Energy Harvesting Random Underlay Cognitive Networks With Power Control

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***Abstract*—Spectrum and energy constraints are fundamental barriers to the future growth of wireless communication networks, and to break this gridlock is the promise of energy harvesting cognitive radio (CR) networks. To this end, this paper investigates the feasibility of energy harvesting underlay CR networks with the primary system employing power control. We consider primary and underlay nodes distributed randomly in $\mathbb{R}^2$ as homogeneous Poisson point processes (PPP). Underlay transmitters scavenge power from primary transmitters, and are able to transmit as long as they are outside a guard region surrounding a primary receiver. The primary and underlay systems are assumed to perform power control based on path loss inversion, and that a underlay transmitter requires N charging slots to fully charge its batteries after depletion. We consider two cases of power depletion after a underlay transmission: 1) full power depletion, and 2) partial power depletion based on distances to the intended receivers. We derive the probability of a successful charge by mapping the PPP of primary transmitters to an equivalent PPP incorporating random transmit powers, and use a Markov chain to derive the probability of a successful transmission while incorporating temporal effects for the two aforementioned power depletion scenarios. We show that the probability of successful transmission is not greatly affected by the guard distance, and that it drops by approximately 10 fold for each $15\,$dB increase in the threshold received power level required for an energy harvest. We further show that energy harvesting is most feasible when the threshold power required for a harvest is lower than the receiver sensitivity of a primary receiver.**

I. Introduction

Cognitive radio (CR) is a promising candidate technology to alleviate spectrum scarcity in fifth generation (5G) cellular wireless networks [1], and it supports several applications including sensor networks, device-to-device networks, and mobile ad-hoc networks. One variant, underlay networks [2] are especially attractive since they allow secondary nodes concurrent access to licensed spectrum. However, this concurrent access must not cause performance degradation to the spectrum licensee (primary user devices) due to co-channel interference. To limit this interference, implementations of guard regions around the primary nodes and power control mechanisms are thus needed.

Apart from spectrum constraints, energy constraints have been recognized as a bottleneck for wireless devices [3]. Therefore, wireless underlay nodes may be powered by energy harvesting [4], which is especially attractive for cognitive sensor nodes [5]. This harvesting from the ambient environment enables greener devices, and extends the lifetime of sensor nodes indefinitely due to the self-sustaining nature of the harvesting process. Practical RF devices have been shown to successfully harvest far field non-directive energy in both the UHF and ISM bands at power densities of $1\,\mu W/cm^2$ or lower [6].

While the energy source can either be a dedicated one or an ambient one [5], energy harvesting from the primary network itself is the most logical choice for an underlay CR network [4]. However, the situation is complicated by the random nature of both primary and underlay node distributions, power control procedures, and guard regions. Therefore, investigating the probabilities of successful energy harvests and resulting underlay transmissions while considering these uncertainties is essential to better understand the feasibility of energy harvesting underlay networks.

A. Related Research

Energy harvesting techniques in CR has received much interest in the research community, and their combination is expected to increase both spectrum and energy efficiency in 5G networks [1]. Thus, [7] proposes a throughput maximization technique for energy harvesting CR devices operating in the hybrid overlay-underlay mode. Cognitive and energy harvesting based device-to-device communication is modeled and analyzed in [8]. Furthermore, [9] proposes a queuing model to analyze energy harvesting CR networks considering multiple underlay users and channel states while [10] investigates optimum spectrum sensing strategies for CR networks subject to energy causality and collision constraints. Moreover, [11] investigates optimal co-operation strategies between a pair of primary and underlay users in an energy harvesting CR set-up, and [4] proposes and analyzes a novel network model for energy harvesting underlay nodes to co-exist with a primary network.

B. Motivation and Contribution

Several studies have focused on the success probability of harvesting energy, and throughput maximization. Although most research consider a static network or a network with only a pair of primary-underlay nodes, node locations and numbers can be random. Reference [4] uses a stochastic model to propose a novel architecture where underlay nodes are powered by harvested energy from the primary network. However, the authors only consider fixed-power primary transmitters. Moreover, the underlay nodes were assumed to fully deplete their energy after a transmission. However, power control procedures employed in practical set-ups significantly alter the zone where a successful harvest may occur, and also ensure that the energy level is not always fully depleted after a transmission.

