Coverage Analysis of Decode-and-Forward Relaying in Millimeter Wave Networks

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***Abstract*—In this paper, we demonstrate the coverage probability improvement of a millimeter wave (mmWave) network due to the deployment of spatially random decode-and-forward (DF) relays. We assume the transmitter and receiver are located at a fixed distance and that the potential relay nodes are spatially distributed as a two dimensional homogeneous Poisson point process (PPP). We first derive the spatial distribution of potential set of relays which meet the required SNR (signal-to-noise ratio) threshold. From this set, we select a relay that has minimum path-loss from the receiver and derive the coverage probability achievable due to this selection. The analysis is based on stochastic geometry and is verified via Monte-Carlo simulation. The coverage probabilities of (a) direct link without relaying and (b) relayed link are compared to show that relaying provides significant coverage improvements.**

I. Introduction

The unprecedented wireless data growth is expected to continue in upcoming years due to the emergence of data demanding services and massive number of connected devices envisioned in the fifth generation of wireless networks [1]. Given that sub-6 GHz bands are already saturated and that the spectral efficiencies of current systems already approaching theoretical limits, millimeter wave (mmWave) frequency bands (20-100 GHz) offer huge bandwidth opportunities [1], [2].

However, unlike the sub-6 GHz bands, mmWave bands suffer very high propagation loss, give directional channels, and exhibit high sensitivity to blockages [3]. The channel measurement results show that path-loss is substantially different in line-of-sight (LOS) and non-line-of-sight (NLOS) regions making it necessary to use different path-loss exponents for LOS and NLOS cases [3]. Because of this disparity, mmWave links are susceptible to outage resulting in poor coverage even at the nearby receivers which fall in NLOS zone [4].

Relay aided transmission has been established as an effective way to increase the coverage, throughput and reliability in the conventional wireless networks [5]–[7]. Authors in [6] have derived the closed-form expressions for the outage probability and average channel capacity and demonstrate that selecting a relay node that maximizes the signal-to-noise ratio (SNR) at the destination provides the full diversity order. In [7], authors investigate the relay selection problem in networks with multiple users and multiple common amplify-and-forward (AF) relays.

In mmWave networks, because of the blockage, the role of relays is considered to be critical to provide seamless coverage to NLOS regions such as the areas blocked by buildings, and also to extend outdoor to indoor coverage [4]. In [8], the authors provide the first multi-hop medium access control protocol for a 60 GHz network, by utilizing the diffracted signals to overcome outage when the direct transmitter-receiver link is not available. The use of densely placed mmWave relays has been investigated to improve the coverage in [9] where the authors propose amplify-and-forward (AF) mmWave relays and evaluate coverage probability by considering spatially random located relays, effect of blockage, and log-normal shadowing.

Majority of works on performance analysis of wireless relay networks consider the fixed network topology, where the locations of users and relay nodes are assumed fixed and known. However, in practice, the location of users as well as the relays are not fixed because of deployment constraints or mobility. Therefore, in our system, we assume the spatially random placement of the relay nodes and model their locations using a homogeneous Poisson point process (PPP). The PPP model is widely used to analyze wireless networks as it allows for the tractable computation of coverage probability [10]. For instance, the PPP model and other stochastic geometry models have been used to study self-backhauled cellular networks [10], ad-hoc networks [11], and multi-tier cellular networks [12]. The PPP has been used to model the locations of base stations and user nodes [10]–[13].

For sub-6 GHz bands, relaying has been widely studied using stochastic geometry [14], [15]. Reference [14] investigates a decode-and-forward (DF) cooperative network by considering PPP distributed relay locations. Reference [15] uses a PPP model for the location of relays to evaluate the DF cognitive relay outage. For mmWave bands, on the other hand, relaying has also been studied using stochastic geometry, albeit not so widely. For example, amplify-and-forward (AF) relays for one-way [9] and two-way [16] relaying show significant improvement in coverage probability and spectral efficiency for mmWave networks impaired by high path loss and blockages.

