Numerical results are also presented to validate the theoretical analysis.**

***Keywords*—Network-coded cooperative (NCC) systems, opportunistic source selection (OSS), relay selection (RS), outage probability, diversity gain, coding gain.**

I. INTRODUCTION

Demands for data services are increasing rapidly. Billions of devices are connected and/or added to global wireless networks. Many of these devices require high data rates to support various applications. It is predicted that the number of devices connected to the Internet grows over 50 billion devices by 2020 of which more than 10 billion are mobile devices. These connected devices will transform the quality of our lives. Smart homes, smart cities, automated transportation, water distribution, environmental monitoring, and urban security are only few examples of the transformative power of wireless networks that improve the quality, efficiency, and safety of our lives.

Two of promising wireless technologies are (i) relay-aided cooperative communication (CC) systems [1] and (ii) network coding (NC) systems [2]. CC systems increase coverage and energy efficiency in wireless networks. Thus, cooperative relaying has been adopted in 3GPP LTE-Advanced, IEEE 802.16j, and IEEE 802.16m mobile standards [3]. On the other hand, NC increases data speed, and provides security and robustness for the data transmission. Due to these reasons, the combination of CC and NC systems referred to as network-coded cooperative (NCC) systems has been proposed [4]–[6]. It has been demonstrated that NCC provides an effective means to improve spectral and power efficiency of CC systems. This improvement is achieved by ensuring that the relays do not utilize orthogonal channels for multiple sources; instead each relay linearly combines the received messages from multiple sources, and then forwards them to the destination. This linear combining remarkably reduces the delay and increases the throughput [7].

NCC, in general, can be classified into two categories namely, repetition-based NCC and relay selection (RS) based NCC. In what follows, we briefly overview these schemes and provide readers with a glimpse of their ideas.

In repetition-based (or classical) NCC all the sources and relays transmit their information to the destination. In particular, in the first phase, the sources transmit their messages to the relays and the destination. In the second phase, the relays decode the data received from the sources, linearly combine the sources' messages using NC, and then transmit them to the destination. If the weighting coefficients at relays are those of a maximum distance separable (MDS) code (hence achieving the Singleton bound), an N-source M-relay network achieves the full diversity gain of $d = M + 1$ [8], [9].

Compared to the repetition-based scheme, the RS-based schemes relax the constraint that all relays need to transmit in orthogonal time-slots. Thus, RS-based NCC improves spectral efficiency. Few recent works have elucidated the concept of RS in NCC [10]–[12]. The results in [10], [11] demonstrate that RS-based NCC achieves the full diversity for specific system parameters. In particular, the full diversity gain of $d = M + 1$ is only achieved if the number of selected relays L is at least equal to the number of sources (i.e., $N \leq L$); otherwise, the diversity gain is limited to the number of selected relays and is equal to $d = L + 1$. The results in [10], [11] have also been extended to generalized RS (GRS) where any arbitrary subset of relays can be selected [12].

In multisource wireless networks, the channel between each source and the destination experiences independent variations due to the fading. This mutual independence can be viewed as a form of diversity gain that can be realized through opportunistic source selection (OSS). Since OSS and RS individually exploit the spatial diversity of wireless channels, combining these two techniques in NCC can considerably improve the system performance. The impact of OSS on NCC, however, has not been thoroughly analyzed and is not fully understood.

This void has motivated our present work. In particular, we propose and analyze an OSS scheme in a multisource multirelay NCC system. For the system under consider-

TABLE I
LIST OF MAIN PARAMETERS

N	Number of sources
M	Number of relays
K	Number of selected sources
L	Number of selected relays
ℓ	Number of relays in the decoding set
R_0	Transmissions rate of each intermediate link
γ_{th}	Threshold SNR
ρ	Transmit power (Transmit SNR)

ation, we derive the exact outage probability and closed-form asymptotic measures such as the diversity and coding gains. The derived analytical results provide an efficient means to evaluate the impact of different system parameters on the system performance. The results reveal that the proposed NCC system greatly exploits the diversity gains in multisource multirelay NCC systems and thus provides a considerable performance improvement. From the derived closed-form diversity gains, we also evaluate the optimal number of selected relays that maximizes the achievable diversity gain. All analytical results are further supported by Monte-Carlo simulations.

