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Ergodic sum rate analysis and efficient power allocation for a massive MIMO two-way relay network

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Abstract: The authors study the transmit power allocation (PA) problem for a network of two multi-antenna terminals (one of which is a massive multiple-input and multiple-output (MIMO) terminal) and a two-way, amplify-and-forward relay. The relay is limited to a single antenna. Using perfect channel state information, the terminals employ beamforming with maximum-ratio-transmission and maximum-ratio-combining for transmission and reception, respectively. The authors investigate two practical problems, namely; (i) maximising the sum rate subject to a total power constraint (ii) maximising the sum rate when one of the terminals must exceed a target signal-to-noise ratio (SNR). For the first case, the authors derive the closed-form optimal PA and for the second, the authors derive a sub-optimal PA. In both cases, the resulting sum rates are a function of instantaneous channel gains. Thus by averaging over the Nakagami-*m* distribution and exploiting the weak law of large numbers, the authors derive the closed-form ergodic sum rates. Finally, the simulation results validate the theoretical analysis and show the sum-rate improvements over uniform PA. For example, to achieve 4 bit/s/Hz, a uniform allocation needs 1 dB more than the authors' optimal allocation. When one of the SNRs must exceed a target value, the gap between the authors' sub-optimal PA and random PA increases to 2 dB.

1 Introduction

Two-way (TW) relaying is a powerful communication protocol to improve the spectral efficiency of wireless networks [1]. Well-known relaying protocols are amplify-and-forward [2–4], decode-and-forward [5], and compress-and-forward [6]. The analogue network coding (ANC) TW relay channel [7–13] requires only two time slots to enable a full bidirectional data exchange. Thus, it overcomes the half-duplex spectral loss inherent in one-way relays [1].

Power allocation (PA) can improve the total data rate or the outage performance of ANC TW relaying, while keeping the total power constant. PA techniques have thus been investigated widely; for example, for Rayleigh fading channels, various PA schemes are developed in [14–19]. One possible PA method is to balance the instantaneous signal-to-noise ratios (SNRs) [14–16]. In [17], a power provisioning strategy is formulated to minimise the total energy with individual outage constraints. In [18], the resource (i.e. time and power) allocation problem for the protocol with perfect receiver-side channel knowledge is studied from an outage perspective. Xu *et al.* [19] proves that the two optimal PA schemes in the sense of minimising the outage probability and maximising the achievable sum rate are equivalent to each other. However, this result is limited for single antenna nodes. The case of relay power optimisation with no power control at the sources is studied under Nakagami-m fading in [20–23]. For one-way decode-and-forward scheme, the problem of joint optimisation of power (at source and relay nodes) and relay location over Nakagami-m fading channels is investigated in [23]. The problem of joint optimal PA among all terminals and relay location for the ANC protocol over Nakagami-m fading channels has been investigated in [24]. In [15], a PA for minimising outage probability is proposed under the assumption of high SNR.

In a non-regenerative TW multiple-input and multiple-output (MIMO) relay system, two fast algorithms for optimising the source covariance matrices and the relay transformation matrix have been proposed in [25]. Based on a space-division scheme, an algorithm has been proposed to maximise the sum rate of a MIMO TW relay channels (TWRC) in [26]. Yang *et al.* [27] have proposed a new eigen-direction alignment precoding technique to enlarge the achievable rate region compared to the existing schemes for a MIMO TWRC. Exploiting generalised singular value decomposition-based precoding and successive interference cancellation decoding for a separated MIMO TWRC, the achievable rate region has been obtained in [28].

In this paper, we consider a TW relay system exploiting ANC with multiple antenna terminals (users) where one terminal is a massive MIMO terminal. Using perfect CSI, the terminals employ beamforming [15] with maximum-ratio-transmission (MRT) and maximum-ratio-combining (MRC) for transmission and reception, respectively. We investigate two practical problems, namely; (i) maximising the sum rate subject to a total power constraint (ii) maximising the sum rate subject to additional constraints on target user SNRs. For these two optimisation problems, we derive closed-form, analytical solutions. By exploiting the weak-law of large numbers, we then derive closed-form expressions for the achievable ergodic sum rate over independent but not necessarily identically distributed Nakagami-m fading channels. Moreover, we show that the derived ergodic sum rate expressions match well with the simulation results. Finally, our results highlight the influence of the proposed PA strategies on the achievable sum rate.

Notations: In this paper, bold, lowercase letters are used to represent vectors and bold, uppercase letters are used to denote matrices. $\boldsymbol{X}^\dagger$ and $\boldsymbol{X}^\mathrm{T}$ denote the Hermitian and the transpose of $\boldsymbol{X}$, respectively. $E(\cdot)$ and $||\cdot||$ denote statistical expectation and the Euclidean norm. Moreover, $\Gamma(x, y)$ is the incomplete Gamma function. The set of all positive integers is $\mathbb{N}$.