However, to the best of our knowledge, the performance of DF mmWave relays considering spatial randomness and small-scale fading has not been investigated thus far. We thus believe that this paper is the first work using stochastic geometry to investigate a DF relay assisted mmWave network and to derive the coverage probability in the presence of small-scale fading. To this end, we consider Nakagami-m distributed channels with different m-parameters for LOS and NLOS cases. The effect of blockage from obstacles such as urban buildings is considered in our analysis.

II. System Model

A. Network Modeling

We consider a mmWave wireless network with a source (S), a receiver (D) and a set of relays distributed in $\mathbb{R}^2$ according to homogeneous PPP of density λ (Fig. 1). The distance between the source and the receiver is fixed and denoted by L. The $S-D$ communication occurs either directly or through opportunistic relaying from the available set of relays. We denote a typical relay by R. The source transmits with power P_{S} and we assume equal transmit power of P_{R} for all the relays. Without loss of generality, the receiver is assumed to be located at the origin.

The spatial distribution of the relays on $\mathbb{R}^2$-plane is denoted by $\Phi = \{x_1, x_2, x_3, ...\}$, where x_j is the location of j-th relay, $j \in \{1, 2, ..., N\}$ and N is a Poisson random variable. In our analysis, we only consider the relay nodes which are within the distance $\mathcal{R}$ from receiver, i.e., the nodes inside the circular disc $\mathcal{S}$. The nodes outside $\mathcal{S}$ are ignored because of very high path loss and increased blockage probability associated with large distances. Therefore, $\mathcal{S}$ is essentially equivalent to entire $\mathbb{R}^2$ [16]. In addition, for notational convenience, we remove the subscript j and just use x to denote the location of typical relay node, R, and x is interchangeably used as (r, θ) in polar coordinate system.

B. Blockage Modeling

Blockages occur when a mmWave signals cannot penetrate certain obstacles such as buildings [17], causing a link to be in either LOS or NLOS condition. Here we use the LOS probability function $p_L(d)$ [18], where each link of length d has a LOS probability $p_L(d) = e^{-\beta d}$ and NLOS probability $1 - p_L(d)$. The constant β depends on the size and density of blocking obstacles [18].

C. Directional Beamforming Modeling

We assume that all the nodes (source, relay and receiver) are capable of directional beamforming. We model the directivity similar to in [10], where the directional gain within the half power beamwidth (ϕ) is $G_{\max}$ and is $G_{\min}$ in all other directions. Mathematically, the gain may be represented as

$$G(\theta) = \begin{cases} G_{\max} & \text{if } |\theta| \leq \frac{\phi}{2} \\ G_{\min} & \text{otherwise.} \end{cases}$$

In our analysis, we consider the perfect beam alignment between the communicating nodes, i.e., $S-R$ or $R-D$, which provides the effective antenna gain, $G_{\mathrm{eq}} = G_{\max}^2$ in a given link and derive coverage probability. The analysis of beam misalignment is out of the scope of this paper and is considered for future work.

D. Small Scale Fading

We model the small scale fading by Nakagami distribution and consider different fading parameters of N_L and N_N for LOS and NLOS links, respectively. When the small scale fading is denoted with $h_l, l \in \{L, N\}$, the fading power $|h_l|^2$ follows a normalized Gamma distribution. In our analysis, we consider positive integer values for N_L and N_N and use large N_L value to approximate LOS scenario where the fading variation is low and small value for N_N to realize fast variation in the NLOS links [13].

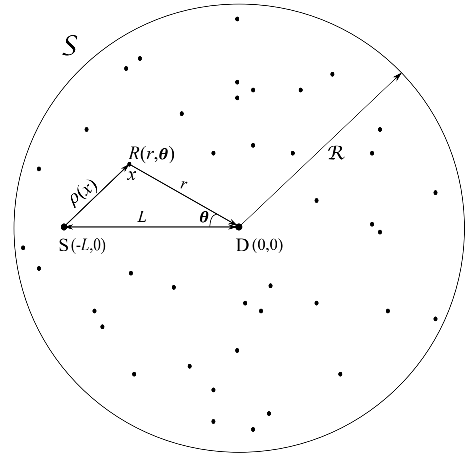

Fig. 1: Geometrical locations of source (S), receiver (D), and a typical relay (R).

