A Novel And Tractable Antenna Selection in Spatial Modulation Systems

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Abstract—A novel opportunistic antenna selection aided spatial modulation, called opportunistic spatial modulation (OSM), is proposed, which exhibits an attractive system reliability enhancement with low complexity. Its unique features enable a comprehensive analytical framework, which is challenging to acquire with existing transmit-antenna-selection-aided spatial modulation (TASS-SM) schemes. Closed-form expression of improved union bound of the average symbol error probability (ASEP) of proposed OSM-MISO system is derived. Furthermore, we compare the proposed OSM with a prevalent existing TASS-SM scheme to confirm the feasibility and effectiveness of our scheme. Simulation results are provided to corroborate the analytical results.

Index Terms—Antenna selection, multiple-input single-output, spatial modulation, symbol error probability.

I. INTRODUCTION

Spatial modulation (SM) is recently emerging as a promising research field to overcome some drawbacks of conventional multiple-input multiple-output (MIMO), such as, inter-antenna synchronization, ICI, high complexity and huge energy consumption, by activating only one of multiple available transmit antennas with a single-RF chain for each transmission. However, for each channel use, SM achieves higher spectral efficiency (SE) than that of conventional single-antenna transmission via transmitting a 3D modulation signal by combining the antenna-index modulation as well as the conventional 2D signal modulation (such as QAM/PSK) [1].

SM is an open-loop scheme, as the active transmit antenna is randomly determined by the spatially modulated information bits [2]. However, the system is likely to transmit with errors if the channel of active antenna is in highly faded condition. To circumvent such scenario, some closed-loop SM schemes have been proposed by exploiting CSI at the transmitter (CSIT) to improve system reliability. Transmit antenna subset selection-aided SM (TASS-SM), i.e., implementing SM based on selected antennas subset instead of all the transmit antennas, is an attractive such adaptive strategy to boost the reliability of SM [3]–[16]. A capacity optimized TASS (COAS) scheme was introduced for SM-MIMO system in [3], which does not offer transmit-diversity but increases the coding gain. An antenna correlation based TASS was also developed in [4]. Euclidean distance (ED) optimized based TASS (EDAS) scheme has shown to be able to offer high transmit diversity gains for SM-MIMO system, which maximizes the minimum ED of the received SM constellation (error performance), by performing exhaustive search over all the possible antenna subset [5]. The diversity order of EDAS-SM was quantified by [6]. Nevertheless, the excellent performance of EDAS-SM scheme is gained at the cost of high computational complexity. Subsequently, a number of work has focused on complexity reduction of EDAS-SM schemes, via either cutting the complexity of evaluating ED [3], [7], [8], [13] or reducing the search complexity burden [4], [9]–[12], [14]. While recently, more practical scenarios of EDAS-SM systems were examined in [15] with a realistic error-infested feedback channel and in [16] by considering frequency selective channels. However, most of existing TASS-SM are heuristic schemes, which, unfortunately, lead to performance analysis intractable or extremely difficult. For example, although the COAS-SM is one of the simplest TASS-SM schemes, its explicit error performance analysis is limited and almost unexplored [3].

To the best of our knowledge, analytical modeling/ frameworks for TASS-SM are rarely studied in the literature to date. To fulfill this research gap, in this paper, we propose a novel low-complex TASS-SM scheme, called Opportunistic Spatial Modulation (OSM), by judiciously combining the opportunistic antenna selection and SM to significantly enhance the system reliability. Its unique features enable us to develop a comprehensive analytical framework of OSM over Rayleigh fading channels. Note that, we focus on MISO systems for simplicity, but it can easily be extended to MIMO systems by conducting a priori single-antenna selection on receiver. The key technical contributions of this paper are: 1). We propose a novel and tractable antenna selection scheme in SM systems called OSM. A complete derivation of explicit expression of error performance of OSM-MISO system, based on accurate 'improved unionbound' method, is presented, which is highly challenging to quantitatively characterize with existing TASS-SM schemes. 2). We also compare the proposed OSM with prevalent low-complex COAS-SM scheme. Our OSM outshines COAS-SM in terms of analytical tractability, reduced CSI feedback bits, low complexity and superior performance as SNR increases, which implies our OSM scheme is a more efficient low-complexity TASS-SM scheme. 3). The proposed OSM analytical framework also opens up a new avenue for possible designing the TASS-SM schemes with all the key merits of tractability, low-complexity and high diversity order.

