Multiuser Diversity in Network-Coded Cooperation: Outage and Diversity Analysis

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***Abstract*— In this letter, we study multiuser diversity (MUD) in network-coded cooperation (NCC) systems. Specifically, we propose a two-step user-relay selection mechanism that selects subsets of users and relays to exploit both MUD and cooperative diversity in a multiuser multirelay NCC system. Assuming independent but not necessarily identically distributed Rayleigh fading channels, we derive closed-form expressions for the system outage probability. We further obtain asymptotic outage expressions to quantify the diversity order as a function of different system parameters. Our results demonstrate the superiority (diversity gains) of the proposed MUD-based NCC compared with NCC schemes without MUD. Monte-Carlo simulations are also presented to verify the accuracy of our theoretical findings.**

***Index Terms*— Multiuser diversity, relay selection, network-coded cooperation, outage probability, diversity order.**

I. Introduction

COOPERATIVE diversity (CD) has emerged as a promising technique to combat channel fading by exploiting spatial diversity due to multiple relays. In conventional cooperative communication (CC) systems, each relay simply repeats the message it receives from the source. Such repetitive relaying, however, is inefficient in terms of time and/or spectrum resources, especially in multi-source wireless networks.

This critical problem of CC is overcome by applying network coding (NC), resulting in network-coded cooperation (NCC) [1]–[9]. Instead of individually re-transmitting the sources' messages, each NCC relay forms a weighted sum of packets received from multiple sources and forwards the resulting sum signal to the destination in a single resource block (a time-slot). If the relays perform linear combinations over a large enough finite field, NCC provides full diversity order with a significantly improved spectral efficiency.

In a multiuser network, an inherent form of diversity — multiuser diversity (MUD) — against channel fading exists, offering potentially large performance improvements. MUD arises because, statistically, among a large number of independently-fading users, a subset of them will have good channels at any time. By exclusively serving this subset, MUD can be exploited to maximize the long-term throughput. MUD can complement CD, and both can be exploited in CC systems [10], [11].

However, the application of MUD to NCC is still limited. Thus, we believe that this work is the first performance analysis of MUD-based NCC. In particular, we propose a two-step user-relay selection scheme for an N-user M-relay network. The relays adopt decode-and-forward (DF) protocol and apply NC using maximum distance separable (MDS) codes. We derive the outage probability of this system over independent but not necessarily identically distributed (i.n.i.d.) Rayleigh fading channels. The asymptotic outage expressions are further derived from which the diversity order of the system is quantified. We show that our scheme outperforms non-MUD NCC (NCC without user selection) in terms of the outage performance and achievable diversity order. We further demonstrate that our derived analytical expressions cover existing results as special cases.

Notations: $\Pr\{\cdot\}$ and $\mathcal{C}_a^b = \frac{b!}{(b-a)!a!}$ represent probability and binomial coefficient, respectively. $\mathbb{F}_q$ represents finite field with size q. $\boxplus$ and $\boxtimes$ denote addition and multiplication in $\mathbb{F}_q$, respectively.

II. System Model

Consider an uplink dual-hop multiuser multirelay NCC network wherein N users $\{u_n\}_{n=1}^N$ communicate with the base station (BS) d through the direct links and through M DF relays $\{r_m\}_{m=1}^M$. All nodes are equipped with a single antenna, transmit with power $\mathcal{P}$, and operate in a half-duplex time-division mode. The channel coefficient h_{ij} of link $i \to j$ is modeled as a complex Gaussian with mean zero and variance λ_{ij}. Also, the additive noise is a complex Gaussian with zero mean and variance N_0. The instantaneous received SNR can then be expressed as $\gamma_{ij} = \bar{\gamma}|h_{ij}|^2$, where $\bar{\gamma} = \mathcal{P}/N_0$ is the transmit SNR.