III. Performance Analysis

In this section, we analyze the coverage probability for the proposed system model of mmWave DF relay network. Since we assume the system to operate in either direct mode or in relaying mode, we derive the coverage probabilities for each of these cases.

A. Direct Mode

In this case, the source and receiver can communicate with each other without the help of a relay. When the transmission distance is short and if the S-D link is in LOS, it is possible to achieve the required SNR or data rate using the direct link. This has additional benefit of using only one time slot compared to two time slots required in relay assisted transmission.

1) Coverage Probability of Direct Link: Coverage probability is defined as the probability that the received SNR is above some predefined threshold γ_{th}.

Lemma 1. *The coverage probability of the direct link (S-D) is given by*

$$P_{\mathrm{c,SD}}(\gamma_{\mathrm{th}}) = p_L(L) P_{\mathrm{SD,L}}(\gamma_{\mathrm{th}}) + (1 - p_L(L)) P_{\mathrm{SD,N}}(\gamma_{\mathrm{th}}), \quad (1)$$

where $P_{\mathrm{SD,L}}(\gamma_{th})$ and $P_{\mathrm{SD,N}}(\gamma_{th})$ are the conditional coverage probabilities given that the links are in LOS and NLOS conditions, respectively, and are given by

$$P_{\mathrm{SD,L}}(\gamma_{\mathrm{th}}) = \sum_{n=1}^{N_L} (-1)^{n+1} \binom{N_L}{n} \exp\Big(-n a_L L^{\alpha_L}\Big), \quad (2)$$

and

$$P_{\mathrm{SD,N}}(\gamma_{\mathrm{th}}) = \sum_{n=1}^{N_N} (-1)^{n+1} \binom{N_N}{n} \exp\Big(-n a_N L^{\alpha_N}\Big), \quad (3)$$

where, $a_L = \frac{\eta_L \gamma_{\rm th} N_0}{P_S \Psi}$, $a_N = \frac{\eta_N \gamma_{\rm th} N_0}{P_S \Psi}$, α_L and α_N are the path loss exponents for LOS and NLOS links, respectively, N_0 is the noise power, and $\eta_L = N_L(N_L!)^{-\frac{1}{N_L}}$ and $\eta_N = N_N(N_N!)^{-\frac{1}{N_N}}$. P_S is the transmit power of S and $\Psi \triangleq G_{\rm eq}\mu^2/(4\pi)^2$, where μ is the wavelength of the operating frequency.

Proof. The proof is given in Appendix A. ∎

B. Relaying Mode

When the direct communication in $S-D$ link is not possible due to excessive path loss or blockage, a relay can assist the transmission from source to the destination. The DF relaying protocol is used and no decoding error is assumed to occur if the receiver SNR is greater than the threshold $\gamma_{\rm th}$. We assume the half duplex relay operation so that it will take two time slots for the information transmission from S to D. The source S can successfully transmit to any candidate relays at which the received SNR is greater than the threshold $\gamma_{\rm th}$.

With path-loss, blockage and random fading, only some relays in Φ will be able to meet the required SNR threshold in the S-R link and thus, can retransmit the successfully decoded message to the receiver D in the second time slot. We define these relays as a set of potential of relays. Since the received SNR at a relay highly depends on it's distance from source and it's probability of being in LOS and NLOS condition, the potential relays will not be distributed uniformly in the $\mathbb{R}^2$-plane. It is therefore critical to know the distribution of potential relays before deriving the coverage probability at receiver with the deployment of relays.

