Generalized User-Relay Selection in Network-Coded Cooperation Systems

Ali Reza Heidarpour†, Masoud Ardakani†, Chintha Tellambura†, and Marco Di Renzo‡
†Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, T6G 1H9, Canada
‡L2S department, CNRS, CentraleSupelec, University of Paris-Sud, 91192 Gif-sur-Yvette, France
Email: marco.direnzo@l2s.centralesupelec.fr, {alirezaheidarpour, ardakani, ct4}@ualberta.ca

***Abstract*—We study the performance of generalized user-relay selection (GURS) scheme in network-coded cooperation systems. In particular, we propose the most general case of user-relay selection mechanism that selects any arbitrary subsets of users and relays subject to any practical constraints such as load balancing conditions, scheduling policy, and other factors. Our results thus can be applied to a large set of situations and include all existing results in the literature as special cases. We develop performance characterizations of the system under consideration in terms of outage probability over non-identically and independently distributed (n.i.i.d.) Rayleigh fading channels. The asymptotic outage expressions at high signal-to-noise ratio (SNR) regime are further derived and then, based on the derived expressions, we quantify the diversity order. The theoretical derivations are validated through Monte-Carlo simulations.**

***Keywords*—Generalized user-relay selection (GURS), network-coded cooperation, outage probability, diversity order.**

I. Introduction

The next generation of cellular systems, known as 5G, is envisioned to provide high data rate, low latency, and improved energy efficiency. Multipath fading—as a result of constructive and destructive interferences of the received signals—is one of the major factors of the performance degradation in cellular wireless networks. A common approach to mitigate the detrimental effects of fading is the use of spatial diversity techniques. Cooperative diversity (CD) has been proposed as a potential candidate to exploit spatial diversity by forming a virtual antenna array among the spatially distributed relay nodes [1]. The deployment of relays in wireless networks thus offers potential performance improvements and has been adopted by several industry standards such as 3GPP long term evolution (LTE)-Advanced, IEEE 802.16j, IEEE 802.16m [2], and it is also an enabler technology for future 5G wireless [3].

Cooperative communication (CC) systems, however, suffer from a substantial loss of spectral efficiency since each relay requires multiple orthogonal resource blocks (e.g., time-slots) to deliver the messages from different users to the base station (BS). This spectral inefficiency is prohibitively impractical, especially in cellular networks serving a large number of users. To overcome this deficiency, while maintaining the CC benefits, network-coded cooperation (NCC) has been proposed (see [4]–[6] and references therein). The basic premise of NCC is that each relay employs network coding (NC) [7] to combine messages from multiple users; generates a network-coded message; and then forwards the resulting message to the BS. This transmission paradigm thus reduces the number of relay transmissions which in turn leads to a significantly improved spectral efficiency.

Multiuser diversity (MUD), an inherent diversity in cellular systems, is another form of diversity against channel fading. The basic idea of MUD is to benefit from the channel variations by opportunistically allocating resources to the users experiencing good channel qualities. The application of MUD to CC has been extensively studied [8]–[10], showing that MUD-based CC with opportunistic relay selection (RS) exploits both MUD and CD and thus provides substantial performance improvement. On the other hand, the analysis of the performance of MUD-based NCC is almost unexplored.

From a practical point of view, opportunistic selection becomes inefficient or even infeasible in some situations: i) the best users/relays may not always be available under some scheduling policy, or load balancing conditions; ii) identifying the best users/relays in the presence of imperfect channel state information (CSI) may be difficult; and iii) the users with the highest channel qualities may not have any data to transmit. Due to these factors, some nodes other than those with the highest signal-to-noise ratio (SNR)s may receive the system resources for data transmission. Motivated by these observations, generalized user selection [11], [12] and generalized RS [13]–[16] have been separately and extensively studied in recent years. However, in this paper, we consider the notion of generalized user-relay selection (GURS). In GURS, any arbitrary subset of the users and any arbitrary subset of the relays may be selected. To the best of our knowledge, the performance of GURS has never been studied in the literature. The goal of this paper is thus to fill this gap.

We consider an uplink dual-hop multiuser multirelay NCC network, where the relays employ NC based on maximum distance separable (MDS) codes. For the system under consideration, we study the most general case of user-relay selection scheme, where any arbitrary subsets of users and relays are selected subject to any practical constraints such as load balancing conditions, scheduling policy, and other factors. Our results thus can be directly applied to a large set of situations and include all existing results in the literature as special cases. We derive exact closed-form outage probability expression of this system, assuming non-identically and independently distributed (n.i.i.d.) Rayleigh fading channels. The high-SNR outage probability approximation is further derived to quantify the achievable diversity order. We also confirm our theoretical findings through Monte-Carlo simulations.

