# Performance Analysis of Massive MIMO Two-Way Relay Networks with Pilot Contamination, Imperfect CSI, and Antenna Correlation

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Abstract-We consider a multi-cell two-way relay network (TWRN) consisting of single-antenna user nodes and amplify-andforward (AF) relay nodes having very large antenna arrays. We investigate the combined impact of co-channel interference (CCI), imperfect channel state information (CSI), pilot contamination, and the antenna correlation at the massive MIMO node. By using a large number of antennas at the relay, we show that the effect of CCI can completely be mitigated. However, the effects of imperfect CSI and pilot contamination degrades the performance even with a large antenna array. Yet, use of massive MIMO allows power scaling at the user nodes and relay and thus, even with channel imperfections, the benefits of employing a massive multiple-input multiple-output (MIMO) enabled relay on transmit power savings are significant. Furthermore, we derive closed-form approximations for the sum rate in our system model for the simplified setup when CCI and pilot contamination are absent and CSI is perfect. This result will be useful to decide the required number of antennas at the relay node to obtain a certain percentage of the asymptotic sum rate. Also, our analysis of antenna correlation shows that the effect of antenna correlation can be mitigated by using a large antenna array. We, also find the optimal pilot sequence length to maximize the sum rate of the system.

# I. INTRODUCTION

Research on massive MIMO, an enabling technology for future fifth generation (5G) wireless [2], [3], has shown very high spectral efficiencies, low transmit powers per bit, and high energy efficiencies [4]. These advantages have greatly excited the research community. However, the main performance limiting factor is pilot contamination, which is the residual interference caused by the reuse of non-orthogonal pilot sequences [4], [5].

In this paper, we consider massive MIMO with Two-way relay networks (TWRNs), which offer two fold increase in the achievable data rate compared to the one-way relay networks (OWRNs) [6]. Multi-pair TWRNs enable mutual data exchanges among multiple pairs of nodes with the aid of an intermediate relay [7]. We are motivated to consider massive MIMO TWRNs due to their wide range of potential applications. For example, the system considered in this paper can be used as a heterogeneous wireless entity for the existing cellular network architecture for reducing the workload of the base-stations [8]. For example, bypassing the base-station and using the two-way relaying instead may be feasible. Service providers can thus use TWRNs to

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improve the data throughput without making drastic changes to their existing infrastructure.

Another potential application scenario is the internet of things (IoT), which connects multiple wireless devices and sensors [9]. For example, the cooling system will require data from temperature sensors while security system may require data from the motion sensors. This scenario fits the model of a multi-pair relay network with a central relay node. Moreover, when multiple IoT networks coexist, the effect of co-channel interference can be a significant impairment. Furthermore, these IoT devices often rely on battery power and thus, the energy and power efficiency is an important factor in IoT communications.

Combining massive MIMO with TWRNs provides the sum rate and energy efficiency performance gains that are required by the above mentioned applications. In the following, we classify existing works into general massive MIMO and massive MIMO TWRNs.

**Related work:** In [4], the asymptotic performance metrics of multi-user massive MIMO base stations (BSs) in non-cooperative cellular networks is investigated. Specifically, [4] concludes that whenever the number of BS antennas increases unbounded, simple linear precoders and decoders become asymptotically optimal. Pilot contamination in multi-cell multi-user massive MIMO systems is investigated by deriving rigorous asymptotic signal-to-interference-plus-noise ratio (SINR) expressions [5].

In [10], the asymptotic performance of multi-pair OWRNs with very large relay antenna arrays is investigated when there is no co-channel interference (CCI) and the CSI is perfect. To this end, the asymptotic SINR and sum rate expressions are derived by considering three transmit power scaling laws. In [11], the sum rate of a MIMO TWRN has been analyzed under ZF beamforming at the relay or user nodes and closed-form results and approximations for sum rate are obtained. Further, in [12], the channel aging effects of multi-cell multi-way massive MIMO relaying are investigated. In [13], the multi-pair TWRNs with massive MIMO is investigated by employing linear precoders and detectors, where again, perfect CSI and CCI/pilot contamination free scenario is assumed. There are some recent publications which analyze multi-pair massive MIMO TWRNs. In [14] and [15], multi-pair massive MIMO TWRNs have been analyzed for maximum ratio combining/transmission (MRC/MRT) beamforming. While [14] assumes perfect CSI, [15] analyzes the system under imperfect CSI scenario. Furthermore, [16] analyzes multi-pair massive MIMO TWRN with imperfect CSI under ZF beamforming. In [17], a full-duplex multi-pair massive MIMO system is analyzed under imperfect CSI with ZF beamforming. However none of these work analyze the system under CCI and pilot contamination in the context of two-way multi-pair massive

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# MIMO relaying.

**Problem statement and our contribution:** With dense deployment of wireless systems, CCI and pilot contamination are dominant performance limiting factors [5]. Further, because precoders/detectors need CSI, channel estimation errors (imperfect CSI) severely degrade the overall performance. However, perfect CSI is assumed in the analysis of [13]–[17]. In contrast, we analyze the effects of CCI, imperfect CSI, pilot contamination, and antenna correlation for multi-pair massive MIMO TWRNs. Although our previous work [1] analyzed these effects separately, this paper provides a complete analysis with a more generalized system model which contains the effects of CCI, imperfect CSI, and pilot contamination simultaneously. Furthermore, this paper provides extended proofs of the SINR results [1].

In this work, we study these imperfections in massive MIMO TWRNs. More specifically, the contributions of this paper can be listed as follows.

- 1) The asymptotic SINR and sum rate expressions are derived for three transmit power scaling laws: namely (i) power scaling at user nodes, (ii) power scaling at the relay, and (iii) power scaling at both the relay and user nodes. We show that for the CCI case, the asymptotic SINR expressions asymptotically become independent of the number of co-channel interferers (L), and consequently, the CCI degradation can be cancelled asymptotically, whenever the relay antenna count grows unbounded. Nevertheless, the asymptotic performance is limited by the residual interference incurred due to pilot contamination, and its impact cannot be completely mitigated even in the limit of infinitely many relay antennas. Notably, the asymptotic performance metrics are independent of the fast fading component of the wireless channel, and hence, the cross-layer resource scheduling becomes simple. Our analysis and Monte-Carlo simulations reveal that substantial sum-rate gains can be achieved via very large relay antenna arrays.
- 2) The asymptotic results are valid only when the number of antennas at the relay is infinite. However, for practical purposes, the sum-rate results for a finite number of antennas is important. Thus, we obtain closed-form sum rate results for the finite relay antenna array. To make analysis tractable we assume perfect channel conditions (i.e. no CCI, no pilot contamination and perfect CSI). The benefits of this result are that it can be used to decide the optimal number of relay antennas to obtain a certain percentage of the asymptotic performance and to determine how fast the performance of the system approaches the asymptotic performance.
- 3) We obtain asymptotic SINR and sum-rates for multipair massive MIMO TWRNs with relay antenna correlation. Its effect on the performance of massive MIMO systems can be significant. Fortunately, our analysis shows that the correlation impact can be mitigated by using a large antenna arrays in TWRNs.

**Notation:**  $\mathbf{Z}^{H}$ ,  $[\mathbf{Z}]_{k}$ , and  $[\mathbf{Z}]_{i,j}$  denote the Hermitian-transpose, the *k*th diagonal element of the matrix,  $\mathbf{Z}$ , and the (i, j) th diagonal element of the matrix,  $\mathbf{Z}$ , respectively. The diagonal matrix  $\mathbf{D}$  with *k*th diagonal element  $d_k$  is denoted as diag  $(d_k)$ .  $\mathbf{I}_{M}$  and  $\mathbf{O}_{M \times N}$  are the  $M \times M$  Identity matrix and  $M \times N$  matrix of all zeros, respectively. A complex Gaussian random variable (RV) X with mean  $\mu$  and standard deviation  $\sigma$  is denoted as  $X \sim C\mathcal{N}(\mu, \sigma^2)$ . Further,  $E_1(z)$ ,  $\Re z > 0$ , is the exponential integral function [18, Eqn. (8.211)].  $\Gamma(z)$  is the Gamma function [18, Eqn. (8.310.1)], and  $\Gamma(a, z)$  is the upper incomplete Gamma function [18, Eqn. (8.350.2)].

