Performance Analysis of Cooperative Beacon Sensing Strategies for Spatially Random Cognitive Users

Sachitha Kusaladharma and Chintha Tellambura, *Fellow, IEEE*

AQ1 AQ2

***Abstract*—Primary user (PU) beacons must be detected by cognitive users (CUs) to access spectrum holes, and misdetection results in interference on PUs. To alleviate this problem, sensing results of spatially separated CUs can be combined to make a final decision. In this paper, we analyze several such cooperative beacon sensing (CBS) strategies given spatial randomness of CU and PU nodes, which is modeled via independent homogeneous Poisson point processes. We consider two cases of beacon emitter placement: 1) at PU-transmitters and 2) at PU-receivers. We analyze three separate local beacon detection schemes and propose five CBS schemes. They require the sharing of CU results via a control channel subject to Rayleigh fading and path loss, and making a final decision via the OR rule. By using stochastic geometry, we derive both the misdetection probability, the false alarm probability, and the primary outage and show that impressive gains are achievable. For example, with PU-receiver beacons, CBS reduces misdetection by a factor of 10^4. In contrast, with PU-transmitter beacons, the reduction diminishes with the increased cell radii, but there exists an optimum cooperation radius.**

AQ3

***Index Terms*—**

I. Introduction

THE MISDETECTION of beacon signals emitted by primary users (PUs) by cognitive users (CUs) is a major problem, leading to interference on PU nodes which reduces their data throughput and increases their outage. Thus, fixing the beacon misdetection problem is critical to the deployment of cognitive radio (CR) networks. The CR paradigm is driven by the scarcity of spectrum and its inefficient use, two of the most critical challenges facing modern wireless networks [2]. For example, traditional static spectrum assignments to individual users/services lead to 85% or more idle licensed spectrum [3]. Thus, unlicensed (i.e., cognitive) opportunistic access to licensed spectrum [4] has been standardized in IEEE 802.22 Wireless Regional Area Network (WRAN) and its amendments, IEEE 802.11af for wireless local area networks, licensed shared access (LSA) for Long Term Evolution (LTE) and others [5]. In particular, the cognitive interweave mode aims to allow opportunistic access to temporary unused space-time-frequency slots (spectrum holes) [6]. However, CU devices must then accurately detect active PU transmissions in real time via matched filtering, cylostationarity, energy, eigenvalues, beacons or other methods [7]–[10].

Of these, PU beacon signaling has the benefits of efficiency and simplicity [11]–[16]. Grant or denial beacons are simply out-of-band, on-off modulated electromagnetic waves [17], proposed for IEEE 802.22.1 [18] and cognitive cellular systems [17], [19]. In this work, we focus on the problem of detecting **denial beacons** of active PU nodes. Beacon misdetection, which leads to interference on the PUs, occurs due to multipath fading, path loss, receiver uncertainty and other factors [20], [21]. Thus, a classical solution is to exploit spatial diversity. We can thus use multiple beacon measurements from spatially separated CUs and combine them into one final decision. This is an instance of cooperative sensing, which can be based on OR, AND, or majority rules [8], [22]. In this paper, we will limit ourselves to the OR rule to determine the presence of a denial beacon, which leads to conservative spectrum access attempts (i.e., ensuring less interference). The reduction in misdetection probability due to cooperative beacon sensing (CBS) depends on the number of cooperating CUs and their locations [23], which are random. Due to this spatial randomness, path loss, and fading, the expected performance improvements of CBS may be severely compromised. To characterize such issues, a comprehensive analysis of the overall beacon misdetection probability (P_{md}) is necessary.

A. Problem Statement and Contribution

In this paper, we analyze the overall P_{md} and false alarm probability (P_f) of several CBS methods as a function of how cooperating CUs are selected, local detection methods, spatial randomness of primary and secondary nodes, channel fading, and the sharing of imperfect decisions. Specifically, we address the following questions: 1) How does a CU device locally process one or more beacons transmitted from multiple PU devices to mitigate the impact of fading and path loss? 2) How do we select a set of CUs for cooperative spectrum sensing when the beacons are sent by PU-receiver nodes or PU-transmitter nodes? What are the rules that specify a suitable

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The authors are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada (e-mail: kusaladh@ualberta.ca; chintha@ece.ualberta.ca).

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set of cooperating CUs? The cooperative sensing phase will be affected by the channel propagation characteristics and spatial randomness of the cooperating CUs. The availability of channel state information (CSI) for the CU-to-CU channels affects the selection of best nodes to cooperate with. Clearly, the cooperating set should be chosen to minimize P_{md}, which will depend on mutual distances and fading conditions. 3) What is the overall performance of CBS?

To investigate all these questions for coexisting cellular (primary) and cognitive networks, we first ensure that the spatial randomness of nodes is fully accounted for. To this end, we use the tools from spatial geometry to model the random locations of PU and CU nodes. Specifically, we model PU-receiver nodes and CUs as Poisson Point Processes (PPPs) [24]. However, the PU-transmitters are fixed at the centers of hexagonal cells. For realistic propagation modeling, we incorporate both power-law path loss and Rayleigh fading. The beacon detection process of a CU is consisted of two distinct phases: 1) local detection, and 2) cooperation. The sharing of detection results is done via a control channel subject to fading and path-loss. Moreover, we consider beacons sent by PU-receivers (Case 1) and by PU-transmitters (Case 2). Our main contributions in this paper are as follows:

i) For phase one, we propose three local beacon processing schemes: 1) aggregating beacon powers, 2) separately sensing multiple beacons, and 3) detecting the best average received beacon signal (i.e., from the closest).
ii) For phase two, we propose three cooperation schemes: 1) nearest scheme, 2) multiple-random scheme, and 3) best received power scheme. For beacons emitted by PU-transmitters, we propose two additional schemes: 1) nearest CU to PU-transmitter scheme and 2) random CU to PU-transmitter scheme.
iii) For all these schemes, we derive P_{md} and P_f from the OR rule fusion in order to characterize the performance improvement of CBS under different system parameters.
iv) We derive the outage probability of a PU-receiver to characterize how its performance is affected by interference due to beacon misdetection.

B. Prior Research

We first review papers that do not focus on beacons signaling but perform general misdetection analysis and interference characterization for CR networks [15], [25]–[30]. For brevity, we denote the aggregate interference by I. In [15], the distribution of I is characterized in terms of sensitivity, transmit power, density of the CUs, the propagation characteristics, and cooperative spectrum sensing. In [30], the theory of truncated stable distributions and power control are studied for a CR network. Reference [25] analyzes the primary coverage probability under misdetections and false alarms, and develops an approximation and bounds for the Laplace transform of I. Statistics of I from a secondary network with an ALOHA based medium access control, spectrum sensing, and power control is derived [26]. Moreover, [27] derives the moment generating function of I for a spectrum sensing CR network, and a scheme is proposed to maximize the transmission powers of multiple active CU transmitters while satisfying I constraints. This scheme leads to significantly higher capacity. Reference [29] analyzes the geometric region allowing CR transmission with the help of cooperative sensors, and finds that the shape of this region is not circular. Furthermore, [31] develops models for bounding interference levels by modeling CUs as a modified Matern process. Co-operating spectrum sensing methods are analyzed over correlated shadow fading environments [28]. The spatial throughput of a CR network is characterized for a two threshold based opportunistic spectrum access protocol in [13].

Several works consider spectrum sensing using beacon detection and also cooperative spectrum sensing [11], [13], [32]–[35]. Reference [11] analyzes capacity-outage probability of a PU due to interference from beacon misdetection. The emission of beacons by PU-receiver nodes leads to higher capacity-outage performance. Furthermore, [34] considers three levels of cooperation under beacon transmissions from the primary users. It is shown that cooperation is vital when the CU node density is high. Threshold based opportunistic spectrum access methods are studied in [13] under PU-transmitter and receiver pilot signals and beacons, and the spatial opportunity (probability that an arbitrary location is discovered as a spectrum hole) is derived. Furthermore, [32] and [33] study the resultant aggregate interference due to misdetection in beacon based CR networks. Moreover, [35] studies the soft combination of spectrum information shared by the cooperating nodes when for multiple beacon signalling, and derives the optimal beacon sequence to reduce misdetection.

The differences among the aforementioned works and this paper are now described. First, spatial randomness of CUs is not considered in [11] and thus the spatial densities of the nodes do not appear in their analysis. Second, the existence of multiple PU-receivers is not considered [32], [33]. Third, the control channel for sharing the sensing result has been assumed perfect [11], [13], [34]. In contrast, in this paper consider the effect of propagation impairments (path loss and fading) on the quality of reception of control signals. Fourth, the availability of channel state information (CSI) has not been considered for cooperating node selection [11], [13], [32]–[35]. However, we CBS strategies depending on the availability of CSI. Fifth, no distinction is made between beacons emitted by PU-transmitters and those by PU-receivers [32], [33]. In contrast, this paper derives the interference statistics of the two cases in detail. Sixth, the impact of spatial locations has not been considered [11], [13], [34], [35]. As such, our paper strives to fill these gaps while investigating the misdetection probability reduction of cooperative sensing.

This paper is organized as follows. Section II introduces the signal model including the spatial model, signal propagation, local detection schemes, and cooperation schemes. The misdetection probability P_{md} is analyzed for PU-receiver and PU-transmitter beacons in Sections III and IV. Section V characterizes the primary system performance. Numerical results are provided in Section VI while Section VII concludes the paper.

AQ4

TABLE I
LIST OF COMMONLY USED PDFS

Name	PDF
$Lin(\alpha)$	$f(t) = \frac{2t}{\alpha^2}, 0 < t < \alpha$
$Ral(\alpha)$	$f(t) = 2\alpha t e^{-\alpha t^2}, 0 < t < \infty$
$TRal(\alpha, \beta)$	$f(t) = \frac{2\alpha t e^{-\alpha t^2}}{1-e^{-\alpha \beta^2}}, 0 < t < \beta$

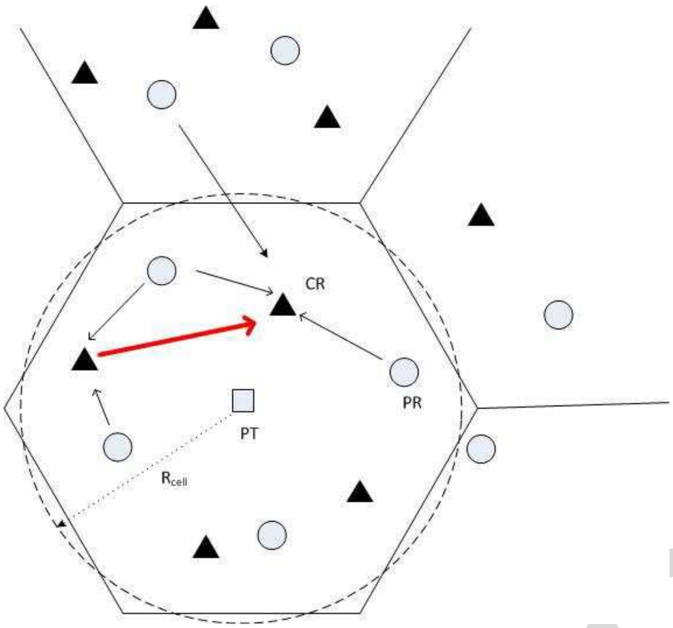

Fig. 1. PU-receiver node emit beacons. Squares, triangles, circles, and solid arrows respectively denote the PU-transmitters, CUs, PU-receivers, and the beacon signals. Each cell is hexagonal with a PU-transmitter at the center. PU-receivers and CUs are distributed randomly in $\mathbb{R}^2$.