A. Related Literature on non-MUD NCC

Many research efforts on the performance analysis of non-MUD NCC (NCC without MUD) have appeared recently. In particular, the network code design for multiuser, multirelay networks with a single BS is studied in [17]. It is shown that a non-binary q-ary Galois field $\mathbb{F}_q$ NC based on MDS codes provides the full diversity order regardless of the number of users and relays. The diversity-multiplexing tradeoff (DMT) of NCC has been further studied in [18]–[21]. Furthermore, [22], [23] studied opportunistic RS-based NCC, where the relay(s) with the highest end-to-end SNRs are selected for cooperation. It is demonstrated that RS-based NCC achieves full diversity order under a restrictive condition, where the number of selected relays must be at least equal to the number of users.

B. Notations and Outline

Throughout this paper, $\Pr\{\cdot\}$ and $\mathcal{C}_n^k = \frac{k!}{(k-n)!n!}$ denote probability and the binomial coefficient, respectively. $\mathbb{F}_q$ represents finite field with size q and $(\cdot)^{\mathrm{T}}$ is the vector transpose operator. Addition and multiplication in $\mathbb{F}_q$ are denoted by $\oplus$ and $\otimes$, respectively.

The rest of this paper is organized as follows. Section II describes the system model. Sections III presents the detailed analysis of exact and asymptotic outage performance of GURS NCC. Numerical results are presented in Section IV. Finally, we conclude in Section V.

II. Preliminaries and Assumptions

A. System and Channel Models

Let us consider a dual-hop multiuser multirelay NCC network that consists of N users $\mathcal{S} = \{S_n | n = 1, 2, ..., N\}$, M decode-and-forward (DF) relays $\mathcal{R} = \{R_m | m = 1, 2, ..., M\}$ and a single BS D (Fig. 1). The users communicate with the BS through direct channels and through dual-hop indirect relays channels. In practice, this setup represents an uplink multiuser cellular system, where some idle users assist some mobile users to communicate with the BS [23]. Each node is equipped with one antenna element and transmits with power ρ. The time-division multiple-access (TDMA) protocol is assumed, where transmissions occur in different orthogonal time-slots. We consider the practical scenario, where the channels suffer from n.i.i.d. frequency-flat Rayleigh fading. In particular, the channel coefficient between any two communicating nodes $i \rightarrow j$ is denoted by h_{ij} and follows $h_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$; a circularly-symmetric complex Gaussian random variable (RV) whose mean is zero and whose variance is equal to σ_{ij}^2. Furthermore, the additive white Gaussian noise (AWGN) term of link $i \rightarrow j$ is denoted by w_{ij} and has mean zero and unit variance i.e., $w_{ij} \sim \mathcal{CN}(0, 1)$. We stress that, although the transmit power ρ and the noise variance are set to be symmetric throughout the network, asymmetry cases of the average SNR and the path-loss can be lumped into the fading variances σ_{ij}^2. The cumulative density function (CDF) of the instantaneous SNR γ_{ij} is then given by

$$F_{\gamma_{ij}}(z) = 1 - e^{-\frac{z}{\rho_{ij}}}, \quad z \geq 0, \tag{1}$$

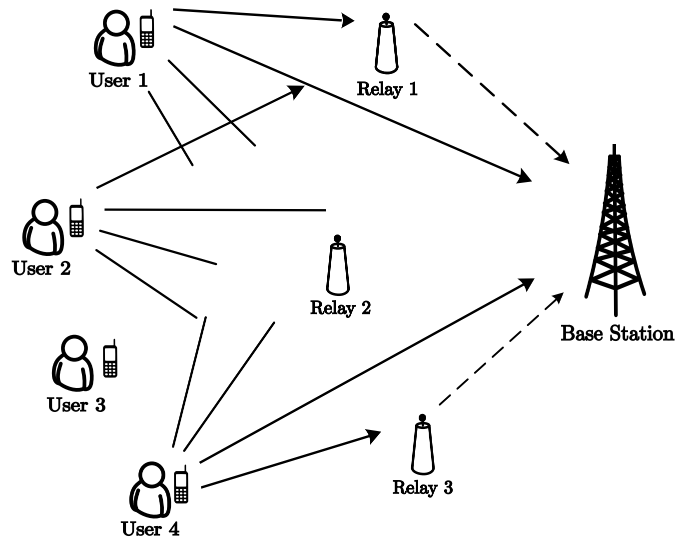

Figure 1. Network-coded cooperation with generalized user-relay selection when $N = 4$, $M = 3$, $K = 3$, and $L = 2$.

where $\rho_{ij} = \rho\sigma_{ij}^2$.