2 System model

We consider a TW relay system including one single antenna relay R and two multiple antenna terminals T_a and T_b with N_a and N_b

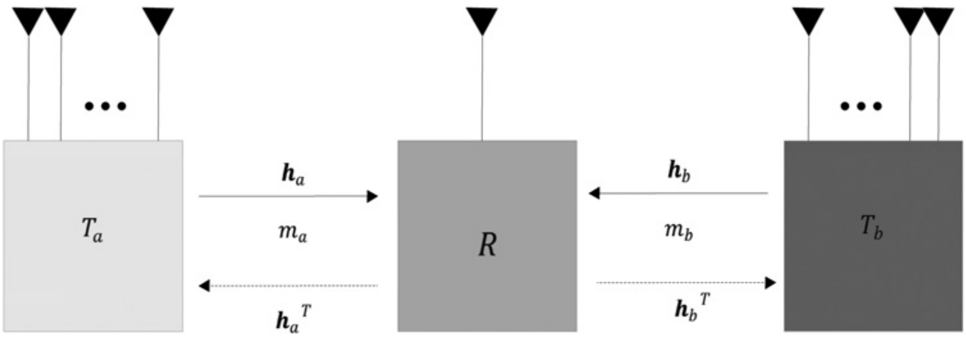

Fig. 1 *System model of a TW relay network using multiple antennas*

antennas, respectively, where $N_a \gg N_b$. This setup, for example, can occur when a massive MIMO base station (T_a) is communicating with a MIMO base station (T_b) using a mobile user (R) as relay. Note that our setup is different from conventional MIMO TW relay systems, where the relay has multiple antennas [26, 27]. Our work, similar to [29–32] assumes a single antenna relay, but the terminals are equipped with multiple antennas.

The channel coefficients $\boldsymbol{h}_a$ and $\boldsymbol{h}_b$ between T_a and T_b, and relay R are assumed to be reciprocal and independent. Moreover, additive white Gaussian noise (AWGN) with mean zero and variance σ^2 are assumed for each link. Let P_a, P_b and P_r denote the transmit powers at the sources T_a, T_b and R, respectively. The system model is shown in Fig. 1.

The bi-directional communication between the two terminals takes two time slots for completion. In the first time slot, both terminals transmit to the relay. In the second time slot, the relay broadcasts the received composite signal to the two terminals. The received signal at R in the first time slot can be expressed as

$$y_\mathrm{r} = \sqrt{P_a}\boldsymbol{h}_a^\mathrm{T}\boldsymbol{w}_a x_a + \sqrt{P_b}\boldsymbol{h}_b^\mathrm{T}\boldsymbol{w}_b x_b + n_\mathrm{r}, \quad (1)$$

where x_a and x_b are unit energy transmit signals at the terminals and n_r is AWGN. With MRT beamforming, the transmit weight vectors are $\boldsymbol{w}_l = \left(\boldsymbol{h}_l^\dagger/\|\boldsymbol{h}_l\|\right)^\mathrm{T}$, $\forall l \in \{a, b\}$. The transmit signal of the relay may be written as $\hat{y}_\mathrm{r} = G y_\mathrm{r}$ where $G = \sqrt{P_\mathrm{r}/(P_a\|\boldsymbol{h}_a\|^2 + P_b\|\boldsymbol{h}_b\|^2 + \sigma^2)}$, where the gain G selected to satisfy the relay power constraint. Finally, the received signals at terminals T_a and T_b, are given by

$$\boldsymbol{y}_l = \boldsymbol{h}_l G\left(\sqrt{P_l}\|\boldsymbol{h}_l\| x_l + \sqrt{P_c}\|\boldsymbol{h}_c\| x_c + n_\mathrm{r}\right) + \boldsymbol{n}_l, \quad \forall l \in \{a, b\} \quad (2)$$

where $c = \{a, b\}\backslash\{l\}$ and $\boldsymbol{n}_l$ denotes the AWGN vector at terminal l After self-interference cancellation and MRC reception with weights $\boldsymbol{w}_l^\mathrm{T} = \left(\boldsymbol{h}_l^\dagger/\|\boldsymbol{h}_l\|\right)$, $\forall l \in \{a, b\}$, the received signals at both terminals can be expressed as

$$\hat{y}_l = G\sqrt{P_c}\|\boldsymbol{h}_l\|\|\boldsymbol{h}_c\| x_c + G\|\boldsymbol{h}_l\| n_\mathrm{r} + \hat{n}_l, \quad \forall l \in \{a, b\} \quad (3)$$

where $\hat{n}_l = \boldsymbol{h}_l^\dagger \boldsymbol{n}_l/\|\boldsymbol{h}_l\|$, $\forall l \in \{a, b\}$. Using (3), the SNRs at the two terminals can be obtained as

$$\gamma_l = \frac{P_c P_\mathrm{r}}{\sigma^2}\left[\frac{\|\boldsymbol{h}_c\|^2\|\boldsymbol{h}_l\|^2}{(P_l + P_\mathrm{r})\|\boldsymbol{h}_l\|^2 + P_c\|\boldsymbol{h}_c\|^2 + \sigma^2}\right], \quad \forall l \in \{a, b\}. \quad (4)$$