Notations: $\Gamma(w, a) = \int_a^\infty t^{w-1} e^{-t} dt$ and $\Gamma(w) = \Gamma(w, 0)$ [36]. $\Pr[A]$ is the probability of event A, $f(\cdot)$ and $F(\cdot)$ are the probability density function (PDF) and the cumulative distribution function (CDF), $M_X(\cdot)$ is the moment generating function (MGF), $E_n(\cdot)$ is the generalized exponential integral, and $E_X[\cdot]$ denotes the expectation over random variable X. The Euclidean distance between two points x and y is denoted by $\|x - y\|$. The following PDFs (Linear, Rayleigh, and truncated Rayleigh) listed in Table I will be used commonly throughout the paper.

II. SYSTEM MODEL

A. Spatial Distribution

We consider coexisting primary and cognitive (secondary) networks. We assume the PU network to be of a conventional cellular type with different cells using the same frequency set (the frequency reuse factor is 1). The area is divided into hexagonal cells with a PU-transmitter (e.g., base-station) at the center of each (Fig. 1), which serves a set of spatially random PU-receivers within each cell. The cognitive network which can be an ad-hoc network or a sensor network [37] utilizes primary spectrum holes to transmit data. To facilitate analysis, we approximate the hexagonal cells with circular cells having a radius of r_{cell} (Fig. 1). The spatial randomness of CUs is also considered.

To model spatial randomness, we will make use of point processes. For our purposes, a point process Φ is a collection of points $\{x_1, x_2, \ldots\}$ where $x_k \in \mathbb{R}^2$ is a point representing the location of a radio node. We say Φ is a Poisson point process with rate $\lambda > 0$ if (1) the number of nodes within a bounded area A denoted by $N(A)$ is a Poisson random variable with $\mathbb{E}[N(A)] = \lambda A$ and (2) the number of nodes in two non-overlapping areas are independently distributed [38]. Poisson processes are widely used to model the locations wireless nodes due to their mathematical tractability and accuracy [30], [39].

In this paper, we model PU-receivers and CU nodes as two independent homogeneous PPPs Φ_p and Φ_s in $\mathbb{R}^2$ with spatial densities λ_p and λ_s. Thus, the number of nodes within the bounded are A is given by

$$\Pr[N(A) = n] = \frac{(\lambda A)^n}{n!} e^{-\lambda A}, n = 0, 1, 2, \ldots \quad (1)$$

where $\lambda \in \{\lambda_p, \lambda_s\}$ [38].

We assume that the CSI of the PU-CU links are not available to individual CUs. This assumption is reasonable and common [11] because of the general commercial and regulatory pressures that push primary and secondary networks to operate independently. However, a CU may or may not know about the CSI of links between itself and other CUs. The degree of availability of this CSI to CUs will impact the development of cooperative spectrum sensing protocols.

In this work, mobility of wireless nodes is not analyzed for two reasons. First, some PU nodes are fixed (e.g., base-stations, TV receivers and others). Second, even if the CUs move randomly (e.g., random walk or the Brownian motion), a snapshot of at any specific time generates a homogeneous PPP. Nevertheless, the impact of the mobility of nodes is a challenging, future topic.

Furthermore, we assume that CUs are always ready to transmit data upon detecting a spectrum hole and that all the PU-receivers are active. There is no loss of generality in these assumptions since activity factors (≤ 1) can easily be incorporated using the Coloring Theorem [24]. That is, if nodes of a PPP Φ with intensity λ are marked independently, and p_t is the probability of a node receiving the t-th color, the set of t-th color nodes forms a PPP Φ_t with intensity $p_t \lambda$. Thus, if a PU-receiver is active with an activity factor of q_p, the set of active PU-receivers follows a thinned PPP with intensity $q_p \lambda_p$. The same argument holds for the CUs.

B. Signal Propagation

The propagation effects are characterized by independent Rayleigh fading and log-distance path loss [40]. With small-scale Rayleigh fading, the channel power gain $|h|^2$ has the Exponential PDF $f_{|h|^2}(t) = e^{-t}, \ 0 < t < \infty$. The log-distance path loss model specifies that the received power $P_R = P r^{-\alpha}$ where r is the distance between the transmitter and the receiver, P is the transmit power and α is the path loss exponent. The path loss exponent is a function of carrier frequency, terrain, obstructions, antenna heights and others. The typical values range from 2 to 8 (at around 1 GHz). Note however that because $g(r) = r^{-\alpha}$ leads to analytical difficulties when $r \to 0$, we will also use $g(r) = \min(1, r^{-\alpha})$. Both forms of $g(r)$ will yield the same results because since spatial densities are small (e.g., $\lambda_p, \lambda_s << 1$), the probability that the distance is small is negligible, $P[r < 1] \to 0$.

Throughout the paper, we assume that all CUs transmit at a fixed power level [41]–[43]. Although CU power control methods are beyond the scope of this paper, they can be easily incorporated if needed [44].

C. Local Detection

As mentioned before, beacon detection process at a CU is divided into 2 phases: the local detection phase, and the cooperative phase. In this paper, we assume the downlink transmission of the cellular network with denial beacons where the PU devices (either PU-transmitters or PU-receivers [16], [17]) transmit a beacon signal. This beacon will have a set number of bits indicating that $\kappa(\kappa \in (1 \ldots K))$ future time-slots will be occupied by the transmitting device. Moreover, the beacon would uniquely identify the transmitting PU device, and would enable synchronization between the primary and secondary network. Furthermore, the beacon signal is transmitted before channel access by the PU device. For example, in the case of PU-transmitter beacons, the device sends the beacon signal before transmitting its data, while for PU-receiver beacons, the beacon is emitted by all active devices before the they begin receiving oncoming data.

Beacons emitted by PU-receivers are more likely to be correctly heard by CUs which can interfere the most. However, PU-receivers (e.g., hand-held user devices), will increase their battery drain because of beacons emissions. To counteract this, beacon signals can be made shorter, their frequency can be reduced, or their power can be reduced. All these options may unfortunately increase the miss detection of beacons. On the other hand, when PU-transmitters emit beacons, the CUs which can potentially interfere the most for cell-edge PU-receivers may miss them. Nevertheless, such beacons can be used under high PU-transmitter densities (lower cell radii), and where PU-receivers are severely power limited [11]. Otherwise, PU-receiver beacons should be used wherever possible.

1) PU-Receiver Beacons: Without the loss of generality, we assume that all PU-receivers are active and transmit beacons. For this to work, we assume synchronization between the different PU-receivers. However, if only a subset of the PU-receiver nodes are active, this can be easily incorporated using the Coloring Theorem [24]. Note that a CU may detect a beacon from a PU-receiver in another cell (Fig. 1). Thus, we suggest three local beacon detection schemes. These schemes are:

i) *Aggregating all beacons in the range:* Each CU simply uses the aggregate beacon power received, which does not require it to differentiate among the different PU-receiver beacons. However, this is a conservative approach in terms of opportunistic spectrum access because the aggregate beacon power may exceed the sensing threshold even when nearby PU-receivers are inactive.

ii) *Sensing beacons separately and OR combining them:* A CU is assumed to differentiate the beacons emitted by various PU-receivers (e.g., each one may use a different orthogonal code [45] or matched filtering may be used [13]). Thus, each distinct beacon is uniquely sensed. However, the implementation of a separate beacon sensing scheme has significant challenges. As the spatial density of PU nodes increases, this schemes requires additional processing. Moreover, longer codewords and thus longer beacons are needed to uniquely identify the different PU-receivers. On a practical point of view, only the PU-receivers within a certain radius from the CU may be considered for local detection instead of all the PU-receivers within the geographical area. The separate sensing scheme is advantageous for CUs because it allows them to access the spectrum whenever a beacon signal from a PU is less than the threshold. This is in contrast with the aggregate scheme where even if the individual beacon powers are far less than the threshold, the aggregate can still be above the threshold, barring a CU from accessing the spectrum.

iii) *Sensing the beacon from the closest PU-receiver only:* The CU must find the closest PU-receiver perhaps by measuring the average received signal power [46]. Moreover, the CU must differentiate among the beacons from different PU-receivers in order to achieve this. This scheme has the advantage of considerable less processing than the separately sensing scheme after the closest PU-receiver has been established. Moreover, it provides the best opportunities for a CU to access the spectrum among the three local detection schemes. However, because only a single PU beacon is considered, there is a high misdetection probability.

2) PU-Transmitter Beacons: We assume that all PU-transmitters become active at the same time. Each CU listens to its own cell's PU-transmitter for beacon signals. It should be noted that while a CU may receive a better instantaneous signal from a neighbouring cell due to a favourable channel, the PU-transmitter of its cell would also be the closest PU-transmitter to a given CU, and thus would provide the best received beacon signal power on average. We assume that the CUs have the ability to uniquely identify its own PU-transmitter from neighbouring PU-transmitters.[1] While beacon signal reception from out-of-cell PU-transmitters can also be considered, we leave this for future work.

D. Co-Operative Sensing

In the cooperative phase, the CU will select one or more other CUs to obtain the sensing results via a single narrow-band control signal. We assume that the CUs can identify each other via the use of separate orthogonal codes or time slots. In our analysis, we will consider distributed cooperation schemes without the involvement of a fusion center, information sharing via decision-fusion, and combination via the OR rule [8]. The OR rule minimizes P_{md} compared to other combining

[1] Separately identifying PU-transmitter beacons may be achieved by using unique codes or time slots.

rules [8]. Because distributed co-operating schemes are used, each individual CU keeps a dynamic database of neighbouring CUs. This database will include details about activity, distance, and CSI if available. Information for the individual databases is obtained via periodic control signals, and updated regularly. We thus propose three cooperation schemes, where the selection is based on the information within each CU's database. They are:

i) *Nearest scheme:* Each CU cooperates with its closest neighbor CU, which provides the best received signal power on average. To implement this, distances among the CUs are needed [47]. These distances may be obtained via a database, shared GPS information or via periodic control signals.

ii) *Multiple random scheme:* Here, M neighbouring CUs are randomly selected within a cooperation radius of $R_c (<< R_e)$. A CU is assumed to only cooperate with a neighbour within this radius. The signals from nodes beyond the outer distance R_e are assumed to have negligible power due to high path loss. If the number of CUs within R_c is less than M, all would be selected. The selected nodes are always available for cooperation.

iii) *Best received power scheme:* In this scheme, each CU cooperates with the neighbouring CU providing the best instantaneous received signal power. This amounts the lowest propagation loss considering both path loss and fading. We assume that each CU knows CSI and the positions of other CUs. Moreover, we further assume that a CU can cooperate with nodes outside its own cell.

We will assume that CUs can differentiate the beacon signals from the PU-receivers and the control signals from other cooperating CUs. For example, this involves using separate orthogonal codes for different CUs and PU-receivers, using different time slots, matched filtering, or having a separate narrow band channel for CU spectrum information sharing [13], [14], [45]. Furthermore, it should be noted that each CU shares its local detection result, but not the final decision of CBS.

With PU-transmitter beacons, we propose two additional schemes based on the intuition that CUs close to the PU-transmitter will have a better chance of correctly detecting the beacon. These schemes are:

i) *Nearest CU to PU-transmitter scheme:* Each CU, $x \in \Phi_s$, selects the closest CU to the PU-transmitter, which has the best probability to detect the beacon signal due to the lowest path loss. Furthermore, selection of distances to a fixed PU-transmitter may be less complex than find all CU-to-CU distance.

ii) *Random CU to PU-transmitter scheme:* A random CU within a distance of R_c from the PU-transmitter is selected. The distance constraint from the PU-transmitter which ensures the cooperating CU has a good chance of detecting the PU beacon. This scheme has the advantage over the previous scheme of not burdening a single CU (the one closest to the PU-transmitter) for sensing data.