The cooperation is composed of two phases, namely i) the broadcasting phase; and ii) the relaying phase. In the broadcasting phase, K users (amongst N users) are selected to transmit their messages to the BS in a round-robin fashion. The user selection might include a set of K highest-SNR users or any other possible selection. This phase lasts K time-slots. Thanks to the broadcast nature of the wireless medium, the M relays also overhear the transmissions. In the relaying phase, any arbitrary subset of relays of size L (out of M available relays) can be selected. More specifically, the selected L relays employ NC to linearly combine K received packets and then are assigned orthogonal channels to sequentially forward the resulting network-coded packets to the BS. This phase thus takes place in L time-slots.

We assume that the user-relay selection process is performed by a central unit (e.g., the BS) which collects all information of instantaneous CSIs and feeds back the results of selection process to the users and relays. The selection depends on load balancing conditions, scheduling policy, and other factors.

B. Signal Model and Transmission Scheme

1) Broadcasting Phase: During the broadcasting phase, the BS selects the $i_1^{\text{th}}, i_2^{\text{th}}, ..., i_K^{\text{th}}$ best users for data transmission. The user selection policy is based on the quality of the user-to-BS channels. We define $\boldsymbol{\gamma}_S = \{\gamma_1, \gamma_2, ..., \gamma_N\}$ as the set of order statistics of SNRs for the user-to-BS channels in a decreasing order of magnitude. Specifically, γ_n can be written as

$$\gamma_n = n^{\text{th}} \max_{1 \leq n \leq N} \{\gamma_{S_n D}\}. \tag{2}$$

Let $\boldsymbol{\gamma}_S^* = \{\gamma_{i_1}, \gamma_{i_2}, ..., \gamma_{i_K}\}$ denote the ordered SNRs of any arbitrary subset of $\boldsymbol{\gamma}_S$ with cardinality of K and $\{i_1, i_2, ..., i_K\}$ being the set of indexes of the elements in $\boldsymbol{\gamma}_S^*$, where $i_1 < i_2 < ... < i_K$. For the spacial case when the selection includes the K highest-SNR users, we have $\{i_1, i_2, ..., i_K\} = \{1, 2, ..., K\}$.

Denote $\epsilon_{S_k} \in \mathbb{F}_q$ as the symbol transmitted by the selected user S_k $k \in \{i_1, i_2, ..., i_K\}$. The received signal at relay R_m $(m = 1, 2, ..., M)$ and the BS can then be written as

$$y_{S_k D} = \sqrt{\rho} h_{S_k D} x_k + w_{S_k D}, \tag{3}$$

$$y_{S_k R_m} = \sqrt{\rho} h_{S_k R_m} x_k + w_{S_k R_m}, \tag{4}$$

where x_k is the modulated version of ϵ_{S_k}.

2) Relaying Phase: The relaying phase is based on the RS policy which minimizes the possible error of network-coded symbols [22]. Under this selection strategy, the equivalent channel for relay R_m is determined by the worst channel in the two-hop user-relay-BS links. Thus, the *equivalent SNR* of the channels between K selected users, relay R_m, and the BS can be expressed as

$$\gamma_m^{\text{eq}} = \min \left\{ \gamma_{S_{i_1} R_m}, \gamma_{S_{i_2} R_m}, ..., \gamma_{S_{i_K} R_m}, \gamma_{R_m D} \right\}. \tag{5}$$

In (5), γ_{ij}'s are independent exponentially distributed RVs. Accordingly, γ_m^{eq} is a exponentially distributed RV and its corresponding CDF can be formulated as

$$F_{\gamma_m^{\text{eq}}}(z) = 1 - e^{-\frac{z}{\rho_m}}, \quad z \geq 0, \tag{6}$$

where

$$\frac{1}{\rho_m} = \frac{1}{\rho_{S_{i_1} R_m}} + ... + \frac{1}{\rho_{S_{i_K} R_m}} + \frac{1}{\rho_{R_m D}}. \tag{7}$$