II. OPPORTUNISTIC SPATIAL MODULATION (OSM)

We consider an OSM-MISO system as shown in Fig.1, which consists of a MISO wireless link with $N_t$ transmit
antennas. The cardinality of the APM signal constellation diagram is denoted by $M$ ($M \geq 2$). We assume both $N_t$ and $M$ are to be power of two. All the channels involved are assumed to be i.i.d Rayleigh fading. The transmitter-to-receiver channel is denoted by $h = [h_1, \ldots, h_{j}, \ldots, h_{N_t}] \in \mathbb{C}^{1 \times N_t}$, where the entry $h_j$ is the channel coefficient between the $j$th transmit antenna to the receiver, and $h_j \sim \mathcal{CN}(0,1)$. The idea of OSM scheme is to opportunistically select the transmit antennas subset as the new spatial-constellation diagram for SM, and is thus called as opportunistic SM (OSM) scheme. More specifically, OSM scheme follows two steps: i) the antenna selection, and ii) the conventional SM implementation. Step I: At the transmitter, $N_t$ transmit antennas are equally split into $K$ groups ($K$ is also assumed to be a power of two). Let $N_g \triangleq \frac{N_t}{K}$. For each Group $k$, $k \in [1,K]$, the antenna with the largest channel gain is selected for the later SM transmission, where the selected antenna index and the corresponding channel are represented by $N(k)$ and $g(k)$, respectively, and given as $N(k) = \arg\max_{i \in [(k-1)N_g+1, kN_g]} |h_i|$ and $g(k) = \max_{i \in [(k-1)N_g+1, kN_g]} |h_i|$. Step II: The conventional SM technique is implemented based on the selected $K$ antennas $\{N(1), \ldots, N(K)\}$ and $M$-ary APM symbols.

Thus, in the OSM scheme, the information bits are conveyed via both the spatial constellation represented by $K$ different antenna index set $\{N(1), \ldots, N(K)\}$, and the $M$-ary APM signal constellation comprised of symbols set $\{s_1, \ldots, s_M\}$ (E[$|s_m|^2$] = 1), given as $B = \log_2(K) + \log_2(M)$ bpcu. The first $\log_2(K)$ bits are used to choose an unique antenna index from $\{N(1), \ldots, N(K)\}$, and the other $\log_2(M)$ bits are used to map an APM symbol from $\{s_1, \ldots, s_M\}$. Then at each transmission, only one of the $K$ selected antennas is activated for transmitting the mapped APM symbol while all the other antennas remain in silent. Example: The OSM mapping principle is shown in Fig. 2 for a transmitter with $N_t = 4$, $K = 2$ and the BPSK modulation. We then have $B = \log_2(2) + \log_2(2) = 2$ bpcu, where one bit is encoded in the antenna group indexes (the bit “0” for Group 1 and “1” for Group 2), and the other bit is encoded in the BPSK symbols (the bit “1” for $s_1 = +1$ and “0” for $s_2 = -1$). Given binary input bits “01”, it means the first antenna (which is the ‘best’ antenna of Group 1) will be activated to emit the $s_1$.

It is easy to find that when $K = N_t$, the OSM-MISO scheme reduces to the conventional SM-MISO system; when $K = 1$, the OSM-MISO scheme reduces to the conventional MISO with the ‘best’ single transmit antenna selection scheme (TAS-MISO). For encoding and decoding purposes, the opportunistic antenna selection rules and the OSM mapping rules are assumed to be known by both the transmitter and receiver a priori. However, the transmitter does not required to access full CSI. This can be achieved by assuming that the receiver can correctly estimate the full CSI and sends the indexes $\{N(1), \ldots, N(K)\}$ to transmitter via a perfect feedback link.

For a given transmission instance, only one transmit antenna is in active. Without loss of generality, the transmission of Group $k$ with symbol $s_m$ is elaborated here. Thus, the transmitted signal can be given as $x_k,m = A \times [x_k, m]$, where $x_k,m \in \mathbb{C}^{N_t \times 1}$; $A \in \mathbb{C}^{N \times N_t}$ is a diagonal matrix denoted as $A = \text{diag}(a_1, \ldots, a_N)$, with $a_1 = 1$ if the $i$th antenna is selected as the ‘best’ antenna in one of the antenna group, i.e. $i \in \{N(1), \ldots, N(K)\}$, otherwise $a_i = 0$. $\Upsilon_k \in \mathbb{C}^{N_t \times 1}$ indicates that the antenna Group $k$ is chosen by first $\log_2(K)$ information bits, which is given as