1) Distribution of Set of Potential Relays: Let $\hat{\Phi}$ denotes the set of potential relays, i.e.,

$$\hat{\Phi} = \{x \in \Phi, {\rm SNR}_{s,x} \geq \gamma_{\rm th}\}.$$

Since the SNRs at candidate relays are independent, the set $\hat{\Phi}$ is formed by independent thinning of the original process Φ, i.e., by selecting a point x of process Φ with probability $p = \mathbb{P}({\rm SNR}_{s,x} \geq \gamma_{\rm th})$ independently of the other points in the process. The density of the thinned point process can be written as [19]:

$$\hat{\lambda}(x) = \lambda \mathbb{P}({\rm SNR}_{s,x} \geq \gamma_{th}). \tag{4}$$

The final expression for $\hat{\lambda}(x)$ is given in (5) on the top of next page, where $\rho(x)$ is the distance of an arbitrary relay R located at x, from the source node. Since we consider the relays to be distributed in a disc of radius $\mathcal{R}$ centered at D (origin), the average number of potential relays in $\mathcal{S}$ can be obtained as

$$\hat{\Lambda}(\mathcal{S}) = \int_{\mathcal{S}} \hat{\lambda}(x)dx = \int_0^{\mathcal{R}} \int_0^{2\pi} \hat{\lambda}(r,\theta) r d\theta dr \tag{6}$$

where we use (r,θ) to represent the location x in polar coordinate system. The analysis of above coverage probability involves the Euclidean distance from randomly located relays to the source and destination. Since the path loss is dependent on the distance, we use polar coordinate to represent the location of relay. We set the coordinate axis to be oriented along the line joining source and destination so that $\rho(x) = \|x - l_s\| = \sqrt{r^2 - 2rL\cos\theta + L^2} = \rho(r,\theta)$. The final expression for $\hat{\Lambda}(\mathcal{S})$ is given in (7) at the top of the next page.

The set of potential relays $\hat{\Phi}$ is the inhomogeneous PPP of density $\hat{\lambda}(x)$. This set can be further divided into two independent processes of densities $p_L(r)\hat{\lambda}(x)$ and $(1-p_L(r))\hat{\lambda}(x)$ to represent the LOS and NLOS sets, respectively, from the receiver (D). We denote the LOS process by $\hat{\Phi}_{\rm L}$ and NLOS process by $\hat{\Phi}_{\rm N}$.

2) Coverage Probability with Relays: It is defined as the probability that the received SNR at receiver D from the selected relay is above a predefined threshold $\gamma_{\rm th}$. To derive this probability, we first need to determine which relay will be selected. A relay is selected to provide smallest path loss at the receiver in R-D link. This means the selected relay can only be either the nearest node in $\hat{\Phi}_{\rm L}$ or nearest one in $\hat{\Phi}_{\rm N}$. To derive the coverage probability, we need to know whether a relay from $\hat{\Phi}_{\rm L}$ or $\hat{\Phi}_{\rm N}$ is selected, and for that, distribution of distance of the nearest relays in $\hat{\Phi}_{\rm L}$ and $\hat{\Phi}_{\rm N}$ from receiver is required.

Lemma 2. *The complimentary cumulative distribution function (CCDF) of the distance from receiver to the nearest LOS relay is given by*

$$\bar{F}_{r_L}(z) = \exp\left(-\int_{r=0}^{z}\int_{\theta=0}^{2\pi} p_L(r)\hat{\lambda}(x) r d\theta dr\right). \tag{8}$$

Proof. The distribution of the distance to the nearest LOS relay from receiver (at origin) can be derived if we know the probability that no LOS relays are available in $\mathcal{B}(0,z)$, where $\mathcal{B}(0,z)$ is the ball centered at 0 and radius z. This is the void probability for a PPP, and can be written as

$$\begin{aligned}\bar{F}_{r_L}(z) &= \mathbb{P}(r_L > z) \\ &= \mathbb{P}\{\text{no LOS relays in } \mathcal{B}(0,z)\} \\ &= \exp\left(-\Lambda_L([0,z))\right)\end{aligned} \tag{9}$$

where $\Lambda_L([0,z))$ is the mean number of LOS relays in $\mathcal{B}(0,z)$, which can be derived as

$$\Lambda_L([0,z)) = \int_{r=0}^{z}\int_{\theta=0}^{2\pi} p_L(r)\hat{\lambda}(x) r d\theta dr \tag{10}$$