Define $\boldsymbol{\gamma}_R = \{g_1, g_2, ..., g_M\}$ as the set of order statistics of the equivalent SNRs of relays in a decreasing order of magnitude. Mathematically, this can be written as

$$g_m = m^{\text{th}} \max_{1 \leq m \leq M} \{\gamma_m^{\text{eq}}\}. \tag{8}$$

In the relaying phase, the $j_1^{\text{th}}, j_2^{\text{th}}, ..., j_L^{\text{th}}$ best relays take part in cooperation. Let $\boldsymbol{\gamma}_R^* = \{g_{j_1}, g_{j_2}, ..., g_{j_L}\}$ denote the ordered SNRs of any arbitrary subset of $\boldsymbol{\gamma}_R$ with cardinality of L, where $j_1 < j_2 < ... < j_L$. For the sake of clarity, assume that the number of relays $M = 10$ and $\boldsymbol{\gamma}_R^* = \{g_1, g_4, g_7, g_9\}$ or equivalently $\{j_1, j_2, j_3, j_4\} = \{1, 4, 7, 9\}$. This implies that four relays out of ten relays are selected whose SNRs are the first, fourth, seventh, and ninth largest SNRs in $\boldsymbol{\gamma}_R$.

The selected relays R_l $l \in \{j_1, j_2, ..., j_L\}$ first decode the estimate $\hat{\epsilon}_{S_k R_l}$ using the maximum likelihood (ML) detector as follows

$$\hat{\epsilon}_{S_k R_l} = \underset{\hat{\epsilon}_{S_k} \in \mathbb{F}_q}{\arg\min} \left\{ \left| y_{S_k R_l} - \sqrt{\rho} h_{S_k R_l} x_k \right|^2 \right\}, \tag{9}$$

and then sequentially transmit their network-coded symbols to the BS. The NC operations is applied to all correct or incorrect received symbols [22]. In particular, relay R_l linearly combines estimated symbols in $\mathbb{F}_q$ using the weighting coefficients $\alpha_{S_k R_l}$ forming an MDS code. The network-coded symbol generated by relay R_l can then be written as

$$\hat{\epsilon}_l = \sum_{k \in \{i_1, i_2, ..., i_K\}} \bigoplus \left(\alpha_{S_k R_l} \bigotimes \hat{\epsilon}_{S_k R_l} \right). \tag{10}$$

Modulating $\hat{\epsilon}_l$ to $\hat{x}_l$, the received signal from relay R_l $l \in \{j_1, j_2, ..., j_L\}$ at D can be expressed as

$$y_{R_l D} = \sqrt{\rho} h_{R_l D} \hat{x}_l + w_{R_l D}. \tag{11}$$

Table I
LIST OF MAIN PARAMETERS

Notation	Description
N	Number of users
M	Number of relays
K	Number of selected users
L	Number of selected relays
$\{i_1^{\text{th}}, i_2^{\text{th}}, ..., i_K^{\text{th}}\}$	Set of selected users
$\{j_1^{\text{th}}, j_2^{\text{th}}, ..., j_L^{\text{th}}\}$	Set of selected relays
ρ	Transmit power (Transmit SNR)
h_{ij}	Fading coefficient of the channel between nodes i and j
σ_{ij}^2	Variance of h_{ij}
γ_{ij}	Instantaneous SNR of the link between nodes i and j
ρ_{ij}	Average received SNR of the link between nodes i and j
γ_m^{eq}	Equivalent SNR of relay R_m
γ_{th}	Threshold SNR

Finally, the BS puts the received packets and network-code coefficients in the matrix forms to solve linear equations over $\mathbb{F}_q$. In particular, the BS requires at least K successful transmissions (out of $K + L$ transmissions) to recover users' packets; otherwise an outage occurs for all users' transmissions.

For convenience, the key parameters of the system model have been summarized in Table I.

III. PERFORMANCE ANALYSIS

A. Exact outage Probability

In this subsection, we derive exact closed-form expressions for the outage probability of GURS NCC system.

Theorem 1. *The outage probability for GURS NCC when* $K > L$ *and* $K \leq L$ *are given by* (12) *and* (13) *on the top of the next page, where* $\Phi(k)$ *and* $\Psi(k)$ *are*

$$\Phi(k) = \sum_{\epsilon=1}^{N-k+1} \mathcal{C}_k^{k+\epsilon-1} (-1)^{\epsilon-1} \mathcal{B}(N, k, \epsilon, \rho_{S_v D}), \tag{14}$$

$$\Psi(k) = \sum_{\epsilon=1}^{M-k+1} \mathcal{C}_k^{k+\epsilon-1} (-1)^{\epsilon-1} \mathcal{B}(M, k, \epsilon, \rho_v), \tag{15}$$

and $\mathcal{B}(\alpha, \beta, \zeta, \lambda)$ *is defined as*

$$\mathcal{B}(\alpha, \beta, \zeta, \lambda) = \sum_{\substack{a_1, a_2, ..., a_{\beta+\zeta-1}=1 \\ a_1 < a_2 < ... < a_{\beta+\zeta-1}}}^{\alpha} \prod_{v=a_1}^{a_{\beta+\zeta-1}} e^{-\frac{\gamma_{th}}{\lambda}}. \tag{16}$$