$$\Upsilon_k \triangleq \begin{bmatrix} 0 & \ldots & 0 & 1 & 0 & \ldots & 0 \end{bmatrix}^T ; \quad (1)$$

and $s_m$ is one of the APM symbols determined by the rest $\log_2(M)$ information bits. Then, the received signal at the receiver of OSM-MISO, denoted as $y$, can be given as

$$y = hA \Upsilon_k x_k s_m + n = \sum_{j=(k-1)N_t+1}^{kN_t} a_j h_j s_m + n = g(k) s_m + n$$

where $n \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise; (a) follows as we consider Group $k$ transmission; (b) follows as Group $k$ has only one active antenna $N_k$ with corresponding channel $g(k)$. OSM-MISO can be treated as conventional SM-MISO with effective channels $g \in \mathbb{C}^{1 \times K} = \{g(1), \ldots, g(K)\}$. Furthermore, similar to conventional SM-MISO, the maximum likelihood (ML) detection criterion is adopted at the receiver of our OSM-MISO to jointly decode both the active antenna group index and the transmitted symbol, given as,

$$(k, m) = \arg\min_{k \in [1,K], m \in [1,M]} |y - g(k) s_m|^2.$$
probability (SEP). Define $\mathbb{X}$ as the set of all possible OSM transmit signal vectors, i.e., $\mathbb{X} = \{\mathbf{x}_{k,m} = \mathbf{A}_k \mathbf{s}_m | k = 1, \ldots, K; \ m = 1, \ldots, M\}$, with the size $|\mathbb{X}| = KM$. We assume all the elements of $\mathbb{X}$ to be equally likely. It is easy to find that each signal vector has only one non-zero element, and the dissimilar between any two signal vectors $\mathbf{x}_{k,m}$ and $\mathbf{x}_{k',m'}$ is either one element or two elements. The first situation occurs when $\mathbf{x}_{k,m}$ and $\mathbf{x}_{k',m'}$ have same active antenna group index (i.e., $k = k'$); while the second situation happens when they have different antenna activation position (i.e., $k \neq k'$).

Based on above mentioned two situations, the average SEP (ASEP), denoted as $P_S$, can be calculated as [17]

$$P_S = \frac{1}{K} \sum_{k} P_{\text{sgn}}(k) + \frac{1}{KM} \sum_{k} \sum_{k' \neq k} \sum_{m} \sum_{m'} \mathbb{E}_h \left[ \Pr \{ \mathbf{x}_{k,m} = \mathbf{x}_{k',m'} | \mathbf{x}_{k,m} \} \right]$$

(2)

where $P_{\text{sgn}}(k) = \frac{1}{M} \sum_{m} \mathbb{P}_{g}(k) \left[ \Pr \{ s_{m} = s_{m'} | s_{m} \} \right]$. We can interpret (2) as follows: (1) The first term $\frac{1}{K} \sum_{k} P_{\text{sgn}}(k)$ corresponds to the ASEP of OSM-MISO when $k = k'$; It means that only the symbol is wrongly detected, and $P_{\text{sgn}}(k)$ can thus be regarded as the ASEP of a conventional TAS-MISO for a given Group $k$. (2) The second term is the ASEP of the OSM-MISO when $k \neq k'$, which implies that the antenna group index is detected incorrectly.

The exact analysis of the $P_S$ in (2) is an arduous task due to the intricate 2nd term. However, due to the low implement complexity of OSM-MISO, the low-moderate SNR performance can be easily obtained via Monte Carlo, and the computational difficulty only exists in getting the high-SNR performance. Therefore, here we focus on obtaining an exact upper bound performance, which is highly accurate in high-SNR. For this purpose, it is natural to consider conventional union-bound. But the challenge in (2) is only the 2nd term, this is known as “Improved Union-bound” [17], a tighter upper bound for ASEP. Denoting such bound as $P_{IU}$, it can be formulated as,

$$P_S \leq P_{IU} = \frac{1}{K} \sum_{k} P_{\text{sgn}}(k) + \frac{1}{KM} \sum_{k} \sum_{k' \neq k} \sum_{m} \sum_{m'} \text{APEP}_{(k,m)\rightarrow(k',m')}$$

(3)

where $\text{APEP}_{(k,m)\rightarrow(k',m')}$ is average pairwise error probability (APEP) of $\mathbf{x}_{k,m}$ being erroneously decoded as $\mathbf{x}_{k',m'} \epsilon \mathbb{X}$.

$$\text{APEP}_{(k,m)\rightarrow(k',m')} \triangleq \mathbb{E}_g \left[ \Pr \{ |y - g(k)s_m| \geq |y - g(k')s_{m'}| | g \} \right]$$

$$= \mathbb{E}_g \left[ Q \left( \frac{\sqrt{|g(k)s_m - g(k')s_{m'}|^2}}{2\sigma^2} \right) \right].$$

(4)

with $Q(\cdot)$ being the Gaussian Q-function.