In this paper, we model primary and underlay nodes stochastically using Poisson point processes (PPPs), and underlay transmitters harvest ambient RF energy from primary transmissions. All signal transmissions undergo Rayleigh fading and log-distance path loss. Path loss inversion based power control is considered for both primary and underlay networks, and the underlay devices are peak power limited being subject to a maximum transmit power threshold. Primary transmitters may transmit to any random receiver within the cell, whereas the underlay transmitters attempt to communicate with the closest available receivers. Moreover, underlay transmitters within the guard regions surrounding primary receivers are barred from transmitting. Our main contributions of this paper are as follows:

- We derive the probability of an underlay node being charged when spatially random primary transmitters employ log-distance power control using stochastic geometry tools such as the Mapping and Marking theorems. The PPP of primary transmitters is transformed to an equivalent inhomogeneous PPP which incorporates the random transmit powers; the lowest distance metric of the new PPP corresponds to the primary transmitter providing the best ambient RF power.
- We derive the steady state transmission probability of an underlay transmitter while considering temporal correlations using a Markov chain based approach. To this end, we consider two transmission scenarios: 1) underlay transmitters fully deplete their power after each transmission, and 2) underlay transmitters only partially deplete their power after each transmission where the amount of depletion is a random quantity depending on the distance to their associated receivers.

II. System Model

A. Spatial Distribution

We consider a system where an underlay network is colocated within an operating area of a primary network spanning $\mathbb{R}^2$. The primary network consists of a random set of transmitters and receivers, where each transmitter connects to one receiver within a given time-frequency slot. While there may be multiple frequency bands used by the primary system, the underlay system is assumed to only access one. The primary transmitters may be interpreted as base stations and primary receivers as users requiring service. The underlay network also consists of transmitter and sink nodes which are also randomly distributed. The number and locations primary and underlay nodes are random, and thus need to be modelled using a stochastic geometry based approach. To this end, we will use independent homogeneous PPPs [12] to model each type of node. In realistic networks, the primary receivers will most often be clustered around transmitters, and are thus best represented by a Poisson cluster process. However, Poisson cluster processes are inherently difficult to analyze, and thus we select homogeneous PPPs for each type of node in order to conduct a tractable analysis. The homogeneous PPP has been extensively used in recent literature to model spatial node distributions in underlay networks [13], [14]. In a homogeneous PPP with density λ, the probability of having k nodes within a given area A ($\Pr[N(A) = k]$) is given by

$$\Pr[N(A) = k] = \frac{(\lambda A)^k}{k!} e^{-\lambda A}. \tag{1}$$

Let the processes of primary transmitters and receivers be denoted as Φ_{pt} and Φ_{pr} with respective densities λ_{pt} and λ_{pr}, while the processes of underlay transmitters and receivers be denoted as Φ_{st} and Φ_{sr} with respective densities λ_{st} and λ_{sr}. Each primary transmitter $\phi_{pt} (\in \Phi_{pt})$ has a probability of being active within a given slot. Let this probability be denoted as p_{pt}. Using the Coloring Theorem [12], the process of active primary transmitters can be modelled as a thinned PPP. If this new PPP is denoted as $\tilde{\Phi}_{pt}$, its intensity $\tilde{\lambda}_{pt}$ is given by $\tilde{\lambda}_{pt} = p_{pt}\lambda_{pt}$.

B. Signal Propagation

We assume that all signals are subject to small scale fading and exponential path loss. We model small scale fading using the Rayleigh distribution, and as such, the small scale channel power gain can be represented using an exponential distribution. If the channel power gain is denoted by $|h|^2$, $f_{|h|^2}(x) = e^{-x}, 0 \leq x \leq \infty$. Moreover, it is assumed that fading between different pairs of nodes is independent. To model path loss, we will utilize the simplified path loss model in which the received power at a distance r from the transmitter averaged over fading is given by $P_r = P_T r^{-\alpha}$, where $\alpha (> 2)$ is termed the path loss exponent. P_T is a value dependent on antenna gains, frequency, and transmit power. However, with all other factors being constant, it may be interpreted without the loss of generality as the "transmit power level".

C. Power Control and Receiver Selection

1) Primary System: Each active primary transmitter $\tilde{\phi}_{pt}$ connects with the primary receiver ϕ_{pr} requiring service. We assume that each $\tilde{\phi}_{pt}$ can connect with a ϕ_{pr} up to a distance of R_C from $\tilde{\phi}_{pt}$. R_c would correspond to the system's maximum transmission distance or the average cell radius. Let us denote the ϕ_{pr} requiring service as $\phi_{pr,i} \in \Phi_{pr}$ and the transmitter node serving it as $\tilde{\phi}_{pt,j} \in \Phi_{pt}$. If the location of $\phi_{pr,i}$ is x_i and the location of $\tilde{\phi}_{pt,j}$ is y_j, the distance between them $r_{i,j}$ is given by $r_{i,j} = ||x_i - y_j||$. Because Φ_{pr} forms a homogeneous PPP, the number of primary receivers increases linearly with the area considered. As such, the CDF of $r_{i,j}$ is obtained as $F_{r_{i,j}}(x) = \frac{\pi x^2}{\pi R_C^2}$, $0 < x < R_C$. Using the CDF, the PDF of $r_{i,j}$ can be written as

$$f_{r_{i,j}}(x) = \begin{cases} 2\frac{x}{R_C^2} & , 0 < x < R_C \\ 0 & , \text{otherwise} \end{cases}. \tag{2}$$