## II. SYSTEM, CHANNEL, AND SIGNAL MODEL

#### A. System and channel model

The system model consists of L adjacent TWRNs having 2K number of users each. The users in the *l*th TWRN are denoted as  $(U_{l,1}, \dots, U_{l,2K})$ , where  $U_{l,k}$  exchange data signals with its paired-user  $U_{l,k'}$  via the half-duplex AF relay  $R_l$  for  $k, k' \in \{1, \dots, 2K\}$  and  $l \in \{1, \dots, L\}$ . Users are singleantenna terminals, and the relays are equipped with N antennas. The number of relay antennas are unbounded with respect to the total number of users  $(N \gg 2K)$ . The channel matrix from 2K users in the *i*th TWRN to the *l*th relay is defined as  $\mathbf{G}_{jl} = \mathbf{F}_{jl} \mathbf{D}_{jl}^{\frac{1}{2}}$ , where  $\mathbf{F}_{jl} \sim \mathcal{CN}_{N \times 2K} (\mathbf{0}_{N \times 2K}, \mathbf{I}_N \otimes \mathbf{I}_{2K})$  accounts for small-scale fading, and  $\mathbf{D}_{jl} = \text{diag}(\eta_{j,l,1}, \cdots, \eta_{j,l,2K})$ represents large-scale fading. As customary, the channel gains are independent and identically distributed (i.i.d.) and are assumed to remain fixed over two consecutive time-slots and reciprocal, and hence, relay-to-user channel matrix becomes  $\mathbf{G}_{il}^T$ . The CCI on the lth TWRN occurs due to the data transmissions of the other L-1 TWRNs with relay  $R_j$ , where  $j \in \{1, \dots, L\}$  and  $j \neq l$ .

## B. Channel Estimation

The *l*th relay estimates  $G_{ll}$  by using the pilot sequences transmitted by the users. To this end, 2K users in each TWRN transmit mutually orthogonal pilot sequences of length  $\tau$ . Yet, due to the unavailability of orthogonal pilot sequences for all users in L TWRNs, same sequence is reused by the L - 1 adjacent cochannel TWRNs. Thus, the channel estimation at the relay not only contains the desired channel, but also the channels belonging to 2K users in each of the adjacent TWRNs. The corresponding minimum mean square error (MMSE) estimation of the users-to-relay channel is written as [19]

$$\hat{\mathbf{G}}_{ll} = \left(\sum_{j=1}^{L} \mathbf{G}_{jl} + \mathbf{V}_l / \sqrt{P_p}\right) \tilde{\mathbf{D}}_{ll}, \text{ for } l \in \{1, \cdots L\}, (1)$$

where  $P_p$  is the transmit power of the pilot sequence, the elements of  $\mathbf{V}_l$  are distributed as independent  $\mathcal{CN}(0,1)$  random variables,  $\tilde{\mathbf{D}}_{ll} = \left(\frac{1}{P_p}\left(\sum_{j=1}^{L} \mathbf{D}_{jl}\right)^{-1} + \mathbf{I}_K\right)^{-1}$ , and  $\mathbf{G}_{jl}$  is defined in the previous section. The estimation error is  $\mathbf{E}_l = \hat{\mathbf{G}}_{ll} - \sum_{j=1}^{L} \mathbf{G}_{jl}$ . The receive-ZF detector and transmit-ZF precoder is constructed at the relay by using the CSI with estimation errors. The elements of the *k*th column of  $\mathbf{E}_l$  is Gaussian distributed with mean zero and variance  $\sum_{j=1}^{L} \eta_{j,l,k} / (P_p \sum_{j=1}^{L} \eta_{j,l,k} + 1)$ . Due to the MMSE properties, the matrices  $\mathbf{E}_l$  and  $\hat{\mathbf{G}}_{ll}$  are statistically independent.

# C. Signal model

During two time-slots, 2K users in each TWRN exchange their information pair-wisely via their assigned relay. Specifically, the paired users  $(U_{l,2i-1}, U_{l,2i})$  exchange their data signals

$$\gamma_{l,k'} = \frac{\hat{\beta}_{l}^{2} P_{S} \left\| \mathbf{g}_{lk'}^{T} \hat{\mathbf{W}}_{l} \mathbf{g}_{lk} \right\|^{2}}{\hat{\beta}_{l}^{2} P_{S} \sum_{m=1, m \neq k}^{2K} \left\| \mathbf{g}_{lk'}^{T} \hat{\mathbf{W}}_{l} \mathbf{g}_{lm} \right\|^{2} + \hat{\beta}_{l}^{2} P_{S} \sum_{j=1, j \neq l}^{L} \left\| \mathbf{g}_{lk'}^{T} \hat{\mathbf{W}}_{l} \mathbf{G}_{jl} \right\|^{2} + \hat{\beta}_{l}^{2} \sigma_{R_{l}}^{2} \sum_{j=1, j \neq l}^{L} \left\| \mathbf{g}_{lk'}^{T} \hat{\mathbf{W}}_{l} \right\|^{2} + \sigma_{n_{l,k'}}^{2}}.$$
(7)

$$\hat{\beta}_{l} = \sqrt{\frac{P_{R}}{P_{S}\sum_{j=1}^{L} \operatorname{Tr}\left(\mathbf{G}_{jl}\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{T}\hat{\mathbf{G}}_{ll}^{*}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^$$

 $(x_{l,2i-1}, x_{l,2i})$ , where  $i \in \{1, \dots, K\}$  and  $l \in \{1, \dots, L\}$ . In the first time-slot, users transmit  $2K \times 1$  signal vector  $\mathbf{x}_l$ , which is the concatenated data signals of 2K users in the *l*th TWRN, towards the relay  $R_l$ . The signal vector  $\mathbf{x}_l$  satisfies  $\mathcal{E}[\mathbf{x}_l \mathbf{x}_l^H] = \mathbf{I}_{2K}$ . The received signal at  $R_l$  is written as

$$\mathbf{y}_{R_l} = \sqrt{P_S} \sum_{j=1}^{L} \mathbf{G}_{jl} \mathbf{x}_j + \mathbf{n}_{R_l}, \qquad (2)$$

where  $\mathbf{n}_{R_l}$  is the  $N \times 1$  additive white Gaussian noise (AWGN) vector at the relay satisfying  $\mathcal{E}[\mathbf{n}_{R_l}\mathbf{n}_{R_l}^H] = \mathbf{I}_N \sigma_{R_l}^2$  and  $P_S$  is the transmit power of the users. During the second-time slot, relay first amplifies and then forwards its received signal towards the users. The transmitted signal from the relay is  $\mathbf{y}'_{R_l} = \hat{\beta}_l \hat{\mathbf{W}}_l \mathbf{y}_{R_l}$ , where  $\hat{\mathbf{W}}_l$  is the concatenated beamforming-and-amplification matrix at the relay and  $\beta_l$  is the amplification factor to satisfy the relay power constraint which is presented in the sequel. Here,  $\hat{\mathbf{W}}_l$  is designed to cancel sub-channel interference within a given TWRN, and hence, it is constructed by using receive-ZF and transmit-ZF precoding and detection concepts as follows [20]:<sup>1</sup>

$$\hat{\mathbf{W}}_{l} = \hat{\mathbf{G}}_{ll}^{*} (\hat{\mathbf{G}}_{ll}^{T} \hat{\mathbf{G}}_{ll}^{*})^{-1} \mathbf{P} (\hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll})^{-1} \hat{\mathbf{G}}_{ll}^{H},$$
(3)

where **P** is the block diagonal permutation matrix for user pairing, constructed as **P** = diag(**P**<sub>1</sub>, ..., **P**<sub>K</sub>) and **P**<sub>i</sub> =  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  for  $i \in \{1, \dots, K\}$ . Further, the relay power constraint is

$$P_R = \operatorname{Tr}\left(\mathbf{y}'_{R_l}\mathbf{y}'^H_{R_l}\right). \tag{4}$$

Next, the received signal at  $U_{l,k'}$  is given as

$$y_{l,k'} = \mathbf{g}_{lk'}^T \mathbf{y}_{R_l}' + n_{l,k'} = \hat{\beta}_l \sqrt{P_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{G}_{ll} \mathbf{x}_l + \hat{\beta}_l \sqrt{P_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{j=1, j \neq l}^L \mathbf{G}_{jl} \mathbf{x}_j + \hat{\beta}_l \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} + n_{l,k'}, \quad (5)$$

where  $U_{l,k}$  and  $U_{l,k'}$  are the paired-users exchanging their data signals with (k,k') = (2i - 1, 2i) for  $i \in \{1, \dots, 2K\}$ . Further,  $\mathbf{g}_{lk'}$  is the k'th column vector of the matrix  $\mathbf{G}_{ll}$  for  $k \in \{1, \dots, K\}$  and  $n_{l,k'}$  is the AWGN at the k'th user of the lth TWRN with variance  $\sigma_{n_{l,k'}}^2$ . (5) is further simplified as

$$y_{l,k'} = \hat{\beta}_l \sqrt{P_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{g}_{lk} x_{l,k} + \hat{\beta}_l \sqrt{P_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{m=1, m \neq k}^{2K} \mathbf{g}_{lm} x_{l,m}$$
$$+ \hat{\beta}_l \sqrt{P_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{j=1, j \neq l}^{L} \mathbf{G}_{jl} \mathbf{x}_j + \hat{\beta}_l \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} + n_{l,k'}, \quad (6)$$

where the first term is the desired signal at  $U_{l,k'}$  and other terms are the interferences and noise. Thus the end-to-end SINR at  $U_{l,k'}$ ,  $\gamma_{l,k'}$  is given in (7) at the top of this page. We derive the asymptotic SINR for three power scaling scenarios in next sections.