### III. Performance Analysis of OSM-MISO

#### A. Explicit Error Performance of OSM-MISO

In this section, we derive the closed-form expression for the improved-upper bound of the ASEP given in (3) with PSK signal modulation. We first derive the exact expression for the first term of (3), i.e., $\frac{1}{K} \sum_{k} P_{\text{sgn}}(k)$. As $P_{\text{sgn}}(k)$ can be considered as the exact ASEP of the conventional TAS-MISO with the selected best channel $g(k)$, according to [18, eq.(30)] [19, eq.(5.66)], $P_{\text{sgn}}(k)$ only depends on the distribution of its channel gain $|g(k)|^2$. Given all the channels are i.i.d Rayleigh fading distributed, according to [20], the pdf distribution of the selected best channel gain for $k$-th antenna group (denoted as $z_k = |g(k)|^2$) has been calculated as [17]

$$f(z_k) = N_g (1 - e^{-z_k}) N_8^{-1} e^{-z_k}, \quad k = 1, \ldots, K$$

(5)

As all $|g(1)|^2, \ldots, |g(K)|^2$ are also i.i.d, we can obtain $P_{\text{sgn}}(1) = \cdots = P_{\text{sgn}}(K) \triangleq P_{\text{sgn}}$, i.e., $\frac{1}{K} \sum_{k} P_{\text{sgn}}(k) = P_{\text{sgn}}$. Then based on [19] and the distribution (5), the closed-form expression for $P_{\text{sgn}}$ with the $M$-PSK modulation is given in the following Proposition 1.

**Proposition 1:** With $M$-PSK modulation, the exact expression of $P_{\text{sgn}} = P_{\text{sgn}}(k), \forall k$ is

$$P_{\text{sgn}} = \frac{M - 1}{M} - \frac{1}{M} \sum_{n=0}^{N_g-1} \left( \frac{N_g}{n+1} \frac{1}{\pi} \sqrt{\frac{\sin^2\left(\frac{n\pi}{M}\right)}{\sigma^2(1+n) + \sin^2\left(\frac{\pi}{M}\right)}} \sqrt{\frac{\sin^2\left(\frac{n\pi}{M}\right)}{\sigma^2(1+n) + \sin^2\left(\frac{\pi}{M}\right)} \cot\left(\frac{\pi}{M}\right)} \right).$$

(6)

**Proof:** The proof can be obtained by applying Binomial theorem and [19, eq.(5A.15)] to [18, eq.(30)] and [19, eq.(5.66)], and is omitted due to space limit.

Note that our Proposition 1 provides a simpler closed-form expression of $P_{\text{sgn}}$ than [18, eq.33]), which involved $N_g$ number of infinite sums and still too complicated to compute.

Now, we focus on deriving the closed-form formulation for the second term of (3). To obtain the analytic expression of $\text{APEP}_{(k,m)\rightarrow(k',m')}$, it is indispensable to know the distribution of $|g(k)s_m - g(k')s_{m'}|$ when $k \neq k'$. This, in general, is a challenging task, but we are able to tackle this impediment. Let $H \triangleq |g(k)s_m - g(k')s_{m'}|$ when $k \neq k'$. The complex random variable $g(k)$ can be written as $g(k) = r_k e^{j\theta_k}, k = 1, \ldots, K$ where $r_k = |g(k)|$ and $\theta_k$ is the phase of $g(k)$. For the $M$-PSK modulation, any symbol can be given as $s_m = e^{j\pi m/M}, \forall m = 1, \ldots, M$.

Then, we have $H = \left| r_k e^{j(\theta_k + \frac{2\pi m}{M})} - r_k e^{j(\theta_k' + \frac{2\pi m'}{M})} \right| = \sqrt{\left| r_k e^{j(\theta_k + \frac{2\pi m}{M})} + r_k e^{j(\theta_k' + \frac{2\pi m'}{M})+\pi} \right|^2}$. The PDFs of $r_k$ and $\theta_k$ are given in the following Lemma 1.