Each $\tilde{\phi}_{pt}$ uses a distance based power control scheme based on path loss inversion to ensure a constant received power level at each ϕ_{pr} when averaged over small scale fading. Let this power level be denoted as P_{rec}. P_{rec} can be interpreted as the receiver sensitivity plus a suitable fade margin. The fade margin is used to negate effects of small scale fading. Each $\tilde{\phi}_{pt}$ estimates the path loss to ϕ_{pr} using signal strength feedback information and sets its transmit power level. Therefore, the transmit power of $\tilde{\phi}_{pt,j}$ ($Pp_{T,j}$) can be written as $P_{T,j} = P_{rec} r_{i,j}^{\alpha}$ [15].

2) Underlay System: Beacause the underlay transmitters scavenge power from the primary system, they have a limited power budget. Let P_{max} be the maximum available power level of any underlay transmitter ϕ_{st}, and $\phi_{st,k}$ be the k-th underlay transmitter located at u_k. The probability that $\phi_{st,k}$ has data to be transmitted at any given slot is denoted as $p_{st,d}$. $\phi_{st,k}$ would initially attempt to transmit its data to the closest underlay receiver $\phi_{sr,k\backslash 1} \in \Phi_{sr}$ located at $v_{k\backslash 1}$ employing a path loss inversion based power control method. Path loss inversion for the underlay network may be carried out using received signal strength feedback from each ϕ_{st} for pilot signals or via prior knowledge of the locations of each ϕ_{sr}. Let the distance between $\phi_{st,k}$ and $\phi_{sr,k\backslash 1}$ be denoted as $r_{k,k\backslash 1}$. Thus $r_{k,k\backslash 1} = ||u_k - v_{k\backslash 1}||$, and its probability distribution can be obtained as [15]

$$f_{r_{k,k\backslash 1}}(x) = 2\pi\lambda_{sr} x e^{-\pi\lambda_{sr}x^2}, 0 < x < \infty. \tag{3}$$

If $P_{rec,s}$ is the average received power level ensured on a underlay receiver, the transmit power level of $\phi_{st,k}$ ($Ps_{T,j}$) can be written as $Ps_{T,k} = P_{rec,s} r_{k,k\backslash 1}^{\alpha}$. However, if $Ps_{T,k} > P_{max}$, transmission cannot occur. Whenever such a situation occurs, $\phi_{st,k}$ would transmit at P_{max}, and other available underlay transmitters able to receive the message will relay it to their nearest ϕ_{sr}. It is assumed that this relaying would be concurrent with the relay node's transmission itself (In other words the message from $\phi_{st,k}$ is relayed only when the relaying node is transmitting its own data). If no other ϕ_{st} is able to receive the transmission from $\phi_{st,k}$, the intended transmission fails. This situation occurs whenever no other ϕ_{st} exists within a distance of $\left(\frac{P_{max}}{P_{rec,s}}\right)^{\frac{1}{\alpha}}$ from $\phi_{st,k}$. Note that we assume all ϕ_{st} and ϕ_{sr} have the same receiver sensitivity. The probability of this occurring is $e^{-\pi\lambda_{st}\left(\frac{P_{max}}{P_{rec,s}}\right)^{\frac{2}{\alpha}}}$.

Due to concurrent spectrum usage, guard regions are required around the primary receivers to limit interference. The guard regions may be implemented using either prior location information or periodic beacon signals. Whenever a underlay transmitter ϕ_{st} falls within the guard region of a primary receiver ϕ_{pr} which is receiving a transmission in the same time-frequency slot, it refrains from transmitting. However, within a given time slot for the frequency band accessed by the underlay network, only a single ϕ_{pr} is served by any primary transmitter ϕ_{pt}. Moreover, there is a probability that this particular time frequency slot remains unoccupied for a particular ϕ_{pt} (with probability $1-p_{pt}$). Therefore, the process of primary receivers occupying the channel can be denoted as $\tilde{\Phi}_{pr}$ having a density $\tilde{\lambda}_{pt}$, which is the density of active primary transmitters.