# D. Calculation of value $\hat{\beta}_l$

By using (2) and  $\mathbf{y}'_{R_l} = \hat{\beta}_l \hat{\mathbf{W}}_l \mathbf{y}_{R_l}$ , we simplify the power constraint (4) as

$$P_{R} = \hat{\beta}_{l}^{2} \operatorname{Tr} \left( \hat{\mathbf{W}}_{l} \left( P_{S} \sum_{j=1}^{L} \mathbf{G}_{jl} \mathbf{G}_{jl}^{H} + \sigma_{R_{l}}^{2} \mathbf{I}_{N} \right) \hat{\mathbf{W}}_{l}^{H} \right)$$
$$= \hat{\beta}_{l}^{2} P_{S} \operatorname{Tr} \left( \hat{\mathbf{W}}_{l} \sum_{j=1}^{L} \mathbf{G}_{jl} \mathbf{G}_{jl}^{H} \hat{\mathbf{W}}_{l}^{H} \right) + \hat{\beta}_{l}^{2} \sigma_{R_{l}}^{2} \operatorname{Tr} \left( \hat{\mathbf{W}}_{l} \hat{\mathbf{W}}_{l}^{H} \right).$$
(8)

By substituting  $\hat{\mathbf{W}}_l$  into (8),  $\hat{\beta}_l$  is obtained in (9) at the top of this page.

#### E. Overall sum rate of the system

The average sum rate for the lth 2K-user TWRN with estimated CSI at the relay is defined as [4]

$$\mathcal{R}_{l} = \frac{(T_{C} - \tau)}{2T_{C}} \sum_{k=1}^{2K} \mathcal{E}\left[\log\left(1 + \gamma_{l,k}\right)\right],\tag{10}$$

where  $T_C$  and  $\tau$  are the coherence time of the wireless channel and length of the pilot sequence used for channel estimation. In particular, the pre-log factor  $(T_C - \tau)/T_C$  accounts for the pilot overhead [4]. The two time-slots required for the data transmission between the paired users results in the pre-log factor of 1/2.

<sup>&</sup>lt;sup>1</sup>The results obtained in this paper can also be obtained for other detection and precoding methods such as matched filter and MMSE. However due to the page restrictions we have only included results for ZF detection and ZF precoding.

$$\lim_{N \to \infty} \frac{\hat{\beta}_l}{\sqrt{N}} = \left[ \frac{E_R}{E_S \sum_{j=1}^L \sum_{i=1}^K \left( \frac{\eta_{j,l,2i-1}^2 + \eta_{j,l,2i}^2}{\tau E_S * \hat{\eta}_{l,2i-1}^2 \hat{\eta}_{l,2i}^2} \right) + 2\sigma_{R_l}^2 \sum_{i=1}^K \left( \tau E_S \hat{\eta}_{l,2i-1} \hat{\eta}_{l,2i} \right)^{-2}} \right]^{\frac{1}{2}}.$$
(12)

$$\sqrt[4]{N} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} = \sqrt[4]{N} \mathbf{g}_{lk'}^T \hat{\mathbf{G}}_{ll}^* (\hat{\mathbf{G}}_{ll}^T \hat{\mathbf{G}}_{ll}^*)^{-1} \mathbf{P} (\hat{\mathbf{G}}_{ll}^H \hat{\mathbf{G}}_{ll})^{-1} \hat{\mathbf{G}}_{ll}^H \mathbf{n}_{R_l} = \frac{\mathbf{g}_{lk'}^T \hat{\mathbf{G}}_{ll}^*}{\sqrt{N}} \left( \frac{\hat{\mathbf{G}}_{ll}^T \hat{\mathbf{G}}_{ll}^*}{\sqrt{N}} \right)^{-1} \mathbf{P} \left( \frac{\hat{\mathbf{G}}_{ll}^H \hat{\mathbf{G}}_{ll}}{\sqrt{N}} \right)^{-1} \frac{\hat{\mathbf{G}}_{ll}^H \mathbf{n}_{R_l}}{\sqrt{N}}.$$
(18)

## F. Energy efficiency of the system

As mentioned in the introduction, for applications with lowpower sensors, the power/energy efficiency of the system will be very important. Thus, we analyze the energy efficiency of the system defined as [13]

$$\rho = \frac{\sum_{k=1}^{2K} \mathcal{E}\left[\log\left(1 + \gamma_{l,k}\right)\right]}{2KP_S + P_R},$$
(11)

where the denominator consists of the total power consumption of the system and the numerator consists of the overall sum rate.

## **III. ASYMPTOTIC PERFORMANCE ANALYSIS**

In this section, we derive asymptotic SINR and sum rate for the three power scaling laws [13] at the user nodes and relay whenever the relay antenna count grows unbounded  $(N \rightarrow \infty)$ . Moreover, we investigate the detrimental impacts of CCI, imperfect CSI, and pilot contamination on the SINR of the system.

#### A. Transmit power scaling at the user nodes

Whenever the transmit power at the user nodes is scaled inversely proportional to the relay antenna count, the power of the pilot sequence is also scaled accordingly. Thus, the overall transmit power can only be scaled inversely proportional to the square-root of the relay antenna count  $(\sqrt{N})$  for estimated CSI. We obtain the normalized relay gain for unlimited number of relay antennas by letting  $P_S = E_S/\sqrt{N}$ ,  $P_P = \tau E_S/\sqrt{N}$  and  $P_R = E_R$  while keeping  $E_S$  and  $E_R$  fixed, as (12) given at the top of this page (see Appendix B for the proof). Here, we define  $\hat{\eta}_{l,k}$  as

$$\hat{\eta}_{l,k} = \sum_{j=1}^{L} \eta_{j,l,k}.$$
(13)

The parameter  $\hat{\eta}_{l,k}$ , is a measure of the pilot contamination experienced by  $U_{l,k}$  and will extensively appear in SINR equations we obtain in the sequel. Further, if pilot contamination is absent, then  $\hat{\eta}_{l,k} = \eta_{l,l,k}$ . We rewrite (6) by dividing both sides by  $\sqrt[4]{N}$  and substituting  $E_S$  values as

$$\frac{y_{l,k'}}{\sqrt[4]{N}} = \frac{\hat{\beta}_l}{\sqrt{N}} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{g}_{lk} x_{l,k} + \frac{\hat{\beta}_l}{\sqrt{N}} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{m=1,m\neq k}^{2K} \mathbf{g}_{lm} x_{l,m} + \frac{\hat{\beta}_l}{\sqrt{N}} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{j=1,j\neq l}^{L} \mathbf{G}_{jl} \mathbf{x}_j + \frac{\hat{\beta}_l}{\sqrt{N}} \sqrt[4]{N} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} + \frac{1}{\sqrt[4]{N}} n_{l,k'}.$$
(14)

Next, we obtain the asymptotic limit of the intended signal term in (14) by using the limits given in Appendix A, as

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$$\lim_{N \to \infty} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{g}_{lk} x_{l,k} 
= \sqrt{E_S} \lim_{N \to \infty} \frac{\mathbf{g}_{lk'}^T \hat{\mathbf{G}}_{ll}^*}{\sqrt{N}} \left( \frac{\hat{\mathbf{G}}_{ll}^T \hat{\mathbf{G}}_{ll}}{\sqrt{N}} \right)^{-1} \mathbf{P} \left( \frac{\hat{\mathbf{G}}_{ll}^H \hat{\mathbf{G}}_{ll}}{\sqrt{N}} \right)^{-1} \frac{\hat{\mathbf{G}}_{ll}^H \mathbf{g}_{lk}}{\sqrt{N}} x_{l,k} 
= \sqrt{E_S} \frac{\eta_{l,l,k'} \eta_{l,l,k}}{\hat{\eta}_{l,k}^2} x_{l,k}.$$
(15)

Similarly we obtain the asymptotic limit of the second term, which represents the inter-user interference in (14), as

$$\lim_{N \to \infty} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{m=1, m \neq k}^{2K} \mathbf{g}_{lm} x_{l,m} = 0.$$
(16)