**Lemma 1:** Given that $g(k) = r_k e^{j\theta_k}, k = 1, \ldots, K, r_k = |g(k)|$, $\forall k$ are iid, with pdf given as

$$f_{r_k}(x) = 2N_g xe^{-x^2} \left(1 - e^{-x^2}\right)^{N_g-1}.$$ (7)

The phase $\theta_k$ is uniformly distributed over the range $[0, 2\pi]$, i.e., $f_{\theta_k}(y) = \frac{1}{2\pi}$; And $r_k$ and $\theta_k$ are independent.

**Proof:** See Appendix A.

From Lemma 1, we can see that $r_k$ and $r_{k'}$ are iid, so do $\theta_k$ and $\theta_{k'}$; $\theta_k + \frac{2\pi m}{M}$ and $\theta_{k'} + \frac{2\pi m'}{M} + \pi$ are also uniformly distributed in $[0, 2\pi]$. This also implies that the $M$-PSK
symbols have no impact on $\Psi$. By using the result of the distribution for the sum of complex random variables [21, Eq. (10)], the PDF of $\Psi$ can be given as

$$f_\Psi(x) = xH_{0x}\{\Lambda(\rho)\}$$ (8)

where $H_{0x}\{\Lambda(\rho)\} = \int_0^\infty \rho J_0(x\rho)\Lambda(\rho)d\rho$ is the zero-order Hankel transform of function $\Lambda(\rho)$, $J_0(\cdot)$ is the zero-order Bessel function of the first kind, and $\Lambda(\rho) = \mathbb{E}_{r_y,\mu_y}\{J_0(r_\rho\mu)J_0(r_\rho\mu)\}$. Based on Lemma 1 and (8), we can acquire the following Lemma 2 and Proposition 2.

**Lemma 2:** The PDF of $\Psi$ can be derived as

$$f_\Psi(x) = \sum_{n=0}^{N_{\beta}-1} \sum_{l=0}^{N_{\gamma}-1} \frac{2(N_g)!}{n!(N_g-1-n)!}(\frac{N_g}{n+1})^{l+1}e^{-\frac{(n+1)(l+1)x^2}{2}}$$ (9)

**Proof:** See Appendix B. 

**Proposition 2:** The closed-form expression for $\text{APEP}_\Psi(k,m)\rightarrow(k',m')$ can be derived as

$$\text{APEP}_\Psi(k,m)\rightarrow(k',m') = \sum_{n=0}^{N_{\beta}-1} \sum_{l=0}^{N_{\gamma}-1} \left(\frac{N_g}{n+1}\right)^{l+1} e^{-\frac{(n+1)(l+1)x^2}{2}} \left[1 - \left(1 + \frac{4(n+1)(l+1)}{n+l+2}x^2\right)^{-\frac{1}{2}}\right]$$ (10)

**Proof:** See Appendix C.

Proposition 2 shows an interesting observation that given $k \neq k'$ and the $M$-PSK modulation, the $\text{APEP}_\Psi(k,m)\rightarrow(k',m')$ in (10) is identical for any given set of $(k, k', m, m')$. This is because i) all selected channels are iid; and ii) the effective phase distributions of channels after absorbing the impact of symbol are all iid uniform distributions. Thus for the presentation simplicity, we denote $\overline{\text{APEP}} = \text{APEP}_\Psi(k,m)\rightarrow(k',m')$, $\forall k \neq k', \forall m, m'$. Therefore, for the proposed OSM-MISO scheme, an explicit improved-union-bound can be derived as

$$P_{IU} = P_{\text{aseal}} + M(K-1)\overline{\text{APEP}}$$ (11)

with closed-form expressions of $P_{\text{aseal}}$ and $\overline{\text{APEP}}$ given in (6) of Proposition 1 and (10) of Proposition 2, respectively.

**B. Comparisons between OSM and other TASS-SM scheme**

We compare our OSM scheme with a popular low-complexity TASS-SM schemes, i.e., COAS-SM, which chooses $K$ out of $N_t$ transmit antennas that corresponding to the first $K$ largest channel gains [3]. Then, the conventional SM scheme is implemented on selected $K$ antennas with $M$-ary APM symbols. Despite distinct antenna selections, the OSM and COAS-SM schemes support for the same data rate, i.e., $B = \log_2\{KM\}$ bpcu, and have identical set-up when $K = 1$ (pure antenna selection TAS) and $K = N_t$ (conventional SM). However when $1 < K < N_t$, with significantly dissimilar set-ups, our OSM outshines COAS-SM with following benefits.