Notations: In this paper, $\Pr(\cdot)$ and $\lceil\cdot\rceil$ denote probability and the ceiling function, respectively. Calligraphic letter $\mathcal{X}$ represents a set of elements, where $|\mathcal{X}|$ denotes its cardinality. $\mathbb{F}_q$ represents finite field with size q, $\bigoplus$ and $\bigotimes$ are addition and multiplication in $\mathbb{F}_q$, respectively. For the convenience of the reader, a list of main parameters is presented in Table I.

The remainder of this paper is organized as follows. In Section II, we describe the network model and channel model. The detailed analysis of the exact and asymptotic outage performance of the proposed NCC scheme is presented in Section III. Section IV provides numerical results. Finally, we conclude in Section V.

II. System Model and Preliminaries

A. System Description

This section describes the network model, channel model, and transmission model in detail.

Consider a generic multisource multirelay NCC network that consists of $N + M + 1$ terminals; a set of N sources $\mathcal{S} = \{S_n | n = 1, 2, ..., N\}$, a single destination, D, and a set of M decode-and-forward (DF) relays $\mathcal{R} = \{R_m | m = 1, 2, ..., M\}$ (Fig. 1). The considered system model is of practical importance (e.g., an uplink of cellular networks, where some active mobile users intend to communicate with the base station with the aid of some relay terminals), which is also considered in [8]–[12]. All nodes are assumed to use a single-antenna, operate in the half-duplex mode, and transmit with power ρ. A time-division multiple-access scheme is assumed such that the transmission of sources and relays occur in non-overlapping time-slots.

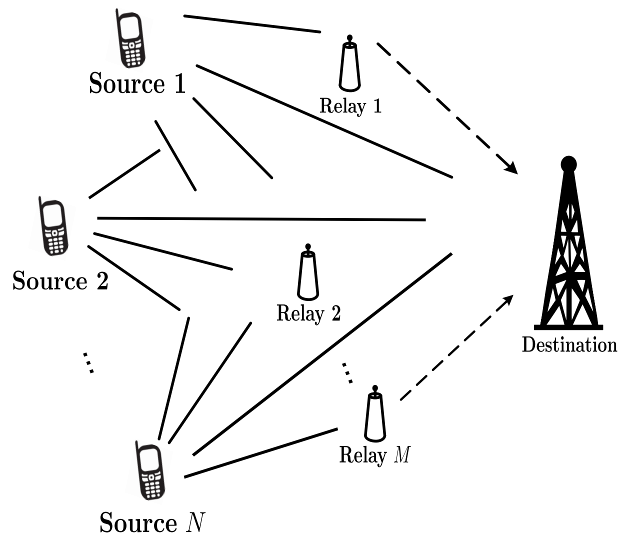

Fig. 1. System Model.

We assume that the channel pertaining to each link undergoes frequency-flat Rayleigh fading. Specifically, the channel coefficient of link $i \to j$ is modeled by $h_{ij} \sim \mathcal{CN}(0, 1)$; a circularly-symmetric complex Gaussian random variable (RV) with mean zero and variance one. Block-wise static fading channels are assumed, where the channel coefficients h_{ij}'s remain constant during a packet duration, and then vary independently from one block to another. Furthermore, the additive white Gaussian noise (AWGN) at each link is a independent and identically distributed (i.i.d.) RV and is modeled by $w_{ij} \sim \mathcal{CN}(0, 1)$.

We denote $\gamma_{ij} = \rho|h_{ij}|^2$ as the instantaneous signal-to-noise ratio (SNR) of $i \to j$ link which is a exponentially distributed RV. The cumulative distribution function (CDF) of γ_{ij} is thus given by

$$F_{\gamma_{ij}}(y) = 1 - e^{-\frac{y}{\rho}}. \tag{1}$$

The communication between the sources and the destination requires $K + L$ time-slots and is split into two phases; the broadcasting phase and the relaying phase. During the broadcasting phase, K sources in $\mathcal{S}$ are selected to transmit their messages to the destination, while the M relays also listen to the transmissions. During the relaying phase, L relay(s) are selected to apply NC on the K received packets and then transmit the network-coded packets to the destination. The detailed information of each phase is described as follows.