Above, inter-user interferences are removed due to ZF receiving and transmission at the relay. Even with a different beamforming method, it can be shown that this value asymptotically goes to zero. Further, we derive the asymptotic limit of the third term in (14), which is the co-channel interference from the nearby jth TWRN as

$$\lim_{N \to \infty} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{G}_{jl} \mathbf{x}_j$$

$$= \sqrt{E_S} \lim_{N \to \infty} \frac{\mathbf{g}_{lk'}^T \hat{\mathbf{G}}_{ll}^*}{\sqrt{N}} \left( \frac{\hat{\mathbf{G}}_{ll}^T \hat{\mathbf{G}}_{ll}^*}{\sqrt{N}} \right)^{-1} \mathbf{P} \left( \frac{\hat{\mathbf{G}}_{ll}^H \hat{\mathbf{G}}_{ll}}{\sqrt{N}} \right)^{-1} \frac{\hat{\mathbf{G}}_{ll}^H \mathbf{G}_{jl}}{\sqrt{N}} \mathbf{x}_j$$

$$= \sqrt{E_S} \frac{\eta_{l,l,k'} \eta_{j,l,k}}{\hat{\eta}_{l,k}^2} x_{j,k}.$$
(17)

Also, we rewrite the fourth term in (14), which represents the added noise at the relay as (18) given at the top of this page. Based on (18), we can obtain the asymptotic distribution of the fourth term as

$$\sqrt[4]{N} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} \xrightarrow[N \to \infty]{} \frac{\eta_{l,l,k'}}{\tau E_S \hat{\eta}_{l,k}^3} \widetilde{n}_l, \tag{19}$$

where  $\tilde{n}_l \sim C\mathcal{N}\left(0, \tau E_S \hat{\eta}_{l,k}^2 \sigma_{R_l}^2\right)$ . Furthermore, we obtain the asymptotic limit of the last term in (14) as

$$\lim_{N \to \infty} \frac{1}{\sqrt[4]{N}} n_{l,k'} = 0.$$
 (20)

By using the above asymptotic values and distributions of the terms in (14), we derive the asymptotic SINR for transmit power scaling at the user nodes as

$$\lim_{N \to \infty} \hat{\beta}_l = \left[ \frac{E_R}{E_S \sum_{j=1}^L \sum_{i=1}^K \frac{\left(\eta_{j,l,2i-1}^2 (1+\tau E_S \hat{\eta}_{l,2i}) + \eta_{j,l,2i}^2 (1+\tau E_S \hat{\eta}_{l,2i-1})\right)}{\tau E_S \hat{\eta}_{l,2i-1}^2 \hat{\eta}_{l,2i}^2}} \right]^{\frac{1}{2}}.$$
(22)

$$\lim_{N \to \infty} \frac{\hat{\beta}_l}{\sqrt[4]{N}} = \left[ \frac{E_R}{E_S \sum_{j=1}^L \sum_{i=1}^K \left( \frac{\eta_{j,l,2i-1}^2 + \eta_{j,l,2i}^2}{\tau E_S * \hat{\eta}_{l,2i-1}^2 \hat{\eta}_{l,2i}^2} \right) + 2\sigma_{R_l}^2 \sum_{i=1}^K \left( \tau E_S \hat{\eta}_{l,2i-1} \hat{\eta}_{l,2i} \right)^{-2}} \right]^{\frac{1}{2}}.$$
(24)

$$\lim_{N \to \infty} \gamma_{l,k'} = \frac{\frac{E_S \eta_{l,l,k'}^2 \eta_{l,l,k}^2}{\hat{\eta}_{l,k}^2}}{\frac{E_S \eta_{l,l,k'}^2 \sum_{j=1, j \neq l}^L \eta_{j,l,k}^2}{\hat{\eta}_{l,k}^1} + \frac{\eta_{l,l,k'}^2}{\tau E_S \hat{\eta}_{l,k}^4} \sigma_{R_l}^2} = \frac{\tau E_S^2 \eta_{l,l,k}^2}{\tau E_S^2 \sum_{j=1, j \neq l}^L \eta_{j,l,k}^2 + \sigma_{R_l}^2} = \gamma_{\infty}.$$
 (21)

## B. Transmit power scaling at the relay

In this case, we derive the asymptotic SINR by scaling transmit power at the relay inversely proportional to the number of relay antennas. Thus, we substitute  $P_R = E_R/N$ ,  $P_S = E_S$  and  $P_p = \tau E_S$  to obtain the asymptotic SINR. We omit the proofs in this section due to their similarity to the results in Section III-A. We obtain the asymptotic value of the normalized relay gain as (22) on the top of this page. Proof of (22) is based on the results in Appendix C.

Similar to the previous case, received signal at  $U_{l,k'}$  is given by (6). We use the limits on each term of (6) to obtain the SINR value at  $U_{l,k'}$  as

$$\lim_{N \to \infty} \gamma_{l,k'} = \frac{\Psi_l^2 E_S \eta_{l,l,k'}^2 \eta_{l,l,k}^2}{\Psi_l^2 E_S \eta_{l,l,k'}^2 \sum_{j=1, j \neq l}^L \eta_{j,l,k}^2 + \hat{\eta}_{l,k}^4 \sigma_{n_{l,k'}}^2}, \quad (23)$$

where  $\Psi_l = \lim_{N \to \infty} \hat{\beta}_l$  and given in (22).

#### C. Transmit power scaling at the user nodes and relay

In this section, we obtain the asymptotic SINR for the transmit power scaling at the relay and user nodes (where  $P_S = E_S/\sqrt{N}$ ,  $P_P = \tau E_S/\sqrt{N}$  and  $P_R = E_R/\sqrt{N}$ ). As in the previous section, we omit the proofs due to their similarity to the results in Section III-A. We obtain the asymptotic value of the normalized relay gain in (24) at the top of this page. Then by substituting  $E_S$  values, we rewrite (6) for the received signal as

$$y_{l,k'} = \frac{\hat{\beta}_l}{\sqrt[4]{N}} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{g}_{lk} x_{l,k} + \frac{\hat{\beta}_l}{\sqrt[4]{N}} \sqrt{E_S} \mathbf{g}_{lk}^T \hat{\mathbf{W}}_l \sum_{m=1,m\neq k}^{2K} \mathbf{g}_{lm} x_{l,m} + \frac{\hat{\beta}_l}{\sqrt[4]{N}} \sqrt{E_S} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \sum_{j=1,j\neq l}^{L} \mathbf{G}_j \mathbf{x}_j + \frac{\hat{\beta}_l}{\sqrt[4]{N}} \sqrt[4]{N} \mathbf{g}_{lk'}^T \hat{\mathbf{W}}_l \mathbf{n}_{R_l} + n_{l,k'}.$$
(25)

By using the same procedure as in the Section III-A, we obtain the SINR as

$$\lim_{N \to \infty} \gamma_{l,k'} = \frac{\tau \Lambda_l^2 E_S^2 \eta_{l,l,k'}^2 \eta_{l,$$

where  $\Lambda_l = \lim_{N \to \infty} \frac{\hat{\beta}_l}{\sqrt[4]{N}}$  and given in (24). **Remark V.1:** We can clearly see that the asymptotic SINRs

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**Remark V.1:** We can clearly see that the asymptotic SINRs (21), (23), and (26) are affected by the pilot contamination. Interestingly, whenever  $\eta_{j,l,k} = 0$  for  $j \neq l$  or when L = 1, the asymptotic SINRs results in (21), (23), and (26) approaches the same SINRs provided in [13]. Furthermore, by substituting asymptotic SINRs (21), (23), and (26) into (10) and (11), overall sum rate and the energy efficiency of the system is obtained.

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## IV. FINDING THE OPTIMAL PILOT SEQUENCE LENGTH

In this section we analyze the optimal pilot sequence length to maximize the sum rate of the system for the case with L = 1 under power scaling at the user nodes scenario. For this case, (21) can be written as

$$\lim_{N \to \infty} \gamma_{l,k'} = \frac{\tau E_S^2 \eta_{l,l,k}^2}{\sigma_{R_l}^2} = \tau \xi_{l,k},$$
(27)

where  $\xi_{l,k} = \frac{E_S^2 \eta_{l,l,k}^2}{\sigma_{R_l}^2}$ . By substituting this value to (10), we obtain the overall sum rate of the system as

$$\mathcal{R}_{l} = \frac{(T_{C} - \tau)}{2T_{C}} \sum_{k=1}^{2K} \log\left(1 + \tau \xi_{l,k}\right).$$
(28)

By taking the partial derivation of (28) with respect to  $\tau$  and by equating it to zero, we obtain the following equation.

$$\sum_{k=1}^{2K} \frac{(T_C - \tau)\xi_{l,k}}{1 + \tau\xi_{l,k}} = \sum_{k=1}^{2K} \ln\left(1 + \tau\xi_{l,k}\right).$$
(29)

Since the total sum rate can be maximized by maximizing the sum rates of individual user nodes, above equation can be rewritten and rearranged as

$$\exp\left(\frac{(T_C - \tau)\xi_{l,k}}{1 + \tau\xi_{l,k}} + 1\right) = (1 + \tau\xi_{l,k})\exp(1), \quad (30)$$

$$\frac{1+T_C\xi_{l,k}}{1+\tau\xi_{l,k}}\exp\left(\frac{1+T_C\xi_{l,k}}{1+\tau\xi_{l,k}}\right) = (1+T_C\xi_{l,k})\exp(1).$$
 (31)

