Energy Detection with Diversity Reception

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***Abstract*—We characterize the performance of energy detector (ED) over square-law, square-law selection, and switch-and-stay diversity combining schemes. The exact average probabilities of a miss ($\overline{P}_m$), and a false alarm ($\overline{P}_f$) are derived in closed-form. To derive $\overline{P}_m$ for versatile Nakagami-*m* and Rician fading channels, a twofold approach, using the probability density function (PDF) and the moment generating function (MGF), is applied. Using the PDF method, the achievable diversity order over the Nakagami-*m* channel is derived. However, this method becomes intractable when analyzing $\overline{P}_m$ of the aforementioned combiners in Rician channels, but the MGF method can handle this case. Our analysis helps to quantify the performance gains of ED due to diversity reception. Theoretical derivations are verified through numerical Monte-Carlo simulation results.**

***Index Terms*—Energy Detection, Non-coherent combining, Square-Law Combining, Switch-and-Stay Combining, Area under the ROC curve, Marcum-*Q* integrals, Cognitive radio, Spectrum sensing**

I. Introduction

Energy detection to determine the absence or presence of a of radio signal finds myriads of applications in ultra wide-band (UWB) [1], [2] and cognitive radio [3]. It is a widely researched, low-complexity detection mechanism and included in the IEEE $802.15.4$a [2] and IEEE 802.22 standards [3], [4]. [5].

To improve the performance of the energy detector in wireless fading, receiver diversity combining schemes have been widely considered [6]–[19]. For example, the maximal ratio (MR) combiner and selection combining under Rayleigh fading [14], [18] and shadowing effects [15] are investigated. In [19], the MR, equal gain (EG) and selection combining schemes in Rician and Nakagami-*m* channels are presented. Diversity reception over cascaded Rayleigh, $\eta-\mu$ and $\kappa-\mu$ fading channels is studied in [6], [20], [21], respectively. The receiver operating characteristic (ROC) curves ($1-\overline{P}_m$ against $\overline{P}_f$) and the area under the ROC curve (AUC) are widely used to quantify the detector performance [15]–[17], [19], [22].

The ROC and AUC analyses show that coherent MR and EG combiners achieve remarkable performance improvements [15], [16], [18], [19]. However, since the philosophy of energy detection is contingent on low-implementation complexity, pilot based estimations of channel state information (CSI) required for MR and EG combining are a drawback [19]. Consequently, low-complexity diversity combining schemes such as square-law (SL), square-law selection (SLS), and switch-and-stay (SS) are vital. In [17], [18], the SL, SLS and SS combining schemes are analyzed for Rayleigh fading. The result of SL combining is extended to Nakagami fading channels in [12], [13]. However, in all these works, the achievable diversity order - an intuitive system performance metric - is not derived. This gap is addressed in this work. We first evaluate the exact ED detection probability in Nakagami-*m* and Rician fading channels for SL, SLS and SS combining schemes. To tackle the mathematical complexities of versatile Rician and Nakagami-*m* fading channels, a two-fold approach, based on the moment generating function (MGF) or the probability density function (PDF), is developed. The MGF method is useful to evaluate the exact detection probabilities over Rician fading which appears difficult with the PDF method. Using the derived results, we then investigate the achievable diversity of SL, SLS and SS combining schemes in both Nakagami-*m* and Rician fading channels. The investigation shows that these low-complexity diversity combining schemes are robust in exploiting the multipath diversity and hence achieve significant performance gains. To the best of our knowledge, the presented results are neither special cases nor deducible readily from the previously reported results such as [7], [8], [10], [12]–[19], and this remark will be further clarified subsequently.

This paper is organized as follows. Section II summarizes the system model. Sections III, IV and V analyze the performance of energy detection with SL, SLS and SS schemes. Section VI provides numerical results. The concluding remarks are given in Section VII.

II. System Model and Diversity Reception

The received signal of single diversity branch $y(t)$ which contains an unknown deterministic band-limited signal $s(t)$ and noise $n(t)$ or noise only, can be modeled as a binary hypotheses (H_0 and H_1) problem, as given in (1)

$$y(t) = b\, hs(t) + n(t) \quad : H_b \tag{1}$$

where h is the complex channel gain and b is a binary hypothesis indicator $b \in \{0,1\}$. Note that in (1), the block fading model is considered. This model is of practical interest because ED is used to obtain a fast (i.e., short duration) sensing decision [3]. The detector decision variable Y is a square sum of $2u$ Gaussian random variables where $u \in \mathbb{Z}^+$ is the time-bandwidth product. The instantaneous signal-to-noise ratio (SNR) is defined by $\gamma = \frac{|h|^2 E_s}{N_{01}}$ where E_s is the observed energy and N_{01} is single sided noise power spectral density. Hence, Y under H_0 and H_1 are, respectively, central and non-central chi-square distributed (χ^2_{2u}) with $2u$ degrees of freedom and non-centrality parameter 2γ. Thus, the PDF of Y is given by

$$f_Y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)}\, y^{u-1} e^{-\frac{y}{2}} & : H_0 \\ \frac{1}{2}\left(\frac{y}{2\gamma}\right)^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}}\, I_{u-1}(\sqrt{2\gamma y}) & : H_1 \end{cases}, \tag{2}$$

where $\Gamma(.)$ is Gamma function and $I_n(.)$ is n^{th} order modified Bessel function of the first kind. Therefore, the average miss probability (missed-detection probability) $\overline{P}_m$ can be computed by averaging the conditional miss probability $P_m(\gamma|h) = 1 - Q_u(\sqrt{2\gamma}, \sqrt{\lambda})$ where $Q_u(.\,,.)$ is the generalized (u^{th} order) Marcum-Q function and λ is the detector threshold, with respect to the PDF of the SNR $f_\gamma(x)$ given in (2) [15], [17], [19]. For brevity, only a summary of the system model is provided. We refer the reader to [17], [19], [23] for more details.