Substituting (10) in (9), we get the desired distribution (8). ∎

Now, using $f_{r_L}(z) = -\dfrac{d\bar{F}_{r_L}(z)}{dz}$, the probability density function (PDF) of r_L is given by

$$f_{r_L}(z) = z p_L(z)\hat{\lambda}(z,\theta) e^{-\lambda \int_{r=0}^{z}\int_{\theta=0}^{2\pi} p_L(z)\hat{\lambda}(x) r d\theta dr} \tag{11}$$

Similarly, we can derive the CCDF of the distance of nearest NLOS relay from the receiver as

$$\bar{F}_{r_N}(z) = \exp\left(-\int_{r=0}^{z}\int_{\theta=0}^{2\pi} (1-p_L(r))\hat{\lambda}(x) r d\theta dr\right), \tag{12}$$

$$\hat{\lambda}(x) = \lambda\Bigg\{p_L(\rho(x))\sum_{n=1}^{N_L}(-1)^{n+1}\binom{N_L}{n}\exp\Big(-na_L(\rho(x))^{\alpha_L}\Big) + (1-p_L(\rho(x)))\sum_{n=1}^{N_N}(-1)^{n+1}\binom{N_N}{n}\exp\Big(-na_N(\rho(x))^{\alpha_N}\Big)\Bigg\} \tag{5}$$

$$\hat{\Lambda}(\mathcal{S}) = \lambda\Bigg\{\sum_{n=1}^{N_L}(-1)^{n+1}\binom{N_L}{n}\int_{r=0}^{\mathcal{R}}\int_{\theta=0}^{2\pi}p_L(\rho(r,\theta))\exp\Big(-na_L(\rho(r,\theta))^{\alpha_L}\Big)rd\theta dr + \sum_{n=1}^{N_N}(-1)^{n+1}\binom{N_N}{n}\int_{r=0}^{\mathcal{R}}\int_{\theta=0}^{2\pi}(1-p_L(\rho(r,\theta)))\exp\Big(-na_N(\rho(r,\theta))^{\alpha_N}\Big)rd\theta dr\Bigg\} \tag{7}$$

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and the corresponding PDF as

$$f_{r_N}(z) = z(1-p_L(z))\hat{\lambda}(z,\theta)e^{-\int_{r=0}^{z}\int_{\theta=0}^{2\pi}(1-p_L(z))\hat{\lambda}(x)rd\theta dr}. \tag{13}$$

Now we derive the probability A_L that a LOS relay will be selected to serve. The selection is based on maximizing the average received power from the candidate relay node or equivalently minimizing the path loss from relay to receiver.

Lemma 3. *The probability that a LOS relay will be selected is given by*

$$A_L = \int_0^\infty \bar{F}_{r_N}(z^{\frac{\alpha_L}{\alpha_N}})f_{r_L}(z)dz \tag{14}$$

where $\bar{F}_{r_N}(z)$ is the CCDF of the distance of nearest NLOS relay from receiver and is given in (12)*.*

Proof. The proof is given in Appendix B. ■

The probability that a NLOS relay will be used to serve, A_N, is given by

$$A_N = 1 - A_L$$

Lemma 4. *Given that a LOS relay is selected to serve, the PDF of its distance from the receiver is*

$$g_{r_L}(z) = \frac{f_{r_L}(z)}{A_L}\exp\left(-\int_{r=0}^{z^{\frac{\alpha_L}{\alpha_N}}}\int_{\theta=0}^{2\pi}(1-p_L(r))\hat{\lambda}(x)rd\theta dr\right), \tag{15}$$

where $z > 0$. Given a NLOS relay is selected to serve, the PDF of its distance from the receiver is

$$g_{r_N}(z) = \frac{f_{r_N}(z)}{A_N}\exp\left(-\int_{r=0}^{z^{\frac{\alpha_N}{\alpha_L}}}\int_{\theta=0}^{2\pi}p_L(r)\hat{\lambda}(x)rd\theta dr\right), \tag{16}$$

where $z > 0$.