We will assume that the guard regions are in the shape of discs around each active primary receivers, and that the radius of each disc is R_g where R_g is termed the guard distance. For $\phi_{pr,i}$, the guard region surrounding it is denoted as $b(x_i, R_g)$. Therefore, if $u_k \in \mathcal{G}$, $\phi_{st,k}$ will be barred from transmission, where $\mathcal{G} \in \mathbb{R}^2$ is the area within all guard zones surrounding active primary receivers. $\mathcal{G}$ can be written as $\mathcal{G} = \bigcup_{i \in \tilde{\Phi}_{pr}} b(x_i, R_g)$. Let $p_{st,g}$ be the probability that $\phi_{st,k}$ doesn't fall within $\mathcal{G}$. $p_{st,g}$ is derived using the void probability of a PPP as $p_{st,g} = e^{-\pi\tilde{\lambda}_{pt}R_g^2}$.

D. Energy Harvesting

The underlay transmitters are assumed to rely on harvesting energy from the primary system for their power requirements. However, practical circuits used to convert energy from ambient radio-frequency signals have specific sensitivity requirements [16]. If the ambient power level falls below P_{γ}, the power conversion circuit in $\phi_{st,k}$ will not be able to harvest energy. Let $r_{k,j} = ||u_k - y_j||$ be the distance between $\phi_{st,k}$ and $\tilde{\phi}_{pt,j}$. We can represent the criterion for a successful energy harvest for $\phi_{st,k}$ from $\tilde{\phi}_{pt,j}$ within a time slot as $Pp_{T,j} r_{k,j}^{-\alpha} > P_{\gamma}$ (note that we neglect the small scale fading within a time slot which average out to 1. In other words, $\phi_{st,k}$ has to be within a harvesting zone of radius $r_{h,j}$ around $\tilde{\phi}_{pt,j}$, where $r_{h,j} = \left(\frac{Pp_{T,j}}{P_{\gamma}}\right)^{\frac{1}{\alpha}}$. Therefore, for an energy harvest from any $\tilde{\phi}_{pt}$, $u_k \in \mathcal{P}$, where $\mathcal{P}$ is the union of all harvesting zones around $\tilde{\phi}_{pt}$. We can denote $\mathcal{P}$ as $\mathcal{P} = \bigcup_{j \in \tilde{\Phi}_{pt}} b(y_j, r_{h,j})$, where $b(y_j, r_{h,j}) \in \mathbb{R}^2$ denotes a disc shaped area of radius $r_{h,j}$ surrounding y_j. However, it is worth keeping in mind that because $Pp_{T,j}$ is a random variable, $r_{h,j}$ is also a random variable.

We assume that each ϕ_{st} needs N time slots of successful energy harvesting to get fully charged, and that transmission would not occur from a ϕ_{st} unless it's fully charged. In other words, the power conversion circuitry has limited performance, and even if the ambient RF power well exceeds P_{γ}, the harvested energy per time slot remains the same. Moreover, whenever a ϕ_{st} conducts a transmission, we consider two cases for power depletion: 1) ϕ_{st} is assumed to be fully depleted of power, and needing the full N charging slots, and 2) ϕ_{st} may require less than N time slots for charging depending on the transmitted power which in turn depends on the location of the nearest ϕ_{sr}.

III. Probability of ϕ_{st} Being Within the Harvesting Region

In this section, we will derive the probability that the k-th underlay transmitter $\phi_{st,k}$ lies within the harvesting region $\mathcal{P} \in \mathbb{R}^2$ which we'll denote as $p_{st,p}$. Our objective is to find $\Pr[u_k \in \bigcup_{j \in \tilde{\Phi}_{pt}} b(y_j, r_{h,j})]$. However, this is complicated by the fact that individual harvesting zones surrounding each $\tilde{\Phi}_{pt}$ have variable radii due to the power control scheme adopted by the primary system. To overcome this, we will employ the Mapping theorem [12] of PPPs. The Mapping theorem is used to transform a given PPP into an alternate PPP of another dimension while providing equivalent statistics. Thus, our objective is to translate the homogeneous PPP of $\tilde{\Phi}_{pt}$ with a variable transmit power and path loss exponent α into an equivalent PPP ($\tilde{\Phi}'_{pt}$) with a constant transmit power of 1 and a path loss exponent of 1 such that the received power at $\phi_{st,k}$ from $\tilde{\Phi}'_{pt}$ is the same on average.