As we will show, when several diversity branches of the form (1) are combined, the overall decision variable will also be chi-square distributed, but the degrees of freedom and the PDF of SNR are modified accordingly. Thus, in general, we encounter an integral involving the Marcum-Q function and the PDF of resulting SNR of the combiner. For this purpose, we follow a two fold approach as described below.

A. Average probability of miss-detection – PDF and MGF based methods

1) PDF based method: The average missed-detection probability for a diversity combining scheme can be computed by averaging the conditional miss probability $P_m(\gamma|h) = 1 - Q_q(\sqrt{2\gamma}, \sqrt{\lambda})$ where $Q_q(.\,,.)$ is the generalized (q^{th} order) Marcum-Q function with respect to the PDF of the SNR $f_\gamma(x)$ [15], [17], [19];

$$\overline{P}_{m,Div} = 1 - \int_0^\infty Q_q(\sqrt{2x}, \sqrt{\lambda}) f_\gamma(x) dx. \tag{3}$$

The order of Marcum-Q function, i.e., $q \in \mathbb{Z}^+$ depends on the diversity combiner and will be specified later.

2) MGF based method: The MGF of γ may be defined as $M(s) = E(e^{-s\gamma})$ where $E(.)$ is the statistical expectation. Using the contour integral representation of generalized Marcum-Q function [24], $\overline{P}_m$ can be expressed as

$$\overline{P}_{m,Div} = 1 - \frac{e^{-\frac{\lambda}{2}}}{2\pi j} \oint_\Delta M\left(1 - \frac{1}{z}\right) \frac{e^{\frac{\lambda}{2}z}}{z^q(1-z)} dz, \tag{4}$$

where Δ is a circular contour of radius r that encloses origin; $0 < r < 1$, $j = \sqrt{-1}$ and $z \in \mathbb{C}$ [19]. The integral in (4) depends only on the residues at the poles of the integrand inside the contour Δ.

Depending on the fading model and the type of diversity combining, the evaluation and tractability of integrals (3) and (4) are different and consequently, the choice between MGF and PDF based methods varies. For example, it appears that the case of diversity combining over Rician fading is intractable via the PDF, whereas the MGF method handles the related integrals readily. In the discussion below, we follow either the MGF or the PDF based method and where possible, results from both methods are provided.

B. Average probability of false-alarm

The channel fading does not affect the detection under noise only hypothesis H_0 (please see (1)). Thus, the average probability of a false alarm $\overline{P}_{f,Div} = \Gamma(q, \lambda/2)/\Gamma(k)$ where $\Gamma(.\,,.)$ represents upper incomplete Gamma function defined by the integral form $\Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt$ and $\Gamma(a,0) = \Gamma(a)$ [17], [19]. For integer a, $\Gamma(a) = (a-1)!$. Using series representation for $\Gamma(1+a,x) = a!\, e^{-x} \sum_{i=0}^{a} \frac{x^i}{i!}$, $a = 0, 1, \ldots$ [25, eq. 8.352-2], $\overline{P}_f$ can be expressed as

$$\overline{P}_{f,Div} = \frac{\Gamma(q, \lambda/2)}{\Gamma(q)} \tag{5a}$$

$$= e^{-\frac{\lambda}{2}} e_{q-1}(\lambda/2) \geq e^{-\frac{\lambda}{2}}, \tag{5b}$$

where $e_n(.)$ is the exponential sum function, i.e., $e_n(x)$ represents the first n terms of the series representation of e^x. The inequality $\overline{P}_{f,Div} \geq e^{-\frac{\lambda}{2}}$ holds as $e_{q-1}(\lambda/2) \geq 1$ with equality when $q = 1$. Note that $\overline{P}_{f,Div}$ is lower bounded by $e^{-\frac{\lambda}{2}}$, i.e., the benchmark performance in terms of average false-alarm probability decays exponentially over $\lambda/2$.

III. Detection with Square-Law (SL) Combining

The SL concept may prove useful in two scenarios where the individual energy measurements are available to the combiner through (i) multipath propagation (ii) through spatially scattered user energy measurements reported to a fusion center [26]. In the distributed setup (scenario (ii)), we consider that the measurements are i.i.d. which is a realistic assumption for the equidistant users. However, the fusion center needs instantaneous individual user energy measurements as well as CSI estimates, which leads to large control channel overheads. Therefore, in scenarios (i) and (ii), non-coherent combiners which exploit the diversity gain without the need for CSI are more preferable.

The SL combiner adds the individual energy measurements without compensating for the channel gains and thus does not need CSI. The output decision variable is thus defined by $Y_{SL} = \sum_{i=1}^{L} Y_i$ where L is the number of diversity branches and Y_i is the decision variable of the i-th branch ($i = 1, \ldots, L$). Under H_0 and H_1, Y_{SL} is χ^2_{2Lu} and $\chi^2_{2Lu}(2\gamma_{SL})$ distributed respectively. The output SNR of the combiner $\gamma_{SL} = \sum_{i=1}^{L} \gamma_i$ and γ_i denotes the SNR of the i^{th} indexed branch. Thus the average false-alarm probability for SL combining ($\overline{P}_{f,SL}$) can be obtained by replacing q by Lu in (5), i.e., $\overline{P}_{f,SL} = \frac{\Gamma(Lu, \lambda/2)}{\Gamma(Lu)}$. Moreover, miss-detection probability can be obtained after replacing q by Lu and evaluating the integrals in (3) and (4) as follows.