Proof. The total received SNRs from K selected users and L selected relays at the BS can be written as

$$\boldsymbol{\gamma}_{\text{tot}}^{(K+L)\times 1} = \left[\gamma_{i_1}, \gamma_{i_2}, ..., \gamma_{i_K}, g_{j_1}, g_{j_2}, ..., g_{j_L}\right]^{\text{T}}. \tag{17}$$

To jointly recover the selected users' packets, at least K elements (out of $K + L$ elements) in (17) must be above the threshold SNR γ_{th}; otherwise, an outage event happens. Thus, the outage probability based on (17) is formulated as

$$\begin{aligned} \mathcal{P}_{out} = &\Pr\{0 \text{ links in } \boldsymbol{\gamma}_{\text{tot}} > \gamma_{th}\} + \Pr\{1 \text{ link in } \boldsymbol{\gamma}_{\text{tot}} > \gamma_{th}\} \\ &+ ... + \Pr\{K-1 \text{ links in } \boldsymbol{\gamma}_{\text{tot}} > \gamma_{th}\}. \end{aligned} \tag{18}$$

$$\mathcal{P}_{out}=\underbrace{\sum_{c=1}^{K-L}\left(\sum_{n=N+1-i_c}^{N-i_{c-1}}\Phi(n)\right)}_{\Pr\{\mathcal{O}_a\}}+\underbrace{\sum_{c=1}^{L}\left\{\sum_{n=N+1-i_{K+1-c}}^{N-i_{K-c}}\Phi(n)\sum_{\ell=1}^{c}\left(\sum_{m=M+1-j_\ell}^{M-j_{\ell-1}}\Psi(m)\right)\right\}}_{\Pr\{\mathcal{O}_b\}},\quad if\quad K>L \tag{12}$$

$$\mathcal{P}_{out}=\underbrace{\sum_{c=1}^{K}\left\{\sum_{n=N+1-i_{K+1-c}}^{N-i_{K-c}}\Phi(n)\sum_{\ell=1}^{c}\left(\sum_{m=M+1-j_\ell}^{M-j_{\ell-1}}\Psi(m)\right)\right\}}_{\Pr\{\mathcal{O}_b\}},\qquad if\quad K\leq L \tag{13}$$

We denote the number of operational (i.e., not in outage) users and relays by $K_{\text{op}} \leq K$, and $L_{\text{op}} \leq L$, respectively. Now, depending on K_{op} and L_{op}, the outage events can be divided into two types, namely Type (a) and Type (b). Type (a) corresponds to the events when $K > L$ and there are not enough operational user-to-BS channels, K_{op}, such that even if $L_{\text{op}} = L$, the BS is still in outage i.e., $K_{\text{op}} + L < K$. On the other hand, Type (b) includes the outage events, where $K_{\text{op}} + L \geq K$, but the sum of all operational channels is less than K i.e., $K_{\text{op}} + L_{\text{op}} < K$.

Denoting the outage events corresponding to Type (a) and Type (b) by $\mathcal{O}_a$ and $\mathcal{O}_b$, the overall outage probability can be formulated as

$$\mathcal{P}_{out}=\begin{cases}\Pr\{\mathcal{O}_a\}+\Pr\{\mathcal{O}_b\}, & if\quad K>L\\ \Pr\{\mathcal{O}_b\}, & if\quad K\leq L\end{cases} \tag{19}$$

The probability that k users are in outage can be written as

$$\Phi(k)=\Pr\Big\{(N-k\text{ values }\gamma_{S_nD}>\gamma_{th})\cap\ (k\text{ values }\gamma_{S_nD}<\gamma_{th})\Big\}. \tag{20}$$

Since, γ_{S_nD} $\forall n \in \{1,2,...,N\}$ are mutually independent RVs, and there are $\mathcal{C}_k^N$ combinations that satisfy the condition in (20), we have

$$\Phi(k)=\sum_{a_1,a_2,\cdots,a_N=1}^{N}\left\{\prod_{n=a_{k+1}}^{a_N}\left(1-F_{\gamma_{S_nD}}(\gamma_{th})\right)\prod_{n'=a_1}^{a_k}F_{\gamma_{S_{n'}D}}(\gamma_{th})\right\}, \tag{21}$$

where $a_1, a_2, \cdots, a_N \in \{1, 2, \cdots, N\}$, $a_1 \neq a_2 \neq \cdots \neq a_N$, $a_1 < ... < a_k$, $a_{k+1} < ... < a_N$, and $F_{\gamma_{ij}}(z)$ is already given by (1).