Without the loss of generality, we will take $\phi_{st,k}$ to be located at the origin. With respect to $\phi_{st,k}$, $\tilde{\Phi}_{pt}$ is a homogeneous PPP in $\mathbb{R}^2$ with intensity $\tilde{\lambda}_{pt}$. Using the Mapping Theorem, we can translate $\tilde{\Phi}_{pt}$ into an inhomogeneous PPP on the positive real axis $\tilde{\Phi}_{pt,1}$. If the intensity of this PPP is $\tilde{\lambda}_{pt,1}$, it is given by

$$\tilde{\lambda}_{pt,1} = 2\pi\tilde{\lambda}_{pt}r, 0 < r < \infty. \tag{4}$$

Using the Mapping theorem further as in [17], we can develop the PPP of primary transmitters which provides the same received power statistics at $\phi_{st,k}$ with a unit path loss exponent and a constant transmit power of 1 ($\tilde{\Phi}'_{pt}$). In other terms, the received power at $\phi_{st,k}$ from $\tilde{\phi}_{pt,j}$ neglecting small scale fading which is $P_{rec}r_{i,j}^{\alpha}r_{k,j}^{-\alpha}$ becomes $\frac{1}{r}$, where r is a distance based metric, and not the true distance[1]. If the intensity of $\tilde{\Phi}'_{pt}$ is given by $\tilde{\lambda}'_{pt}$, it can be obtained as follows using product space representation, the Mapping theorem, and the Marking theorem [12].

$$\begin{aligned}\tilde{\lambda}'_{pt} &= E_{r_{i,j}}\left[P_{rec}r_{i,j}^{\alpha}\frac{2\pi\tilde{\lambda}_{pt}}{\alpha}(rP_{rec}r_{i,j}^{\alpha})^{\frac{2}{\alpha}-1}\right] \\ &= \frac{2\pi\tilde{\lambda}_{pt}}{\alpha}r^{\frac{2}{\alpha}-1}P_{rec}^{\frac{2}{\alpha}}E_{r_{i,j}}[r_{i,j}^2] \\ &= \frac{\pi\tilde{\lambda}_{pt}}{\alpha}r^{\frac{2}{\alpha}-1}P_{rec}^{\frac{2}{\alpha}}R_C^2, 0 < r < \infty. \end{aligned} \tag{5}$$

Now, the $\tilde{\Phi}_{pt}$ providing the highest ambient RF power to $\phi_{st,k}$ would have the lowest distance metric r within the new PPP $\tilde{\Phi}'_{pt}$. Let's denote this primary transmitter as $\tilde{\phi}_{pt,h}$, and its distance metric from $\phi_{st,k}$ as r_h. Thus, the received power at $\phi_{st,k}$ from $\tilde{\phi}_{pt,h}$ can be written as $\frac{1}{r_h}$. Therefore, if $\frac{1}{r_h} < P_\gamma$, $\phi_{st,k} \notin \mathcal{P}$. Conversely, if $\frac{1}{r_h} > P_\gamma$, $\phi_{st,k}$ lies within the harvesting region of at least one $\tilde{\phi}_{pt}$.

By definition, there will be 0 nodes with a distance metric of less than r_h. As such, the CDF of r_h is found out using (5) and (1) as

$$\begin{aligned}F_{r_h}(x) &= 1 - e^{-\int_0^x \frac{\pi\tilde{\lambda}_{pt}}{\alpha}r^{\frac{2}{\alpha}-1}P_{rec}^{\frac{2}{\alpha}}R_C^2 dr} \\ &= 1 - e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(xP_{rec})^{\frac{2}{\alpha}}R_C^2}. \end{aligned} \tag{6}$$

Now, we can obtain the final expression for $p_{st,p}$ as

$$\begin{aligned}p_{st,p} &= \Pr[\frac{1}{r_h} > P_\gamma] \\ &= 1 - e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(\frac{P_{rec}}{P_\gamma})^{\frac{2}{\alpha}}R_C^2}. \end{aligned} \tag{7}$$

[1]This distance metric now includes information about the path loss exponent and the transmit power, and thus is not the actual distance.

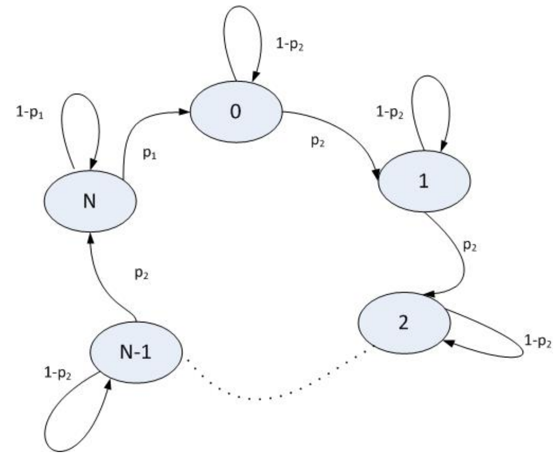

Fig. 1: Markov chain model for full power depletion after each transmission with 0 being the uncharged state and N being the fully charged state.