By using the Lambert-W function [21], (31) is solved as

$$\mathbb{W}\left([1+T_C\xi_{l,k}]\exp(1)\right) = \frac{1+T_C\xi_{l,k}}{1+\tau\xi_{l,k}}.$$
(32)

The optimal value for  $\tau$  to maximize the sum rate can be derived as

$$\tau_k^* = \left\lfloor \frac{1}{\xi_{l,k}} \left( \frac{1 + T_C \xi_{l,k}}{\mathbb{W}\left( [1 + T_C \xi_{l,k}] \exp(1) \right)} - 1 \right) \right\rfloor, \qquad (33)$$

$$\bar{\beta}_{l}^{2} = \frac{\left(\left(N-2K\right)^{2}-1\right)\left(N-2K\right)P_{R}}{\left(\left(N-2K\right)^{2}-1\right)P_{S}\sum_{i=1}^{2K}\eta_{l,l,i}^{-1}+2\left(N-2K\right)\sigma_{R_{l}}^{2}\sum_{i=0}^{K}\left(\eta_{l,l,2i-1}\eta_{l,l,2i}\right)^{-1}}.$$
(40)

where  $\lfloor . \rfloor$  is the floor function. Analysis on the second partial derivative of  $R_l$  with respect to  $\tau$  shows that the solution in (33), maximizes the sum rate of the system.

## V. ANALYSIS FOR A FINITE NUMBER OF ANTENNAS

In this section, we derive the approximate closed-form sum rate for the three power scaling laws at the user nodes and relay with a finite number of antennas. Further, for mathematical tractability we limit our analysis for CCI free, perfect CSI and no pilot contamination case. Furthermore, we obtain the cumulative distribution function (CDF) and the probability distribution function (PDF) of the end-to-end SINR at  $U_{l,k'}$ .

For this case, the beamforming matrix  $\hat{\mathbf{W}}_l$  can be rewritten as

$$\hat{\mathbf{W}}_{l} = \mathbf{W}_{l} = \mathbf{G}_{ll}^{*} (\mathbf{G}_{ll}^{T} \mathbf{G}_{ll}^{*})^{-1} \mathbf{P} (\mathbf{G}_{ll}^{H} \mathbf{G}_{ll})^{-1} \mathbf{G}_{ll}^{H}, \qquad (34)$$

By using  $\mathbf{W}_l$ , the amplification factor  $\beta_l$  is given as

$$\beta_{l} = \sqrt{\frac{P_{R}}{P_{S} \operatorname{Tr}\left(\left[\mathbf{G}_{ll}^{T} \mathbf{G}_{ll}^{*}\right]^{-1}\right) + \sigma_{R_{l}}^{2} \operatorname{Tr}\left(\left[\mathbf{G}_{ll}^{H} \mathbf{G}_{ll}\right]^{-1} \mathbf{P}\left[\mathbf{G}_{ll}^{T} \mathbf{G}_{ll}^{*}\right]^{-1} \mathbf{P}\right)}.(35)$$

Using  $\mathbf{W}_l$  and the absence of co-channel interfering TWRNs, (5) is further simplified as

$$y_{l,k'} = \beta_l \sqrt{P_S} x_{l,k} + \beta_l \mathbf{1}_{k'} \mathbf{P} (\mathbf{G}_{ll}^H \mathbf{G}_{ll})^{-1} \mathbf{G}_{ll}^H \mathbf{n}_{R_l} + n_{l,k'},$$
(36)

where  $1_{k'}$  represents  $1 \times 2K$  vector with value 1 at the k' location and zeros in all other places. Thus the end-to-end SINR at  $U_{l,k'}$ ,  $\gamma_{l,k'}$  is derived as

$$\gamma_{l,k'} = \frac{\beta_l^2 P_S}{\beta_l^2 \sigma_{R_l}^2 \left[ \left( \mathbf{G}_{ll}^H \mathbf{G}_{ll} \right)^{-1} \right]_{k'} + \sigma_{n_{l,k'}}^2}$$
(37)

where  $\left[ \left( \mathbf{G}_{ll}^{H} \mathbf{G}_{ll} \right)^{-1} \right]_{k'}$  is the k'th diagonal entry of the matrix  $\left( \mathbf{G}_{ll}^{H} \mathbf{G}_{ll} \right)^{-1}$ . To begin the analysis of SINR value we use the long term power amplification factor  $\beta_l$ . Here, we use the following matrix identities from [22].

$$\operatorname{Tr}\left(\mathcal{E}\left[\left(\mathbf{G}_{ll}^{H}\mathbf{G}_{ll}\right)^{-1}\right]\right) = \frac{\operatorname{Tr}\left(\mathbf{D}_{ll}^{-1}\right)}{N-2K} = \frac{\sum_{i=1}^{2K} \eta_{l,l,i}^{-1}}{N-2K}.$$
(38)  
$$\operatorname{Tr}\left(\mathcal{E}\left[\left(\mathbf{G}_{ll}^{H}\mathbf{G}_{ll}\right)^{-1}\mathbf{P}\left(\mathbf{G}_{ll}^{T}\mathbf{G}_{ll}^{*}\right)^{-1}\mathbf{P}\right]\right)$$
$$= \frac{\operatorname{Tr}\left(\mathbf{P}\mathbf{D}_{ll}^{-1}\mathbf{P}\mathbf{D}_{ll}^{-1}\right)}{(N-2K)^{2}-1} + \frac{\operatorname{Tr}\left(\mathbf{P}\operatorname{Tr}\left(\mathbf{P}\mathbf{D}_{ll}^{-1}\right)\mathbf{D}_{ll}^{-1}\right)}{((N-2K)^{2}-1)(N-2K)}$$
$$= \frac{2\sum_{i=0}^{K}\left(\eta_{l,l,2i-1}\eta_{l,l,2i}\right)^{-1}}{(N-2K)^{2}-1}.$$
(39)

By using the above equations the value of  $\bar{\beta}_l$  is rewritten as (40) at the top of this page. By substituting the value of  $\bar{\beta}_l$  to (37), we obtain

$$\gamma_{l,k'} = \frac{\alpha_{l,k'} A}{\eta_{l,k'} X + \zeta_{l,k'}},\tag{41}$$

where  $X = \left( \left[ \left( \mathbf{G}_{ll}^{H} \mathbf{G}_{ll} \right)^{-1} \right]_{k',k'} \right)^{-1}$  and other symbols are given as follows:

$$\alpha_{l,k'} = \left( (N - 2K)^2 - 1 \right) (N - 2K) P_R P_S.$$
(42)

$$\zeta_{l,k'} = \left( (N - 2K)^2 - 1 \right) (N - 2K) P_R \sigma_{R_l}^2.$$
 (43)

$$\eta_{l,k'} = \left( (N - 2K)^2 - 1 \right) P_S \sigma_{n_{l,k'}}^2 \sum_{i=1}^{2K} \eta_{l,l,i}^{-1} + 2 \left( N - 2K \right) \sigma_{n_{l,k'}}^2 \sigma_{R_l}^2 \sum_{i=0}^K \left( \eta_{l,l,2i-1} \eta_{l,l,2i} \right)^{-1}.$$
(44)

By using distribution of the *k*th diagonal element of the inverse Wishart matrix [23], the CDF of  $\gamma_{l,k'}$  was obtained as [11]:

$$F_{\gamma_{l,k'}}(x) = \begin{cases} 1 - \frac{\Gamma\left(N - 2K + 1, \frac{\zeta_{l,k'}x}{\alpha_{l,k'} - \eta_{l,k'}x}\right)}{\Gamma(N - 2K + 1)}, \ 0 < x < \frac{\alpha_{l,k'}}{\eta_{l,k'}} \\ 1, \qquad x \ge \frac{\alpha_{l,k'}}{\eta_{l,k'}}. \end{cases}$$
(45)

By differentiating (45) by using the Leibniz integral rule the PDF is obtained as

$$\begin{split} \hat{r}_{\gamma_{l,k'}}(x) &= \frac{d}{dx} \bigg[ \frac{\zeta_{l,k'x}}{\alpha_{l,k'} - \eta_{l,k'x}} \bigg] \bigg( \frac{\zeta_{l,k'x}}{\alpha_{l,k'} - \eta_{l,k'x}} \bigg)^{N-2K} \frac{e^{-\frac{\zeta_{l,k'x}}{\alpha_{l,k'} - \eta_{l,k'x}}}}{\Gamma(N-2K+1)} \\ &= \frac{\alpha_{l,k'}(\zeta_{l,k'})^{N-2K+1} x^{N-2K} e^{-\frac{\zeta_{l,k'x}}{\alpha_{l,k'} - \eta_{l,k'x}}}}{\Gamma(N-2K+1)(\alpha_{l,k'} - \eta_{l,k'x})^{N-2K+2}}, \end{split}$$
(46)

where  $0 \le x < \frac{\alpha_{l,k'}}{\eta_{l,k'}}$ . The average sum rate can be approximated by solving  $\bar{R}_l = \frac{1}{2\ln(2)} \int_0^\infty \ln(1+x) f_{\gamma_{l,k'}}(x) \, dx$  as

$$\bar{\mathcal{R}}_{L^*} \approx \frac{1}{2\ln(2)} \frac{1}{(N-2K)!} \mathbb{I}_1,$$
 (47)

where  $\mathbb{I}_1$  is defined as follows:

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$$\mathbb{I}_{1} = \alpha_{l,k'} \zeta_{l,k'}^{N-2K+1} \int_{0}^{\frac{\alpha_{l,k'}}{\eta_{l,k'}}} \frac{x^{N-2K}}{(\alpha_{l,k'} - \eta_{l,k'}x)^{N-2K+2}} \\
\times \exp\left(-\frac{\zeta_{l,k'}x}{\alpha_{l,k'} - \eta_{l,k'}x}\right) \ln(1+x) \, \mathrm{d}x,$$
(48)

By substituting the dummy variable  $t = \zeta_{l,k'} x / (\alpha_{l,k'} - \eta_{l,k'} x)$ into (48) the integral  $\mathbb{I}_1$  can be simplified as

$$\mathbb{I}_{1} = \int_{0}^{\infty} t^{N-2K} e^{-t} \ln\left(\frac{\zeta_{l,k'} + (\alpha_{l,k'} + \eta_{l,k'})t}{\zeta_{l,k'} + \eta_{l,k'}t}\right) dt, \qquad (49)$$

Next,  $\mathbb{I}_1$  in (49) can be solved in closed-form as follows:

$$\mathbb{I}_{1} = \mathbb{J}(N - 2K, \zeta_{l,k'}, \alpha_{l,k'} + \eta_{l,k'}) - \mathbb{J}(N - 2K, \zeta_{l,k'}, \eta_{l,k'})$$
(50)

$$\mathbb{J}(x,y,z) = \int_0^\infty \lambda^x \exp(-\lambda) \ln(y+z\lambda) \, \mathrm{d}\lambda = \Gamma(x+1) \left( \ln(y) + \sum_{p=0}^x \frac{1}{\Gamma(x-p+1)} \left( \left(\frac{-y}{z}\right)^{x-p} \exp\left(\frac{y}{z}\right) E_1\left(\frac{y}{z}\right) + \sum_{q=1}^{x-p} \Gamma(q)\left(\frac{-y}{z}\right)^{x-p-q} \right) \right).$$
(51)

where the function  $\mathbb{J}(x, y, z)$  is defined in (51) at the top of this page. By substituting (50) into (47), an approximation of the sum rate can be derived in closed-form as in (52).

$$\bar{\mathcal{R}}_{L^*} = \frac{\mathbb{J}\left(N - 2K, \zeta_{l,k'}, \alpha_{l,k'} + \eta_{l,k'}\right) - \mathbb{J}\left(N - 2K, \zeta_{l,k'}, \eta_{l,k'}\right)}{2\ln(2) \left(N - 2K\right)!}.$$
 (52)

The sum rates under different power scaling scenarios can be obtained by substituting the  $P_S$  and  $P_R$  values to  $\alpha_{l,k'}$ ,  $\zeta_{l,k'}$  and  $\eta_{l,k'}$ . The results obtained in (52) will be useful to identify the number of antennas required to obtain a certain percentage of the asymptotic sum rate.

## VI. Asymptotic analysis for antenna correlation at the relay

In this section, we derive asymptotic results for the SINR and sum-rate under the antenna correlation at the relay nodes under the three power scaling scenarios identified in section III. Further, for mathematical tractability we limit our analysis for CCI free, perfect CSI and no pilot contamination case.

When there is antenna correlation at the relay the channel vector to the relay from the user k is written as  $\bar{\mathbf{g}}_{ll,k}^T = (\Psi_{l,k})^{\frac{1}{2}} \mathbf{f}_{ll,k}^T d_{lk}^{\frac{1}{2}}$ , where  $\Psi_{l,k}$  is the  $N \times N$  correlation matrix at the relay. Furthermore,  $\mathbf{f}_{ll,k}^T$  is the kth column vector in the matrix  $\mathbf{F}_{ll}$  and  $d_{lk}$  is the kth diagonal entry of the matrix  $\mathbf{D}_{ll}$  that is given in II-A. Accordingly the channel matrix between all relay and all the users is given as  $\bar{\mathbf{G}}_{ll} = \left[ \mathbf{g}_{ll,1}^T \ \mathbf{g}_{ll,2}^T \ \cdots \ \mathbf{g}_{ll,2K}^T \right]$ . This corresponds to the max-semi-correlated Rayleigh fading scenario presented in [24]. For this case, the beamforming matrix  $\hat{\mathbf{W}}_l$  can be rewritten as

$$\bar{\mathbf{W}}_{l} = \bar{\mathbf{G}}_{ll}^{*} (\bar{\mathbf{G}}_{ll}^{T} \bar{\mathbf{G}}_{ll}^{*})^{-1} \mathbf{P} (\bar{\mathbf{G}}_{ll}^{H} \bar{\mathbf{G}}_{ll})^{-1} \bar{\mathbf{G}}_{ll}^{H},$$
(53)

By using  $\mathbf{W}_l$ , the amplification factor  $\beta_l$  is given as

$$\bar{\beta}_{l} = \sqrt{\frac{P_{R}}{P_{S} \mathrm{Tr}\left(\left[\bar{\mathbf{G}}_{ll}^{T} \bar{\mathbf{G}}_{ll}^{*}\right]^{-1}\right) + \sigma_{R_{l}}^{2} \mathrm{Tr}\left(\left[\bar{\mathbf{G}}_{ll}^{H} \bar{\mathbf{G}}_{ll}\right]^{-1} \mathbf{P}\left[\bar{\mathbf{G}}_{ll}^{T} \bar{\mathbf{G}}_{ll}^{*}\right]^{-1} \mathbf{P}\right)}.(54)$$

By using the similar steps as in section V, we obtain the endto-end SINR at  $U_{l,k'}$ ,  $\gamma_{l,k'}$  as

$$\gamma_{l,k'} = \frac{\bar{\beta}_l^2 P_S}{\bar{\beta}_l^2 \sigma_{R_l}^2 \left[ \left( \bar{\mathbf{G}}_{ll}^H \bar{\mathbf{G}}_{ll} \right)^{-1} \right]_{k'} + \sigma_{n_{l,k'}}^2}$$
(55)

where  $\left[\left(\bar{\mathbf{G}}_{ll}^{H}\bar{\mathbf{G}}_{ll}\right)^{-1}\right]_{k'}$  is the k'th diagonal entry of the matrix  $\left(\bar{\mathbf{G}}_{ll}^{H}\bar{\mathbf{G}}_{ll}\right)^{-1}$ .

To analyse the asymptotic performance of antenna correlation at the relay, we first look at the limit results relevant to the channel matrix  $\bar{\mathbf{G}}_{ll}$ . Here, we obtain

$$\left[\frac{\bar{\mathbf{G}}_{ll}^{H}\bar{\mathbf{G}}_{ll}}{N}\right]_{i,j} = d_{li}^{1/2} \frac{\mathbf{f}_{ll,i}^{H} \left(\boldsymbol{\Psi}_{l,i}^{H} \boldsymbol{\Psi}_{l,j}\right)^{1/2} \mathbf{f}_{ll,j}}{N} d_{lj}^{1/2}.$$
 (56)



Fig. 1. Spectral efficiency versus the number of relay antennas of an 12-user TWRN with different L values. The channels in  $\mathbf{G}_{jl}$  are i.i.d. Rayleigh RVs with  $\mathbf{D}_{ll} = \mathbf{I}_{2K}$  and  $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$ , where  $j, l \in \{1, \dots, L\}$  and  $j \neq l$ .

By using the limit results, it can be shown that if  $i \neq j$ , then the value of (65) goes to zero. If i = j then the above value equals  $Tr(\Psi_{l,j})$  which is equal to N for correlation matrices. Based on this, the limit result can be given as

$$\left[\frac{\mathbf{G}_{ll}^{H}\mathbf{G}_{ll}}{N}\right] \xrightarrow[N \to \infty]{a.s.} \mathbf{D}_{ll}, \tag{57}$$

and coincidently the asymptotic results for the case with antenna correlation is equal to the results obtained for the case without any antenna correlation. This shows that by using massive MIMO, the degenerative effect of antenna correlation can be removed in our system model.