The whole transmission process consists of two phases. During the first phase, K (out of N) users with the highest SNRs of $u_n \to d$ links transmit their messages to the BS in K orthogonal time-slots. The M relays overhear the transmissions. Denote $\mathbf{x} \in \mathbb{F}_q^{K\times 1}$ as the $K \times 1$ vector of transmitted messages. The kth best user u_{k^*} emits $x_{k^*} \in \mathbb{F}_q$ (kth element of $\mathbf{x}$) in the kth time-slot, where

$$k^* = arg \max_{n=1,2,\ldots,N}^{k\text{th}} \gamma_{u_n d}, \quad 1 \le k \le K. \tag{1}$$

We define the decoding set $\mathcal{S}_r$ with cardinality of ℓ as the set of relays that successfully decode all K messages. Let L denote a predefined number of relays that we wish to select. If $0 \le \ell \le L$, then all the ℓ members of $\mathcal{S}_r$ participate. Hence, no relay selection (RS) is performed. This relaying phase thus lasts for ℓ orthogonal time-slots. In particular, at the $(K + m)$th time-slot, relay r_m $(m \in \mathcal{S}_r)$ transmits weighted sum of all K users' messages $x_m = \boxplus_{k^*=1}^K \beta_{mk^*} \boxtimes x_{k^*}$ to the BS, where $\beta_{mk^*} \in \mathbb{F}_q$ is a network code coefficient drawn from a sufficiently large field size. Thus, the network-coded messages are distinct and independent, each includes K messages. Here, one round of cooperation lasts for $K + \ell$ time-slots.

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On the other hand, if $L < \ell \leq M$, then L best relays out of ℓ relays in $\mathcal{S}_r$ are selected based on the channel quality of $r_m \rightarrow d$ channels. Here, the relaying phase takes place in L orthogonal time-slots. More precisely, at the $(K+l)$th time-slot, the lth best relay r_{l^*} transmits $x_{l^*} = \boxplus_{k^*=1}^{K} \beta_{l^*k^*} \boxtimes x_{k^*}$ to the BS, where

$$l^* = arg \max_{m\in\mathcal{S}_r}^{l\text{th}} \gamma_{r_m d}, \quad 1 \leq l \leq L. \tag{2}$$

This cooperative period thus takes place in $K+L$ time-slots.

III. Exact Outage Probability

In this section, we derive closed-form expressions for the outage probability of the proposed MUD-based NCC.

Let $\gamma_{1^*} \geq \gamma_{2^*} \geq \cdots \geq \gamma_{N^*}$ denote the order statistics of SNRs for $u_n \rightarrow d$ links in a decreasing order of magnitude, where $\gamma_{k^*} = \max_{n=1,2,\cdots,N}^{k\text{th}}\{\gamma_{u_n d}\}$ is the SNR of the kth best user u_{k^*}. Based on order statistics, we have

$$\begin{aligned}\mathsf{Pr}\{\gamma_{k^*} < y, \gamma_{k-1^*} > y\} = \mathsf{Pr}\{&(k-1 \text{ of } \gamma_{u_n d}\text{'s} > y)\\ &\cap (N-k+1 \text{ of } \gamma_{u_n d}\text{'s} < y)\}.\end{aligned} \tag{3}$$

Since $\gamma_{u_n d}$'s are mutually independent random variables, (3) can be expressed as

$$F_{\gamma_{k^*}}(y) = \sum_{i_1,i_2,\cdots,i_N}^{N} \left\{ \prod_{n=i_1}^{i_{k-1}} \left(1 - F_{\gamma_{u_n d}}(y)\right) \prod_{n'=i_k}^{i_N} F_{\gamma_{u_{n'} d}}(y) \right\}, \tag{4}$$

where $i_1, i_2, \ldots, i_N \in \{1, 2, \ldots, N\}$, $i_1 < i_2 < \ldots < i_{k-1}$, and $i_k < i_{k+1} < \ldots < i_N$. Also, $F_{\gamma_{ij}}(y) = 1 - e^{-\frac{y}{\bar{\gamma}_{ij}}}$, where $\bar{\gamma}_{ij} = \bar{\gamma}\lambda_{ij}$.