IV. Transmission Probability of ϕ_{st}

We will now derive the steady-state transmission probability for the k-th underlay transmitter $\phi_{st,k}$.

A. Power is fully depleted during a transmission

We will first consider the case when ϕ_{st} fully depletes it's power resource during a transmission; thus needing the full N charging slots. Let's denote the transmission probability of $\phi_{st,k}$ as p_{st}. p_{st} depends on the probability that $\phi_{st,k}$ is outside a guard zone ($p_{st,g}$), the probability that $\phi_{st,k}$ is fully charged at the start of the given time slot (denoted as $p_{st,c}$), and the probability that $\phi_{st,k}$ has data to be transmitted at the beginning of a time slot ($p_{st,d}$). We assume that these probabilities are mutually independent. Moreover, it is assumed that there are no spatial correlations on node locations between different time slots[2]. Thus, we can write p_{st} as

$$p_{st} = p_{st,c}p_{st,g}p_{st,d}. \tag{8}$$

However, there exists a temporal correlation with regards to $p_{st,c}$ as the current state of $\phi_{st,k}$ regarding power levels depends on events which occurred before. Therefore, we will use a Markov chain to analyze $p_{st,c}$.

$\phi_{st,k}$ needs N successful slots of charging before each transmission. Therefore, the Markov chain will have $N+1$ states with 0 being the uncharged state, N being the fully charged state, and 1 to $N-1$ being the intermediate states (Fig. 1). The probability of transitioning from state N $p_1 = p_{st,g}p_{st,d}$, while the probability of transitioning from the other states

[2]In reality, a correlation occurs such that whenever $\phi_{st,k}$ is within the guard region or the harvesting region within a particular time slot T, it is more likely to be in the same region within the subsequent time slots $T+1, T+2, \ldots$.

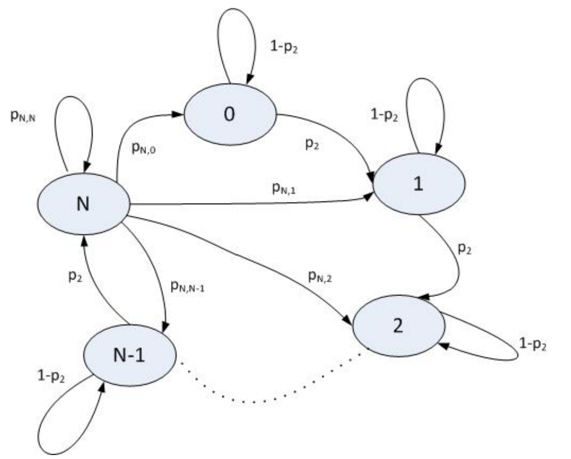

Fig. 2: Markov chain model for partial power depletion with 0 being the uncharged state and N being the fully charged state.

$p_2 = p_{st,p}$. Let P denote the state transition matrix of the Markov chain. We can write P as follows:

$$P = \begin{bmatrix} 1-p_2 & p_2 & 0 & . & . & 0 \\ 0 & 1-p_2 & p_2 & . & . & 0 \\ 0 & 0 & 1-p_2 & . & . & 0 \\ . & . & . & . & . & . \\ p_1 & 0 & 0 & . & . & 1-p_1 \end{bmatrix}.$$

Now, $p_{st,c}$ is simply the steady state probability of state N. Let ω be the steady state probability vector of P, where $\omega = [\omega_0\ \omega_1\ .\ .\ \omega_N]$. At steady state, we can write

$$\omega = P\omega. \tag{9}$$

Solving this equation, we obtain

$$\begin{aligned} p_{st,c} &= \omega_N = \frac{p_2}{p_2 + Np_1} \\ &= \frac{1-e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(\frac{P_{rec}}{P_\gamma})^{\frac{2}{\alpha}}R_C^2}}{1-e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(\frac{P_{rec}}{P_\gamma})^{\frac{2}{\alpha}}R_C^2} + N\left(p_{st,d}e^{-\pi\tilde{\lambda}_{pt}R_g^2}\right)}. \end{aligned} \tag{10}$$