A. Nakagami-m Fading - PDF based method

The PDF of a Gamma $\mathcal{G}(\alpha, \delta)$ random variable is given by $f(x) = \frac{1}{\delta^\alpha \Gamma(\alpha)} x^{\alpha-1} \mathrm{e}^{-x/\delta}$, $x \geq 0$, where the shape parameter $\alpha > 0$ and the scale parameter $\delta > 0$. When the received signal amplitude follows Nakagami-*m* fading, the SNR in each branch γ_i is a $\mathcal{G}(m, \bar{\gamma}/m)$ random variable where $\bar{\gamma}$ is the average SNR, and $m \geq \frac{1}{2}$ is the fading severity index. In this case, γ_{SL} is a $\mathcal{G}(Lm, \bar{\gamma}/m)$ random variable.

Using (3) and the alternative representation of $Q_u(.\,,.)$ [27, eq. 4.63], the average miss probability over SL ($\overline{P}_{m,SL,Nak}$)

can be expressed as

$$\overline{P}_{m,SL,Nak} = 1-\left(\frac{m}{\bar{\gamma}}\right)^{Lm}\frac{1}{\Gamma(Lm)} \times \int_0^\infty \gamma^{Lm-1}\, e^{-\frac{m\gamma}{\bar{\gamma}}}\, Q_{Lu}\left(\sqrt{2\gamma},\sqrt{\lambda}\right)\, d\gamma, \tag{6a}$$

$$= \left(\frac{m}{\bar{\gamma}}\right)^{Lm}\frac{e^{-\frac{\lambda}{2}}}{\Gamma(Lm)}\sum_{n=Lu}^{\infty}\left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \times \int_0^\infty \gamma^{Lm-\frac{n}{2}-1}\, e^{-(1+\frac{m}{\bar{\gamma}})\gamma}\, I_n\left(\sqrt{2\lambda\gamma}\right)\, d\gamma. \tag{6b}$$

Evaluating (6b) by [25, eq. 6.643-2] and using the relation between Whittaker function and Hypergeometric function ${}_1F_1(.;.;)$ [25, eq. 9.220-2], $\overline{P}_{m,SL,Nak}$ can be expressed as in (7) where $\beta = \frac{\bar{\gamma}}{\bar{\gamma}+m}$.

$$\overline{P}_{m,SL,Nak} = (1-\beta)^{Lm}\, e^{-\frac{\lambda}{2}}\sum_{n=Lu}^{\infty}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^{n} \times {}_1F_1\left(Lm; n+1; \frac{\lambda\beta}{2}\right) \tag{7}$$

The ${}_1F_1(.;.;.)$ is a special case of generalized Hypergeometric function given in (8)

$$ {}_vF_w(a_1,a_2,...,a_v;b_1,b_2,...,b_w;x) = \sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n...(a_v)_n}{(b_1)_n(b_2)_n...(b_w)_n}\frac{x^n}{n!}, \tag{8}$$

where $(.)_n$ denotes the Pochhammer symbol; $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$ [28]. By expanding ${}_1F_1(.;.;.)$ in (7) using (8) and constructing the Hypergeometric function of two variables of the form given in (9) [28, pp. 25], $\overline{P}_{m,SL,Nak}$ is derived as in (10).

$$\Phi_2(\mu,\nu;\rho;x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\mu)_m(\nu)_n}{(\rho)_{m+n}}\frac{x^m}{m!}\frac{y^n}{n!} \quad |\, x\,| < \infty, |\, y\,| < \infty \tag{9}$$

$$\overline{P}_{m,SL,Nak} = (1-\beta)^{Lm}\, e^{-\frac{\lambda}{2}}\left[\Phi_2\left(Lm,1;1;\frac{\lambda\beta}{2},\frac{\lambda}{2}\right) - \sum_{n=0}^{Lu-1}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^{\frac{n}{2}}{}_1F_1\left(Lm;n+1;\frac{\lambda\beta}{2}\right)\right] \tag{10}$$

The convergence of the infinite series in (7) is evident from (10). Although $\Phi_2(.,.;.;.,.)$ can easily be implemented in software packages, the series truncation is necessary. The error in truncating (7) by N terms $|E_{SL}|$ can be upper bounded as

$$|E_{SL}| < (1-\beta)^{Lm}{}_1F_1\left(Lm;N+1;\frac{\lambda\beta}{2}\right)\left(1-\frac{\Gamma(N+1,\frac{\lambda}{2})}{N!}\right). \tag{11}$$

This bound is derived by using the monotonically decreasing property of ${}_1F_1\left(Lm;n+1;\frac{\lambda\beta}{2}\right)$ over n for given values of $L,m,\lambda,\bar{\gamma}$ and [25, eq. 8.352-4]. The $|E_{SL}|$ in (11) is used to find the minimum number of terms required in calculating $\overline{P}_{m,SL,Nak}$ to a given accuracy figure (Table I).

The special case of $L=1$ (i.e., no-diversity reception) (10) simplifies to [19, eq. (8)]. It should be noted that, the PDF of the SNR for maximal ratio (MR) combiner γ_{MR} is also the same as $\mathcal{G}(Lm,\bar{\gamma}/m)$. However, the decision statistic under H_1 with MR combining of L i.i.d. branches is $\chi^2_{2u}(2\gamma_{MR})$, i.e., $q=u$ [19]. For this reason, it is easily seen that the SL combiner miss-detection probability is neither a special case nor deducible from that of the MR combining (cf. [19, eq. (25)]).