3 Problem statement and proposed method

3.1 Problem formulation

Restricting the total power consumed below a threshold can control the total interference of this network on neighbouring networks. Thus, the total power constraint has also been considered in [14, 33–35]. Therefore, we maximise the achievable sum rate of the system subject to a total power constraint, which can be formulated as

$$\max_{P_a, P_b, P_\mathrm{r}} \boldsymbol{R} \quad \text{s.t} \quad P_a + P_b + P_\mathrm{r} = P_\mathrm{t}, \quad \text{(P1)}$$

where $\boldsymbol{R} = (1/2)\log_2(1 + \gamma_a) + (1/2)\log_2(1 + \gamma_b)$. Since $\log(x)$ is an increasing function, by combining the two log terms, we can reformulate (P1) as

$$\max_{P_a, P_b, P_\mathrm{r}} (1 + \gamma_a)(1 + \gamma_b) \quad \text{s.t} \quad P_a + P_b + P_\mathrm{r} = P_\mathrm{t}. \quad \text{(P2)}$$

3.2 Optimal PA

In the following, we give the exact solution to (P2):

Theorem 1: For massive antenna users, the optimal PA of (P2) is

$$P_a = \frac{P_\mathrm{t}}{2(\sqrt{\nu} + 1)}, \quad P_b = \frac{P_\mathrm{t}\sqrt{\nu}}{2(\sqrt{\nu} + 1)}, \quad P_\mathrm{r} = \frac{P_\mathrm{t}}{2} \quad (5)$$

where $\nu = \gamma_{ar}/\gamma_{br}$, $\gamma_{ar} = (P_\mathrm{t}/\sigma^2)\left|\left|\boldsymbol{h}_a\right|\right|^2$ and $\gamma_{br} = (P_\mathrm{t}/\sigma^2)\left|\left|\boldsymbol{h}_b\right|\right|^2$.

Proof: We rewrite the total power constraint $P_a + P_b + P_\mathrm{r} = P_\mathrm{t}$ with two auxiliary variables α and β such $P_a = \alpha\beta P_\mathrm{t}$, $P_b = (1 - \alpha)\beta P_\mathrm{t}$ and $P_\mathrm{r} = (1 - \beta)P_\mathrm{t}$ $(0 \le \alpha, \beta \le 1)$. We now need to find the optimal value of α and β. For this purpose, we can use the arithmetic geometric mean inequality. Hence, (P2) becomes a special case of [36, Theorem 1] with $n = 2$, $X_1 = 1 + \gamma_a$, $X_2 = 1 + \gamma_b$ and $a_1 = a_2 = 1$ and [36, Proposition 1] – which indicates that for $\nu \gg 1$, if $\gamma_a^\mathrm{opt} = \gamma_b^\mathrm{opt} = \gamma_{br}/(2 + 4\sqrt{1/\nu})$ is in the feasibility region then the corresponding PA is optimal – we only need to show that

$$\gamma_a^\mathrm{opt} = \gamma_b^\mathrm{opt} = \frac{\gamma_{br}}{2 + 4\sqrt{1/\nu}} \quad (6)$$

is feasible. This is equivalent to showing that $0 \le \alpha^\mathrm{opt}, \beta^\mathrm{opt} \le 1$. The corresponding α and β can be obtained as

$$\alpha^\mathrm{opt} = \frac{1}{\sqrt{\nu} + 1}, \quad \beta^\mathrm{opt} = \frac{1}{2}, \quad (7)$$

which both satisfy $0 \le \alpha^\mathrm{opt}, \beta^\mathrm{opt} \le 1$. □

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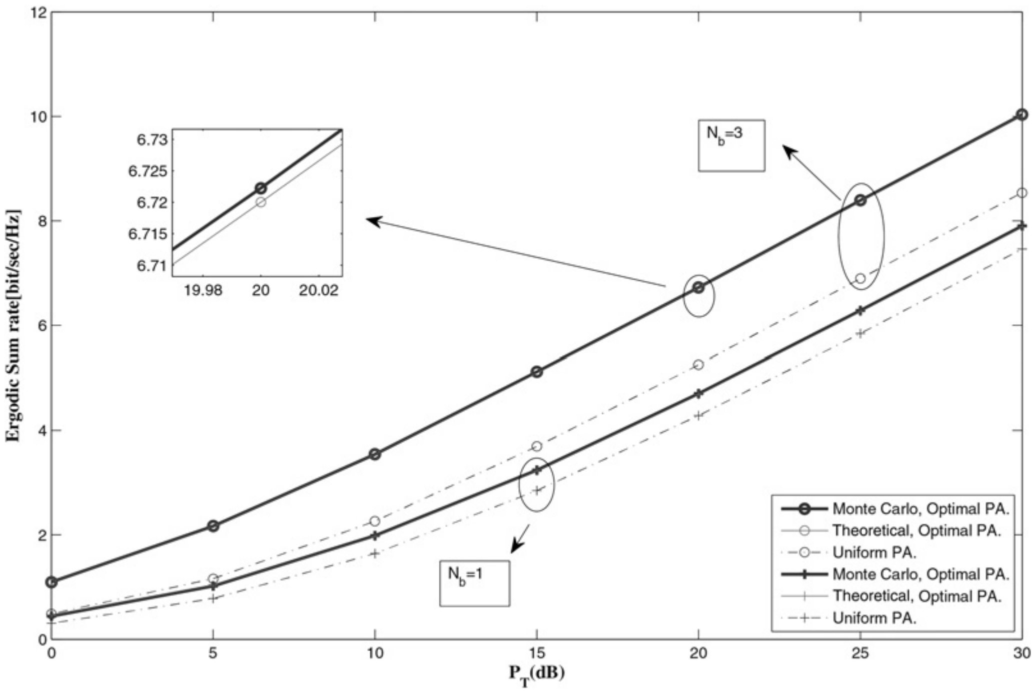