The broadcasting phase is performed over K time-slots, $\{T_k|, k = 1, 2, ..., K\}$. Specifically, at the beginning of each time-slot T_k, the best source $S_{(k)}$ with the maximum SNR of the source-to-destination links is selected to transmit its data symbol $\theta_{S_{(k)}} \in \mathbb{F}_q$ within T_k and it is then discarded from $\mathcal{S}$ at the end of T_k. This procedure continues until K distinct sources out of N sources are selected for data transmission.

The relaying phase lasts for L time-slots, $\{T_l|, l = 1, 2, ..., L\}$, in which L relay(s) are selected to participate in cooperation. Let $\mathcal{D}$ denote a set of relays that successfully decoded the sources' messages in the broadcasting phase. Mathematically, $\mathcal{D}$ can be written as

$$\mathcal{D}_{\subseteq\mathcal{R}} \triangleq \{R_m \in \mathcal{R} : \gamma_{S_{(k)}R_m} > \gamma_{th},\ \forall k\}. \tag{2}$$

At the beginning of time-slot T_l ($\forall l$) the destination selects the best relay $R_{(l)}$ in $\mathcal{D}$ that maximizes the SNR of the relay-to-destination channels. The selected relay $R_{(l)}$ then generates a linear combination of the K sources' symbols using network-code coefficients $\mu_{S_{(k)}R_{(l)}}$'s drawn from an MDS code as follows $\theta_{R_{(l)}} = \sum_{k=1}^{K} \bigoplus \left(\mu_{S_{(k)}R_{(l)}} \bigotimes \theta_{S_{(k)}}\right)$ and transmits $\theta_{R_{(l)}} \in \mathbb{F}_q$ to the destination.

In the next subsection, some definitions are provided for later use in this paper.

B. Definitions

Definition 1. In wireless slow fading channel, the outage probability is one of the most important performance metrics. Namely, when the instantaneous mutual information $\underset{i\to j}{\mathcal{I}}$ of the intermediate link $i \to j$ is below the transmission rate R_0, an outage event will occur. The instantaneous mutual information of link $i \to j$ can be expressed as

$$\underset{i\to j}{\mathcal{I}} = \log_2(1+\gamma_{ij}). \tag{3}$$

The outage probability of $i \to j$ link can be written as

$$\underset{i\to j}{\mathcal{O}} = \Pr\left(\log_2(1+\gamma_{ij}) < R_0\right). \tag{4}$$

Noting that the CDF of γ_{ij} is given by (1), $\underset{i\to j}{\mathcal{O}}$ in (4) is computed as

$$\underset{i\to j}{\mathcal{O}} = 1 - e^{-\frac{\gamma_{th}}{\rho}}, \tag{5}$$

where $\gamma_{th} = 2^{R_0} - 1$ is the threshold SNR.

Definition 2. The outage probability of the system in high SNRs is approximated as

$$\lim_{\rho\to\infty} \text{OP} \overset{\rho\to\infty}{\approx} (\mathcal{C}.\rho)^{-d}, \tag{6}$$

where $\mathcal{C}$ and d are referred to as the "*diversity gain*", and "*coding gain*", respectively. In particular, $\mathcal{C}$ represents the horizontal shift of the outage cure compared to the benchmark curve ρ^{-d} and d is defined as the slope of outage curve at high SNR regime and is expressed as

$$d = \lim_{\rho\to\infty} -\frac{\log(\text{OP}(\rho))}{\log(\rho)}. \tag{7}$$

III. Performance Analysis

In this section, we study the performance of the proposed NCC scheme in terms of the outage probability. In particular, we derive the exact system outage probability. The asymptotic outage expression is further derived to quantify the diversity and coding gains at high-SNR regime.

A. Exact Outage Probability

Here, we derive the outage probability of the system under consideration over Rayleigh fading channels.