We can further re-write (4) in a more compact form as

$$F_{\gamma_{k^*}}(y) = \sum_{v=1}^{k} \mathcal{C}_{N-k+1}^{N-k+v} (-1)^{v-1} \underset{u_n\rightarrow d}{\mathcal{H}'}(N, k, v, y), \tag{5}$$

where $\underset{i\rightarrow j}{\mathcal{H}'}(\mu, \eta, \epsilon, z)$ is given by

$$\underset{i\rightarrow j}{\mathcal{H}'}(\mu, \eta, \epsilon, z) = \sum_{\substack{i_1,i_2,\cdots,i_{\mu-\eta+\epsilon}=1\\ i_1<i_2<\cdots<i_{\mu-\eta+\epsilon}}}^{\mu} \prod_{n=i_1}^{i_{\mu-\eta+\epsilon}} F_{\gamma_{ij}}(z). \tag{6}$$

In the case of $0 \leq \ell \leq L$, all ℓ relays in $\mathcal{S}_r$ forward network-coded symbols to the BS without using RS. The BS is capable of recovering the users' packets if it receives at least K correct packets either from K selected users or ℓ relays. The vector of all the SNRs can then be written as

$$\Gamma_{\mathcal{A}} = \{\gamma_{1^*}, \gamma_{2^*}, \ldots, \gamma_{K^*}, \gamma_{r_1 d}, \gamma_{r_2 d}, \ldots, \gamma_{r_\ell d}\}. \tag{7}$$

If less than K elements (out of $K+\ell$ elements) in (7) are above the threshold SNR γ_{th}, the system is in outage. Thus, the system outage probability can be expressed as

$$\begin{aligned}\mathcal{O}_{\mathcal{A}} = \mathsf{Pr}\{&0 \text{ links in } \Gamma_{\mathcal{A}} > \gamma_{th}\} + \mathsf{Pr}\{1 \text{ link in } \Gamma_{\mathcal{A}} > \gamma_{th}\}\\ &+ \ldots + \mathsf{Pr}\{K-1 \text{ links in } \Gamma_{\mathcal{A}} > \gamma_{th}\}.\end{aligned} \tag{8}$$

Based on (8), the outage probability for $K > L$ can be computed as

$$\mathcal{O}_{\mathcal{A}}^{(1)} = \sum_{m=0}^{\ell} \left(\mathcal{M}(\ell, m) \sum_{k=1}^{K-m} F_{\gamma_{k^*}}(\gamma_{th}) \right), \tag{9}$$

where $F_{\gamma_{k^*}}(y)$ is already given by (5) and $\mathcal{M}(\ell, m)$ is the probability that m (out of ℓ relays in $\mathcal{S}_r$) be operational (i.e., not in outage). $\mathcal{M}(\ell, m)$ can be derived as

$$\mathcal{M}(\ell, m) = \sum_{v=1}^{m+1} \mathcal{C}_{\ell-m}^{\ell-m-1+v} (-1)^{v-1} \underset{r_m\rightarrow d}{\mathcal{H}'}(\ell, m+1, v, \gamma_{th}). \tag{10}$$

Furthermore, the outage probability for $K \leq L$ can be derived as

$$\mathcal{O}_{\mathcal{A}}^{(2)} = \underbrace{\sum_{m=0}^{\ell} \left(\mathcal{M}(\ell, m) \sum_{k=1}^{K-m} F_{\gamma_{k^*}}(\gamma_{th}) \right)}_{\mathcal{O}_{\mathcal{A}}^{(2')}} + \underbrace{\sum_{m=0}^{K-1} \left(\mathcal{M}(\ell, m) \sum_{k=1}^{K-m} F_{\gamma_{k^*}}(\gamma_{th}) \right)}_{\mathcal{O}_{\mathcal{A}}^{(2'')}}. \tag{11}$$

On the other hand, if $L < \ell \leq M$, then RS is performed and L relays (out of ℓ relays in $\mathcal{S}_r$) are selected. Let $\gamma_{l^*|\ell} = \max_{m\in\mathcal{S}_r}^{l\text{th}}\{\gamma_{r_m d}\}$ and $\Gamma_{\mathcal{B}}$ denote the SNR of the lth best relay r_{l^*} conditioned on ℓ and the vector of all the SNRs, respectively. $\Gamma_{\mathcal{B}}$ can be written as

$$\Gamma_{\mathcal{B}} = \left\{\gamma_{1^*}, \gamma_{2^*}, \ldots, \gamma_{K^*}, \gamma_{1^*|\ell}, \gamma_{2^*|\ell}, \ldots, \gamma_{L^*|\ell}\right\}. \tag{12}$$