Fig. 2 *Achievable sum rate performance using PPA and UPA with* $\nu = 100$, $\sigma_a^2 = \sigma_b^2 = 1$, $m_b = 1$ *and* $N_0 = 1$

It is interesting to see that the optimal allocation requires that half of the total power be allocated to the relay and the remaining half is divided according to the ratio $1{:}\sqrt{\nu}$. Since ν is large, more power is thus allocated to T_b. Note that ν (5) is a random variable. However, since one of the terminals is massive MIMO, by using the weak-law of large numbers, we find that it converges to $\nu \simeq N_a\sigma_a^2/N_b\sigma_b^2$. The reasons for the approximation are detailed in Appendix 1, and numerical evidence for its accuracy is given in Section 5. This constant value is used for the ergodic sum rate analysis next.

3.3 Ergodic sum rate

While the optimal solution derived (5) yields the instantaneous total sum rate as a function of instantaneous channel gains, the ergodic sum rate, a far more important performance measure, is derived by averaging over all channel statistics. For this purpose, all the entries in the channel vectors $\boldsymbol{h}_a$ and $\boldsymbol{h}_b$ between T_a and T_b, and relay R are assumed to be independent and Nakagami-m distributed with parameters $m_a, m_b \in \mathbb{N}$ and average fading powers σ_a and σ_b, respectively. Therefore, our results also include Rayleigh fading channels as a special case when $m_a = m_b = 1$.

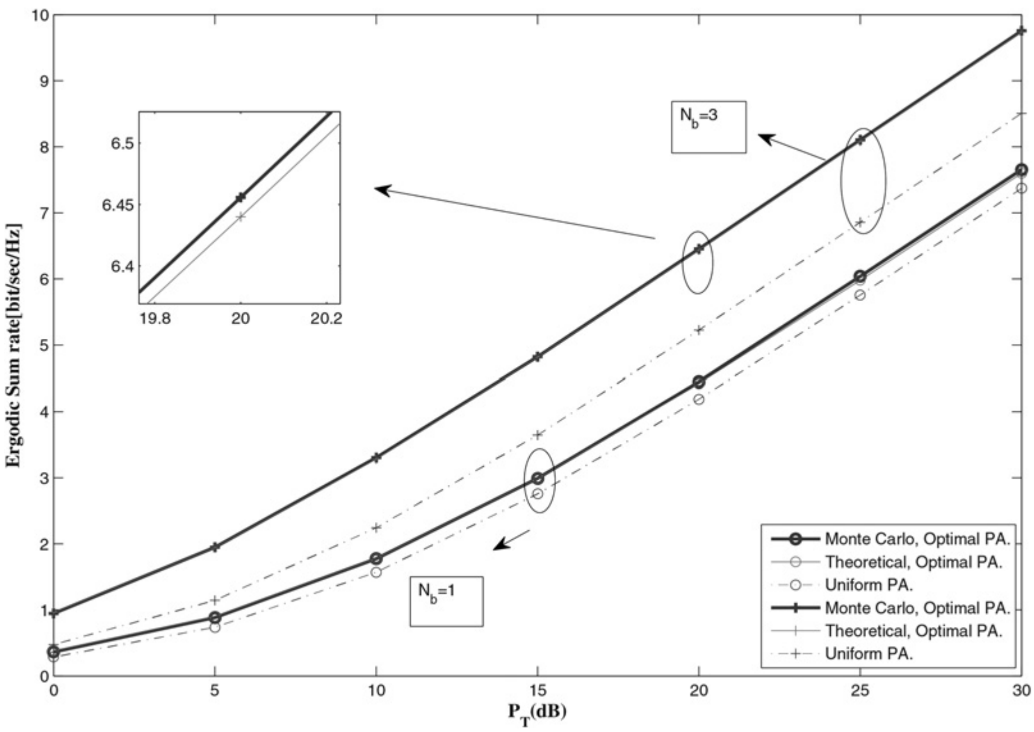

Fig. 3 *Achievable sum rate performance using PPA and UPA with* $\nu = 20$, $\sigma_a^2 = \sigma_b^2 = 1$ *and* $m_b = 1$

Theorem 2: The ergodic sum rate for the optimal PA obtained in Theorem 1 can be expressed as

$$\bar{\boldsymbol{R}} = s\left(2 + 4\sqrt{\frac{1}{\nu}}\right)^{m_b N_b} \exp\left(\frac{m_b\left(2 + 4\sqrt{1/\nu}\right)}{\bar{\gamma}_{\rm br}}\right) \times \sum_{k=1}^{m_b N_b} \frac{\Gamma\left(k - m_b N_b, m_b\left(2 + 4\sqrt{1/\nu}\right)/\bar{\gamma}_{\rm br}\right)}{\left(m_b\left(2 + 4\sqrt{1/\nu}\right)/\bar{\gamma}_{\rm br}\right)^k} \tag{8}$$

where $s = ((m_b N_b - 1)!/\ln 2\Gamma(m_b N_b))(m_b/\bar{\gamma}_{\rm br})^{m_b N_b}$.