The probability that ℓ relays succeed to recover K sources' messages can be written as

$$\Pr\{|\mathcal{D}| = \ell\} = \binom{M}{\ell} \Pr(\mathfrak{S})^{\ell}[1-\Pr(\mathfrak{S})]^{M-\ell}, \tag{8}$$

where

$$\Pr(\mathfrak{S}) = e^{-\frac{K\gamma_{th}}{\rho}}. \tag{9}$$

Denote $\mathcal{A}_{\subseteq\mathcal{S}} \triangleq \{S_{(k)} \in \mathcal{S} : \gamma_{S_{(k)}D} > \gamma_{th}\}$ and $\mathcal{E}_{\subseteq\mathcal{D}} \triangleq \{R_{(l)} \in \mathcal{D} : \gamma_{R_{(l)}D} > \gamma_{th}\}$ as the sets of selected sources and relays which are not in outage (i.e., they are *effective*). The probability that η sources (out of K selected sources) are effective can then be formulated as

$$\Pr\{|\mathcal{A}| = \eta\} = \sum_{\substack{1\le a_1<a_2<\ldots<a_\eta\le K \\ 1\le a_{\eta+1}<a_{\eta+2}<\ldots<a_K\le K}} \left\{ \prod_{k=a_1}^{a_\eta} [1 - \underset{S_{(k)}\to D}{\mathcal{O}}] \prod_{k'=a_{\eta+1}}^{a_K} \underset{S_{(k')}\to D}{\mathcal{O}} \right\}, \tag{10}$$

where $a_1 \neq a_2 \neq \cdots \neq a_K$ and

$$\underset{S_{(k)}\to D}{\mathcal{O}} = \left(1 - e^{-\frac{\gamma_{th}}{\rho}}\right)^{N-k+1}. \tag{11}$$

Furthermore, the probability that ζ relays (out of L selected relays) are effective given that $|\mathcal{D}| = \ell$ is given by

$$\Pr\{|\mathcal{E}| = \zeta|\ell\} = \binom{L}{\zeta} [1 - \underset{R_{(l)}\to D}{\mathcal{O}_\ell}]^{\zeta} \underset{R_{(l)}\to D}{\mathcal{O}_\ell}{}^{L-\zeta}, \tag{12}$$

where

$$\underset{R_{(l)}\to D}{\mathcal{O}_\ell} = \left(1 - e^{-\frac{\gamma_{th}}{\rho}}\right)^{\ell}. \tag{13}$$

In NCC system, the destination solves the linear equations transmitted by the selected sources and relays to recover the original sources' packets. Thus, at least K (out of $K + L$) successful transmissions are required. An outage occurs if fewer than K packets are received by the destination. Define $\Gamma_{S_{(k)}}$ as the received SNR of the selected source in time-slot T_k ($\forall k$) and $\Gamma_{R_{(l)}}$ as the received SNR of the selected relay in time-slot T_l ($\forall l$). The vector of the received SNRs at the destination during the broadcasting and relaying phases can be written as

$$\boldsymbol{\gamma} = \left\{\Gamma_{S_{(1)}}, ..., \Gamma_{S_{(K)}}, \Gamma_{R_{(1)}}, ..., \Gamma_{R_{(L)}}\right\}, \tag{14}$$

where $\Gamma_{S_{(k)}}$ is

$$\Gamma_{S_{(k)}} = \max_{S_n \in \mathcal{S} - \{S_{(1)}, S_{(2)}, ..., S_{(k-1)}\}} \gamma_{S_n D},\ \ \forall k \tag{15}$$

and $\Gamma_{R_{(l)}}$ is given by

$$\Gamma_{R_{(l)}} = \max_{R_m \in \mathcal{D}} \gamma_{R_m D},\ \ \forall l \tag{16}$$

Then, the outage probability of the system can be formulated as

$$\mathsf{OP} = \mathsf{Pr}(\text{at most } K-1 \text{ elements of } \boldsymbol{\gamma} \geq \gamma_{th}) = \mathsf{Pr}\left(\bigcup_{v=0}^{K-1} v \text{ elements of } \boldsymbol{\gamma} \geq \gamma_{th}\right). \quad (17)$$

In other words, the system is in outage if $|\mathcal{A}| + |\mathcal{E}| \leqslant K-1$. Thus, the overall outage probability conditioned on ℓ can be expressed as

$$\mathsf{OP}_\ell = \mathsf{Pr}\{\eta + \zeta|\ell \leqslant K-1\}. \quad (18)$$