Choosing other CU nodes to cooperate with based on distances to PU nodes is most suitable when PU-transmitters emit beacons. PU-transmitters would generally be fixed, and their locations would thus not change dynamically. As such, choosing CU nodes within a certain distance from the PU-transmitter is relatively straightforward. On the other hand, PU-receivers may be fluid in their activity, and multiple PU-receivers will be transmitting (with PU-transmitters, we assume the CU only listens to the PU-transmitter of its own cell) their beacons. As such, choosing cooperating CU nodes satisfying distance requirements from PU-receivers is more cumbersome, and such schemes are not considered in this paper.

III. P_{md} Analysis for PU-Receiver Beacons

A. Local Primary Beacon Detection

In this section, we analyze P_{md} for the local spectrum sensing methods in Section II-C.

1) Aggregating Beacon Power: Consider the CU node $x \in \Phi_s$ and the PU-receiver node $y \in \Phi_p$. The distance between them is $\|x - y\|$. However, as this distance becomes large, $g(\|x-y\|) \to 0$. As such, the beacons emitted by PU-receiver nodes y such that $\|x-y\| > R_e$ are considered to be negligible, where R_e is an outer distance. Since x and y are two random points from two independent PPPs, we need the distribution of the distance $\|x - y\|$. However, because a homogeneous Poisson process is considered for Φ_p, its points are distributed randomly. Moreover, due to the outer distance, the area of node distribution is annular. Therefore, the CDF of $\|x - y\|$ can be obtained as [43]

$$F_{\|x-y\|}(t) = \frac{t^2}{R_e^2}, \quad 0 < t < R_e. \tag{2}$$

Thus, $\|x - y\|$ is distributed with PDF $Lin(R_e)$.

All PU-receiver nodes $y \in \Phi_p$ transmit a beacon signal of constant power level P_b. As the CU will aggregate these beacons, the received beacon power at CU x is given by

$$P_R = P_b \sum_{y \in \Phi_p} |h_{x,y}|^2 g(\|x - y\|), \tag{3}$$

where $h_{x,y}$ is the channel between nodes x and y, and this incorporates both path loss and small scale fading. The received signal to noise ratio (SNR) γ at CU $x \in \Phi_s$ becomes $\gamma = \frac{P_R}{\sigma_b^2}$, where σ_b^2 is the additive noise variance. The CUs can employ energy detection of the beacon channel or use a received power threshold. However, as shown in [11], even an energy detection based scheme can be approximated as a simple received power threshold based scheme with an appropriate threshold. Therefore, in our analysis, a beacon is detected whenever the received beacon power $P_R > P_{th}$, where P_{th} is the reception threshold.

Let $P_{md}(x)$ be the probability of PU beacon misdetection by the CU $x \in \Phi_s$ in its local-detection phase. This probability is given by

$$P_{md}(x) = \Pr[P_R < P_{th}] = F_{P_R}(P_{th}),$$

which is the CDF of P_R. This can be evaluated using an MGF based approach [41], [48]–[50]. Let $M_{P_R}(s)$ be the MGF of the received beacon power at $x \in \Phi_s$, which is defined as $M_{P_R}(s) = E\left[e^{-sP_R}\right]$. If $M_{P_{R,y}}(s)$ is the MGF of the received beacon power from $y \in \Phi_p$, and N is a Poisson random variable with mean $\pi R_e^2 \lambda_p$, we can write $M_{P_R}(s)$ as [41], [43]

$$M_{P_R}(s) = E_N\Big[\big(M_{P_{R,y}}(s)\big)^N\Big] = e^{\pi R_e^2 \lambda_p \left(M_{P_{R,y}}(s) - 1\right)}. \quad (4)$$

$M_{P_{R,y}}(s)$ is obtained as follows.

$$\begin{aligned} M_{P_{R,y}}(s) &= E\Big[e^{-sP_b |h_{x,y}|^2 g(\|x-y\|)}\Big] \\ &= \int_0^1 \frac{1}{1+sP_b}\frac{2t}{R_e^2}dt + \int_1^{R_e} \frac{1}{1+sP_b t^{-\alpha}}\frac{2t}{R_e^2}dt. \quad (5) \end{aligned}$$

A closed-form expression for the second integral (5) appears intractable. However, using the expansion $(1+t)^{-1} = \sum_{k=0}^{\infty}(-t)^k$, $|t| < 1$, we derive a simplified expression as

$$M_{P_{R,y}}(s) = \frac{1}{R_e^2}\left(\frac{1}{1+sP_b} + \sum_{l=0}^{\infty} 2(-sP_b)^l \frac{R_e^{2-\alpha l} - 1}{2 - \alpha l}\right). \quad (6)$$

$F_{P_R}(t)$ can be obtained through the inverse Laplace transform by $F_{P_R}(t) = \mathcal{L}^{-1}(\frac{M_{P_R}(s)}{s})$, and replacing t with P_{th} gives $P_{md}(x)$, $x \in \Phi_s$. Note that because a closed-form solution is not apparent for $P_{md}(x)$, where $x \in \Phi_s$, numerical techniques and approximations must be used.

Although aggregating beacon power decreases P_{md}, viable spectrum access opportunities are also lost due to detecting aggregated beacons even when there may not be any PU-receivers close by to be hindered by interference.

2) Separately Sensing Primary Beacons: Misdetection occurs only when all beacon sensing outputs fall below the threshold. Thus we have $P_{md}(x) = (\Pr[P_{R,y} < P_{th}])^N$, where $x \in \Phi_s$ and $P_{R,y}$ is the beacon power from $y \in \Phi_p$ received at $x \in \Phi_s$, and N is a Poisson random variable with $\mathbb{E}[N] = \lambda_p \pi R_e^2$. The misdetection of the beacon from $y \in \Phi_p$ may be written as $\Pr[P_{R,y} < P_{th}] = E_{(\|x-y\|)}[1 - e^{-\frac{P_{th}}{P_b g(\|x-y\|)}}]$. Thus, denoting $\|x-y\| = t$, the local misdetection probability may be expressed as

$$P_{md}(x) = e^{-\pi R_e^2 \lambda_p \left(\frac{e^{-\frac{P_{th}}{P_b}}}{R_e^2} + \frac{2}{R_e^2}\int_1^{R_e} e^{-\frac{P_{th}}{P_b t^{-\alpha}}} t\,dt\right)}. \quad (7)$$

Because a closed-form solution for (7) appears impossible, we numerically evaluate this. A series summation based simplification can be used to simplify (7) which results in

$$P_{md}(x) = e^{-\pi R_e^2 \lambda_p \left(\frac{e^{-\frac{P_{th}}{P_b}}}{R_e^2} + \frac{2}{R_e^2}\sum_{k=0}^{\infty}\frac{\left(-\frac{P_{th}}{P_b}\right)^k}{k!}\left(\frac{R_e^{2+\alpha k} - 1}{2+\alpha k}\right)\right)}. \quad (8)$$

However, more resources are required for separate sensing, and is invariably more complex. Furthermore, the PU-receivers need to be co-ordinated to send separately identifiable beacons. This may not be practical for certain PU-receiver types such as digital terrestrial television subscribers.

3) Closest PU-Receiver Selection: Each CU, $x \in \Phi_s$, senses the beacon emitted by the closest PU-receiver. The closest PU-receiver may be found in practice by measuring the average received signal power [46]. Moreover, the CU must then have the ability to differentiate among different beacons.

Let $y^* = \arg\min_{y \in \Phi_p} \|y - x\|$ ($y^* \in \Phi_p$) be the nearest PU-receiver to $x \in \Phi_s$, and the distance $r^* = \|y^* - x\|$. The distribution of r^* is derived via the void probability of a PPP (probability of no nodes within a given radius from the origin) [51], [52], and is found out to be $Ral(\pi\lambda_p)$.

However, as the beacons from node $y \in \Phi_p$ at a distance more than R_e are neglected due to path loss, there may be an occasion where there is no closet PU-receiver within R_e. The probability of this event is $p_0 = e^{-\pi\lambda_p R_e^2}$. Whenever this occurs, the CU $x \in \Phi_s$ will misdetect with probability 1. However, conversely, because of the high path loss in such a scenario, the interfering signals will also have a negligible effect on the primary system. Let r_1^* be the truncated distance from x to y^* whenever $r^* < R_e$. Thus, r_1^* is distributed according to $TRal(\pi\lambda_p, R_e)$.

Let $|h_{x,y*}|^2$ be the channel power gain between x and y^*. Therefore, when a PU-receiver exists, the received beacon power (P_R) at x from y^* is given by $P_R = P_b|h_{x,y*}|^2 g(r_1^*)$, where $g(r_1^*)$ is the path-loss factor between x and y^*.

$P_{md}(x)$ can thus be written as

$$\begin{aligned} P_{md}(x) &= e^{-\pi\lambda_p R_e^2} + \left(1 - e^{-\pi\lambda_p R_e^2}\right) \times \Pr[R_b < P_{th}] \\ &= e^{-\pi\lambda_p R_e^2} + \left(1 - e^{-\pi\lambda_p R_e^2}\right) \times \Pr\left[|h_{x,y*}|^2 < \frac{P_{th}}{P_b g\left(r_1^*\right)}\right] \\ &= e^{-\pi\lambda_p R_e^2} + \left(1 - e^{-\pi\lambda_p R_e^2}\right)\Bigg(1 - e^{-\frac{P_{th}}{P_b}}\left(\frac{1 - e^{-\pi\lambda_p}}{1 - e^{-\pi\lambda_p R_e^2}}\right) \\ &\quad - \int_1^{R_e} \frac{2\pi\lambda_p t}{1 - e^{-\pi\lambda_p R_e^2}} e^{-\frac{P_{th}}{P_b t^{-\alpha}}} e^{-\pi\lambda_p t^2} dt\Bigg), \quad (9) \end{aligned}$$

and the integration in (9) can be performed numerically.

B. Co-Operative Spectrum Sensing

In this section, we analyze P_{md} when each CU employs the CU selection schemes proposed in Section II-D. The total P_{md} depends on both: 1) beacon misdetection, and 2) control channel misdetection.

1) Nearest Scheme: Let the closest neighbour from CU $x \in \Phi_s$ be denoted as x^* ($x^* \in \Phi_s$) with $x^* = \arg\min_{z \in \Phi_s} \|z - x\|$, located at a distance $\tilde{r}^*$ from x. Because the signals from x^* with $\tilde{r}^* > R_e$ are neglected due to path loss, there may be an occasion where a node x^* does not exist for cooperation. This probability ρ_0 is obtained as $\rho_0 = e^{-\pi\lambda_s R_e^2}$ using the void probability of a PPP. Let $\tilde{r}_1^*$ be the distance from x to x^* whenever $\tilde{r}^* < R_e$. Thus $\tilde{r}_1^*$ is distributed as $TRal(\pi\lambda_s, R_e)$.

Node x^* senses the presence of primary receiver beacons, and passes that information in the form of binary information in a narrow band channel using another control signal. Let $P_{b,s}$ be the power of this control signal, and $|h_{x,x^*}|^2$ be the channel power gain between x and x^*. Therefore, if the received control signal power ($P_{R,s}$) at x from x^* is given by $P_{R,s} = P_{b,s}|h_{x,x^*}|^2 g(\tilde{r}_1^*)$, where $g(\tilde{r}_1^*)$ is the path loss gain between x and x^*.