At large SNR $\bar{\gamma}$ values, $\overline{P}_{m,SL,Nak}$ can be approximated using (10) as

$$\overline{P}_{m,SL,Nak} \approx \left\{m^{Lm}e^{-\frac{\lambda}{2}}\left[\Phi_2\left(Lm,1;1;\frac{\lambda}{2},\frac{\lambda}{2}\right) - \sum_{n=0}^{Lu-1}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^{\frac{n}{2}}{}_1F_1\left(Lm;n+1;\frac{\lambda}{2}\right)\right]\right\}\bar{\gamma}^{-Lm} \tag{12}$$

$$= g^{SL,Nak}(m,u,\lambda,L)\,\bar{\gamma}^{-Lm} \tag{13}$$

where $g^{SL,Nak}(m,u,\lambda,L)$ is the term inside the curly brackets which depends on parameters m,u,λ and L. Hence $\overline{P}_{m,SL,Nak}$ approaches 0 in the order of Lm on average SNR $\bar{\gamma}$. Thus, Lm can be defined as the *diversity order* or *detection diversity gain*. Thus, SL combining in L path Nakagami-m fading channel has a diversity order of Lm and the Nakagami-m fading (with no-diversity combining) has a diversity order of m.

B. Nakagami-m Fading - MGF based method

The application of MGF based method to evaluate miss-detection probability under Nakagami-*m* fading gives an alternative analytical expression for $\overline{P}_{m,SL,Nak}$ and can be used to verify the result (10) derived from the PDF. Moreover, we show that, under certain special cases, the MGF based result can be used to evaluate the AUC as well.

The MGF of the $\mathcal{G}(Lm,\bar{\gamma}/m)$ is

$$M_{\gamma_{SL,Nak}}(s) = \left(1+\frac{\bar{\gamma}s}{m}\right)^{-Lm}, \; m \geq \frac{1}{2}, Lm \in \mathbb{Z}. \tag{14}$$

Using (4), $\overline{P}_{m,SL,Nak}$ can be written as

$$\overline{P}_{m,SL,Nak} = 1-(1-\beta)^{Lm}\, e^{-\frac{\lambda}{2}} \times \frac{1}{2\pi j}\oint_{\Delta} f(z)dz \tag{15}$$

where $f(z) = \frac{e^{\frac{\lambda}{2}z}}{(z-\beta)^{Lm}\, z^{L(u-m)}\,(1-z)}$. The contour integral in (15) is evaluated for integer values of Lm. We assume that the function $f(z)$ has a pole of order $k \geq 1$ at $z = z_0$. We count the residues corresponding to the poles inside the contour $|z_0| < 1$. The residue of the poles at $z = z_0$ of order $k \geq 1$ is given by

$$\text{Res}(f;z_0,k) = \frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}[f(z)(z-z_0)^k]\Big|_{z=z_0}. \tag{16}$$

When $u > m$, integrand in (15) has Lm poles at β and $L(u-m)$ poles at 0. Thus, from residue calculus, $\overline{P}_{m,SL,Nak}$ can be calculated as

$$\overline{P}_{m,SL,Nak} = 1 - (1-\beta)^{Lm} e^{-\frac{\lambda}{2}} \left[\mathrm{Res}(f;\beta,Lm) + \mathrm{Res}(f;0,L(u-m))\right]. \tag{17}$$

When $u \leq m$, poles at 0 disappear. Thus $\overline{P}_{m,SL,Nak}$ is obtained by evaluating residues at β only.

Note that $f(z)$ and consequently the residues in (17) consist of exponential and rational functions only. Therefore, unlike (10) which involves special functions such as ${}_1F_1(.;.;.)$, the $\overline{P}_{m,SL,Nak}$ in (17) yields a simpler expression of rationals and exponentials only. As an illustrative example, detector performance under Rayleigh fading channel with $u = 1$ can be shown to have simple expression, i.e., when $L = 1, m = 1, u = 1$, the average probability of a miss ($\overline{P}_{m,Ray,1}$) can be written as

$$\overline{P}_{m,Ray,1} = 1 - e^{-\frac{\lambda}{2(1+\bar{\gamma})}} = 1 - \left(\overline{P}_{f,1}\right)^{\frac{1}{1+\bar{\gamma}}}, \tag{18}$$

where $\overline{P}_{f,1} = e^{-\frac{\lambda}{2}}$ denotes the $\overline{P}_f$ when $u = 1$ (please see (5)). As $u = 1$ corresponds to the benchmark performance of the detector [17], [19], the relationship given in (18) fully characterizes the optimum detector performance under Rayleigh fading.

The AUC represents the probability that the detector decides on the correct decision more likely than the incorrect decision [16]. The AUC provides a single figure of merit of detection capability which allows us to compare the performances of different detection mechanisms. However, if the ROC curves of two detectors cross each other, the AUC values themselves might fail to provide a fair comparison. In such cases, the ROC analysis or alternatively the partial-AUC (i.e. the area within two false alarm thresholds $0 \leq P_f^1 \leq P_f^2 \leq 1$) is necessary. We refer the reader to [16] and references therein for more details. As the partial-AUC measure appears intractable in closed-form analysis, in [16], numerical techniques are proposed. To elaborate the evaluation of partial-AUC in closed-form, in the following, we utilize the MGF method result obtained above.