Proof: See Appendix 1. □

4 Additional quality of service (QoS) constraints

4.1 Problem formulation

Suppose, in comparison to the SNRs they have achieved in (6), one of the terminals needs a higher SNR (e.g. for better quality of service) while the other one does not. This scenario is of interest in wireless cellular networks where some mobile users may require higher SNRs due to limitations such as hardware requirements. Next, we treat the case where it is terminal T_b that needs higher SNR out of the two terminals.

In this scenario, we maximise the sum rate subject to the constraints that the total power of the network is $P_{\rm t}$ and the SNRs at both terminals must exceed target threshold values. Therefore, the optimisation problem (P2) can be reformulated as

$$\begin{aligned} &\max_{P_a, P_b, P_{\rm r}} \boldsymbol{R} \\ \text{s.t} \quad & P_a + P_b + P_{\rm r} = P_{\rm t} \\ & \gamma_a \geq \hat{\gamma}_a \\ & \gamma_b \geq \hat{\gamma}_b \end{aligned} \tag{P3}$$

where $\hat{\gamma}_b > \gamma_b^{\rm opt}$.

4.2 Sub-optimal solution

To solve (P3), we first note that with the optimal PA (5), both terminals reach SNRs approximately $\gamma_{br}/2$ as $\nu \to \infty$. From a practical point of view, since T_a and T_b can be considered as base station and mobile user, respectively, it makes sense to increase the SNR of the mobile terminal (T_b), who has the fewer number of antennas. To quantify the SNR improvment, we define an improvement coefficient $\hat{\gamma}_b = \zeta(\gamma_{br}/2)$ where $1 \leq \zeta \leq 2$.

To achieve a sub-optimal solution for this case, we set the SNR of T_b, which requires higher SNR, to the target value and maximise the SNR of T_a. Hence, let $\gamma_b = \alpha\beta(1-\beta)\gamma_{ar}/(\alpha\beta\nu + 1) = \hat{\gamma}_b$ then we have

$$\alpha = \frac{\hat{\gamma}_b}{\beta(1-\beta)\gamma_{\rm ar} - \beta\nu\hat{\gamma}_b}. \tag{9}$$

To simplify analysis, let $\nu \gg 1$ which is equivalent to exploiting the large scale antenna arrays at terminal T_a. Therefore, we have

$$\gamma_a(\beta) \simeq \beta\gamma_{br} - \frac{\hat{\gamma}_b \gamma_{\rm br}}{\gamma_{\rm ar}\left(1 - \beta - (\hat{\gamma}_b/\gamma_{\rm br})\right)}, \tag{10}$$

By taking derivative of $\gamma_a(\beta)$ with respect to β, we obtain

$$\beta^{s-{\rm opt}} = 1 - \sqrt{\frac{\hat{\gamma}_b}{\gamma_{\rm ar}}} - \frac{\hat{\gamma}_b}{\gamma_{\rm br}}, \tag{11}$$

Because $0 < \beta^{s-{\rm opt}} < 1$, we have $0 < \sqrt{\hat{\gamma}_b/\gamma_{ar}} + (\hat{\gamma}_b/\gamma_{br}) < 1$. The second inequality should be considered as the feasibility condition. By substituting $\beta^{s-{\rm opt}}$ in (9), one can easily show that $0 < \alpha < 1$. Using $\beta^{s-{\rm opt}}$, we obtain

$$\gamma_a^{s-{\rm opt}} = \gamma_{\rm br} - \hat{\gamma}_b - 2\gamma_{br}\sqrt{\frac{\hat{\gamma}_b}{\gamma_{\rm ar}}}. \tag{12}$$