We can further write (21) in a simple form of (14). With similar steps above, the probability that k relays out of M relays are in outage $\Psi(k)$ can be derived as (15).

Then, by computing the probability of outage events $\mathcal{O}_a$ and $\mathcal{O}_b$ with the aid of (14) and (15), the outage probability in (19) can be derived as (12) and (13). Thus, we complete the proof. ■

Although the derived outage probability expressions in (12) and (13) are exact in all SNR regime, it is hard to obtain insights into the effects of the system parameters. In the following subsection, we derive the diversity order of the system as a function of different system parameters by deriving the asymptotic outage expressions in high SNRs (i.e., $\rho \to \infty$).

B. Asymptotic Analysis and Diversity Order

Theorem 2. *The diversity order of GURS NCC for $K > L$ and $K \leq L$ are given by*

$$d=\begin{cases}N-i_{K-L}+1, & if\quad K>L\\ N+M-\max\{\delta_1,\delta_2,...,\delta_K\}+2, & if\quad K\leq L\end{cases} \tag{22}$$

where $\delta_v = i_{K-v+1} + j_v$.

Proof. The diversity order indicates the slope of the outage probability in high SNRs and is defined as

$$d=-\lim_{\rho\to\infty}\frac{\log\left(\mathcal{P}_{out}(\rho)\right)}{\log(\rho)}. \tag{23}$$

To find the asymptotic expressions in the high SNR regime, we use Taylor series expansion of the exponential function given by $e^{-x} = \sum_{k=0}^{\infty} \frac{(-x)^k}{k!}$. Plugging this expression in (14) and then retaining the dominant terms, $\Phi(k)$ can be approximated as

$$\Phi^{\infty}(k)=\sum_{\substack{a_1,a_2,...,a_k=1\\ a_1<a_2<...<a_k}}^{N}\prod_{n=a_1}^{a_k}\left(\frac{\gamma_{th}}{\rho_{S_nD}}\right). \tag{24}$$

Similarly, the asymptotic expression of $\Psi(k)$ in (15) can be derived as

$$\Psi^{\infty}(k)=\sum_{\substack{a_1,a_2,...,a_k=1\\ a_1<a_2<...<a_k}}^{M}\prod_{m=a_1}^{a_k}\left(\frac{\gamma_{th}}{\rho_m}\right). \tag{25}$$

Substituting (24) and (25) in (12) and setting $n = N + 1 - i_{K-L}$ in the second summation and $n = N + 1 - i_{K+1-c}$, $m = M + 1 - j_c$ in the forth and sixth summations, (12) can be approximated as (26) on the top of the next page. Keeping the dominant terms and then invoking (23), the diversity order d for the case of $K > L$ can be derived in the closed-form expression as given in (22).

With similar steps above, the outage expression in high SNRs for $K \leq L$ can be obtained as (27). Using (23), the diversity order for the case of $K \leq L$ can be derived. This concludes the proof. ■

$$\mathcal{P}_{out}^{\infty} \approx \sum_{\substack{a_1,a_2,\dots,a_{N+1-i_{K-L}}=1 \\ a_1<a_2<\dots<a_{N+1-i_{K-L}}}}^{N} \prod_{n=a_1}^{a_{N+1-i_{K-L}}} \left(\frac{\gamma_{th}}{\rho_{S_n D}}\right)$$

$$+ \sum_{c=1}^{L} \left\{ \sum_{\substack{a_1,a_2,\dots,a_{N+1-i_{K+1-c}}=1 \\ a_1<a_2<\dots<a_{N+1-i_{K+1-c}}}}^{N} \prod_{n=a_1}^{a_{N+1-i_{K+1-c}}} \left(\frac{\gamma_{th}}{\rho_{S_n D}}\right) \sum_{\substack{a_1,a_2,\dots,a_{M+1-j_c}=1 \\ a_1<a_2<\dots<a_{M+1-j_c}}}^{M} \prod_{m=a_1}^{a_{M+1-j_c}} \left(\frac{\gamma_{th}}{\rho_m}\right) \right\}, \quad if \quad K > L \tag{26}$$