Proof. The proof follows similar to in [13] and is omitted. ■

Theorem 1. *The total SNR coverage probability at the receiver using the selected relay is given by*

$$P_{\text{cov,R}}(\gamma_{th}) = A_L P_{\text{c,L}}(\gamma_{th}) + A_N P_{\text{c,N}}(\gamma_{th}), \tag{17}$$

where $P_{\text{c},l}(\gamma_{th}), l \in \{L, N\}$ is the conditional coverage probability given that a relay from $\hat{\Phi}_l$ is selected, which is given by

$$P_{\text{c,L}}(\gamma_{th}) \approx \sum_{k=1}^{N_L}(-1)^{k+1}\binom{N_L}{k} \times \int_{\theta=0}^{2\pi}\int_{z=0}^{\infty}e^{-ka_L z^{\alpha_L}}g_{r_L}(z)zdzd\theta, \tag{18}$$

and

$$P_{\text{c,N}}(\gamma_{th}) \approx \sum_{k=1}^{N_N}(-1)^{k+1}\binom{N_N}{k} \times \int_{\theta=0}^{2\pi}\int_{z=0}^{\infty}e^{-ka_N z^{\alpha_N}}g_{r_N}(z)zdzd\theta, \tag{19}$$

where α_L and α_N are the path loss exponents for LOS and NLOS links, respectively, and a_L and a_N are same as in Lemma 1.

Proof. Next we derive the conditional coverage probability when a relay from LOS relays is selected. Since the relay is selected from $\hat{\Phi}_{\text{L}}$ which is closest to receiver, coverage can be written as

$$P_{\text{c,L}}(\gamma_{th}) = \mathbb{P}\left(\frac{P_R\Psi|h_{x,l_d}|^2 r_L^{-\alpha_L}}{N_0} > \gamma_{th}\right) = 1 - \mathbb{E}\left[\mathbb{P}\Big(|h_{x,l_d}|^2 < \frac{\gamma_{th}N_0 r_L^{\alpha_L}}{P_R\Psi}\Big)\right]$$

where r_L is the distance between the closest LOS relay and the receiver. Now, using the similar approximation as in (21), we can write

$$P_{\text{c,L}}(\gamma_{th}) \approx \mathbb{E}\left[\sum_{k=1}^{N_L}(-1)^{k+1}\binom{N_L}{k}e^{-ka_L r_L^{\alpha_L}}\right] = \sum_{k=1}^{N_L}(-1)^{k+1}\binom{N_L}{k} \times \int_{\theta=0}^{2\pi}\int_{z=0}^{\infty}e^{-ka_L z^{\alpha_L}}g_L(z)zd\theta dz, \tag{20}$$

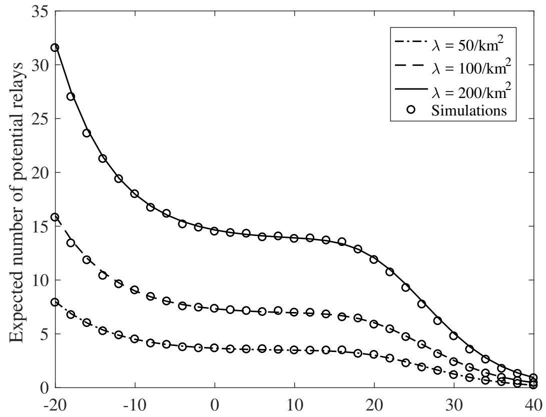

Fig. 2: Expected number of potential relays vs SNR threshold, L=300 m.

where $a_L = \frac{\eta_L \gamma_{\rm th} N_0}{P_R \Psi}$. Similarly we can derive the expression for $P_{\rm c,N}(\gamma_{th})$. ■

IV. Simulation and Analytical Results

Here we validate our analysis with Monte Carlo simulations by averaging over 10^5 independent realizations. We set $\alpha_L = 2$, $\alpha_N = 3.3$, $N_L = 3$, $N_N = 2$. The analytical results (curves) and the simulations (markers) match closely, verifying the correctness of our analysis.