In (12), if K (out of $K+L$) elements are above γ_{th}, the users' messages can be jointly recovered; otherwise an outage occurs and the transmission fails. The outage probability can then be expressed as

$$\begin{aligned}\mathcal{O}_{\mathcal{B}} = \mathsf{Pr}\ \{&0 \text{ links in } \Gamma_{\mathcal{B}} > \gamma_{th}\} + \mathsf{Pr}\ \{1 \text{ link in } \Gamma_{\mathcal{B}} > \gamma_{th}\}\\ &+ \ldots + \mathsf{Pr}\{K-1 \text{ links in } \Gamma_{\mathcal{B}} > \gamma_{th}\}.\end{aligned} \tag{13}$$

For $K > L$, the outage probability is derived as

$$\mathcal{O}_{\mathcal{B}}^{(1)} = \sum_{k=1}^{K-L} F_{\gamma_{k^*}}(\gamma_{th}) + \sum_{k=1}^{L} \left(F_{\gamma_{K-k+1^*}}(\gamma_{th}) \sum_{l=1}^{k} F_{\gamma_{l^*|\ell}}(\gamma_{th}) \right). \tag{14}$$

On the other hand, if $K \leq L$; then the outage probability can be computed as

$$\mathcal{O}_{\mathcal{B}}^{(2)} = \sum_{k=1}^{K} \left(F_{\gamma_{K-k+1^*}}(\gamma_{th}) \sum_{l=1}^{k} F_{\gamma_{l^*|\ell}}(\gamma_{th}) \right), \tag{15}$$

where $F_{\gamma_{l^*|\ell}}(\gamma_{th})$ is given by

$$F_{\gamma_{l^*|\ell}}(\gamma_{th}) = \sum_{v=1}^{l} \mathcal{C}_{\ell-l+1}^{\ell-l+v} (-1)^{v-1} \underset{r_m\rightarrow d}{\mathcal{H}'}(\ell, l, v, \gamma_{th}). \tag{16}$$

Finally, applying the law of total probability, the overall outage probability of the system can be written as (17) and (18), shown at the top of the next page, where $\mathcal{B}(M, \ell)$ is the probability that ℓ relays succeed to decode K users' messages. This can be written as

$$\mathcal{B}(M, \ell) = \sum_{v=1}^{\ell+1} \mathcal{C}_{M-\ell}^{M-\ell-1+v} (-1)^{v-1} \underset{u_{k^*}\rightarrow r_m}{\mathcal{H}''}(M, \ell+1, v), \tag{19}$$

$$\mathcal{O}^{(1)} = \sum_{\ell=0}^{L} \mathcal{O}_{\mathcal{A}}^{(1)} \mathcal{B}(M,\ell) + \sum_{\ell=L+1}^{M} \mathcal{O}_{\mathcal{B}}^{(1)} \mathcal{B}(M,\ell) \quad if\ K > L \tag{17}$$

$$\mathcal{O}^{(2)} = \sum_{\ell=0}^{K-1} \mathcal{O}_{\mathcal{A}}^{(2')} \mathcal{B}(M,\ell) + \sum_{\ell=K}^{L} \mathcal{O}_{\mathcal{A}}^{(2'')} \mathcal{B}(M,\ell) + \sum_{\ell=L+1}^{M} \mathcal{O}_{\mathcal{B}}^{(2)} \mathcal{B}(M,\ell) \quad if\ K \leq L \tag{18}$$