**Analytical tractability:** The performance analysis of OSM scheme is tractable, as shown in Section III, benefit from its independent group antenna selection setting. On the contrary, although also being a low-complexity TASS-SM scheme, no much explicit analysis of the COAS-SM has been carried out [3]. And such analysis is challenging for COAS-SM, due to the coupling of order statistic distribution of selected $K$ antennas, and the problem is greatly exacerbated by increasing the number of $K$.

**Reduced feedback bits for CSI:** For both schemes, the receiver only needs to feedback the selected antenna indices to the transmitter. The COAS-SM scheme requires $\log_2\frac{N_t}{K}$ feedback bits, which is much larger than that of OSM scheme $K\log_2\frac{N_t}{K}$. E.g., when $N_t = 32$ and $K = 8$, the OSM and COAS-SM schemes will feedback at least 16 and 24 bits, respectively.

**Lower complexity:** (1) For antenna selection - while the OSM scheme requires $K$ times of sorting $N_t$ elements, the COAS-SM requires sorting $N_t$ elements which costs higher computations, e.g., when $N_t = 32$ and $K = 8$, the worst-case complexity ($O(n^2)$) of the COAS-SM scheme is eight times higher than that of OSM scheme. (2) For analytical computation - Since there is no rigorous analytical expressions to calculate error rate of the COAS-SM scheme, we always need to perform Monte-Carlo simulations. Although the COAS-SM scheme may has low implement complexity, it consumes significant time for numerical evaluation, especially in high-SNR region. And the problem is dramatically aggravated by increasing the number of $N_t$ and/or feedback bits $B$. Conversely, the high-SNR performance of the OSM scheme can be instantly computed by the derived explicit expressions with a high accuracy.

**Superior performance at high SNR:** While the both scheme have similar performance at low SNR, the benefit of OSM scheme becomes more pronounced as SNR increases. Performance comparison is shown in Section IV.

**IV. NUMERICAL RESULT**

In this section, we validate the derived theoretical expressions, and evaluate the performance of the proposed OSM-MISO scheme via numerical simulations. The performance is also compared with COAS-SM-MISO. It is important to note that, for a given number of transmit antennas $N_t$, in order to convey fixed $B$ information bits, we may have different combinations of $K$ (number of antenna groups) and $M$ (modulation size), to achieve $\log_2KM = B$ bits.

Fig. 3 shows the ASEP performance of OSM-MISO scheme versus the average SNR when $N_t = 8$ and $B = 6$ bits by considering some possible set of $(K, M)$ combinations such as $(K, M) = \{(1, 64), (2, 32), (8, 8)\}$ ( $(K, M) = (4, 16)$ case is omitted to make the figure less busy). It illustrates the accuracy of the improved union-bound (analytical expression (11)), by comparing with exact ASEP (Monte Carlo simulations) and the conventional union-bound. For all three cases, figure shows that the improved union-bound is more accurate than the union-bound as expected. Furthermore, improved union-bound is completely identical to the Monte Carlo results of $K = 1$ case (i.e., conventional TAS-MISO case) for the entire SNR region. It also well overlap with the exact performance of $K \geq 2$ cases for the moderate or high SNR region. These validate the explicit expressions of improved union-bound of OSM-MISO in (11), which thus can be employed to instantly and efficiently obtain the ASEP of OSM-MISO.
scheme at moderate-to-high SNR instead of Monte Carlo simulations. Fig.4 demonstrates the performance comparison between OSM-MISO and COAS-SM-MISO, for $N_t = 16$ with 4, 6, 8 bits data rate ($1 < K < N_t$) ($K = 4$ cases are omitted to keep figure clear), respectively. From the Fig.4, we can see that for any given cases with $1 < K < N_t$, at low SNR, the performance of two schemes looks very similar, however, as SNR increases, OSM-MISO clearly exhibits pronounced advantage and outshines COAS-SM-MISO, especially for small $K$ cases. Although, at high SNR, both schemes fail to offer the transmit diversity, the coding gain of our OSM scheme surpasses that of the COAS-SM scheme. This confirm that OSM-MISO is a more effective low-complexity TASS-SM scheme than the existing COAS-SM-MISO.