Mathematically, the partial-AUC for the two false alarm thresholds $0 \leq P_f^1 \leq P_f^2 \leq 1$ is defined as $\overline{A}(\bar{\gamma}) = \int_{P_f^1}^{P_f^2} \overline{P}_d(\bar{\gamma})\, d\overline{P}_f$ where $\overline{P}_d(\bar{\gamma}) = 1 - \overline{P}_m$ and the total-AUC is given by the limits $P_f^1 = 0$ and $P_f^2 = 1$. Thus, from (18), the partial-AUC over Rayleigh fading $\overline{A}_{Ray}$ can be easily be obtained as

$$\overline{A}_{Ray} = \frac{1}{\xi}\left[\left(P_f^2\right)^{\xi} - \left(P_f^1\right)^{\xi}\right] \tag{19}$$

where $\xi = \frac{2+\bar{\gamma}}{1+\bar{\gamma}}$. Hence, the total-AUC is $\frac{1+\bar{\gamma}}{2+\bar{\gamma}}$.

C. Rician Fading - MGF based method

For the performance with the SL combiner in Rician fading channels, we find that the PDF based method intractable. Therefore, we try the MGF method. The MGF of the output SNR of a SL combined L i.i.d. Rician branches is given by

$$M_{\gamma_{SL,Ric}}(s) = \left[\frac{1+K}{(1+K+s\bar{\gamma})}\right]^L \exp\left[-\frac{KL\bar{\gamma}\, s}{(1+K+s\bar{\gamma})}\right], \tag{20}$$

where K is the Rice factor. Using (4) with $k = Lu$, the average probability of a miss over SL combining under Rician fading $\overline{P}_{m,SL,Ric}$ can be derived as

$$\overline{P}_{m,SL,Ric} = 1 - \left[\frac{\theta_r(1+K)}{\bar{\gamma}}\right]^L e^{-\left(\frac{\lambda}{2}+KL\theta_r\right)} \times \frac{1}{2\pi j}\oint_{\Delta} \frac{e^{\left(\frac{a_r}{z-\theta_r}+\frac{\lambda}{2}z\right)}}{(z-\theta_r)^L (1-z)\, z^{L(u-1)}}\, dz \tag{21}$$

where $\theta_r = \frac{\bar{\gamma}}{(\bar{\gamma}+K+1)}$ and $a_r = KL\theta_r(1-\theta_r)$. It is easy to verify that the special case of Rayleigh fading, i.e., (21) with $K = 0$ reduces to (15) with $m = 1$. Applying Laurent series expansion to the term $\frac{\exp(\frac{a_r}{z-\theta_r})}{(z-\theta_r)^L}$ when $K \neq 0$ and integrating terms by Residue theorem, $\overline{P}_{m,SL,Ric}$ when $u > 1$ can be derived as

$$\overline{P}_{m,SL,Ric} = 1 - \left[\frac{\theta_r(1+K)}{\bar{\gamma}}\right]^L e^{-\left(\frac{\lambda}{2}+KL\theta_r\right)} \sum_{n=0}^{\infty} \frac{a_r^n}{n!}\left[\frac{1}{(L(u-1)-1)!}\frac{d^{L(u-1)-1}}{dz^{L(u-1)-1}}\left(\frac{e^{\frac{\lambda}{2}z}}{(z-\theta_r)^{n+L}(1-z)}\right) + \frac{1}{(n+L-1)!}\frac{d^{n+L-1}}{dz^{n+L-1}}\left(\frac{e^{\frac{\lambda}{2}z}}{z^{L(u-1)}(1-z)}\right)\right]. \tag{22}$$

When $u = 1$, the poles at 0 disappear and therefore, $\overline{P}_{m,SL,Ric}$ can be obtained by setting the value of the first derivative in (22) to 0. The special case of Rician fading (without diversity combining) can be obtained by setting $L = 1$ in (22). For given L, u values, $\overline{P}_{m,SL,Ric}$ in (22) is a function of rationals and exponentials only. It appears difficult to derive the error result in truncating the infinite series in (22). Using the derived result in (22), the diversity order of the Rician fading channel with SL combining will be discussed in Section VI.

IV. Detection with Square-Law Selection (SLS) Combining

The SLS diversity combiner picks up the branch with the highest value in decision variable, i.e., $Y_{SLS} = \text{maximum}\,(Y_1, Y_2, ..., Y_L)$, where L is the number of branches in the combiner. Note this combiner does not require CSI and hence is a potential solution for application scenarios discussed in Section III.

When the branch statistics are independent, the false-alarm probability for the SLS combiner $\overline{P}_{f,SLS}$ can be expressed as

$$\begin{aligned}\overline{P}_{f,SLS} &= Pr\left\{Y_{SLS} > \lambda | H_0\right\} \\ &= 1 - \prod_{j=1}^{L} Pr\left\{Y_j < \lambda | H_0\right\} \\ &= 1 - \left(1 - \overline{P}_f\right)^L\end{aligned} \tag{23}$$

where $\overline{P}_f$ is given in (5) with $q = 1$.