$$\delta^{(2)} = \sum_{\ell=0}^{K-1} \left(\sum_{m=0}^{\ell} \hat{\mathcal{H}'}_{\underset{r_m \to d}{\infty}}(\ell, m+1) \hat{\mathcal{H}'}_{\underset{u_n \to d}{\infty}}(N, K-m) \right) \underset{u_{k^*} \to r_m}{\hat{\mathcal{H}}''_{\infty}}(M, \ell+1) + \sum_{\ell=K}^{L} \left(\sum_{m=0}^{K-1} \hat{\mathcal{H}'}_{\underset{r_m \to d}{\infty}}(\ell, m+1) \hat{\mathcal{H}'}_{\underset{u_n \to d}{\infty}}(N, K-m) \right)$$
$$\times \underset{u_{k^*} \to r_m}{\hat{\mathcal{H}}''_{\infty}}(M, \ell+1) + \sum_{\ell=L+1}^{M} \left(\sum_{k=1}^{K} \hat{\mathcal{H}'}_{\underset{u_n \to d}{\infty}}(N, K-k+1) \underset{r_m \to d}{\hat{\mathcal{H}}'_{\infty}}(\ell, k) \right) \underset{u_{k^*} \to r_m}{\hat{\mathcal{H}}''_{\infty}}(M, \ell+1). \tag{29}$$

where

$$\underset{u_{k^*} \to r_m}{\mathcal{H}''}(M, \ell+1, v) = \sum_{\substack{i_1, i_2, \cdots, i_{M-\ell-1+v}=1 \\ i_1 < i_2 < \cdots < i_{M-\ell-1+v}}}^{M} \prod_{m=i_1}^{i_{M-\ell-1+v}} \mathsf{Pr}\{w_m\}. \tag{20}$$

In (20), $\mathsf{Pr}\{w_m\} = 1 - \mathsf{Pr}\{\bar{w}_m\}$, where $\mathsf{Pr}\{\bar{w}_m\} = \prod_{k^*=1}^{K} \left(1 - F_{\gamma_{u_{k^*} r_m}}(\gamma_{th})\right)$ is the probability that relay r_m succeed to decode all K users' massages.

IV. Asymptotic Analysis

In this section, we derive the asymptotic outage probability expressions in high SNR regime to quantify the achievable diversity order of the proposed MUD-based NCC system. For high SNRs (when $\bar{\gamma} \to \infty$), we have $\lim_{\bar{\gamma} \to \infty} F_{\gamma_{ij}}(\gamma_{th}) = F_{\gamma_{ij\,\infty}}(\gamma_{th}) = \gamma_{th}/\bar{\gamma}_{ij}$. Substituting this expression into (5), and omitting higher-order terms, we obtain

$$\lim_{\bar{\gamma} \to \infty} F_{\gamma_{k^*}}(\gamma_{th}) = \underset{u_n \to d}{\mathcal{H}'_{\infty}}(N, k), \tag{21}$$

where $\underset{i \to j}{\mathcal{H}'_{\infty}}(\mu, \eta) = \lim_{\bar{\gamma} \to \infty} \underset{i \to j}{\mathcal{H}'}(\mu, \eta, \epsilon, z)$ and is given by

$$\underset{i \to j}{\mathcal{H}'_{\infty}}(\mu, \eta) = \sum_{\substack{i_1, i_2, \cdots, i_{\mu-\eta+1}=1 \\ i_1 < i_2 < \cdots < i_{\mu-\eta+1}}}^{\mu} \prod_{n=i_1}^{i_{\mu-\eta+1}} \left(\frac{\gamma_{th}}{\bar{\gamma}_{ij}} \right). \tag{22}$$

Also, $\mathcal{M}(\ell, m)$ and $F_{\gamma_{l^*|\ell}}(\gamma_{th})$ in (10) and (16) can be approximated as

$$\lim_{\bar{\gamma} \to \infty} \mathcal{M}(\ell, m) = \underset{r_m \to d}{\mathcal{H}'_{\infty}}(\ell, m+1), \tag{23}$$

and

$$\lim_{\bar{\gamma} \to \infty} F_{\gamma_{l^*|\ell}}(\gamma_{th}) = \underset{r_m \to d}{\mathcal{H}'_{\infty}}(\ell, l). \tag{24}$$