The probability of misdetecting the control signal transmitted by x^*, $q_{s,i}$, is obtained as

$$q_{s,i} = \Pr\big[P_{R,s} < P_{th}\big] = E_{\tilde{r}_1^*}\left[1 - e^{-\frac{P_{th}}{P_{b,s} g\left(\tilde{r}_1^*\right)}}\right]. \quad (10)$$

After performing the averaging with respect to $\tilde{r}_1^*$, the simplified expression for $q_{s,i}$ is

$$q_{s,i} = 1 - e^{-\frac{P_{th}}{P_{b,s}}}\left(\frac{1-e^{-\pi\lambda_s}}{1-e^{-\pi\lambda_s R_e^2}}\right) - \int_1^{R_e} \frac{2\pi\lambda_s t}{1-e^{-\pi\lambda_s R_e^2}} e^{-\frac{P_{th}}{P_b t^{-\alpha}}} e^{-\pi\lambda_s t^2} dt \quad (11)$$

Let P_{md}^1 be the final misdetection probability of x when cooperating with its closest neighbor. We will assume that x uses an OR rule [11] where P_{md}^1 becomes the product of the separate primary beacon and secondary control signal misdetecting probabilities. However, the probability that there is no CU within R_e must be considered. P_{md}^1 is composed of the following events: (1) x^* does not exist, and x misdetects, (2) x^* does exist, but both x^* and x misdetect the primary beacons, and (3) x^* does exist, and detects the primary beacon correctly, but x misdetects both the primary system beacons and the control signal from x^*. After combining these three events, we can write P_{md}^1 as

$$P_{md}^1 = P_{md}(x)\Big(e^{-\pi\lambda_s R_e^2} + \Big(1-e^{-\pi\lambda_s R_e^2}\Big) \times \Big(P_{md}(x) + (1-P_{md}(x))q_{s,i}\Big)\Big). \quad (12)$$

We have used the fact that correct secondary control signal reception due to double errors (x^* misdetects the primary beacons but x detects a secondary control signal when it's not present) are negligible. Moreover, spatial correlations have not been taken into account in the derivation of (12).

2) Multiple Random Scheme: Let x_r ($x_r \in \Phi_s$) be any CU within a cooperating distance of R_c from x, and r_r be the distance from x to x_r. Using similar arguments as the derivation of $\|x-y\|$, the distribution of r_r is shown to be distributed according to $Lin(R_c)$.

Similar to the nearest scheme, whenever an x_r detects the primary beacons, this information is sent via a control signal to x. We assume that x can differentiate the control signals coming from the M associated CUs, which can be easily achieved via orthogonal codes serving as an identifier of each CU within Φ_s. If $|h_{x,x_r}|^2$ and $g(r_r)$ are the small scale channel gain and path loss gain between x_r and x, the received signal power $P_{R,s}$ from x_r is given by $P_{R,s} = P_{b,s}|h_{x,x_r}|^2 g(r_r)$.

If $q_{s,i}$ is the probability of x misdetecting the control signal from x_r, it is obtained as

$$\begin{aligned} q_{s,i} &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,s}}}}{R_c^2} - \frac{2}{R_c^2}\int_1^{R_c} e^{-\frac{P_{th}}{P_{b,s}t^{-\alpha}}} t dt \\ &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,s}}}}{R_c^2} + \frac{2}{\alpha} E_{1-\frac{2}{\alpha}}\left(\frac{P_{th}R_c^\alpha}{P_{b,s}}\right). \end{aligned} \quad (13)$$

Let P_{md}^2 be the final misdetection probability of $x \in \Phi_s$. Although M is fixed beforehand, due to spatial randomness, the available number of CUs may be less than M. Thus, P_{md}^2 is the sum of several probability components corresponding to the number of cooperating nodes. Let q be the probability of misdetection arising from a single cooperating node (sum of the primary beacon misdetection probability by x_r and the probability that the control signal of x_r is misdetected by x when x_r correctly detects the primary beacons). It can be written as $q = (P_{md}(x) + (1-P_{md}(x))q_{s,i})$. Whenever a given $k(\leq M)$ cooperating nodes are present, the final misdetecting probability of $x \in \Phi_s$ becomes $P_{md}(x)q^k$. As such, $P_{md}^2 = E_k[P_{md}(x)q^k]$, where $0 \leq k \leq M$. After averaging with respect to k using (1), P_{md}^2 becomes

$$P_{md}^2 = P_{md}(x)\left(e^{-\pi\lambda_s R_c^2(1-q)}\frac{\Gamma\big(M,\pi\lambda_s R_c^2 q\big)}{\Gamma(M)} + \left(1-\frac{\Gamma\big(M,\pi\lambda_s R_c^2\big)}{\Gamma(M)}\right)q^M\right). \quad (14)$$

3) Best Received Power Scheme: Let the neighbouring CU of $x \in \Phi_s$ having the best instantaneous received signal power be denoted as x_h. In order to evaluate the secondary control signal misdetection probability ($q_{s,i}$), the Mapping theorem [24] is used on the PPP Φ_s. Furthermore, for convenience, we will use the path loss function $g(r_h) = r_h^{-\alpha}$ where $r_h = \|x - x_h\|$ is the distance between x and x_h. Moreover, we denote the channel gain between x and x_h as $|h_{x,x_h}|^2$. The mapping procedure is as follows. With respect to $x \in \Phi_s$, the process of CUs is homogeneous in $\mathbb{R}^2$ with it at the center. It is shown that an inhomogeneous PPP $\Phi_{s,h}$ with intensity $\lambda_{s,h}$, an exponential path loss with a path loss exponent of 1 and no fading generates the equivalent received power to that from a homogeneous PPP, and exponential path loss with an exponent α and Rayleigh fading [53], where $\lambda_{s,h}$ is written as (see the Appendix)

$$\lambda_{s,h} = \frac{2\pi}{\alpha}\lambda_s r_{s,h}^{\frac{2}{\alpha}-1}\Gamma\left(\frac{2}{\alpha}+1\right), 0 < r_{s,h} < \infty. \quad (15)$$

Note that $r_{s,h}$ is a distance based metric of the PPP and not any physical distance. In $\Phi_{s,h}$, the node having the smallest distance metric from x is x_h. Thus, using (15), the PDF of the distance metric to x_h (denoted by r_h^*) can be obtained as

$$f_{r_h^*}(t) = \frac{2\pi}{\alpha}\lambda_s\Gamma\left(\frac{2}{\alpha}+1\right)t^{\frac{2}{\alpha}-1}e^{-\pi\lambda_s\Gamma\left(\frac{2}{\alpha}+1\right)t^{\frac{2}{\alpha}}}, 0 < t < \infty. \quad (16)$$

With these results, the received secondary control signal power at x is written as $P_{R,s} = P_{b,s}(r_h^*)^{-1}$. Thus, $q_{s,i}$ is obtained as

$$\begin{aligned} q_{s,i} &= \Pr\Big[P_{b,s}\big(r_h^*\big)^{-1} < P_{th}\Big] \\ &= e^{-\pi\lambda_s\Gamma\left(\frac{2}{\alpha}+1\right)\left(\frac{P_{b,s}}{P_{th}}\right)^{\frac{2}{\alpha}}}. \end{aligned} \quad (17)$$

The final misdetection probability of $\phi_{s,i}$ (P_{md}^3) is composed of two components. First x and x_h may both misdetect the primary beacon. Second, while x_h detects the primary beacon, x may misdetect the control channel between x and x_h. Thus, P_{md}^3 is obtained as

$$P_{md}^3 = P_{md}(x)\big(P_{md}(x) + (1-P_{md}(x))q_{s,i}\big). \quad (18)$$

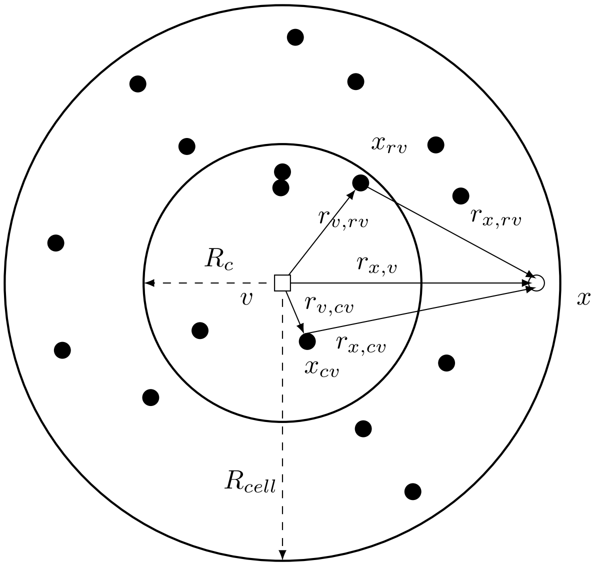

Fig. 2. The PU-transmitter v located at (0,0) sends the beacon. The cell radius is denoted by R_{cell}, the cooperating radius is denoted by R_c, while the black dots denote the CUs. The CU x located at a distance $r_{x,v}$ from v can cooperate with either the closest CU to v (x_{cv}), or cooperate with a random CU within a distance of R_c from v (x_{rv}).

IV. P_{md} Analysis for PU-Transmitter Beacons

This case is depicted in Fig. 2. In primary cellular networks where the transmitter is a base station, and receivers are user equipment, this approach provides wide benefits as base stations are not power limited and avoids PU-receiver power drain.

A. Local Primary Beacon Detection

Each CU ($x \in \Phi_s$) listens for the beacon of the PU-transmitter (v) of its cell. Let R_{cell} be the cell radius, and $P_{b,p}$ be power level of the beacon. Let $r_{x,v} = \|x - v|$. This is the distance between a fixed point and a random point from Φ_s. The $r_{x,v}$ will be distributed as $Lin(R_{cell})$ (we assume that $R_{cell} << R_e$). If $|h_{x,v}|^2$ and $g(r_{x,v})$ are the small scale channel gain and path loss gain between x and v, the received beacon power at x (P_R) is given by $P_R = P_{b,p}|h_{x,v}|^2 g(r_{x,v})$. Whenever it falls below the threshold, the detection fails. Thus, the probability of misdetection is given by

$$\begin{aligned} P_{md}(x) &= \Pr\Big[P_{b,p}|h_{x,v}|^2 g\big(r_{x,v}\big) < P_{th}\Big] \\ &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,p}}}}{R_{cell}^2} - \frac{2}{R_{cell}^2}\int_1^{R_{cell}} e^{-\frac{P_{th}}{P_{b,p}t^{-\alpha}}} t dt \\ &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,p}}}}{R_{cell}^2} + \frac{2}{\alpha}E_{1-\frac{2}{\alpha}}\left(\frac{P_{th}R_{cell}^{\alpha}}{P_{b,p}}\right). \qquad (19) \end{aligned}$$

B. Co-Operative Sensing

For PU-transmitter emitted beacons, we will now analyze the two additional schemes proposed.

1) Nearest CU to PU-Transmitter Scheme: Let x_{cv} be the closest CU ($\in \Phi_s$) to v ($x_{cv} = \arg\min_{z\in\Phi_s} \|z - v\|$), with $r_{v,cv} = \|v - x_{cv}\|$ and $r_{x,cv} = \|x - x_{cv}\|$. If $r_{v,cv} > R_{cell}$, a cooperating node does not exist. The probability of this scenario occurring (ρ_1) is given by $\rho_1 = e^{-\pi\lambda_s R_{cell}^2}$. Thus, the variable $r_{v,cv}$ is distributed according to $TRal(\pi\lambda_s, R_{cell})$. This distribution is obtained by removing x from Φ_s. This removal does not significantly affect the statistics of Φ_s.