Using (10) and (12), OP_ℓ for $K > L$ can be written as

$$\mathsf{OP}_\ell^{(1)} = \sum_{\zeta=0}^{L}\left(\mathsf{Pr}\{|\mathcal{E}| = \zeta|\ell\}\sum_{\eta=0}^{K-\zeta-1}\mathsf{Pr}\{|\mathcal{A}| = \eta\}\right). \quad (19)$$

On the other hand, OP_ℓ for $K \leq L$ can be computed as

$$\mathsf{OP}_\ell^{(2)} = \sum_{\zeta=0}^{K-1}\left(\mathsf{Pr}\{|\mathcal{E}| = \zeta|\ell\}\sum_{\eta=0}^{K-\zeta-1}\mathsf{Pr}\{|\mathcal{A}| = \eta\}\right). \quad (20)$$

The overall outage probability can then be obtained by taking average of the conditional outage probabilities $\mathsf{OP}_\ell^{(v)}$ $(v = 1, 2)$ over all possible size of the decoding set. Thus, the outage probability of the proposed NCC system can be obtained as

$$\mathsf{OP}^{(v)} = \sum_{\ell=0}^{M}\mathsf{OP}_\ell^{(v)}\mathsf{Pr}\{|\mathcal{D}| = \ell\}, \ (v = 1, 2) \quad (21)$$

where $\mathsf{Pr}\{|\mathcal{D}| = \ell\}$ is given by (8).

B. Asymptotic Analysis

In the following, we analyze the outage probability at high-SNR regime $(\rho \to \infty)$ to quantify the diversity and coding gains of the proposed NCC system.

The high-SNR approximation of (10) is given by

$$\lim_{\rho\to\infty}\mathsf{Pr}\{|\mathcal{A}| = \eta\} = f(K, \eta, \underset{S_{(K)}\to D}{\mathcal{O}^\infty}), \quad (22)$$

where

$$f(\delta, \beta, z_k) \triangleq \sum_{\substack{i_1, i_2, \ldots, i_{\delta-\beta} \\ i_1 \neq i_2 \neq \ldots \neq i_{\delta-\beta}}}^{\delta}\prod_{k=i_1}^{i_{\delta-\beta}} z_k, \quad (23)$$

and

$$\underset{S_{(K)}\to D}{\mathcal{O}^\infty} = \left(\frac{\gamma_{th}}{\rho}\right)^{N-k+1}. \quad (24)$$

In (23), we use the following shorthand notation

$$\sum_{\substack{i_1, i_2, \ldots, i_k \\ i_1 \neq i_2 \neq \ldots \neq i_k}}^{\delta}\prod_{k'=i_1}^{i_k} z_{k'} = \sum_{i_1=1}^{\delta-k+1}\cdots\sum_{i_k=i_{k-1}+1}^{\delta}(z_{i_1}\cdots z_{i_k}). \quad (25)$$

In addition, (8) and (12) are approximated as

$$\lim_{\rho\to\infty}\mathsf{Pr}\{|\mathcal{D}| = \ell\} = \binom{M}{\ell}[\mathrm{Pr}^\infty(\bar{\mathfrak{S}})]^{M-\ell}, \quad (26)$$

and

$$\lim_{\rho\to\infty}\mathsf{Pr}\{|\mathcal{E}| = \zeta|\ell\} = \binom{L}{\zeta}\underset{R_{(l)}\to D}{\mathcal{O}^\infty}{}^{L-\zeta}, \quad (27)$$

where $\mathrm{Pr}^\infty(\bar{\mathfrak{S}}_m)$ and $\underset{R_{(l)}\to D}{\mathcal{O}^\infty}$ in (26) and (27) are respectively given by

$$\underset{R_{(l)}\to D}{\mathcal{O}^\infty} = \left(\frac{\gamma_{th}}{\rho}\right)^\ell, \quad (28)$$

and

$$\mathrm{Pr}^\infty(\bar{\mathfrak{S}}_m) = \frac{K\gamma_{th}}{\rho}. \quad (29)$$

Inserting (22), (26), and (27) in (21) and then keeping the dominant terms, the asymptotic outage probability for $K > L$ can be derived as (30) and (31).