B. Partial power depletion

We now consider the situation where ϕ_{st} may not fully deplete its power when a transmission is conducted. When the k-th ϕ_{st} ($\phi_{st,k}$) attempts to associate with its closest ϕ_{sr} ($\phi_{sr,k\backslash 1}$), it must transmit at a power level of $Ps_{T,k}$ whenever $Ps_{T,k} < P_{max}$. Therefore, after the transmission has been conducted, the power level of $\phi_{st,k}$ lies within an intermediate state (1 to $N-1$). Let $p_{N,\nu}$ be the transition probability from state N to state ν ($\nu \in (1,\ .\ .\ N-1)$). However, if $Ps_{T,k} < P_{max}$, $\phi_{st,k}$ transmits at full power (P_{max}), and thus transitions to state 0 with probability $p_{N,0}$. The probability of not conducting a transmission at state N is similar to the previous case, and is given by $p_{N,N} = 1 - p_{st,g}p_{st,d}$. The Markov chain for this scenario is depicted in Fig. 2. If Q denotes the state-transition matrix, we can write Q as

$$Q = \begin{bmatrix} 1-p_2 & p_2 & 0 & . & . & 0 \\ 0 & 1-p_2 & p_2 & . & . & 0 \\ 0 & 0 & 1-p_2 & . & . & 0 \\ . & . & . & . & . & . \\ p_{N,0} & p_{N,1} & p_{N,2} & . & . & p_{N,N} \end{bmatrix}.$$

Let Ω be the steady state probability vector of Q. We can write Ω as $\Omega = [\Omega_0\Omega_1\ .\ .\ \Omega_N]$. At steady state, $\Omega = Q\Omega$. Thus, we obtain $p_{st,c} = \Omega_N$ as

$$p_{st,c} = \frac{1-e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(\frac{P_{rec}}{P_\gamma})^{\frac{2}{\alpha}}R_C^2}}{1-e^{-\frac{\pi\tilde{\lambda}_{pt}}{2}(\frac{P_{rec}}{P_\gamma})^{\frac{2}{\alpha}}R_C^2} + \sum_{l=0}^{N-1}(N-l)p_{N,l}}. \tag{11}$$

In order to evaluate (11), we need to obtain $p_{N,l}$ where $l \in (0,\ .\ .\ N-1)$. $p_{N,0}$ is the probability that $\phi_{st,k}$ fully depletes its power. This occurs whenever the required power needed to transmit to $\phi_{sr,k\backslash 1}$ is greater than the maximum possible transmit power P_{max}, given that $\phi_{st,k}$ does have data to transmit and that $\phi_{st,k} \notin \mathcal{G}$. We can thus derive $p_{N,0}$ as

$$\begin{aligned} p_{N,0} &= p_{st,g}p_{st,d}\Pr[Ps_{T,k} > P_{max}] \\ &= p_{st,g}p_{st,d}e^{-\pi\lambda_{sr}\left(\frac{P_{max}}{P_{rec,s}}\right)^{\frac{2}{\alpha}}}. \end{aligned} \tag{12}$$

We assume that the threshold transmit power level differences to be within each of the $N-1$ intermediate states are equal. Moreover, irrespective of the exact power level within the ν-th ($\nu \in (1,\ .\ .\ N-1)$) intermediate state, it takes $N-\nu$ charging slots for $\phi_{st,k}$ to get fully charged. The transmit power level $Ps_{T,k}$ thus has to satisfy $\frac{P_{max}(N-(\nu+1))}{N-1} < Ps_{T,k} < \frac{P_{max}(N-\nu)}{N-1}$ in order for $\phi_{st,k}$ to transition from state N to state ν. Using this fact, $p_{N,\nu}$ is obtained for $1 \le \nu \le N-1$ as

$$p_{N,\nu} = p_{st,g}p_{st,d}\left(e^{-\pi\lambda_{sr}\left(\frac{P_{max}(N-(\nu+1))}{P_{rec,s}(N-1)}\right)^{\frac{2}{\alpha}}} - e^{-\pi\lambda_{sr}\left(\frac{P_{max}(N-\nu)}{P_{rec,s}(N-1)}\right)^{\frac{2}{\alpha}}}\right) \tag{13}$$

V. Numerical Results

This section provides numerical results for the underlay user transmission probability (p_{st}) for both full and partial power depletion. Throughout the section, we will use the values of $P_{rec} = 1 \times 10^{-9}$, $P_{rec,s} = 1 \times 10^{-9}$, $P_{max} = -10\,\text{dBm}$, $\alpha = 3$, $p_{pt} = 1$, $p_{st,d} = 1$, $R_C = 1000$, and $\lambda_{pt} = \frac{1}{\pi} \times 10^{-4}$.

Fig. (3) plots p_{st} with respect to P_γ under different N and R_g. For higher P_γ values, we observed that p_{st} drops by 10 fold approximately for each increments of 15 dB. On the other hand, for lower P_γ values, a smaller guard region slightly increases p_{st}, but this characteristic diminishes as P_γ increases. Furthermore, this increase reduces with N. The plots flatten out for very low P_γ and the respective levels are determined by R_g, N, and λ_{pt}. The value of P_γ for which the plots flatten out roughly corresponds to -90 dBm which is the value of the underlay receiver sensitivity $P_{rec,s}$. As such, when $\frac{P_{rec}}{P_\gamma} > 1$, the feasibility of energy harvesting is high, and is further increased when the required number of charging slots (N) is less.