We now need to find the probability that x_{cv} misdetects the PU-transmitter's beacon ($P_{md}(x_{cv})$) for this scenario. Let $|h_{v,cv}|^2$ and $g(r_{v,cv})$ be the small scale channel gain and path loss gain between v and x_{cv}. The received beacon power at x_{cv} ($P_{R,s}$) is given by $P_{R,s} = P_{b,p}|h_{v,cv}|^2 g(r_{v,cv})$. $P_{md}(x_{cv})$ is thus obtained as

$$\begin{aligned} P_{md}(x_{cv}) = 1 - e^{-\frac{P_{th}}{P_{b,p}}}\left(\frac{1 - e^{-\pi\lambda_s}}{1 - e^{-\pi\lambda_s R_{cell}^2}}\right) \\ - \int_1^{R_{cell}} \frac{2\pi\lambda_s t}{1 - e^{-\pi\lambda_s R_{cell}^2}} e^{-\frac{P_{th}}{P_{b,p}t^{-\alpha}}} e^{-\pi\lambda_s t^2} dt. \qquad (20) \end{aligned}$$

We now derive the probability that $x \in \Phi_s$ misdetects secondary control signal from x_{cv} whenever it (x_{cv}) detects the PU transmitter's beacon. The small scale channel gain and path loss gain between x and x_{cv} are denoted by $|h_{x,cv}|^2$ and $g(r_{x,cv})$ respectively. The received power of the secondary control at $x \in \Phi_s$ is given by $P_{R,s} = P_{b,s}|h_{x,cv}|^2 g(r_{x,cv})$, and the probability of x misdetecting the control signal ($q_{s,i}$) is then given by

$$\begin{aligned} q_{s,i} &= \Pr\big[P_{R,s} < P_{th}\big] \\ &= E_{r_{x,cv}}\left[1 - e^{-\frac{P_{th}}{P_{b,s}g\left(r_{x,cv}\right)}}\right]. \qquad (21) \end{aligned}$$

In order to evaluate this, the distribution of $r_{x,cv}$ is needed. From the cosine rule, $r_{x,cv}$ can be written as $r_{x,cv} = \sqrt{r_{v,cv}^2 + r_{x,v}^2 - 2r_{x,v}r_{v,cv}\cos\theta}$, where θ is a uniform between 0 and 2π, with $f_\theta(x) = \frac{1}{2\pi}$, $0 \le x < 2\pi$. Furthermore, for mathematical convenience, we will take $g(r_{x,cv}) = r_{x,cv}^{-\alpha}$. Thus, $q_{s,i}$ becomes

$$\begin{aligned} q_{s,i} = 1 - \int_0^{R_{cell}}\int_0^{R_{cell}}\int_0^{2\pi} e^{-\frac{P_{th}}{P_{b,s}\left(r_{v,cv}^2 + r_{x,v}^2 - 2r_{x,v}r_{v,cv}\cos\theta\right)^{-\frac{\alpha}{2}}}} \\ \times \frac{2\lambda_s r_{x,v} r_{v,cv}}{R_{cell}^2\left(1 - e^{-\pi\lambda_s R_{cell}^2}\right)} e^{-\pi\lambda_s r_{v,cv}^2} d\theta dr_{v,cv} dr_{x,v}. \qquad (22) \end{aligned}$$

Let P_{md}^4 be the overall misdetection probability of $x \in \Phi_s$. Similar to the previous analysis, it is necessary to consider probability of no cooperating node (ρ_1). Thus, P_{md}^4 is composed of three events: (1) x misdetects beacon and no cooperating node exists, (2) x and x_{cv} both misdetect beacon, and (3) x misdetects the beacon and x_{cv} detects it but x misdetects the control signal from x_{cv}. Considering these three events, we can write

$$\begin{aligned} P_{md}^4 = P_{md}(x)\Big(e^{-\pi\lambda_s R_{cell}^2} + \Big(1 - e^{-\pi\lambda_s R_{cell}^2}\Big) \\ \times \big(P_{md}(x_{cv}) + (1 - P_{md}(x_{cv}))q_{s,i}\big)\Big). \qquad (23) \end{aligned}$$

2) Random CU to PU-Transmitter Scheme: Let the randomly selected CU be x_{rv}, its distance from v be $r_{v,rv}$, and its distance from x be $r_{x,rv}$. We assume that $R_c < \min(R_{cell}, R_e)$.

If no such CU exists within a distance of R_c of v, no cooperation occurs. The probability of it is $\rho_2 = e^{-\pi\lambda_s R_c^2}$.

The probability that x_{rv} misdetects the beacon from v is obtained next. We denote this probability as $P_{md}(x_{rv})$, and the small scale channel gain and path loss gain between v and x_{rv} respectively as $|h_{v,rv}|^2$ and $g(r_{v,rv})$. The received beacon power at x_{rv} ($P_{R,s}$) is given by $P_{R,s} = P_{b,p}|h_{v,rv}|^2 g(r_{v,rv})$. We can now write $P_{md}(x_{rv})$ as

$$\begin{aligned} P_{md}(x_{rv}) &= \Pr\Big[P_{b,p}|h_{v,rv}|^2 g\big(r_{v,rv}\big) < P_{th}\Big] \\ &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,p}}}}{R_c^2} - \frac{2}{R_c^2}\int_1^{R_c} e^{-\frac{P_{th}}{P_{b,p}t^{-\alpha}}} t dt \\ &= 1 - \frac{e^{-\frac{P_{th}}{P_{b,p}}}}{R_c^2} + \frac{2}{\alpha}E_{1-\frac{2}{\alpha}}\left(\frac{P_{th}R_c^\alpha}{P_{b,p}}\right). \end{aligned} \tag{24}$$

We will now derive the probability that x misdetects the secondary control signal from x_{rv} (denoted by $q_{s,i}$), whenever a secondary control signal is transmitted. The small scale channel gain and path loss gain between x and x_{rv} are respectively denoted as $|h_{x,rv}|^2$ and $g(r_{x,rv})$. Similar to the previous scheme, we will use $g(r_{x,rv}) = r_{x,rv}^{-\alpha}$ for mathematical convenience. Using the cosine rule, $r_{x,rv}$ is written as $r_{x,rv} = \sqrt{r_{v,rv}^2 + r_{x,v}^2 - 2r_{x,v}r_{v,rv}\cos\theta}$. Thus, $q_{s,i}$ is written as

$$\begin{aligned} q_{s,i} &= \Pr\Big[P_{b,s}|h_{x,rv}|^2 r_{x,rv}^{-\alpha} < P_{th}\Big] \\ &= 1 - \int_0^{R_{cell}}\int_0^{R_c}\int_0^{2\pi} e^{-\frac{P_{th}}{P_{b,s}\left(r_{v,rv}^2+r_{x,v}^2-2r_{x,v}r_{v,rv}\cos\theta\right)^{-\frac{\alpha}{2}}}} \\ &\quad\times \frac{2}{\pi R_c^2 R_{cell}^2} r_{x,v}r_{v,rv}d\theta dr_{v,rv}dr_{x,v}. \end{aligned} \tag{25}$$

The final misdetection probability of x (denoted as P_{md}^5) is comprised of 3 terms as the previous scheme (Nearest CU to PU-transmitter scheme). Thus, P_{md}^5 is obtained as

$$\begin{aligned} P_{md}^5 = P_{md}(x)\Big(e^{-\pi\lambda_s R_c^2} + \Big(1 - e^{-\pi\lambda_s R_c^2}\Big) \\ \times \big(P_{md}(x_{rv}) + (1 - P_{md}(x_{rv}))q_{s,i}\big)\Big). \end{aligned} \tag{26}$$

This scheme can be generalized where a cooperates with up to M CUs within a distance of R_c from the PU-transmitter.

V. P_f Analysis

For completeness, we will conduct an analysis of the false alarm probability P_f. First, we will analyze P_f for the different local detection schemes for PU-receiver and PU-transmitter beacons.

A. P_f for Local Detection Schemes

1) Aggregating Beacon Power: A false alarm occurs when the CU detects the presence of a beacon when none are present. In this scenario, the received power is purely composed of noise. Thus

$$P_R = w, \tag{27}$$

where $w = \mathcal{N}(0, \sigma^2)$, and σ^2 is the noise variance (it should be noted that because a narrowband channel is used for beacons and control signals, σ^2 is very small). Let $P_f(x)$ be the probability of falsely detecting PU beacons by the CU $x \in \Phi_s$ in its local detection phase. P_f can be written as $P_f = \Pr[P_R > P_{th}]$. As such

$$P_f(x) = Q\left(\frac{P_{th}}{\sigma}\right), \tag{28}$$

and $Q(\cdot)$ is the Q function.

2) Separately Sensing Primary Beacons: When separately detecting primary beacons, a false alarm can occur even if a single stream from a PU is detected in error. Thus, we have

$$P_f(x) = \mathbb{E}\big[1 - (\Pr[P_R < P_{th}])^N\big] = \mathbb{E}\left[1 - \left(1 - Q\left(\frac{P_{th}}{\sigma}\right)\right)^N\right]. \tag{29}$$

After averaging with respect to N,

$$P_f(x) = 1 - e^{-\pi R_e^2 \lambda_p Q\left(\frac{P_{th}}{\sigma}\right)}. \tag{30}$$

3) Closest PU-Receiver Selection: $P_f(x)$ for this scenario is identical to (28), and $P_f(x) = Q(\frac{P_{th}}{\sigma})$.

4) PU-Transmitter Beacons: As each CU ($x \in \Phi_s$) listens to the beacon of the primary transmitter of its own cell, $P_f(x)$ is simply written similar to (28) as $P_f(x) = Q(\frac{P_{th}}{\sigma})$.

B. P_f After Co-Operation

Using the local false alarm probabilities derived above, we now derive the final false alarm probability after co-operation for the different schemes.

1) PU-Receiver Beacons (Nearest Scheme): For this scheme, false alarm occurs even if one of the following cases occur: 1) x falsely detect beacons, 2) x properly detects beacons, the nearest neighbour x^* properly detects, but x improperly detects the control channel, and 3) x properly detects beacons, the nearest neighbour x^* falsely detects, and x detects the control channel. After combining these events, we can write P_f^1 as

$$P_f^1 = P_f(x) + \big(1 - P_f(x)\big)\big((1 - P_f(x)\big)P_f(x) + P_f(x)\big(1 - q_{s,i}\big)\big). \tag{31}$$

It should be noted that the probability of falsely detecting the control channel also follows (28), and that $q_{s,i}$ follows (11).

2) PU-Receiver Beacons (Multiple Random Scheme): In this scheme, even a single false alarm from one of the co-operating nodes triggers a false alarm after combination. Let p be the probability that there is **no** false alarm from a co-operating node x_r. p can be written as $p = (1 - P_f(x))(1 - P_f(x)) + P_f(x)(q_{s,i})$. The final false alarm probability can thus be written as $P_f^2 = 1 - (1 - P_f(x))p^k$, for a given $k(\leq M)$. Averaging with respect to k (1) results in

$$\begin{aligned} P_f^2 = 1 - \big(1 - P_f(x)\big)\Bigg(e^{-\pi\lambda_s R_c^2(1-p)}\frac{\Gamma\big(M, \pi\lambda_s R_c^2 p\big)}{\Gamma(M)} \\ + \left(1 - \frac{\Gamma\big(M, \pi\lambda_s R_c^2\big)}{\Gamma(M)}\right)p^M\Bigg). \end{aligned} \tag{32}$$

3) PU-Receiver Beacons (Best Received Power Scheme): The final false alarm probability P_f for this scheme follows (31) with $q_{s,i}$ following (17).

4) PU-Transmitter Beacons (Nearest CU to PU-Transmitter Scheme): The final P_f for this scheme follows (31) with $q_{s,i}$ following (22).

5) PU-Transmitter Beacons (Random CU to PU-Transmitter Scheme): The final P_f for this scheme also follows (31) with $q_{s,i}$ following (25).