In addition, $\mathcal{B}(M, \ell)$ in (19) is simplified as

$$\lim_{\bar{\gamma} \to \infty} \mathcal{B}(M, \ell) = \underset{u_{k^*} \to r_m}{\mathcal{H}''_{\infty}}(M, \ell+1), \tag{25}$$

where

$$\underset{u_{k^*} \to r_m}{\mathcal{H}''_{\infty}}(M, \ell+1) = \sum_{\substack{i_1, i_2, \cdots, i_{M-\ell}=1 \\ i_1 < i_2 < \cdots < i_{M-\ell}}}^{M} \prod_{m=i_1}^{i_{M-\ell}} \left[\sum_{i_1=1}^{K} \left(\frac{\gamma_{th}}{\bar{\gamma}_{u_{i_1} r_m}} \right) \right]. \tag{26}$$

Finally, plugging (21), (23), (24) and (25) into (17) and then retaining the dominant term, the asymptotic outage probability expression for $K > L$ is derived as

$$\mathcal{O}_{\infty}^{(1)} = \delta^{(1)} \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{N-K+L+1} \quad if\ \ K > L \tag{27}$$

where $\delta^{(1)} = \hat{\mathcal{H}'}_{\underset{u_n \to d}{\infty}}(N, K-L)$ with $\hat{\mathcal{H}'}_{\underset{u_n \to d}{\infty}}(N, k) = (\gamma_{th}/\bar{\gamma})^{k-N-1} \underset{u_n \to d}{\mathcal{H}'_{\infty}}(N, k)$.

By repeating given steps, the asymptotic outage expression for $K \leq L$ can be derived as

$$\mathcal{O}_{\infty}^{(2)} = \delta^{(2)} \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{N-K+M+1} \quad if\ \ K \leq L \tag{28}$$

where $\delta^{(2)}$ is given by (29), shown at the top of the this page, and its corresponding parameters are given by $\hat{\mathcal{H}'}_{\underset{r_m \to d}{\infty}}(\ell, m+1) = (\gamma_{th}/\bar{\gamma})^{m-\ell} \underset{r_m \to d}{\mathcal{H}'_{\infty}}(\ell, m+1)$, $\hat{\mathcal{H}'}_{\underset{r_m \to d}{\infty}}(\ell, l) = (\gamma_{th}/\bar{\gamma})^{l-\ell-1} \underset{r_m \to d}{\mathcal{H}'_{\infty}}(\ell, l)$, and $\underset{u_{k^*} \to r_m}{\hat{\mathcal{H}}''_{\infty}}(M, \ell+1) = (\gamma_{th}/\bar{\gamma})^{\ell-M} \underset{u_{k^*} \to r_m}{\mathcal{H}''_{\infty}}(M, \ell+1)$.

Remark 1: The diversity order of the proposed MUD-based NCC is equal to $d^{(1)} = N - K + L + 1$ when $K > L$ and $d^{(2)} = N - K + M + 1$ when $K \leq L$. This indicates that MUD-based NCC is capable of exploiting both MUD and CD. The diversity order increases linearly with the number of users N and it linearly decreases as the number of selected users K increases.

Remark 2: The derived diversity order covers existing results as special cases. In particular, for $K = N$ and $L = M$, it reduces to the diversity order of "*non-MUD non-RS NCC*" (NCC without user-relay selection) [1], [2]. For $K = N$ and $L < M$, the diversity order coincides with that of "*non-MUD RS NCC*" [6], [7].[1]

V. Comparisons With Conventional Schemes

In non-MUD non-RS NCC, N users and M relays transmit their messages sequentially, and no user-relay selection is performed [1], [2]. In non-MUD RS NCC proposed in [6] and [7], the BS selects L best relays (out of M relays) based on the

[1] It can be shown that the outage performance of the RS based on local CSI is identical to that of RS based on "max-min" criterion [6], [7].

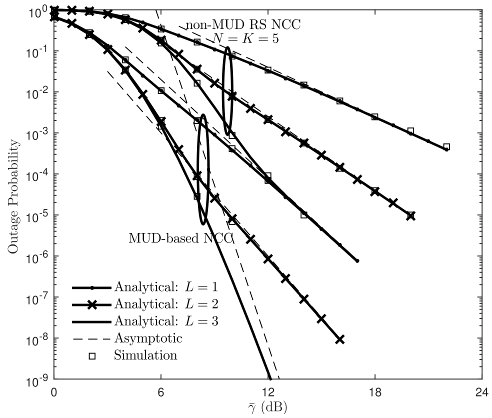

Fig. 1. Outage probability for $N = 5$, $K = 3, 5$, $M = 10$, $L = 1, 2, 3$.