Fig. 2 plots the average number of potential relays in (7) which meet the required SNR threshold for different relay densities. As expected, when the required SNR threshold increases, the potential relay density decreases. This is because only the relays which are closer to S and fall in LOS region can achieve the required SNR threshold. For example, for a moderate relay density of 100 relays/km^2, seven nodes can act as potential relays at a SNR threshold of 20 dB.

Fig. 3 plots and compares the SNR coverage probability without relaying in (1) and with relay in (17) when S-D distance L is set to 300 meters. Note that coverage probability improves significantly with relays. Since the LOS probability of the direct link is very small for this distance, the direct link coverage probability remains close to 5% for practical range of SNR thresholds in 0-20 dB. Also we observe significant coverage improvement with increasing relay density. For example, when the density increases from 100 to 200 relays/km^2, coverage increases from 20% to nearly 45% for $\gamma_{\rm th}$ = 10 dB.

To study the effect of S-D distance, in Fig. 4, we compare th coverage probability for $L = 200, 300$ meters at a fixed relay density of 200/km^2. As expected, coverage probability is higher for the shorter distance for whole range of SNR thresholds. Also the coverage improves significantly when decreasing L, i.e., from 40% to 80% if γ_{th} is set to 10 dB when decreasing L from 300 meters to 200 meters. Also, the coverage from direct link is significantly less than that with relays being deployed in both the cases.

We plot the coverage probability of the direct link and that with relays along the S-D separation distance (L) in Fig. 5. Coverage probability decreases with distance because both

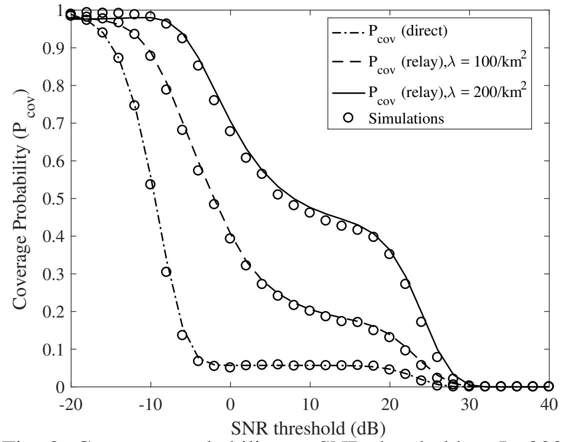

Fig. 3: Coverage probability vs SNR thresholds – L=300 m.

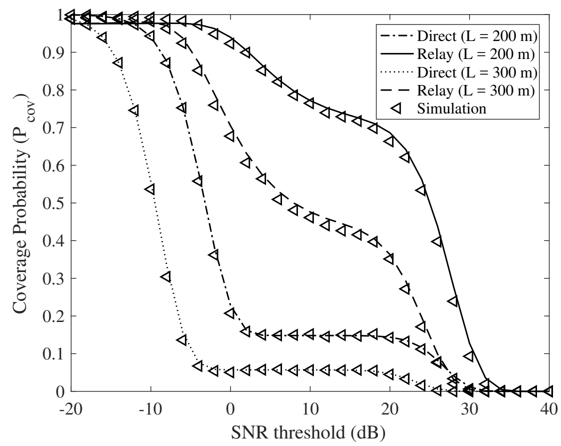

Fig. 4: Coverage probability vs SNR threshold – λ=200/km^2.

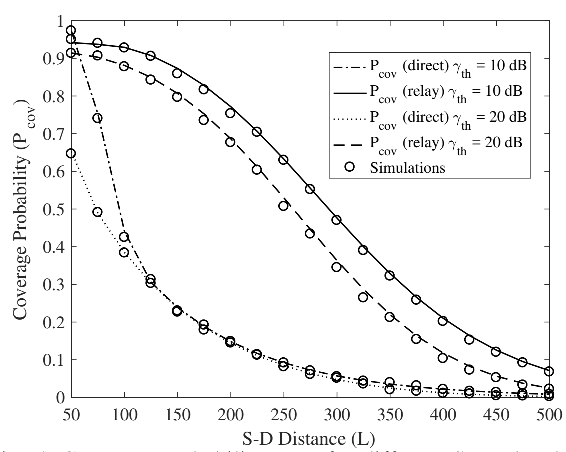

Fig. 5: Coverage probability vs L for different SNR thresholds.