VI. Primary System Performance

The misdetection of beacons by a set of CUs will cause interference, which will degrade the received SINR, $\gamma_{p,y}$, at PU-receiver $y \in \Phi_p$. Thus, let I be the aggregate interference from the CUs, $P_{R,p}$ be the received primary signal power at $y \in \Phi_p$, and σ_n^2 be the noise power spectral density at the PU-receiver. We assume that different PU-transmitters use orthogonal codes, and do not pose significant interference to PU-receivers within other cells. $P_{R,p}$ is written as $P_{R,p} = P_p|h_{v,y}|^2 g(r_{v,y})$, where P_p, $|h_{v,y}|$ and $r_{v,y}$ are respectively the PU transmit power, channel power gain and distance between the PU-transmitter v and y. We can thus write the SINR as $\gamma_y = \frac{P_{R,p}}{I+\sigma_n^2}$. An outage occurs whenever $\gamma_y < \gamma_{th}$ where γ_{th} is a threshold. Note that we are more interested in the SINR falling below a threshold for the primary signals as opposed to the received signal falling below a threshold used for beacon detection. The primary signals would be transmitting data whereas the beacon signals only indicate the channel occupation for which the received signal level was sufficient. Thus, the outage probability of γ_y may be written as

$$P_{Out,y} = \Pr\left[\gamma_y < \gamma_{th}\right].$$

We can write

$$\begin{aligned} P_{Out,y/I,r_{v,y}}(x) &= \Pr\left[\frac{P_p|h_{v,y}|^2 g(r_{v,y})}{I+\sigma_n^2} \le \gamma_{th}\right] \\ &= \Pr\left[\left|h_{v,y}\right|^2 \le \frac{\gamma_{th}\left(I+\sigma_n^2\right)}{P_p g\left(r_{v,y}\right)}\right] \\ &= 1 - e^{\left(-\frac{\gamma_{th}\left(I+\sigma_n^2\right)}{P_p g\left(r_{v,y}\right)}\right)}. \end{aligned}$$

$P_{Out,y/I,r_{v,y}}(x)$ can be further averaged with respect to I as

$$\begin{aligned} P_{Out,j/r_{v,y}}(x) &= 1 - e^{\left(-\frac{\gamma_{th}\sigma_n^2}{P_p g\left(r_{v,y}\right)}\right)} E_I\left[e^{-I\left(\frac{\gamma_{th}}{P_p g\left(r_{v,y}\right)}\right)}\right] \\ &= 1 - e^{\left(-\frac{\gamma_{th}\sigma_n^2}{P_p g\left(r_{v,y}\right)}\right)} M_I\left(\frac{\gamma_{th}}{P_p g\left(r_{v,y}\right)}\right). \end{aligned} \tag{33}$$

Equation (33) provides the outage probability of node y given $r_{v,y}$ is known. However, if averaging over all PU-receivers is needed, we need the PDF of $r_{v,y}$, which is the distance from a fixed point to a random point from Φ_p, which can be shown to be $Lin(R_{cell})$. Thus, the average outage can be expressed as

$$\begin{aligned} P_{Out,y}(x) &= 1 - \frac{e^{\left(-\frac{\gamma_{th}\sigma_n^2}{P_p}\right)}}{R_{cell}^2} M_I\left(\frac{\gamma_{th}}{P_p}\right) \\ &\quad - \int_1^{R_{cell}} 2\frac{t}{R_{cell}^2} e^{\left(-\frac{\gamma_{th}\sigma_n^2}{P_p t^{-\alpha}}\right)} M_I\left(\frac{\gamma_{th}}{P_p t^{-\alpha}}\right) dt. \end{aligned} \tag{34}$$

To evaluate this, the MGF of the aggregate interference at y ($M_I(s)$) needs to be obtained. However, the exact expressions for interference is a function of each individual cooperating scheme, and thus complex. But, for completion, we suggest the following approximate approach. $M_I(s)$ is written as $M_I(s) = E[e^{-sI}]$. Let $r_{x,y} = \|x - y\|$ for any interfering CU $x \in \Phi_s$, which is distributed as $Lin(R_e)$. Note that similar to Section III, we do not consider the interference from x whenever $r_{x,y} > R_e$. When $P_{md}(x)$ is the final misdetection probability of $x \in \Phi_s$ with CBS, the Coloring theorem [24] suggests that the intensity of the interfering CUs is $P_{md}(x)\lambda_s$. $M_I(s)$ is thus obtained as

$$M_I(s) = e^{\pi R_e^2 P_{md}(x)\lambda_s\left(M_{I_x}(s)-1\right)}, \tag{35}$$

where $M_{I_x}(s)$ is the MGF of the interference from x. It is given by $M_{I_x}(s) = E[e^{-sP_s|h_{x,y}|^2 g(r_{x,y})}]$, where P_s is the CU transmit power, and $|h_{x,y}|^2$ and $g(r_{x,y})$ are respectively the small scale channel gain and the path loss gain between y and x. $M_{I_x}(s)$ is derived as

$$M_{I_x}(s) = \frac{1}{R_e^2}\left(\sum_{k=0}^{\infty}(-sP_s)^k + \sum_{l=0}^{\infty} 2(-sP_s)^l \frac{R_e^{2-\alpha l} - 1}{2 - \alpha l}\right). \tag{36}$$

VII. Numerical Results

We will provide numerical results on the total misdetection and false alarm probabilities for the different cooperation and local primary beacon detection schemes. We used MATLAB for the simulation, with 10^4 topologies, and 10^4 transmissions for each topology; thus 10^8 simulations for each plot point. Note that because simulation results match with the theoretical results, we have not used separate marker styles.

A. Beacons Emitted by PU-Receiver Nodes

We will first investigate the case of PU-receiver beacons. The parameters are $R_e = 1500$, $R_c = 500$, $\alpha = 3$, $P_{b,s} = -40$ dBm, and $P_b = -50$ dBm. P_b has been set 10 dB lower than $P_{b,s}$ because it makes sense that the energy of a PU-receiver node should not be used excessively for beacon signaling. Moreover, P_{th} is chosen as -110 dBm, which is the minimum signal reception thresholds for several mobile standards [54].

Fig. 3 plots the total misdetection probability P_{md} (eqs. (14), (18), and (12)) and the false alarm probability P_{md} with respect to the CU detection threshold (P_{th}). While the performance improvement due to CBS is slight for higher P_{th}, it is significant when P_{th} is small. For example, when $P_{th} = -120$ dBm and using multiple random cooperation with 10 nodes, P_{md} decreases by a 10^4 fold. This decrease is even higher for best received power cooperation when separately

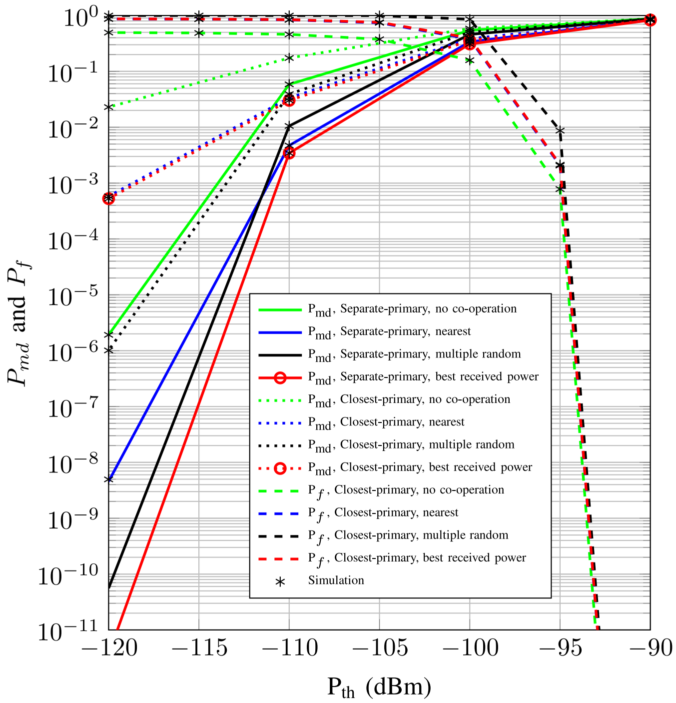

Fig. 3. P_{md} and P_f for PU-receiver beacons as a function of P_{th} for different cooperation schemes. $\lambda_p = 0.0001$, $\sigma^2 = 10^{-10}$, $\lambda_s = 0.0001$, and $M = 10$.

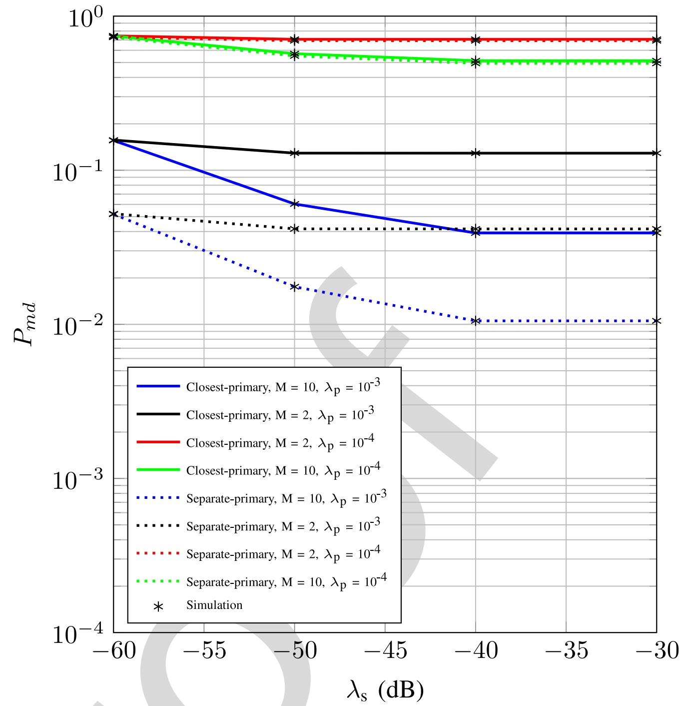

Fig. 4. P_{md} for PU-receiver beacons as a function of CU receiver density λ_s for multiple random cooperation. $P_{th} = -110$ dBm.

detecting all PU-receiver beacons. Furthermore, the latter performs better than sensing the beacon from the closest PU-receiver. However, as mentioned before, this comes at the cost of additional complexity and resources. It is also interesting to note that while the nearest scheme performs better than the multiple random scheme when P_{th} is higher, the converse is true for lower P_{th}. Moreover, while the best received power cooperation scheme always has better performance than the nearest scheme, the difference is only slight when detecting the closest PU-receiver's beacon. Contrary to the misdetection probability, the false alarm probability is very high for low P_{th} values and drops sharply as P_{th} increases. As expected, co-operation slightly increases the false alarm probability. The multiple random scheme with PU-receiver beacons has the worst performance because this scheme takes input from multiple CUs; even a single false alarm from one CU makes the final decision a false alarm. Moreover, the nearest and best received power co-operation schemes show almost identical performance with respect to the false alarm probability. It should also be noted that as unlicensed users, CUs should err in the side of false alarm rather than misdetection.

The behaviour of P_{md} for the multiple random scheme (eq. (14)) is investigated in Fig. 4 under different values of M and primary node density λ_p. For both separate and closest methods of primary beacon detection, the misdetection probability approaches 1 when λ_p is low. Increasing the number of cooperating nodes M does not help significantly. However, when λ_p increases to 10^{-3}, increasing M has some effect. Furthermore, the performance gap between these two methods becomes apparent. Moreover, all curves flatten out indicating that the effect of λ_s becomes negligible beyond -40 dB.

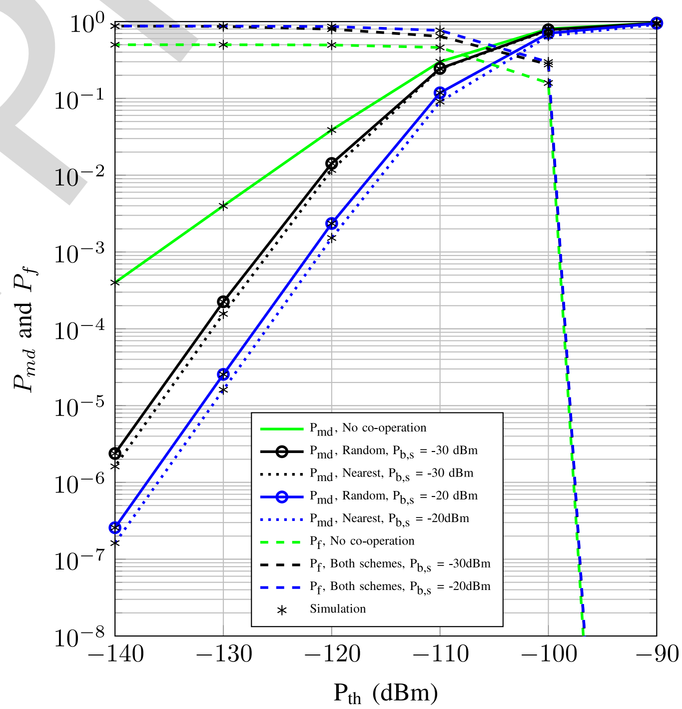

Fig. 5. P_{md} and P_f as a function of P_{th} for PU-transmitter beacons. $\lambda_s = 0.0001$, $R_{cell} = 1000$, $R_c = 500$, $\sigma^2 = 10^{-10}$ and $P_{b,p} = -20$ dBm.