V. CONCLUSIONS AND POSSIBLE EXTENSIONS TO OSM-MIMO SYSTEMS

In this paper, a novel and tractable low-complexity TASS-SM scheme, OSM, is proposed to enhance system reliability. We develop a closed-form expression for error performance of OSM-MISO system over Rayleigh fading channels. By comparing to a prevalent TASS-SM scheme (COAS-SM), OSM shows excellent and appealing performance-complexity trade-off. The concept and the results of OSM-MISO can be easily extended to MIMO systems, i.e., more than one receive antenna, with a priori best single-antenna selection on the receiver side as well.

APPENDIX

A. Proof of Lemma 1

Let $h_i = b_i e^{j\phi_i}$, $\forall i = 1, \ldots, N_t$, where $b_i = |h_i|$ is the magnitude of $h_i$ and $\phi_i$ is the phase. Since $h_i \sim CN(0, 1)$, $b_i$ follows the Rayleigh distribution as $f_{b_i}(u) = 2ue^{-u^2}$, and $b_i^2$ follows the exponential distribution as $f_{b_i^2}(v) = e^{-v}$. The phase $\phi_i$ is uniformly distributed over the range $[0, 2\pi]$, i.e., $f_{\phi_i}(t) = \frac{1}{2\pi}$, and it is independent of $b_i$. For Group $k, \forall k = 1, \ldots, K$, as $N_k = \arg \max_{k(N_k-1)+1 \leq i \leq kN_k} b_i^2$, the joint CDF of the magnitude and phase of $g_i(k)$ is given as

$$F_{(r_x, \theta_x)}(x, y) = \Pr \left \{ b_{N_k}^2 | x, \phi_{N_k} \leq y \big| b_{N_k}^2 = \max_{k(N_k-1)+1 \leq i \leq kN_k} b_i^2 \right \}$$

$$= \Pr \left \{ b_{N_k}^2 \leq x, \phi_{N_k} \leq y \big| b_{N_k}^2 = \max_{k(N_k-1)+1 \leq i \leq kN_k} b_i^2 \right \}$$

$$= \int_{y}^{\min(\infty, y)} \int_{0}^{\infty} \frac{f_{b_k^2}(v) f_{\theta_k}(t)}{2\pi} e^{-v} d\theta_k$$

$$= \int_{y}^{\infty} \frac{1}{2\pi} \left( \frac{e^{-v}}{v} \right)$$

Thus, the CDF of $r_k$ is given as $F_{r_k}(x) = F_{(r_x, \theta_x)}(x, 2\pi) = 1 - e^{-x^2}$ and the CDF of $\theta_k$ is given as $F_{\theta_k}(y) = F_{(r_x, \theta_x)}(\infty, y) = \frac{y}{2\pi}$. Since we have $F_{(r_x, \theta_x)}(x, y) = F_{r_k}(x)F_{\theta_k}(y)$, random variables $r_k$ and $\theta_k$ are independent. The PDFs of $r_k$ and $\theta_k$ can be obtained respectively as $f_{r_k}(x) = \frac{\partial F_{r_k}(x)}{\partial x} = 2N_g (1 - e^{-x^2}) N_k^{-1} x e^{-x^2}$, and $f_{\theta_k}(y) = \frac{\partial F_{\theta_k}(y)}{\partial y} = \frac{1}{2\pi}$.

B. Proof of Lemma 2

$r_k$ and $r_k$ are i.i.d. with the pdf given in Lemma 1, then,

$$\Lambda(\rho) = \mathbb{E}_{r_k} \left[ J_0(r_k\rho) \right]^2$$

$$= \left( \int_{0}^{\infty} J_0(r_k\rho) 2N_g (1 - e^{-r_k^2}) N_k^{-1} r_k e^{-r_k^2} dr_k \right)^2$$

$$= \left( \frac{N_g}{2\pi} \int_{0}^{\infty} J_0(\sqrt{t}\rho) e^{-(n-1)t} dt \right)^2$$

$$= \left( \frac{N_g}{2\pi} \sum_{n=0}^{N_g-1} \frac{(-1)^n}{n!(N_g - 1 - n)!} \int_{0}^{\infty} J_0(\sqrt{t}\rho) e^{-(n-1)t} dt \right)^2$$

$$= \left( \frac{N_g}{2\pi} \sum_{n=0}^{N_g-1} \frac{(-1)^n}{(n+1)!(N_g - 1 - n)!} \frac{1}{2\pi} \int_{0}^{\infty} J_0(\sqrt{t}\rho) e^{-(n+1)t} dt \right)^2$$

$$= \frac{N_g}{2\pi} \sum_{n=0}^{N_g-1} \sum_{l=0}^{N_g-1} \frac{(N_g)^2(-1)^{n+l} e^{-\frac{(n+1)(n+2)+l(l+1)}{4\pi N_g^2}}}{(n+1)!(N_g - 1 - n)!(l+1)!(N_g - 1 - l)!}$$