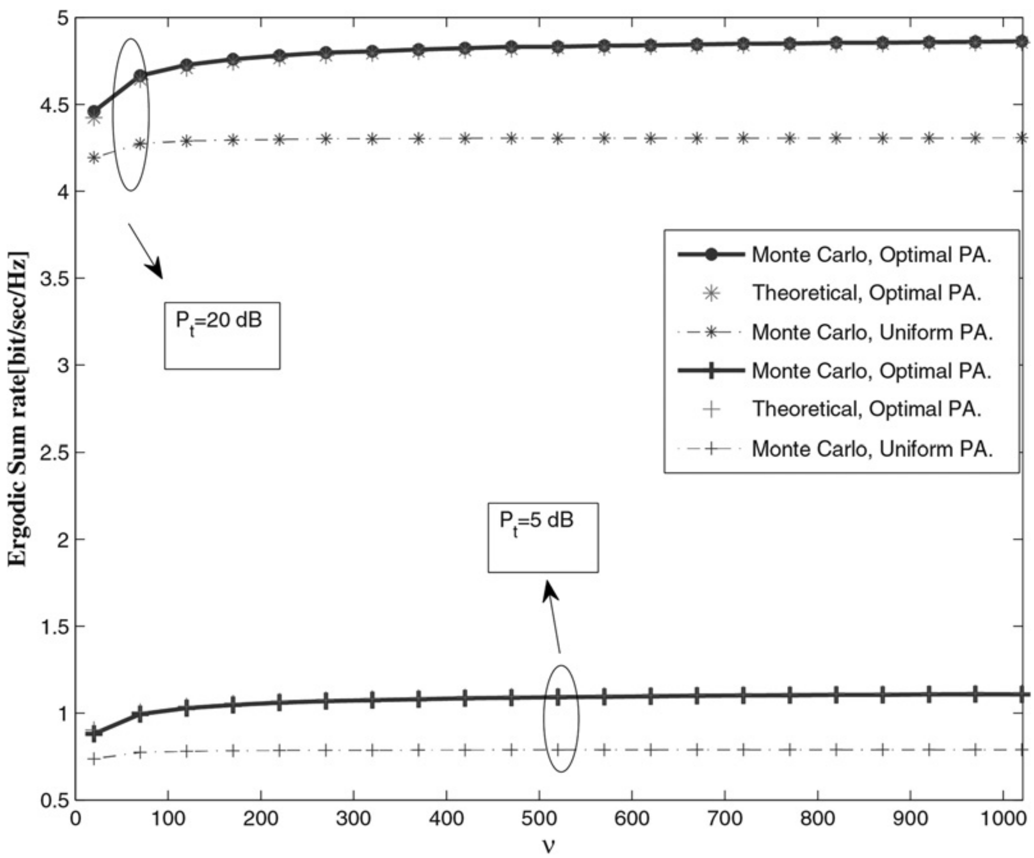

Fig. 4 *Impact of ν on the achievable sum rate with $\sigma_a^2 = \sigma_b^2 = 1$, $m_b = 1$, $N_0 = 1$ and $N_b = 1$*

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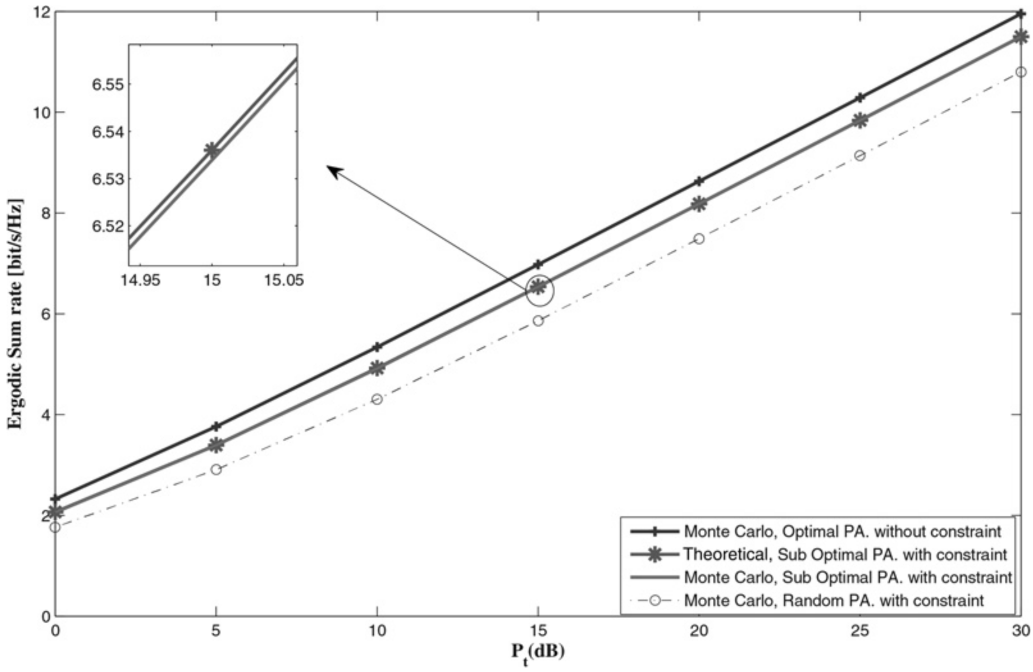

Fig. 5 *Achievable sum rate of sub-optimal PA for a feasible system with $\sigma_a^2 = \sigma_b^2 = 1$, $m_b = 1$, $N_0 = 1$, $N_b = 10$, $\zeta = 1.4$ and $\nu = 100$*

4.3 Ergodic sum rate

We next provide the closed-form ergodic sum rate over Nakagami-m fading channels for the solutions given in (9) and (11).