A. *Nakagami-*m *and Rician fading - PDF and MGF based methods*

The average miss probability of SLS combining $\overline{P}_{m,SLS}$ over independent fading branches is derived as

$$\begin{aligned}\overline{P}_{m,SLS} &= \int_{\gamma_1=0}^{\gamma_1=\infty} \dots \int_{\gamma_L=0}^{\gamma_L=\infty} \prod_{i=1}^{L} \left\{1 - Q_u\left(\sqrt{2\gamma_i}, \sqrt{\lambda}\right)\right\} \\ &\qquad f_{\gamma_i}(\gamma_i)\, d\gamma_i \\ &= \prod_{i=1}^{L} \left\{1 - \int_0^{\infty} Q_u\left(\sqrt{2\gamma_i}, \sqrt{\lambda}\right) f_{\gamma_i}(\gamma_i)\, d\gamma_i\right\} \\ &= \prod_{i=1}^{L} \overline{P}_{m,i},\end{aligned}$$

where γ_i is the instantaneous SNR in the i^{th} indexed branch, $f_{\gamma_i}(\gamma_i)$ is the PDF of γ_i and $\overline{P}_{m,i}$ denotes the average probability of a miss of the i^{th} independent path and $i \in \{1, 2, \dots, L\}$. Hence, the miss probability with SLS combining over Nakagami-*m* fading ($\overline{P}_{m,SLS,Nak}$) and Rician fading ($\overline{P}_{m,SLS,Ric}$) can be obtained by (24) after substituting the miss probability over the respective fading channels. For this purpose, MGF and PDF based methods' results derived in (10), (17) (for Nakagami-*m* fading) and in (22) (for Rician fading) with the substitution $L = 1$ are used. Also note that (24) is valid for non-identical branch statistics.

At large SNR $\bar{\gamma}$ values, the average probability of a miss over SLS combining in Nakagami-*m* fading ($\overline{P}_{m,SLS,Nak}$) can be approximated by

$$\begin{aligned}\overline{P}_{m,SLS,Nak} &\approx \left\{\left[m^m e^{-\frac{\lambda}{2}} \left(\Phi_2\left(m, 1; 1; \frac{\lambda}{2}, \frac{\lambda}{2}\right)\right.\right.\right. && (24) \\ &\left.\left.\left. - \sum_{n=0}^{u-1} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} {}_1F_1\left(m; n+1; \frac{\lambda}{2}\right)\right)\right]^L\right\} \bar{\gamma}^{-Lm} \\ &= g^{SLS,Nak}(m, u, \lambda, L)\, \bar{\gamma}^{-Lm} && (25)\end{aligned}$$

where $g^{SLS,Nak}(m, u, \lambda, L)$ is the term inside the curly-brackets. Thus, L branch SLS combiner has a diversity order of Lm over Nakagami-*m* channel. The diversity order of the Rician fading channel with SLS combining will be discussed in Section VI.

V. Detection with Switch-and-Stay (SS) Combining

The dual branch SS combiner, switches to the other branch, if the SNR of the currently connected branch falls below a predetermined threshold value (γ_T) and stays with that branch, irrespective of whether the branch SNR is below or above γ_T. This implementation is simpler than the dual branch selection combining considered in [14], [19]. Therefore, in the following, we investigate the diversity benefit offered by SS combining over Nakagami-*m* fading.

A. *Nakagami-*m *Fading - PDF based method*

The PDF of output SNR of the SS combining scheme (γ_{SS}) is given in [27, eq. 9.276], and thus the average miss probability is $\overline{P}_{m,SS} = 1 - (I_1 + I_2)$ where I_1 and I_2 are defined by the integral forms as in (26a) and (26b) respectively.

$$I_1 = \left(1 - \frac{\Gamma(m, \frac{m\gamma_T}{\bar{\gamma}})}{\Gamma(m)}\right) \left(\frac{m}{\bar{\gamma}}\right)^m \frac{1}{\Gamma(m)} \int_0^{\infty} \gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}}}\, Q_u(\sqrt{2\gamma}, \sqrt{\lambda})\, d\gamma \quad (26a)$$

$$I_2 = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{1}{\Gamma(m)} \int_{\gamma_T}^{\infty} \gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}}}\, Q_u(\sqrt{2\gamma}, \sqrt{\lambda})\, d\gamma \quad (26b)$$

By means of $Q_u(.,.)$ [27, eq. 4.63], [25, eq. 9.220-2] and [25, eq. 6.643-2] and constructing the $\Phi_2(.,.;.;.,.)$ given in (9), I_1 can be calculated for integer $m \geq 1$ as

$$I_1 = \left(1 - \frac{\Gamma(m, \frac{m\gamma_T}{\bar{\gamma}})}{\Gamma(m)}\right) \left\{1 - e^{-\frac{\lambda}{2}} (1-\beta)^m \left[\Phi_2\left(m, 1; 1; \frac{\lambda\beta}{2}, \frac{\lambda}{2}\right) - \sum_{n=0}^{u-1} \left(\frac{\lambda}{2}\right)^n \frac{{}_1F_1\left(m; n+1; \frac{\lambda\beta}{2}\right)}{\Gamma(n+1)}\right]\right\},$$

where $\beta = \frac{\bar{\gamma}}{\bar{\gamma}+m}$. Similar to (11), the error result in truncation of I_1 by N terms $|E_{SS,I_1}|$ can be upper bounded as $|E_{SS,I_1}| < \{1 - [\Gamma(m, \frac{m\gamma_T}{\bar{\gamma}})/\Gamma(m)]\} |E_{SL,L=1}|$ where $|E_{SL,L=1}|$ is the $|E_{SL}|$ when $L = 1$ given by (11). Using the alternative representation of $Q_u(.,.)$ in [27, eq. 4.74], [25, eq. 8.352-2] and [25, eq. 3.351-2], for integer $m \geq 1$, I_2 is evaluated as

$$\begin{aligned}I_2 = &\frac{(1-\beta)^m}{\Gamma(m)} \sum_{n=0}^{\infty} \frac{\beta^n}{n!\,(n+u-1)!} \\ &\times \Gamma\left(n+m, \frac{\gamma_T}{\beta}\right) \Gamma\left(n+u, \frac{\lambda}{2}\right).\end{aligned}$$