$\rho$)

where (a) is obtained by applying the Binomial theorem $(1 + x)^n = \sum_{l=0}^{n} \binom{n}{l} x^l$, and (b) is due to $\int_{0}^{\infty} J_0(b\sqrt{t}) e^{-ax} dt = \frac{1}{a} e^{-\frac{b^2}{4a}}$ [22, eq 6.614]. Substituting (13) into (8), we have,

$$f_{\phi}(t) = x \int_{0}^{\infty} \rho J_0(x \rho) \Lambda(\rho) d\rho$$

$$= \frac{N_g}{2\pi} \sum_{n=0}^{N_g-1} \sum_{l=0}^{N_g-1} \frac{(N_g)^2(-1)^{n+l} x \int_{0}^{\infty} \rho J_0(x \rho) e^{-\frac{(n+1)(n+2)+l(l+1)}{4\pi N_g^2}} d\rho}{(n+1)!(N_g - 1 - n)!(l+1)!(N_g - 1 - l)!}$$

$$= \frac{N_g}{2\pi} \sum_{n=0}^{N_g-1} \sum_{l=0}^{N_g-1} \frac{2(N_g)^2(-1)^{n+l} x \int_{0}^{\infty} \frac{e^{-\frac{(n+1)(n+2)+l(l+1)}{4\pi N_g^2}}}{(n+1)!(N_g - 1 - n)!(l+1)!(N_g - 1 - l)!(n+l+2)!}}$$

$\rho^2$}

(14)
According to [22, eq 6.614], we can get (13) into (15), we have,

\[
 f(a) = \text{due to applying the integral representation of}
\]

**C. Proof of Proposition 2**

From (4), we have,

\[
 \text{AEP}(k, m) \rightarrow (k', m') = E_k \left[ Q \left( \sqrt{\frac{g(k) s_m - g(k') s_{m'}}{2\sigma^2}} \right) \right]
\]

\[
 = \int_0^\infty Q \left( \frac{x}{\sqrt{2\sigma^2}} \right) f_q(x) dx
\]

\[
 = \int_0^\infty Q \left( \frac{x}{\sqrt{2\sigma^2}} \right) x \int_0^\infty \rho J_0(x \rho) \Lambda(\rho) d\rho dx
\]

\[
 \frac{1}{2} \int_0^\infty e^{-\beta} \left[ I_0(\beta) - I_1(\beta) \right] A \left( \frac{\sqrt{23}}{\sigma} \right) d\beta
\]

(15)

(a) is due to applying the integral representation of \( f_q(x) \) from (8) and (b) is obtained by applying [23, eq.(20)]. Substituting (13) into (15), we have,

\[
 \text{AEP}(k, m) \rightarrow (k', m') = \frac{1}{2} \sum_{n=0}^{N} \sum_{l=0}^{N} \frac{(N!)^2 (-1)^{n+l}}{(n+l)! (N - 1 - n)! (l+1)! (N - 1 - l)!}
\]

\[
 \int_0^\infty \left[ I_0(\beta) - I_1(\beta) \right] e^{-\beta} \left( \frac{\sqrt{23}}{\sigma} \right) d\beta
\]

(16)

According to [22, eq 6.611.4], we can get \( \int_0^\infty I_0(\beta) e^{-c \beta} d\beta = \frac{1}{\sqrt{c^2 - 1}} \), \( c > 1 \) and \( \int_0^\infty I_1(\beta) e^{-c \beta} d\beta = \frac{\sqrt{c^2 - 1}}{c} \). Thus we have,

\[
 \int_0^\infty \left[ I_0(\beta) - I_1(\beta) \right] e^{-c \beta} d\beta = 1 - \frac{c - 1}{c + 1}, \quad c > 1
\]

(17)

Applying (17) into (16), the closed-form expression of \( \text{AEP}(k, m) \rightarrow (k', m') \) in (10) is obtained.

**References**