Theorem 3: The ergodic sum rate for the sub-optimal solutions given in (9) and (11), can be expressed as

$$\bar{\boldsymbol{R}} = \sum_{i \in C} s i^{m_b N_b} \exp\left(\frac{m_b i}{\bar{\gamma}_{\text{br}}}\right) \sum_{k=1}^{m_b N_b} \frac{\Gamma\left(k - m_b N_b, m_b i/\bar{\gamma}_{\text{br}}\right)}{\left(m_b i/\bar{\gamma}_{\text{br}}\right)^k}, \quad (13)$$

where $C = \left\{1/(1 - (\zeta/2) - 2\sqrt{\zeta/2\nu}), 2/\zeta\right\}$ and s is given in (8).

Proof: See Appendix 2.

5 Numerical and simulation results

In this section, Monte Carlo simulation results and theoretical analyses given in (5) and (13) are compared for verification.

Figs. 2 and 3 show the ergodic sum rate of both optimal PA and uniform power allocation (UPA) ($P_a = P_b = P_r = P_t/3$) for different values $\nu = 100, 20$ and $N_b = 1, 3$. These figures show the following:

(i) The analytical result (8) agrees well with the Monte Carlo simulations, even for lower values of $\nu = 20$.

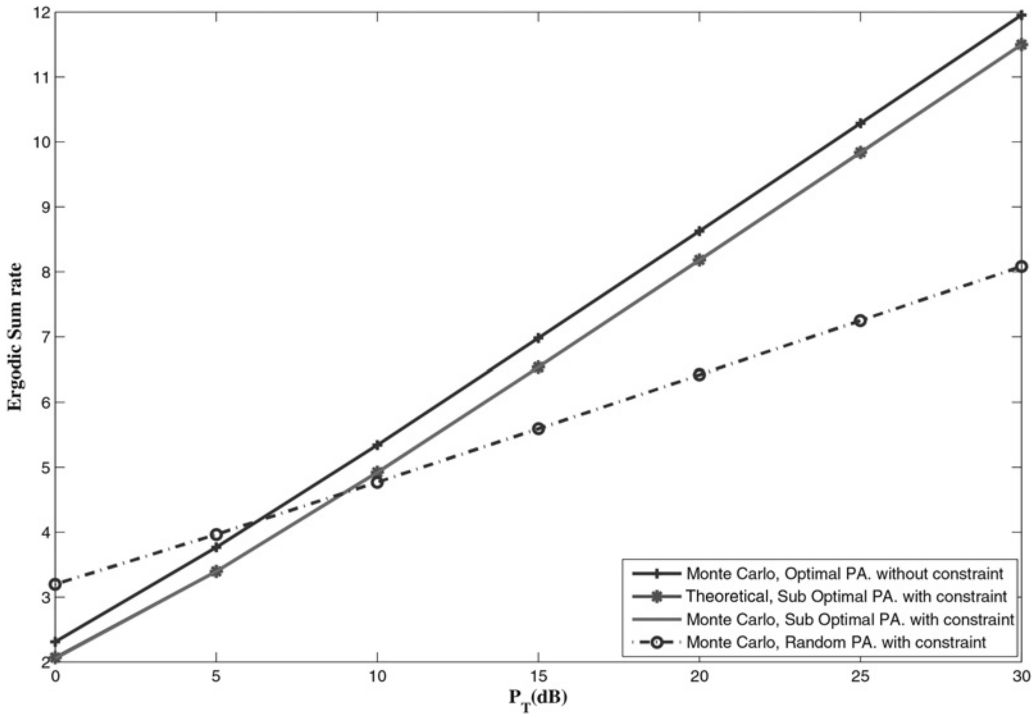

Fig. 6 *Achievable sum rate of sub-optimal PA for an infeasible system with $\sigma_a^2 = \sigma_b^2 = 1$, $m_b = 1$, $N_0 = 1$, $N_b = 10$, $\zeta = 1.4$ and $\nu = 100$*

(ii) The optimal PA offers better ergodic sum rates over the UPA. For example, for $\nu = 100$, $N_b = 1$ and $\bar{\boldsymbol{R}} = 4\,\text{bit/s/Hz}$, a 1 dB gap exists between (8) and UPA. Moreover, as the antennas of T_b increases to 3, this gap increases to about 5 dB.

Fig. 4, illustrates the impact of ν on the system ergodic sum rate for both (8) and UPA. As can be seen, optimal PA outperforms UPA for different values of ν.

Fig. 5 shows that the simulation results match well with theoretical expression (13). Furthermore, the proposed sub-optimal PA outperforms random PA which satisfies considered QoS constraints. For example, for $\nu = 100$, $N_b = 10$ and $\bar{\boldsymbol{R}} = 4\,\text{bit/s/Hz}$, the sub-optimal PA strategy saves the total power about 2 dB in comparison with random PA.

In Fig. 6, $\hat{\gamma}_a$ is set so that the system becomes infeasible. It is clear that the sum rate for random PA is even greater than the PPA in low powers. In this case, the system cannot provide the target SNR due to total power constraint. However, when the systems become feasible, the presented sub-optimal PA strategy outperforms the random PA.