$$\lim_{\rho\to\infty}\mathrm{OP}_a^{(1)} = \left(\frac{\gamma_{th}}{\rho}\right)^{(L+1)\left(N-K+\frac{L}{2}+1\right)}, \quad (30)$$

and

$$\lim_{\rho\to\infty}\mathrm{OP}_b^{(1)} = \varpi^{(1)}\left(\frac{\gamma_{th}}{\rho}\right)^{N-K+M+1}. \quad (31)$$

In (31), $\varpi^{(1)}$ is given by

$$\varpi^{(1)} = \begin{cases} \sum_{\ell=0}^{M}\binom{M}{\ell}K^\ell, & \textbf{\textit{if}} \quad L = 1 \\ K^M, & \textbf{\textit{if}} \quad L \neq 1 \end{cases} \quad (32)$$

Recalling the definition of diversity gain in (7), the diversity of the proposed NCC scheme for $K > L$ can be derived as

$$d^{(1)} = \begin{cases} (L+1)\left(N-K+\frac{L}{2}+1\right), \\ \quad \textbf{\textit{if}} \quad M > L\left(N-K+\frac{L+3}{2}\right) \\ N-K+M+1, \\ \quad \textbf{\textit{if}} \quad M \leq L\left(N-K+\frac{L+3}{2}\right) \end{cases} \quad (33)$$

Furthermore, based on (6) the coding gains can be obtained as

$$\mathcal{C}^{(1)} = \begin{cases} \frac{1}{\gamma_{th}}, & \textbf{\textit{if}} \ M > L\left(N-K+\frac{L+3}{2}\right) \\ \frac{\varpi^{(1)^{-\frac{1}{N-K+M+1}}}}{\gamma_{th}}, & \textbf{\textit{if}} \ M < L\left(N-K+\frac{L+3}{2}\right) \\ \frac{\left(1+\varpi^{(1)}\right)^{-\frac{1}{N-K+M+1}}}{\gamma_{th}}, & \\ & \textbf{\textit{if}} \ M = L\left(N-K+\frac{L+3}{2}\right) \end{cases} \quad (34)$$

Using the same procedure above, the asymptotic outage expression for $K \leq L$, can be obtained and is equal to

$$\lim_{\rho\to\infty}\mathrm{OP}^{(2)} = \varpi^{(2)}\left(\frac{\gamma_{th}}{\rho}\right)^{N-K+M+1}, \quad (35)$$

where $\varpi^{(2)} = K^M$.

Invoking (6) and (7) the diversity and coding gains for $K \leq L$ are respectively given by

$$d^{(2)} = N - K + M + 1, \tag{36}$$

$$\mathcal{C}^{(2)} = \frac{\varpi^{(2)^{-\frac{1}{N-K+M+1}}}}{\gamma_{th}}. \tag{37}$$

C. Remarks and Guidelines

The diversity gain of the proposed NCC system is highly dependent on the system parameters. In the following, we provide some remarks and guidelines based on our diversity analyses.

Remark 1. The diversity gain is always a function of the number of sources N and the number of selected sources K. It increases with N and decreases with K.

Remark 2. The impact of N and K on the the diversity is scaled by the factor of $L+1$ when $K > L$ and $M > L\left(N - K + \frac{L+3}{2}\right)$. Therefore, increasing N by Δ, improves the diversity gain by a factor of $(L+1)\Delta$ which significantly improves the system performance.

Remark 3. Interestingly but counter intuitively, the diversity is independent of the number of selected relays L when $M < L\left(N - K + \frac{L+3}{2}\right)$. On the other hand, the number of relays M does not change the diversity gain when $M > L\left(N - K + \frac{L+3}{2}\right)$. This contradicts our intuition that selecting more relays or increasing the number of relays improves the diversity gain.

Remark 4. The optimal number of selected relays that maximizes the achievable diversity gain is a function of all the system parameters (i.e., N, K, and M) and is equal to (38). Therefore, the number of selected relays can improve the diversity gain if the condition $L \leq L^*$ is satisfied; otherwise, selecting more than L^* relays (i.e., $L > L^*$) does not change the diversity.

$$L^* = \left\lceil K - N - \frac{3}{2} + \sqrt{\left(N - K + \frac{3}{2}\right)^2 + 2M} \right\rceil. \tag{38}$$

Remark 5. The diversity gain and the coding gain of $K > L$ when $M < L\left(N - K + \frac{L+3}{2}\right)$ and $L \neq 1$ are identical to that of $K \leq L$.