## VII. SIMULATION RESULTS

This section presents our simulation results and comparisons with the derived asymptotic results. The power at the user nodes and the relay nodes is taken as  $E_S = E_R = 10$ , and the noise powers at user and relay nodes is taken as  $\sigma_{n_{l,k}}^2 = \sigma_{n_R}^2 = 1$ . The pathloss exponent  $\eta$  is assumed to be two. The normalized pilot sequence power factor  $\tau/T_C$  is 0.8. Spectral and energy efficiencies under different power scaling scenarios, different K values, and different L values are presented in the sequel.

What is the effect of having multiple TWRNs on the spectral efficiency and the energy efficiency? In Fig. 1 and Fig. 2, the spectral efficiency and the energy efficiency are presented for power scaling at user nodes (case 1), for L = 2, L = 4 and L = 8 values, when K = 6, respectively. The obtained asymptotic values (21) are also plotted for comparison. We can see in Fig. 1 that for all L values, the spectral efficiency asymptotically reaches our analytical results, validating our derived results. Furthermore, as the number of TWRNs (L) in the system is increased, the



Fig. 2. Energy efficiency versus the number of relay antennas of 12-user, L TWRNs. The channels in  $\mathbf{G}_{jl}$  are i.i.d. Rayleigh RVs with  $\mathbf{D}_{ll} = \mathbf{I}_{2K}$  and  $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$ , where  $j, l \in \{1, \dots, L\}$  and  $j \neq l$ .

achievable spectral efficiency and the energy efficiency of a single TWRN decreases due to the interference and pilot contamination introduced by other TWRNs. As an example, a L = 2 system can obtain 4.9bps/Hz efficiency while an L = 8 system can only achieve a spectral efficiency of 3bps/Hz. However, if we consider the spectral efficiency of the whole system (by multiplying the spectral efficiency of a single TWRN by L), we can conclude that the bandwidth can be utilized further by increasing the number of TWRNs. According to the values obtained in Fig. 1, the total spectral efficiency is 9.8 bps/Hz when L = 2 and approximately 15 bps/Hz when L = 8. Thus we can conclude that by using multipair massive MIMO TWRNs for pairwise communications between nodes, that the limited bandwidth can be utilized effectively.

We analyze the effect of number of users in a single system (2K) in Fig. 3. Specifically, the sum rate is plotted for a system with eight relay networks under power scaling at the relay nodes for K = 2 and K = 6 (case 2). A four-user TWRN achieves 0.62bps/Hz while a 12-user TWRN obtains 1.28 bps/Hz. Once again, we have plotted our analytical results (23), which match the simulated values. Note that the spectral efficiency increases as the number of users increases, as the same bandwidth is used by the additional users. Thus, a massive MIMO pairwise TWRN improves bandwidth utilization by serving more users. However, there will be countering factors that will limit the number of user pairs in a network, such as the number of available orthogonal pilot sequences which is limited by the coherence time of the system.

We compare the spectral efficiency gains and energy efficiency gains of different power scaling scenarios in Fig. 4 and Fig. 5 for 12-user TWRNs (L = 8). The analytical results ((21, (23), and (26)) are also plotted for comparison purposes. Power scaling at the user nodes has the highest asymptotic of 7.2 bps/Hz out of all the three cases while case-2 and case-3 achieved asymptotically 5.2 bps/Hz and 1.8 bps/Hz, respectively. Moreover,



Fig. 3. Spectral efficiency versus the number of relay antennas of a L = 4 TWRN systems with different number of user pairs. The channel gains of  $\mathbf{G}_{jl}$  are i.i.d. Rayleigh RVs. with  $\mathbf{D}_{ll} = \mathbf{I}_{2K}$  and  $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$ , where  $j, l \in \{1, \dots, L\}$  and  $j \neq l$ .



Fig. 4. Spectral efficiency versus the number of relay antennas of 12-user, L = 8 TWRNs. The channels in  $\mathbf{G}_{jl}$  are i.i.d. Rayleigh RVs with  $\mathbf{D}_{ll} = \mathbf{I}_{2K}$  and  $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$ , where  $j, l \in \{1, \dots, L\}$  and  $j \neq l$ .

in Fig. 5 power scaling at both user and relay nodes obtains the highest energy efficiency as expected. Furthermore, the energy efficiency of case-2 (power scaling at the relay only) is very low compared to other two cases. This result is expected as the power of user nodes are kept unchanged in this scenario. Thus the numerator in (11) is relatively constant for different N values, and thus the power efficiency will be low.

How accurate are our analytical results when the number of antennas is finite? In Fig. 6, we answer this question by plotting the sum rate results. The simulated sum rate values match with our closed-form result in (52), justifying the accuracy of our approximation. For example, as few as 14 relay antennas yields about 85% of the asymptotic performance ( $N = \infty$ ). Moreover,



Fig. 5. Energy efficiency versus the number of relay antennas of 12-user, 8 TWRNs. The channels in  $\mathbf{G}_{jl}$  are i.i.d. Rayleigh RVs with  $\mathbf{D}_{ll} = \mathbf{I}_{2K}$  and  $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$ , where  $j, l \in \{1, \dots, L\}$  and  $j \neq l$ .



Fig. 6. Sum rate versus the number of relay antennas of a 4-user TWRN (L = 1) under different power scaling scenarios. The channel gains of  $G_{ll}$  are i.i.d. Rayleigh RVs.

increasing to N = 22 relay antennas yields a 92%. This is good news because more or less the performance of massive MIMO is possible with a finite number of antennas.

How much degradation of the sum rate occurs due to antenna correlation? In Fig. 7, we have used the antenna correlation model given in [25, Eqn. (4)]. Here, the (p,q)th element of correlation matrix is given as  $e^{-j2\pi(p-q)l\cos(\theta)}e^{-\frac{1}{2}(2\pi(p-q)l\sin(\theta)\sigma^2)}$  [24], where l is the relative antenna spacing,  $\theta$  is the average angle of arrival/departure, and  $\sigma$  is the standard deviation of the angle of arrival/departure. We have plotted the sum rate of the system under low correlation. As seen from Fig. 7, although antenna correlation degrades the sum rate, as the number of relay antennas increases, the sum rate approaches to that of the uncorrelated



Fig. 7. Sum-rate versus the number of relay antennas of a 4-user TWRN (L = 1) under relay antenna correlation.

antenna case. This observation corroborates our analysis that the effect of antenna correlation can be mitigated by a massive relay antenna array.

# VIII. CONCLUSION

For multi-pair massive MIMO TWRNs, we have investigated the impact of several key impairments, namely co-channel interference, imperfect CSI, antenna correlation and pilot contamination. We derived the asymptotic SINR, sum rate, and energy efficiency for the three transmit power scaling laws. Importantly, we show that the user transmit power can be scaled down inversely proportional to the square-root of the number of relay antennas. Notably, if power scaling is limited to the relay node, power can be scaled down inversely proportional to the number of relay antennas without any performance penalty. Our analytical and simulation results reveal that substantial sum-rate gains and energy-efficiency gains can be achieved via a massive antenna array at the relay. Also, the closed-form results obtained for a finite number of antennas at the relay help wireless engineers to compute the number of antennas required to obtain a certain percentage of the asymptotic sum rate. Further, our results show that massive MIMO mitigates the degenerative effects of antenna correlation.

In terms of future research, further work is required to rigorously analyze (by considering all imperfections) the sum-rate results for multiple massive MIMO TWRNs with not so large number of antennas, especially in a practical range between 100 to 200.

# APPENDIX A PROOF OF LIMITS

In this section, several important limit results are provided. We begin with expressing the following three results [26].

$$\frac{\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}}{N} \xrightarrow[N \to \infty]{a.s.} E_{P} \operatorname{diag}\left(\frac{\eta_{j,l,1} \sum_{j=1}^{L} \eta_{j,l,1}}{1 + E_{P} \sum_{j=1}^{L} \eta_{j,l,1}}, \cdots, \frac{\eta_{j,l,2K} \sum_{j=1}^{L} \eta_{j,l,2K}}{1 + E_{P} \sum_{j=1}^{L} \eta_{j,l,2K}}\right), \quad \text{for} \quad P_{P} = E_{P}.$$
(64)

$$\frac{\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}} \xrightarrow[N \to \infty]{} E_P \operatorname{diag}\left(\eta_{j,l,1} \sum_{j=1}^{L} \eta_{j,l,1}, \cdots, \eta_{j,l,2K} \sum_{j=1}^{L} \eta_{j,l,2K}\right), \quad \text{for} \quad P_P = \frac{E_P}{\sqrt{N}}.$$
(65)