$$\mathcal{P}_{out}^{\infty} \approx \sum_{c=1}^{K} \left\{ \sum_{\substack{a_1,a_2,\dots,a_{N+1-i_{K+1-c}}=1 \\ a_1<a_2<\dots<a_{N+1-i_{K+1-c}}}}^{N} \prod_{n=a_1}^{a_{N+1-i_{K+1-c}}} \left(\frac{\gamma_{th}}{\rho_{S_n D}}\right) \sum_{\substack{a_1,a_2,\dots,a_{M+1-j_c}=1 \\ a_1<a_2<\dots<a_{M+1-j_c}}}^{M} \prod_{m=a_1}^{a_{M+1-j_c}} \left(\frac{\gamma_{th}}{\rho_m}\right) \right\}, \quad if \quad K \le L \tag{27}$$

Corollary 1. *The diversity order for $K > L$ only depends on the number of users N and the i_{K-L}^{th} best user, no matter how the RS process proceeds.*

Corollary 2. *When $K \le L$, the diversity order depends on N, M, the $i_1^{\text{th}}, i_2^{\text{th}}, \dots, i_K^{\text{th}}$ best users and the $j_1^{\text{th}}, j_2^{\text{th}}, \dots, j_K^{\text{th}}$ best relays. Therefore, the $j_{K+1}^{\text{th}}, j_2^{\text{th}}, \dots, j_L^{\text{th}}$ best relays does not change the diversity order.*

Corollary 3. *Based on* (22)*, the maximum achievable diversity order is given by* (28)*. The diversity order of $d^* = N - K + L + 1$ can be obtained if and only if $i_{K-L} = K - L$ in* (22)*. This implies that the set of selected users must include $K - L$ highest-SNR users. On the other hand, when $K \le L$, the maximum diversity order of $d^* = N - K + M + 1$ can be achieved if and only if* $\max\{\delta_1, \delta_2, \dots, \delta_K\} = K + 1$*. This indicates that the selection must include K best users and K best relays.*

$$d^* = \begin{cases} N - K + L + 1, & if \quad K > L \\ N - K + M + 1, & if \quad K \le L \end{cases} \tag{28}$$

Note that the term $N - K + 1$ in (28) *corresponds to the MUD and the remaining terms L and M correspond to the CD. It can be readily checked that d^* increases as the number of users N increases. Also, it linearly decreases as the number of selected users K increases. Obviously, when $K = N$ the MUD gain diminishes and only the CD gain can be achieved.*

The derived diversity order (22) is the most generic expression in the literature and includes all existing results as special cases. For $K = N$ and $L = M$ (NCC without user-relay selection), it reduces to $d = M + 1$ [17], [19]. When $K = N$ and L highest-SNR relays are selected, the diversity order for $K > L$ and $K \le L$ reduces to $d = L + 1$ and $d = M + 1$. This coincides with the diversity order reported in [22], [23]. When $K = N$ and any arbitrary relays are selected, it reduces to the diversity order of $d = L + 1$ and $d = N + M - j_N + 1$ for $K > L$ and $K \le L$, respectively [15].

IV. Numerical Results and Discussions

In this section, we present simulation results to corroborate the theoretical expressions derived in the previous sections by assuming $h_{ij} \sim \mathcal{CN}(0,1)$ and $\gamma_{th} = 0$ dB.

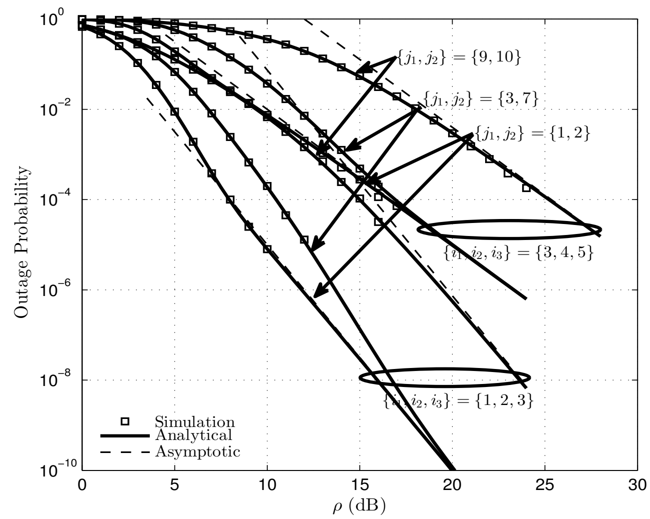

Figure 2. Outage probability versus SNR for $N = 5$, $M = 10$, $K = 3$, $L = 2$, and different user-relay selections ($K > L$).