IV. Numerical Results and Discussions

In this section, numerical results are presented to support the theoretical analysis.

Fig. 2 illustrates the outage probability versus ρ for $N = 2$, 4, 6, $M = 10$, $K = 2$, and $L = 1$. These system parameters satisfy the condition $K > L$ and $M > L\left(N - K + \frac{L+3}{2}\right)$ in (33). It can be seen that analytical curves are the same as the Monte-Carlo simulations, which confirm the accuracy of our derivations. Furthermore, We observe that the slope of the curves is identical to that of asymptotic lines and is equal to $(L+1)\left(N - K + \frac{L}{2} + 1\right)$. In particular, the diversity gain for $N = 2$ is equal to 3. As the number of sources increases form $N = 2$ to $N = 4$ and $N = 6$, the diversity increases to 7 and 11, respectively. This indicates that the diversity gain increases by a factor of $(L+1)\Delta = 4$.

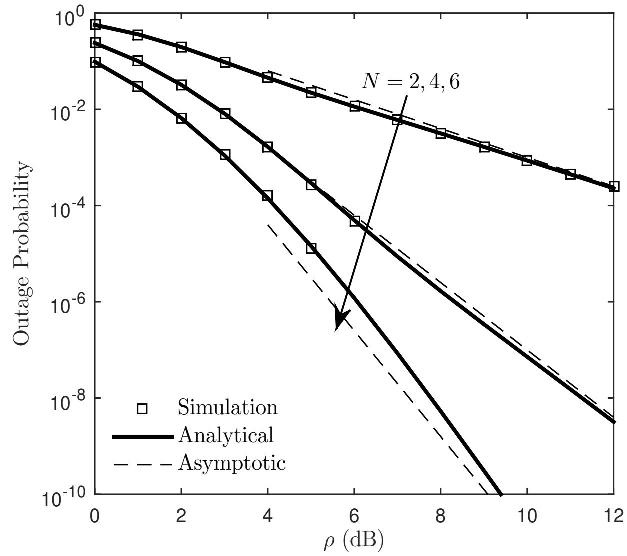

Fig. 2. Outage probability versus ρ for $N = 2$, 4, 6, $M = 10$, $K = 2$, and $L = 1$.

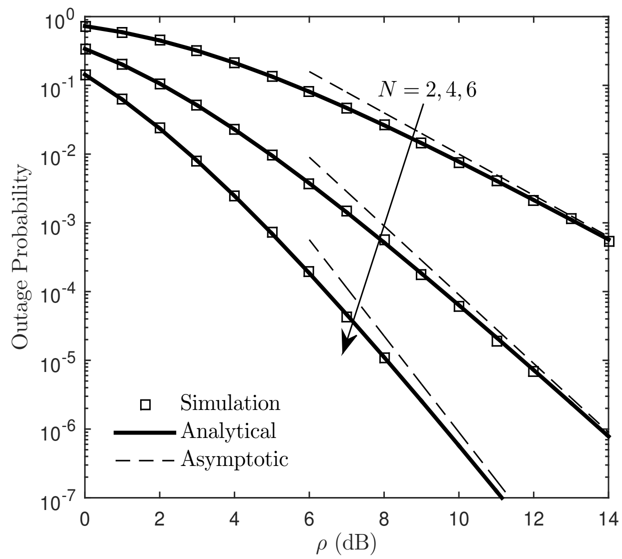

Fig. 3. Outage probability versus ρ for $N = 2$, 4, 6, $M = 2$, $K = 2$, and $L = 1$.

In Fig. 3, we plot the outage probability, assuming $N = 2$, 4, 6, $M = 2$, $K = 2$, and $L = 1$. This set of system parameters satisfies the condition $M \leq L\left(N - K + \frac{L+3}{2}\right)$ in (33). It is observed that the diversity gain of the system is equal to 3, 5, 7 for $N = 2$, 4, 6, respectively. Therefore, we conclude that the diversity gain at high SNR regime is determined by $N - K + M + 1$.