We now investigate the behaviour of p_{st} vs. P_γ under partial power depletion in Fig. (4). While decreasing N only shows a marginal effect at increasing p_{st} when $\lambda_{sr} = 1 \times 10^{-3}$, the effect is more pronounced for higher λ_{sr}. It is interesting to note that while the plots for $\lambda_{sr} = 1 \times 10^{-4}$ and $\lambda_{sr} = 1 \times 10^{-5}$ almost overlap when $N = 3$ and $N = 10$, there is a slight difference when $N = 5$. Furthermore, it is observed from the plots of $N = 5$ that when λ_{sr} is increased from 1×10^{-5} to 1×10^{-3}, p_{st} actually drops before increasing again. Similar to the full power depletion scenario, energy harvesting is most feasible for $\frac{P_{rec}}{P_\gamma} > 1$, when $0.1 < p_{st} < 1$ for lower N and higher λ_{sr}.

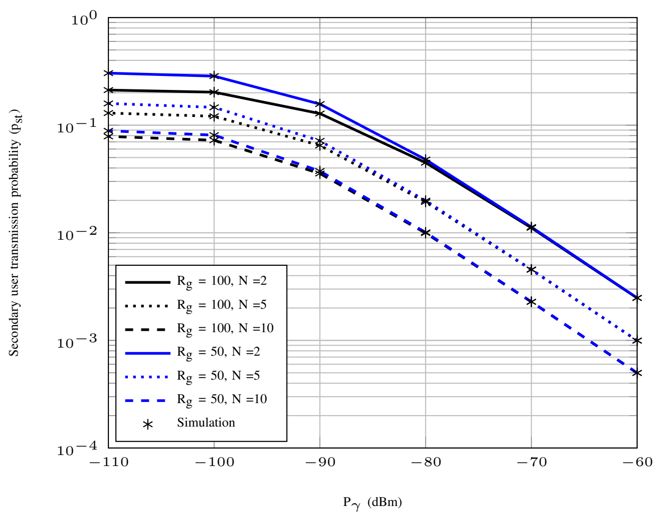

Fig. 3: Secondary user transmission probability p_{st} vs. the energy harvesting threshold P_γ for full power depletion.

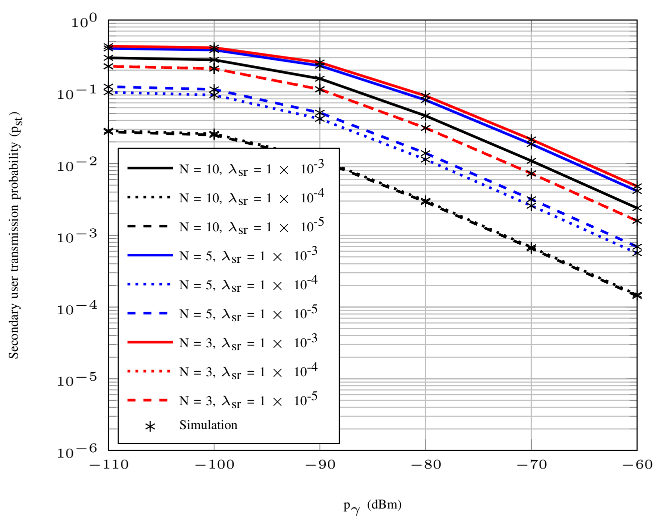

Fig. 4: Secondary user transmission probability p_{st} vs. the energy harvesting threshold P_γ under partial power depletion. $R_g = 50$.

VI. Conclusion

This paper investigated the probability of charging and the probability of successful transmission for a underlay node in an energy harvesting underlay CR setup. PPPs were considered to represent all types of nodes, and path loss inversion based power control schemes were considered for both primary and underlay networks, with an added maximum power constraint for the underlay network. From a fully depleted state, it was assumed that underlay nodes require N charging cycles. Two scenarios were considered where the underlay nodes deplete their total power, and where they only partially deplete their power after a transmission. It was observed that the probability of a successful transmission dropped by 10 fold with each 15 dB increase in P_γ. Moreover, the guard region has negligible impact for high P_γ. Furthermore, under partial power depletion, there exists a underlay receiver density value which provides the lowest successful transmission probability. To conclude, a ratio of $\frac{P_{rec}}{P_\gamma} > 1$ and a low N is required to achieve a high probability of successful transmission.

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