TABLE I
Minimum number of terms required to evaluate the infinite series expressions to a five figure accuracy ($\tilde{N}$)

$\lvert E_{SL} \rvert$	u	1	5	1	1	1	1
	$\overline{P}_{f,SL}$	0.01	0.01	0.01	0.01	0.0001	0.01
	SNR(dB)	10	10	20	10	10	10
	L	2	2	2	2	2	4
	m	1	1	1	4	1	1
	$\tilde{N}$	17	35	12	13	25	18
$\lvert E_{SS,I_1} \rvert$	u	1	5	1	1	1	1
	$\overline{P}_{f,SS}$	0.01	0.01	0.01	0.01	0.0001	0.01
	SNR(dB)	10	10	20	10	10	10
	m	1	1	1	4	1	1
	γ_T(dB)	5	5	5	5	5	10
	$\tilde{N}$	14	25	10	10	21	14
$\lvert E_{SS,I_2} \rvert$	SNR(dB)	5	5	10	10	15	15
	m	1	4	1	4	1	4
	$\tilde{N}$	41	21	120	54	369	155

The error result in truncating the I_2 by N terms $|E_{SS,I_2}|$ is upper bounded as

$$|E_{SS,I_2}| \leq (1-\beta)^m \left[{}_1F_0\left(m;;\beta\right) - \sum_{n=0}^{N} \frac{(m)_n \beta^n}{n!} \right]. \quad (27)$$

The bound $|E_{SS,I_2}|$ is derived by using the inequality $\Gamma(n,x) \leq \Gamma(n)$ and constructing the Hypergeometric function of the form ${}_1F_0(.;.;.)$ which is a special case of ${}_pF_q(.;.;.)$ with $p=1, q=0$ given in (8). At a given instance, the SS detector operates on a single branch. Thus, the average false alarm probability is the same as (5), i.e., $\overline{P}_{f,SS} = \frac{\Gamma(u,\lambda/2)}{\Gamma(u)}$.

At high SNR (i.e. $\lim \bar{\gamma} \rightarrow \infty$), we have $\lim_{\bar{\gamma}\rightarrow\infty} \Gamma(m, \frac{m\gamma_T}{\bar{\gamma}}) = \Gamma(m)$ and thus $\lim_{\bar{\gamma}\rightarrow\infty} I_1 = 0$. Furthermore, from (6a) and (26b), we note that $I_2 < 1 - \overline{P}_{m,Nak}$ where $\overline{P}_{m,Nak}$ is the probability of missed-detection over Nakagami-*m* channel without diversity reception, i.e., (6) with $L=1$. Hence,

$$\overline{P}_{m,SS} \approx 1 - I_2 > \overline{P}_{m,Nak} \quad (28)$$

Therefore, at high SNR, we observe that the SS combiner has a diversity order of m over the Nakagami-*m* channel. However, analyzing its performance over Rician fading channels appears intractable.

VI. Numerical and Simulation Results

Derived analytical expressions (average probability of a miss, average probability of a false alarm and error bounds) can easily and directly be evaluated in software packages such as Mathematica. The minimum number of terms ($\tilde{N}$) required to calculate the error bounds $|E_{SL}|$, $|E_{SS,I_1}|$ and $|E_{SS,I_2}|$ to an accuracy of 10^{-5} is given in Table I. From Table I, one can observe that reasonably low number of terms evaluates the average miss probability of $\overline{P}_m$ to a high accuracy. The results of the MGF and PDF methods are numerically equivalent. Monte-Carlo (MC) simulations are provided to verify the theoretical derivations.

How do the low-complexity combining schemes exploit the diversity in energy detection? What are the performances of the SL and SLS combining schemes which have the same diversity order? These questions are answered in the following discussion. Fig. 1 plots the average miss-detection probability ($\overline{P}_m$) over $\bar{\gamma}$ for a fixed average false-alarm probability $\overline{P}_f = 0.1$ and $u=2$. The detector without diversity combining requires an average SNR of 11.02dB to reach the target sensing performance specified in cognitive radio standard IEEE 802.22, i.e., $\overline{P}_m = \overline{P}_f = 0.1$ [3]. The SL, SLS and SS combining schemes reach the target performance, respectively, at SNRs 7.57dB, 7.97dB and 9.40dB. Therefore, the SL, SLS and SS combining schemes respectively show 3.45dB, 3.05dB and 1.62dB performance gains compared to the no-diversity fading channel. More importantly, non-coherent SL and SLS demonstrate more than 3dB gains.

Our analysis in Sections III, IV and V reveals that the diversity orders of ED over Nakagami-*m* fading channel for SL, SLS and SS combining schemes are respectively Lm, Lm and m (see (12), (24) and (28)). To verify this analysis, Fig. 2 plots the $\overline{P}_m$ over $\bar{\gamma}$ for $\overline{P}_f = 0.1$ and different values of u.