Fig. 2 illustrates the outage probability versus SNR ρ for GURS NCC when $K > L$. We assume $N = 5$, $M = 10$, $K = 3$, $L = 2$. The outage probability of the best user-relay selection i.e., when $\{i_1, i_2, i_3\} = \{1, 2, 3\}$ and $\{j_1, j_2\} = \{1, 2\}$ is also plotted as a benchmark. It can be seen that analytical curves (12) are in excellent agreement with simulation results. This agreement confirms the correctness of the derived analytical expressions. As expected, the best user-relay selection achieves the full diversity order of $d^* = N - K + L + 1 = 5$ (28) and outperforms the other selections in all SNR regime. On the other hand, the worst outage performance corresponds to the user-relay selection with the lowest SNRs i.e., $\{i_1, i_2, i_3\} = \{3, 4, 5\}$ and $\{j_1, j_2\} = \{9, 10\}$. In addition, the slope of the curves reveals that the diversity order is either equal to 3 or 5, regardless of the order of the selected relays. This indicates that the diversity order is determined by $d = N - i_{K-L} + 1$ (22) which only depends on the number of users N and the i_{K-L}^{th} best user. It is also observed that the order of the selected relays only manifests its effect on the coding gain, rather than the diversity.

In Fig. 3, we plot the outage probability versus SNR ρ for

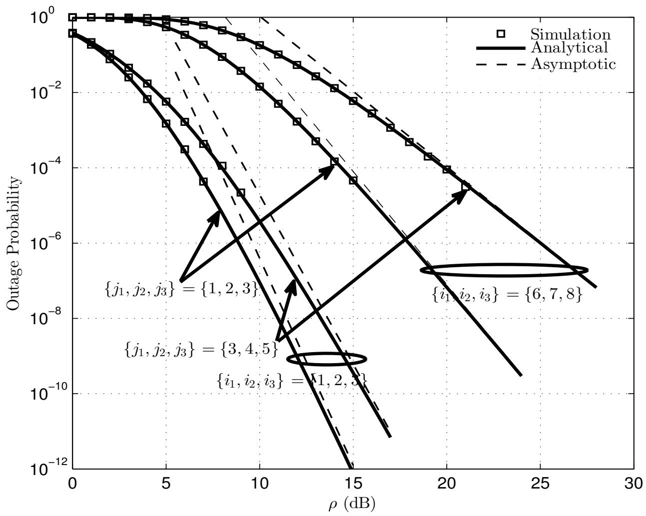

Figure 3. Outage probability versus SNR for $N = 8$, $M = 5$, $K = 3$, $L = 3$, and different user-relay selections ($K \leq L$).

different user-relay selections, assuming $N = 8$, $M = 5$, $K = 3$, $L = 3$ ($K \leq L$). It can be seen that the best and worst outage performance corresponds to the best and worst uers-relay selections. In addition, unlike $K > L$, where the diversity order is only dominated by the i^{th}_{K-L} best user, the diversity order for $K \leq L$ depends on both users and relays selections. More precisely, the diversity order of $\{i_1, i_2, i_3\} = \{1, 2, 3\}$ with $\{j_1, j_2, j_3\} = \{1, 2, 3\}$ (i.e., the best user-relay selection) is equal to $d^* = N - K + M + 1 = 11$ (28); the maximum possible diversity order that can be achieved. Furthermore, the diversity order of $\{i_1, i_2, i_3\} = \{1, 2, 3\}$ with $\{j_1, j_2, j_3\} = \{3, 4, 5\}$ is equal to $d = 9$. On the other hand, the diversity orders of $\{i_1, i_2, i_3\} = \{6, 7, 8\}$ with $\{j_1, j_2, j_3\} = \{1, 2, 3\}$ and $\{j_1, j_2, j_3\} = \{3, 4, 5\}$ are respectively equal to 6 and 4 confirming (22).

V. Conclusions

In this paper, we proposed GURS for multiuser multirelay NCC systems. In particular, we consider N users, M relays and a single BS, where the $i^{\text{th}}_1, i^{\text{th}}_2, ..., i^{\text{th}}_K$ best users and the $j^{\text{th}}_1, j^{\text{th}}_2, ..., j^{\text{th}}_L$ best relays are selected subject to any practical considerations. The performance of the system under consideration was quantified by deriving the outage probability over n.i.i.d Rayleigh fading channels. The diversity orders were further derived by using the high-SNR approximations of the outage probability. Numerical results were presented to show the system performance, and thereby, to obtain valuable insights into the performance of NCC systems under realistic operating conditions.

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