$$\operatorname{Tr}\left(\mathbf{G}_{jl}\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{T}\hat{\mathbf{G}}_{ll}^{*}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\hat{\mathbf{G}}_{ll}^{H}\right) = \frac{1}{\sqrt{N}}\operatorname{Tr}\left(\frac{\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}}\right]^{-1}\mathbf{P}\left[\frac{\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}}\right]^{-1}\mathbf{P}\left[\frac{\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}}\right]^{-1}\hat{\mathbf{G}}_{ll}^{H}\mathbf{G}_{ll}\right]^{-1}\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll} = \frac{1}{\sqrt{N}}\operatorname{Tr}\left(\frac{\mathbf{G}_{jl}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}}\right]^{-1}\mathbf{P}\left[\frac{\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}}\right]^{-1}\hat{\mathbf{G}}_{ll}^{H}\mathbf{G}_{jl}\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\hat{$$

$$\sqrt{N} \operatorname{Tr} \left( \mathbf{G}_{jl} \mathbf{G}_{jl}^{H} \hat{\mathbf{G}}_{ll} \left[ \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll} \right]^{-1} \mathbf{P} \left[ \hat{\mathbf{G}}_{ll}^{T} \hat{\mathbf{G}}_{ll}^{*} \right]^{-1} \mathbf{P} \left[ \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll} \right]^{-1} \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll} \right]^{-1} \hat{\mathbf{G}}_{ll}^{H} \underbrace{\mathbf{G}}_{ll} \frac{\mathbf{G}_{ll}^{H} \hat{\mathbf{G}}_{ll}}{\mathbf{G}_{ll}^{-1} \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll}} \xrightarrow{a.s.} \frac{1}{E_{P}} \sum_{i=1}^{K} \left( \frac{\eta_{j,l,2i-1}^{2}}{\eta_{l,2i-1}^{2} \hat{\eta}_{l,2i-1}} + \frac{\eta_{j,l,2i-1}^{2}}{\hat{\eta}_{l,2i-1} \hat{\eta}_{l,2i}^{2}} \right).$$
(67)

$$N \operatorname{Tr} \left( \mathbf{G}_{jl} \mathbf{G}_{jl}^{H} \hat{\mathbf{G}}_{ll} \left[ \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll} \right]^{-1} \mathbf{P} \left[ \hat{\mathbf{G}}_{ll}^{T} \hat{\mathbf{G}}_{ll}^{*} \right]^{-1} \mathbf{P} \left[ \hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll} \right]^{-1} \mathbf{Q} \left[ \hat{\mathbf{G}}_{ll$$

For two independent vectors,  $\mathbf{p} \sim \mathcal{CN}_{N \times 1}(0, \sigma_p^2)$  and  $\mathbf{q} \sim \mathcal{CN}_{N \times 1}(0, \sigma_q^2)$ , the following identities are valid.

$$\mathbf{p}^{H}\mathbf{p}/N \xrightarrow[N \to \infty]{} \sigma_{p}^{2} \quad \text{and} \quad \mathbf{p}^{H}\mathbf{q}/N \xrightarrow[N \to \infty]{} 0, \quad (58)$$

$$\mathbf{p}^{H}\mathbf{q}/\sqrt{N} \xrightarrow[N \to \infty]{d} \mathcal{CN}\left(0, \sigma_{p}^{2}\sigma_{q}^{2}\right),$$
(59)

where subscripts a.s. and d stands for almost sure convergence and the convergence of distributions, respectively. By using the aforementioned identities, it can be shown that

$$\frac{\mathbf{G}_{jl}^{H}\mathbf{G}_{jl}}{N} = \mathbf{D}_{jl}^{\frac{1}{2}} \left(\frac{\mathbf{F}_{jl}^{H}\mathbf{F}_{jl}}{N}\right) \mathbf{D}_{jl}^{\frac{1}{2}} \xrightarrow[N \to \infty]{a.s.} \mathbf{D}_{jl}, \quad \text{and}$$
(60)

$$\frac{\mathbf{G}_{jl}^{H}\mathbf{G}_{ml}}{N} = \mathbf{D}_{jl}^{\frac{1}{2}} \left( \frac{\mathbf{F}_{jl}^{H}\mathbf{F}_{ml}}{N} \right) \mathbf{D}_{ml}^{\frac{1}{2}} \xrightarrow[N \to \infty]{a.s.} \mathbf{0}_{2K}, \text{ for } j \neq m.$$
(61)

Furthermore, by using the above two results, following limit can be obtained.

$$\frac{\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}}{N} = \frac{1}{N} \left[ \left( \sum_{j=1}^{L} \mathbf{G}_{jl} + \frac{\mathbf{V}_{l}}{\sqrt{P_{p}}} \right) \tilde{\mathbf{D}}_{ll} \right]^{H} \left( \sum_{j=1}^{L} \mathbf{G}_{jl} + \frac{\mathbf{V}_{l}}{\sqrt{P_{p}}} \right) \tilde{\mathbf{D}}_{ll} \\
= \tilde{\mathbf{D}}_{ll}^{H} \left( \sum_{j=1}^{L} \sum_{m=1}^{L} \frac{\mathbf{G}_{jl}^{H}\mathbf{G}_{ml}}{N} + \frac{\mathbf{V}_{l}^{H}\sum_{j=1}^{L}\mathbf{G}_{jl}}{\sqrt{P_{p}}N} + \frac{\sum_{j=1}^{L}\mathbf{G}_{jl}^{H}\mathbf{V}_{l}}{\sqrt{P_{p}}N} + \frac{\mathbf{V}_{l}^{H}\mathbf{V}_{l}}{N} \right) \tilde{\mathbf{D}}_{ll} \\
= \tilde{\mathbf{D}}_{ll}^{H} \left( \sum_{j=1}^{L} \mathbf{D}_{jl} + \mathbf{I}_{2K} \right) \tilde{\mathbf{D}}_{ll} \xrightarrow{a.s.}{N \to \infty} \mathbf{H}_{ll}, \quad (62)$$

where  $\mathbf{H}_{ll}$  is a diagonal matrix with kth diagonal entry given by  $\frac{P_P\left(\sum_{j=1}^{L} \eta_{j,l,k}\right)^2}{1+P_P \sum_{j=1}^{L} \eta_{j,l,k}}$ . Furthermore when transmit power scaling is done at the users (ie.  $P_P = E_P / \sqrt{N}$ ), the following limit can be obtained.

$$\frac{\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}}{\sqrt{N}} \xrightarrow[N \to \infty]{a.s.} \hat{\mathbf{H}}_{ll}, \tag{63}$$

where  $\hat{\mathbf{H}}_{ll}$  is a diagonal matrix with *k*th diagonal entry given by  $E_P\left(\sum_{j=1}^L \eta_{j,l,k}\right)^2$ . Furthermore, the limit results (64) and (65) that are shown at the top of this page are obtained using the same procedure.

# APPENDIX B

## PROOF OF LIMITS FOR POWER SCALING AT THE USER NODES

This section provides a sketch of the proof of SINR for the transmit power scaling scenario. First we prove the limits for the power normalizing factor  $\hat{\beta}_l$ . The first term in the denominator in (9) can be written as shown in (66) at the top of this page. By using the limit results (62) and (63) given in Appendix

A on each term in the above equation, following result is formulated as (67) shown at the top of this page. Here in (67),  $\hat{\eta}_{l,k} = \sum_{j=1}^{L} \eta_{j,l,k}$ . Similarly, the limit of the second term in the denominator in (9) is derived as

$$N \operatorname{Tr}\left(\left[\hat{\mathbf{G}}_{ll}^{H} \hat{\mathbf{G}}_{ll}\right]^{-1} \mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{T} \hat{\mathbf{G}}_{ll}^{*}\right]^{-1} \mathbf{P}\right) \xrightarrow[N \to \infty]{a.s.} 2 \sum_{i=1}^{K} \frac{1}{\left(E_{P} \hat{\eta}_{l,2i-1} \hat{\eta}_{l,2i}\right)^{2}}.$$
 (68)

By using the above two results, (12) is obtained.

# APPENDIX C

# PROOF OF LIMITS FOR POWER SCALING AT THE RELAY NODES

This section provides a sketch of the proof of SINR for the transmit power scaling at the relay. The first term in the denominator in (9) can be written in a similar way by replacing  $\sqrt{N}$  by N in (66) in Appendix B. By using the limit results (62) and (64) given in Appendix A, the limit result (69) shown at the top of this page is obtained. Similarly, the limit of the second term in the denominator in (9) is derived as

$$N^{2} \operatorname{Tr}\left(\left[\hat{\mathbf{G}}_{ll}^{H}\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^{T}\hat{\mathbf{G}}_{ll}^{*}\right]^{-1}\mathbf{P}\right) \xrightarrow[N \to \infty]{a.s.}{N \to \infty}$$
$$2\sum_{i=1}^{K} \left(\frac{1+P_{P}\hat{\eta}_{l,2i-1}}{P_{P}\hat{\eta}_{l,2i-1}}\right)^{2} \left(\frac{1+P_{P}\hat{\eta}_{l,2i}}{P_{P}\hat{\eta}_{l,2i}}\right)^{2}.$$
 (70)

By using the above two results, (22) is obtained.

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