B. Beacons Emitted by PU-Transmitter Nodes

We now focus on nearest CU-to-PU and random CU-to-PU schemes (eqs. (23) and (26)). Parameter values of $\alpha = 3$ and $P_{b,p} = -20$ dBm are used. The latter reflects the fact that the PU-transmitters can manage high power levels. Fig. 5 shows how P_{md} and P_f of the two CBS schemes varies with

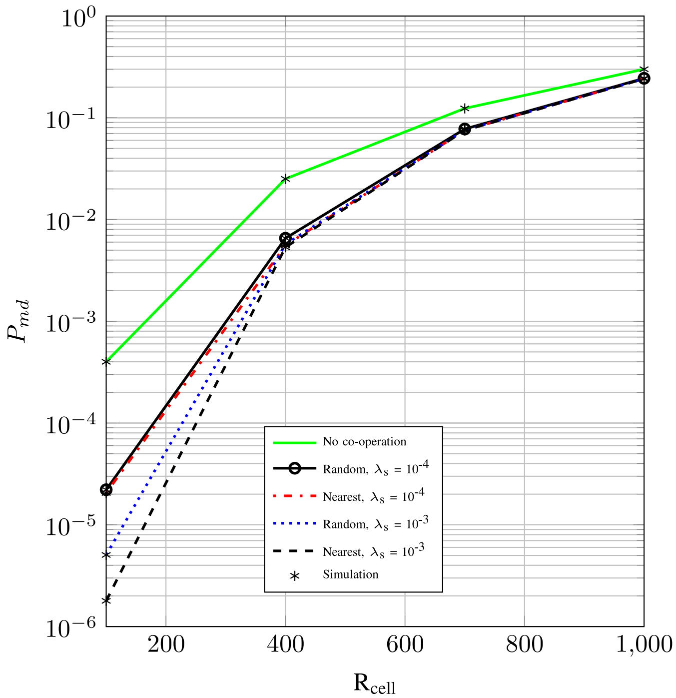

Fig. 6. P_{md} for PU-transmitter beacons as a function of R_{cell}. $R_c = 100$, $P_{b,p} = -20\,\text{dBm}$, $P_{b,s} = -30\,\text{dBm}$, and $P_{th} = -110\,\text{dBm}$.

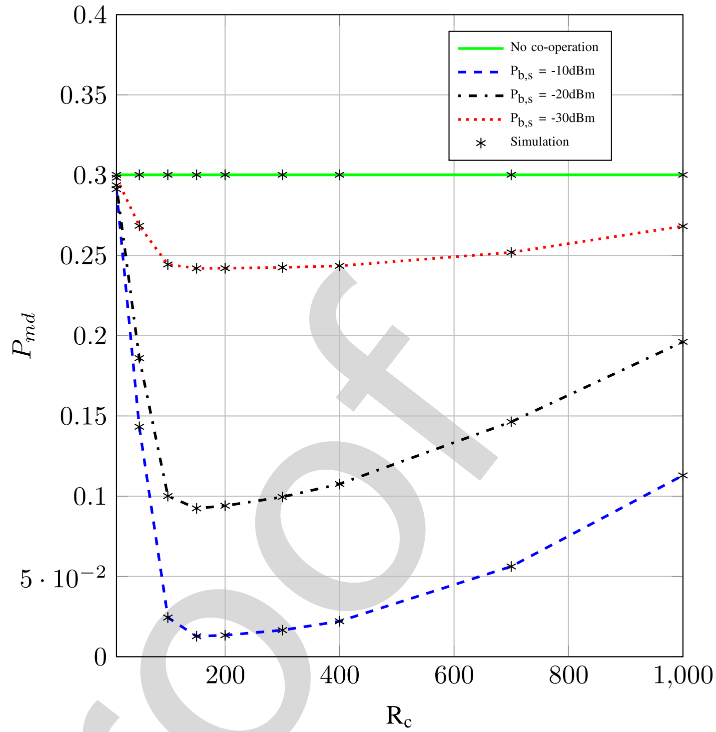

Fig. 7. P_{md} for PU-transmitter beacons as a function of R_c for random cooperation. $R_{cell} = 1000$, $P_{b,p} = -20\,\text{dBm}$, and $P_{th} = -110\,\text{dBm}$.

the detection threshold P_{th}. The effect of cooperation is more pronounced for low P_{th} values in terms of misdetection. The impact of the control channel is also seen. For example, a 10 dB increase in the control power $P_{b,s}$ results in order of magnitude reduction of misdetection. For both $P_{b,s}$ values, the nearest CU to PU-transmitter scheme has a slightly lower misdetection probability compared with the random CU to PU-transmitter scheme. In terms of false alarm, co-operation slightly increases P_f, and both co-operation schemes show very similar performance. When P_{th} increases beyond $-100\,\text{dBm}$, there is a sudden drop in the false alarm probability. Furthermore, as expected, when the control channel power increases, the false alarm probability is slightly higher as erroneous information is more readily received from co-operating devices.

In Fig. 6, we study the impact of the cell size; P_{md} (eqs. (23) and (26)) versus cell radius R_{cell} is plotted. The most important insight from this graph is that the effect of cooperation decreases as cell radius increases when other parameters are kept constant, and that both CBS schemes converge in performance. This is due to a high R_{cell} outweighing the effect from other parameters, and the overall performance gain diminishing. With a high R_{cell} and a low cooperation radius R_c, the distance from the given CU to its cooperating node is high irrespective of the cooperation scheme causing similar secondary control channel misdetection probabilities. As expected, increasing the CU node spatial density λ_s decreases P_{md}. This is especially important for the nearest CU to PU-transmitter scheme. For the random CU to PU-transmitter scheme, increasing λ_s ensures that there is a CU available for cooperation within R_c. The effect of increasing λ_s are mainly seen for lower cell radius values. Furthermore, the nearest CU to PU-transmitter scheme shows a slightly better performance than the random CU to PU-transmitter scheme for both λ_s values. However, the performance increase is higher when $\lambda_s = 10^{-3}$.

The effect of the cooperation radius R_c on the misdetection for the random CU to PU-transmitter scheme (26) is investigated in Fig. 7 for various levels of control signal power, $P_{b,s}$. A best-case cooperation radius can be observed, which ensures the lowest misdetection probability. When the cooperation radius approaches 0, random cooperation approaches converge no cooperation as expected. However, as R_c increases, the misdetection probability drops steeply to the best-case value. Furthermore, it is observed that the steepness of this reduction increases with the control signal power. Subsequent increases in R_c up to R_{cell} only result in a gradual increase in misdetection.

VIII. Conclusion

This paper investigated the overall misdetection and false alarm probabilities of an interweave CU using several cooperative beacon sensing strategies. We captured the spatial randomness of PU and CU nodes via independent PPPs. The propagation effects included path loss and Rayleigh fading. Moreover, beacons emitted by both PU-receivers and PU-transmitters were considered. For the former, when sensing beacons emitted by the closest PU-receiver, multiple random CBS performs better when the reception threshold P_{th} is lower; e.g., misdetection decreases by 10^4 fold for thresholds as low as $-120\,\text{dBm}$. However, the best received power scheme works slightly better for higher P_{th}. Moreover, the spatial density of PU-receiver nodes varies inversely with detection performance. Furthermore, the best received power scheme outperforms the nearest and multiple random cooperation schemes significantly when CUs sense primary beacons separately. When PU-transmitters send the beacons, a 10 dB increase in $P_{b,s}$ decreases the misdetection probability by 10 fold for both cooperation schemes. Furthermore, the effect of cooperation decreases for higher cell radii, and there exists a best case cooperation distance R_c which provides the lowest misdetection probability for random cooperation. For PU-transmitter beacons, nearest cooperation provides slightly better results than random cooperation. In addition, it was seen that co-operation slightly increases the false alarm probability. Future research ideas extending this work include considering spatial and temporal correlation, considering other detection rules at the CU, and investigating the energy efficiency of cooperation schemes for CR networks.

Appendix
Proof of Equation (15)

The density function associated with a PPP in $\mathbb{R}^2$ can be transformed to polar coordinates using the Mapping theorem [24] (This is used to convert the 2-D PPP to a 1-D PPP). Thus, the density of the 1-D PPP of CUs with respect to x ($\lambda_{r,1}(r)$) can be obtained as

$$\lambda_{r,1}(r) = \int_0^{2\pi} \lambda_s \; r_{r,1} d\theta = 2\pi \lambda_s r_{r,1}, \quad 0 < r_{r,1} < \infty. \tag{37}$$

The received power at a CU x from a cooperating CU at distance r is given by $Pr^{-\alpha}|h|^2$, where P is the transmit power and $|h|^2$ is the channel gain between a cooperating CU and x. Our first objective is to use the Mapping theorem to obtain a new equivalent PPP which generates a received power identical to what is generated by the above PPP with intensity $\lambda_{r,1}$, but with a path loss exponent of 1. The intensity function of the new PPP $\lambda_{r,2}(r)$ is derived as [53]

$$\lambda_{r,2}(r) = \frac{2\pi \lambda_s r_{r,2}^{\frac{2}{\alpha}-1}}{\alpha}, \quad 0 < r_{r,2} < \infty. \tag{38}$$

In the next step, we use the Marking theorem [24] and the Mapping theorem to obtain a new PPP which generates the identical received power, but with a path loss exponent of 1 and no fading. The intensity function of the new PPP $\lambda_{s,hp}(r)$ can be derived as [53]

$$\lambda_{s,h}(r) = E_{|h|^2}\left[|h|^2 \lambda_{r,2}\left(r_{s,h}|h|^2\right)\right], \quad 0 < r_{s,h} < \infty. \tag{39}$$

When the fading is modelled as Rayleigh, (39) can be simplified as

$$\begin{aligned} \lambda_{s,h}(r) &= \frac{2\pi}{\alpha} \lambda_s r_{s,h}^{\frac{2}{\alpha}-1} E_{|h_i|^2}\left[\left(|h_i|^2\right)^{\frac{2}{\alpha}}\right], 0 < r < \infty \\ &= \frac{2\pi}{\alpha} \lambda_s r_{s,h}^{\frac{2}{\alpha}-1} \Gamma\left(\frac{2}{\alpha}+1\right), 0 < r_{s,h} < \infty, \end{aligned} \tag{40}$$

which is equation (15). It should be noted that the limits of r do not change because the CUs are distributed in a 2-D field.

References

[1] S. Kusaladharma and C. Tellambura, "Interweave cognitive networks with co-operative sensing," in *Proc. IEEE GLOBECOM*, San Diego, CA, USA, Dec. 2015, pp. 1–6.

[2] X.-L. Huang *et al.*, "Intelligent cooperative spectrum sensing via hierarchical Dirichlet process in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 5, pp. 771–787, May 2015.

[3] M. T. Masonta, M. Mzyece, and N. Ntlatlapa, "Spectrum decision in cognitive radio networks: A survey," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 3, pp. 1088–1107, 3rd Quart., 2013.