Fig. 4 shows the outage performance for $N = 4$,

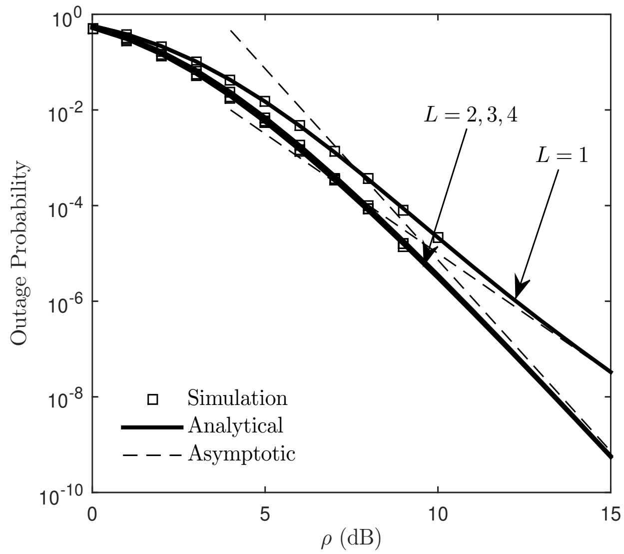

Fig. 4. Outage probability versus ρ for $N = 4$, $M = 6$, $K = 3$, and $L = 1, 2, 3, 4$.

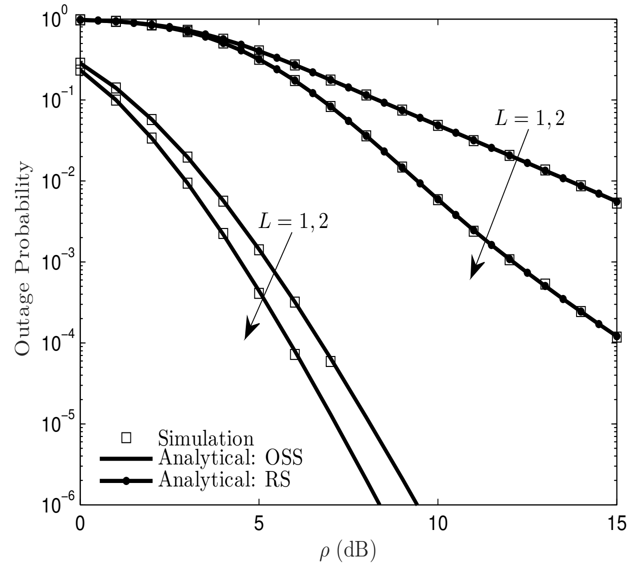

Fig. 5. Outage probability versus ρ for $N = 4$, $M = 6$, $K = 2$, and $L = 1, 2$.

$M = 6$, $K = 3$, and different values of number of selected relays. From (38), we can easily check that the optimal number of selected relays that maximizes the diversity gain is equal to $L^* = 2$. Therefore, as L increases from 1 to 2 the diversity increases from 5 to its maximum value 8. However, when $L = 3$, 4 the diversity gain is identical to that of $L^* = 2$ and the outage performance is almost the same as that of $L^* = 2$ in all SNR regime. Therefore, selecting more than L^* relays does not provide further improvement over the configuration with L^*.

In Fig. 5, we compare the performance of the proposed NCC system with that of RS-based NCC [10], [11] when $N = 4$, $M = 6$, $K = 2$, and $L = 1, 2$. We observe that the proposed NCC system archives the diversity gain of 7 and 9 when $L = 1$ and 2, while, RS-based NCC achieves the diversity gain of 2 and 3, only. This superiority in diversity gain leads to an impressive improvement in the outage performance. In particular, for the outage probability of 10^{-4}, RS-based NCC requires an SNR=15 dB for $L = 2$. However, NCC with OSS requires SNR=6 dB, indicating an SNR gain of 9 dB.

V. Conclusion

In this paper, we proposed and analyzed OSS in multisource multirelay NCC systems. The exact outage probability expression was presented and then validated by simulations. The asymptotic expressions were also presented which clearly indicate the diversity gain and the coding gain. The results revealed that OSS significantly improves the outage performance of NCC systems. Furthermore, based on our diversity analysis, the optimal number of selected relays that maximizes the diversity was evaluated. We also compared the outage performance of the proposed NCC scheme with that of RS-based NCC and demonstrated that our proposed NCC scheme yields a superior performance compared to RS-based NCC.

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