blockage probability and path loss increase with distance. When S-D distance is 200 meters, coverage probability increases from about 15% to about 70% and 80% for SNR thresholds of 10 dB and 20 dB, respectively. Also, relays achieve significantly higher coverage probability than that for the direct link for the entire range of link lengths, and the only instance that direct link coverage exceeds that with relay is when S-D distance is 50 meters for an SNR threshold of 10 dB. The link distance leads to high LOS probability.

V. Conclusion

In this paper, we analyzed coverage probability of a DF mmWave relay. We considered blockage, directional antenna, and directional gain. We first derived the potential relay set which follows inhomogeneous PPP, and from this set selected a relay which provides minimum path loss to the receiver. Our analysis shows that significant coverage improvements can be achieved by deploying mmWave relays compared to that without relaying.

Appendix A
Derivation of Equation (2):

Equation (1) is obtained using the law of total probability, where $p_L(L)$ and $1-p_L(L)$ represent the LOS and NLOS probabilities of a link of length L. We next derive the conditional coverage probability $P_{\mathrm{SD,L}}$ in (2).

$$\begin{aligned} P_{\mathrm{SD,L}}(\gamma_{th}) &= \mathbb{P}\Big(\mathrm{SNR}_{\mathrm{SD,L}} > \gamma_{\mathrm{th}}\Big) \\ &= \mathbb{P}\left(\frac{|h_{\mathrm{L}}|^2 L^{-\alpha_L} P_S \Psi_s}{N_0} > \gamma_{\mathrm{th}}\right) \\ &= 1 - \mathbb{P}_r\left(|h_{\mathrm{L}}|^2 < \frac{\gamma_{\mathrm{th}} N_0 L^{\alpha_L}}{P_S \Psi_s}\right) \\ &\overset{(a)}{\approx} 1 - \left(1 - \exp\Big(-\frac{\eta_L \gamma_{\mathrm{th}} N_0 L^{\alpha_L}}{P_S \Psi_S}\Big)\right)^{N_L} \\ &\overset{(b)}{=} \sum_{n=1}^{N_L} (-1)^{n+1} \binom{N_L}{n} \exp\Big(-n a_L L^{\alpha_L}\Big) \end{aligned} \tag{21}$$

where (a) is using upper bound for normalized gamma random variable [13], (b) is obtained from binomial expansion, and α_L is the path loss exponent for LOS link. We define $a_L \triangleq \frac{\eta_L \gamma_{\mathrm{th}} N_0}{P_S \Psi_S}$, where $\eta_L = N_L (N_L!)^{-\frac{1}{N_L}}$, N_0 is the noise power, P_S is the transmit power of S and $\Psi_S \triangleq G_{\mathrm{eq}} \mu^2/(4\pi)^2$ is a constant that includes the directional gain and reference path loss at a 1 m distance, where μ is the wavelength of the operating frequency. $P_{\mathrm{SD,N}}$ in (3) can be derived similarly.

Appendix B
Proof of Lemma 2:

We define

$$\begin{aligned} A_L &\triangleq \mathbb{P}(P_s \Psi_s r_L^{-\alpha_L} > P_s \Psi_s r_N^{-\alpha_N}) \\ &= \mathbb{P}\left(r_N > r_L^{\left(\frac{\alpha_L}{\alpha_N}\right)}\right) \\ &= \int_0^\infty \mathbb{P}\left(r_N > r_L^{\left(\frac{\alpha_L}{\alpha_N}\right)} | r_L\right) f_{r_L}(z) dz \\ &= \int_0^\infty \bar{F}_{r_N}\left(r_L^{\left(\frac{\alpha_L}{\alpha_N}\right)}\right) f_{r_L}(z) dz \end{aligned} \tag{22}$$

where $\bar{F}_{r_N}(z)$ is given in (12).

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