[4] J. Mitola, "Cognitive radio for flexible multimedia communications," in *Proc. IEEE Int. Workshop MoMuC*, San Diego, CA, USA, Nov. 1999, pp. 3–10.

[5] L. Gavrilovska, D. Denkovski, V. Rakovic, and M. Angjelichinoski, "Medium access control protocols in cognitive radio networks: Overview and general classification," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 2092–2124, 4th Quart., 2014.

[6] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, 2006. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1389128606001009

[7] S. Atapattu, C. Tellambura, and H. Jiang, *Energy Detection for Spectrum Sensing in Cognitive Radio*. New York, NY, USA: Springer, 2014. AQ5

[8] S. Atapattu, C. Tellambura, and H. Jiang, "Energy detection based cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1232–1241, Apr. 2011.

[9] A. Al-Dulaimi, J. Cosmas, and A. Mohammed, *Self-Organization and Green Applications in Cognitive Radio Networks*. Hershey, PA, USA: IGI Glob., 2013.

[10] S. Kusaladharma, P. Herath, and C. Tellambura, "An overview of cognitive radio networks," in *Wiley Encyclopedia of Electrical and Electronics Engineering*. AQ6

[11] M. Derakhshani and T. Le-Ngoc, "Aggregate interference and capacity-outage analysis in a cognitive radio network," *IEEE Trans. Veh. Technol.*, vol. 61, no. 1, pp. 196–207, Jan. 2012.

[12] K. Bian, J.-M. Park, L. Chen, and X. Li, "Addressing the hidden terminal problem for heterogeneous coexistence between TDM and CSMA networks in white space," *IEEE Trans. Veh. Technol.*, vol. 63, no. 9, pp. 4450–4463, Nov. 2014.

[13] X. Song, C. Yin, D. Liu, and R. Zhang, "Spatial throughput characterization in cognitive radio networks with threshold-based opportunistic spectrum access," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 11, pp. 2190–2204, Nov. 2014. AQ7

[14] A. M. Wyglinski, M. Nekovee, and Y. T. Hou, *Cognitive Radio Communications and Networks: Principles and Practice*. Amsterdam, The Netherlands: Elsevier, 2010.

[15] A. Ghasemi and E. S. Sousa, "Interference aggregation in spectrum-sensing cognitive wireless networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 41–56, Feb. 2008.

[16] H. Venkataraman and G.-M. Muntean, *Cognitive Radio and Its Application for Next Generation Cellular and Wireless Networks*. Dordrecht, The Netherlands: Springer, 2012.

[17] L. Barlemann and S. Mangold, *Cognitive Radio and Dynamic Spectrum Access*. Hoboken, NJ, USA: Wiley, 2009.

[18] Z. Lei and F. Chin, "A reliable and power efficient beacon structure for cognitive radio systems," *IEEE Trans. Broadcast.*, vol. 54, no. 2, pp. 182–187, Jun. 2008.

[19] F. Paisana, N. Marchetti, and L. A. DaSilva, "Radar, TV and cellular bands: Which spectrum access techniques for which bands?" *IEEE Commun. Surveys Tuts.*, vol. 16, no. 3, pp. 1193–1220, 3rd Quart., 2014.

[20] S. M. Mishra, A. Sahai, and R. W. Brodersen, "Cooperative sensing among cognitive radios," in *Proc. IEEE ICC*, vol. 4. Istanbul, Turkey, Jun. 2006, pp. 1658–1663.

[21] C. Jiang *et al.*, "Cognitive radio networks with asynchronous spectrum sensing and access," *IEEE Netw.*, vol. 29, no. 3, pp. 88–95, May/Jun. 2015.

[22] I. F. Akyildiz, B. F. Lo, and R. Balakrishnan, "Cooperative spectrum sensing in cognitive radio networks: A survey," *Phys. Commun.*, vol. 4, no. 1, pp. 40–62, Mar. 2011. [Online]. Available: http://dx.doi.org/10.1016/j.phycom.2010.12.003

[23] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE DySPAN*, Baltimore, MD, USA, Nov. 2005, pp. 131–136.

[24] J. F. Kingman, *Poisson Processes*. Oxford Univ. Press, 1993. AQ8

[25] P. Madhusudhanan, Y. Liu, and T. X. Brown, "On primary user coverage probabilities and faulty cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6207–6218, Nov. 2014.

[26] A. Babaei, P. Agrawal, and B. Jabbari, "Statistics of aggregate interference in cognitive wireless ad hoc networks," in *Proc. IEEE ICNC*, Jan. 2012, pp. 397–401.

[27] Y. Sun and B. L. Mark, "Interference model for spectrum sensing with power control," in *Proc. IEEE CISS*, Baltimore, MD, USA, Mar. 2013, pp. 1–6.

[28] M. Di Renzo, F. Graziosi, and F. Santucci, "Cooperative spectrum sensing in cognitive radio networks over correlated log-normal shadowing," in *Proc. IEEE VTC*, Barcelona, Spain, Apr. 2009, pp. 1–5.

[29] S.-C. Hung and K.-C. Chen, "Geometric design of cooperative spectrum sensing for cognitive radios," in *Proc. IEEE PIMRC*, London, U.K., Sep. 2013, pp. 2496–2501.

[30] A. Rabbachin, T. Q. S. Quek, H. Shin, and M. Z. Win, "Cognitive network interference," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 480–493, Feb. 2011.

[31] A. Busson, B. Jabbari, A. Babaei, and V. Veque, "Interference and throughput in spectrum sensing cognitive radio networks using point processes," *J. Commun. Netw.*, vol. 16, no. 1, pp. 67–80, Feb. 2014.

[32] S. Kusaladharma and C. Tellambura, "Impact of beacon misdetection on aggregate interference for hybrid underlay-interweave networks," *IEEE Commun. Lett.*, vol. 17, no. 11, pp. 2052–2055, Nov. 2013.

[33] S. Kusaladharma, P. Herath, and C. Tellambura, "Aggregate interference analysis for interweave cognitive networks," in *Proc. IEEE VTC*, Vancouver, BC, Canada, Sep. 2014, pp. 1–5.

[34] P. Madhusudhanan, J. G. Restrepo, Y. Liu, T. X. Brown, and K. Baker, "Modeling of interference from cooperative cognitive radios for low power primary users," in *Proc. IEEE GLOBECOM*, Dec. 2010, pp. 1–6.

[35] A. Patel, S. Biswas, and A. K. Jagannatham, "Multiple beacon based robust cooperative spectrum sensing in MIMO cognitive radio networks," in *Proc. IEEE VTC*, Las Vegas, NV, USA, Sep. 2013, pp. 1–5.

[36] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press, 2007.

[37] F. Adelantado, A. A. Juan, H. Skianis, and C. Verikoukis, "Sensing users selection with overhead reduction for cognitive wireless ad-hoc networks," in *Proc. IEEE GLOBECOM*, Miami, FL, USA, Dec. 2010, pp. 1–5.

[38] A. Baddeley, I. Barany, R. Schneider, and W. Weil, *Spatial Point Processes and Their Applications*. Heidelberg, Germany: Springer, 2007.

[39] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 550–560, Apr. 2012.

[40] A. Goldsmith, *Wireless Communications*. Cambridge Univ. Press, 2005.

[41] L. Vijayandran, P. Dharmawansa, T. Ekman, and C. Tellambura, "Analysis of aggregate interference and primary system performance in finite area cognitive radio networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1811–1822, Jul. 2012.

[42] M. J. Rahman and X. Wang, "Probabilistic analysis of mutual interference in cognitive radio communications," in *Proc. IEEE GLOBECOM*, Dec. 2011, pp. 1–5.

[43] S. Kusaladharma and C. Tellambura, "Aggregate interference analysis for underlay cognitive radio networks," *IEEE Wireless Commun. Lett.*, vol. 1, no. 6, pp. 641–644, Dec. 2012.

[44] S. Kusaladharma, P. Herath, and C. Tellambura, "Impact of transmit power control and receiver association on interweave network interference," in *Proc. IEEE VTC*, Vancouver, BC, Canada, Sep. 2014, pp. 1–5.

[45] E. Krouk and S. Semenov, *Coding Techniques in Wireless Communications*. Hoboken, NJ, USA: Wiley, 2010.

[46] P. Herath, C. Tellambura, and W. A. Krzymien, "Stochastic geometry modeling of cellular uplink power control under composite Rayleigh-lognormal fading," in *Proc. IEEE VTC*, Boston, MA, USA, Sep. 2015, pp. 1–5.

[47] S. Kusaladharma, P. Herath, and C. Tellambura, "Impact of transmit power control on aggregate interference in underlay cognitive radio networks," in *Proc. IEEE ICC*, Sydney, NSW, Australia, Jun. 2014, pp. 1–6.

[48] C. Tellambura, A. J. Mueller, and V. K. Bhargawa, "Analysis of M-ary phase-shift keying with diversity reception for land-mobile satellite channels," *IEEE Trans. Veh. Technol.*, vol. 46, no. 4, pp. 910–922, Nov. 1997.

[49] C. Tellambura, A. Annamalai, and V. K. Bhargava, "Closed form and infinite series solutions for the MGF of a dual-diversity selection combiner output in bivariate Nakagami fading," *IEEE Trans. Commun.*, vol. 51, no. 4, pp. 539–542, Apr. 2003.

[50] C. Tellambura, "Evaluation of the exact union bound for trellis-coded modulations over fading channels," *IEEE Trans. Commun.*, vol. 44, no. 12, pp. 1693–1699, Dec. 1996.

[51] D. Moltchanov, "Distance distributions in random networks," *Ad Hoc Netw.*, vol. 10, no. 6, pp. 1146–1166, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1570870512000224

[52] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and It's Applications*. Chichester, U.K.: Wiley, 1995.

[53] P. Madhusudhanan, J. G. Restrepo, Y. Liu, T. X. Brown, and K. R. Baker, "Downlink performance analysis for a generalized shotgun cellular system," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6684–6696, Dec. 2014.

[54] P. Sharma and R. Singh, "Cell coverage area and link budget calculations in GSM system," *Int. J. Modern Eng. Res.*, vol. 2, no. 2, pp. 170–176, Mar. 2012.

Sachitha Kusaladharma received the B.Sc. (First-Class Hons.) degree in electrical and telecommunication engineering from the University of Moratuwa, Moratuwa, Sri Lanka, in 2010 and the M.Sc. degree in wireless communications from the University of Alberta, Edmonton, AB, Canada, in 2013, where he is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering. His research interests include cognitive radio networks, communication theory, multiple-input multiple-output systems, and wireless sensor networks. He was a recipient of the Alberta Innovates Technology Futures Graduate Student Scholarship in 2013.

Chintha Tellambura (F'11) received the B.Sc. (First-Class Hons.) degree from the University of Moratuwa, Moratuwa, Sri Lanka, in 1986, the M.Sc. degree in electronics from the University of London, London, U.K., in 1988, and the Ph.D. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 1993.

From 1993 to 1994, he was a Post-Doctoral Research Fellow with the University of Victoria and, from 1995 to 1996, with the University of Bradford, Bradford, U.K. From 1997 to 2002, he was with Monash University, Melbourne, VIC, Australia. He is currently a Professor with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada. He has authored or co-authored over 500 journal and conference publications with over 12 000 Google Scholar citations and an H-index of 59. His current research interests include the design, modeling, and analysis of cognitive radio networks, heterogeneous cellular networks, and multiple-antenna wireless networks.

Dr. Tellambura was a co-recipient of the Prestigious McCalla Professorship and the Killam Annual Professorship from the University of Alberta, the Communication Theory Symposium Best Paper Award at the 2012 IEEE International Conference on Communications, Ottawa, ON, Canada. He served as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and as an Area Editor of Wireless Communications Systems and Theory for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2007 to 2012.

AUTHOR QUERIES

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