As shown in Figs. 5 and 6, the derived sub-optimal solution is close to the optimal solution of the first scenario which is a relaxed version of the second scenario. Hence, the proposed sub-optimal solution is even closer to the optimal solution of its own setup.

Finally, all the figures verify that the approximation $\nu \simeq N_a\sigma_a^2/N_b\sigma_b^2$ results in the ergodic sum rate expressions that match well with the simulation results.

6 Conclusion

The paper investigated a network of two MIMO terminals and a single-antenna relay. One of the terminals is a massive MIMO device. Subject to the total power constraint, we derived the exact closed-form optimal PA to maximise the sum rate. We also derived a sub-optimal PA to maximise sum rate when the SNRs at both terminals must exceed target values. The resulting sum rates are function of instantaneous channel gains. By exploiting the weak law of large numbers, we then derived the ergodic sum rates in closed-form. To provide a degree of generality, we used the Nakagami-*m* fading model. Both feasible and infeasible systems were simulated. Simulation results showed both the accuracy of the derived theoretical expressions and the efficiency of proposed PA strategies. Note that our results remain valid even with two massive MIMO terminals, as long as the number of antennas in one terminal is much larger than that of the other. The extension of this paper to the more general case of multiple-antenna relay is an interesting idea for future work.

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8 Appendices

8.1 Appendix 1

Using the weak-law of large numbers, $\sum_{i=1}^{N_l} |h_{li}|^2/N_l \xrightarrow{p} \boldsymbol{E}(|h_l|^2) = \sigma_l^2$ as $N_l \to \infty, \forall l \in \{a, b\}$ where $(\xrightarrow{p})$ denotes the convergence in probability. Hence

$$\nu = \frac{N_a}{N_b} \frac{\sum_{i=1}^{N_a} |h_{ai}|^2/N_a}{\sum_{k=1}^{N_b} |h_{bk}|^2/N_b} = \frac{N_a \sigma_a^2}{N_b \sigma_b^2} \tag{14}$$

However, even for small number of antennas, this approximation is good (see Section 5). To evaluate the ergodic sum rate, $\bar{\boldsymbol{R}} = (1/2)E[\log_2(1+\gamma_a) + \log_2(1+\gamma_b)] = E[\log_2(1+\gamma_a)]$, we have (see (15))

where:

(a) Follows from the fact that two channel hops are independent.
(b) Follows from the fact that γ_a is independent from γ_{ar}.
(c) The reason is that channel coefficients follow Nakagami-m distribution.
(d) Follows from the fact that $\int_0^\infty \ln(1+x)x^{n-1}\exp(-tx)\,\mathrm{d}x = (n-1)!e^t \sum_{k=1}^{n} \Gamma(-n+k, t)/t^k$ for $t > 0, n = 1, 2, \ldots$ [37, Appendix B].

8.2 Appendix 2

First, one should note that $1 \le \zeta < 2$ where the upper bound comes from the domain of logarithm function. Using this fact and (9) and (11), the ergodic sum rate expression for the presented sub-optimal PA can be proven similar to Appendix 1.

$$\begin{aligned}
\bar{\boldsymbol{R}} &= E[\log_2(1+\gamma_a)] = \int_0^\infty \int_0^\infty \log_2(1+\gamma_a) f(\gamma_{ar}, \gamma_{br})\,\mathrm{d}\gamma_{ar}\,\mathrm{d}\gamma_{br} \\
&\overset{(a)}{=} \int_0^\infty \int_0^\infty \log_2(1+\gamma_a) f_{\gamma_{ar}}(\gamma_{ar}) f_{\gamma_{br}}(\gamma_{br})\,\mathrm{d}\gamma_{ar}\,\mathrm{d}\gamma_{br} \\
&\overset{(b)}{\simeq} \int_0^\infty \log_2(1+\gamma_a) f_{\gamma_{br}}(\gamma_{br})\,\mathrm{d}\gamma_{br} \\
&\overset{(c)}{=} \frac{1}{\ln 2\Gamma(m_b N_b)} \left(\frac{m_b}{\bar{\gamma}_{br}}\right)^{m_b N_b} \int_0^\infty \ln\left(1 + \frac{\gamma_{br}}{2+4\sqrt{1/\nu}}\right) \gamma_{br}^{m_b N_b - 1} \exp\left(\frac{-m_b \gamma_{br}}{\bar{\gamma}_{br}}\right) \mathrm{d}\gamma_{br} \\
&\overset{(d)}{=} \frac{(2+4\sqrt{1/\nu})^{m_b N_b} (m_b N_b - 1)!}{\ln 2\Gamma(m_b N_b)} \left(\frac{m_b}{\bar{\gamma}_{br}}\right)^{m_b N_b} \exp\left(\frac{m_b(2+4\sqrt{1/\nu})}{\bar{\gamma}_{br}}\right) \sum_{k=1}^{m_b N_b} \frac{\Gamma\left(k - m_b N_b, m_b(2+4\sqrt{1/\nu})/\bar{\gamma}_{br}\right)}{\left(m_b(2+4\sqrt{1/\nu})/\bar{\gamma}_{br}\right)^k}
\end{aligned} \tag{15}$$