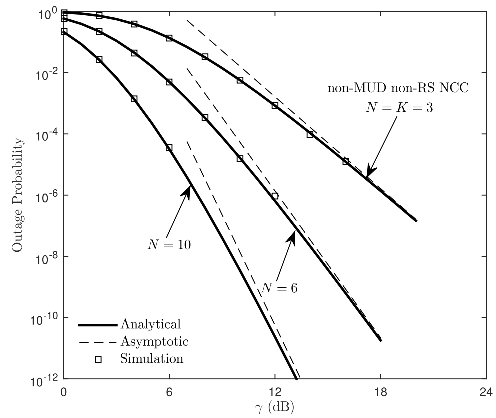

Fig. 2. Outage probability for $N = 3, 6, 10$, $K = 3$, $M = 4$, $L = 4$.

"max-min" criterion which requires channel state information (CSI) of all the user-to-relay and relay-to-BS channels. The CSI of the user-to-relay channels must be sent to the BS, which creates a significant RS signaling overhead. In contrast, in our proposed MUD-based NCC, the BS selects K (out of N) users based on the user-to-BS channels. On the other hand, when $\ell > L$ the RS is performed and L (out of ℓ) best relays are selected based on the local CSI of the relay-to-BS channels. Accordingly, the BS computes the channel quality of users and relays based on the SNRs of its already received signals.

VI. Numerical Results and Discussions

In this section, we provide numerical results to evaluate the performance of the proposed MUD-based NCC and corroborate the validity of our theoretical analysis. All numerical results are plotted by setting $\gamma_{th} = 0$ dB and assuming i.i.d. fading channels i.e., $\bar{\gamma}_{u_n d} = \bar{\gamma}_{u_n r_m} = \bar{\gamma}_{r_m d} = \bar{\gamma}$, $\forall n, m$.

Fig. 1 depicts the exact outage probability of the proposed NCC, where $N = 5$, $K = 3$, $M = 10$, and $L = 1, 2, 3$. We observe that the derived outage probability expressions (17), (18) match well with simulation results, validating our theoretical analysis. Also, the asymptotic outage curves (27), (28) perfectly predict the slope of the curves in high SNRs. We note that the slope of the curves at high SNRs is equal to 4, 5 for $L = 1, 2$ and it dramatically increases to 13 when $L = 3$. This indicates that the achievable diversity order of MUD-based NCC is determined by $d^{(1)} = N - K + L + 1$ when $K > L$, while it is $d^{(2)} = N - K + M + 1$ when $K \leq L$. Furthermore, the proposed NCC significantly outperforms non-MUD RS NCC. This is because the former exploits both MUD and CD gains, while the latter relies on CD only.

Fig. 2 compares the outage probability of MUD non-RS NCC $(N > K,\ L = M)$ against the benchmark non-MUD non-RS NCC $(N = K,\ L = M)$. We assume $N = 3, 6, 10$, $K = 3$, $M = 4$, and $L = 4$. The analytical curves are in excellent agreement with Monte-Carlo simulations. Moreover, the asymptotic curves touch the exact results in the medium-to-high SNR regime. We also observe that the outage performance of MUD-based NCC improves when the number of users N increases. For example, for an SNR of 12 dB, when the number of users increases from three to six, the outage probability decreases form 10^{-3} to 10^{-6}. This performance gain is dramatic for $N = 10$, where the outage is 10^{-11}. In addition, the slope of the asymptotic curves confirms that our proposed scheme exploits both MUD and CD and achieves the diversity order of 8 and 12 for $N = 6$ and $N = 10$, respectively. On the other hand, non-MUD non-RS NCC achieves a much lower diversity order of 5.

VII. Conclusions

In this letter, we proposed a MUD-based N-user M-relay NCC system. We derived closed-form outage probability expressions and quantified the diversity order as a function of different system parameters. Our results revealed that both MUD and CD are exploited in MUD-based NCC. Therefore, the proposed MUD-based NCC significantly outperforms conventional NCC, which relies on CD only.

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