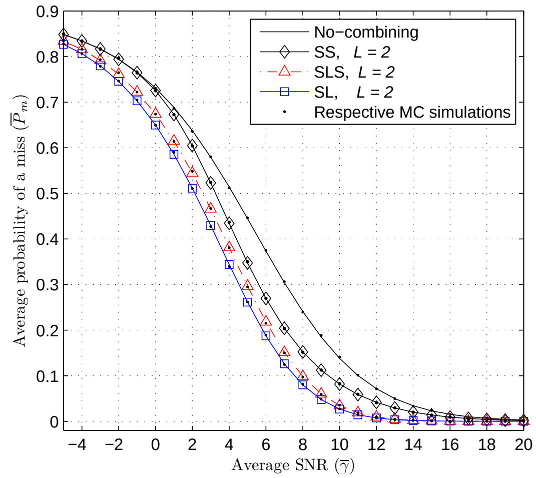

Fig. 1. Average miss probability over average SNR ($\overline{P}_m$ vs. $\bar{\gamma}$) in Nakagami-*m* fading ($\overline{P}_f = 0.1, L=2, m=2, u=2, \gamma_T = 5$ dB)

One can observe that SL and SLS combining schemes achieve approximately diversity order of 4 for $L=2$ and $m=2$ while the SS with $L=2$ and no-combining schemes have approximately diversity order of 2, regardless of the value u. These results clearly agree with our analytical findings. Moreover, observe that for given set of parameters SL combining achieves the highest performance. Note that lower value of u gives better performance [17], [19] and for both SL and SLS combining schemes, the SNR gap between $u=1$ and $u=5$ is approximately remain constant at around 2.3dB. On the other hand, for SS combining with $L=2$ and no-combining schemes show 3.1dB and 2.3dB SNR gap between $u=1$ and $u=5$ curves. Furthermore, from our analytical results in Sections III and IV for Rician fading channels, one can study the diversity benefits without performing time-consuming MC simulations. Fig. 3 plots $\overline{P}_m$ over $\bar{\gamma}$ for $\overline{P}_f = 0.1$ and different values of u and K. The results are similar to that observed for Nakagami-*m* fading and SL combining achieves better performance. As evidenced by Figs. 1, 2 and 3, clearly the SL combining exploits the multipath diversity in the absence of CSI, achieving remarkable performance improvements.

To illustrate the diversity gain achieved by SL combining over different number of faded energy measurements, i.e., varying L, the ROC curves over the Nakagami-*m* and Rician fading channels are plotted in Figs. 4 and 5 respectively. Within the $\overline{P}_f$ interval $(0.01, 0.1)$, the corresponding $\overline{P}_m$ values are, respectively, around 10^{-3} and 10^{-4} in Figs. 4 and 5 for $L=4$. In a practical setup where multiple faded energy measurements are available for combining (through multi-path fading or multiple user measurements reported to a fusion center), such significant gains are achievable through SL combining in the absence of CSI at the combiner.

VII. Conclusion

We comprehensively characterize the performance of energy detection in Nakagami-*m* and Rician fading channels for square-law (SL), square-law selection (SLS) and switch-and-

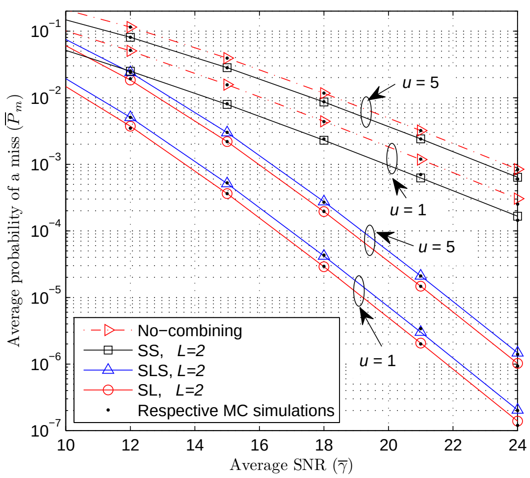

Fig. 2. Average probability of a miss over average SNR ($\overline{P}_m$ vs. $\bar{\gamma}$) in Nakagami-*m* fading ($\overline{P}_f = 0.1, L = 2, m = 2, u = \{1, 5\}, \gamma_T = 5\,\text{dB}$)

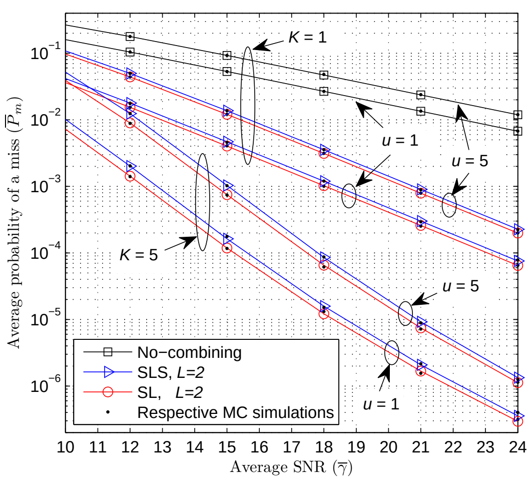

Fig. 3. Average probability of a miss over average SNR ($\overline{P}_m$ vs. $\bar{\gamma}$) in Rician fading ($\overline{P}_f = 0.1, L = 2, K = \{1, 5\}, u = \{1, 5\}$)

stay (SS) combining schemes. In particular, their exact miss and false-alarm probabilities are derived. No diversity reception is a special case of the derived results. The derivations are based on the moment generating function (MGF) and probability density function (PDF). Using the PDF method, the non-coherent SL and SLS combining schemes are shown to achieve a diversity order of Lm in Nakagami-*m* fading channels, while the SS combiner is shown to gain a diversity order of m. The MGF based method can solve several cases analytically intractable from the PDF based method including the Rician fading case. The analysis helps to understand and quantify the energy detector performance improvements achievable using the low-complexity diversity schemes.

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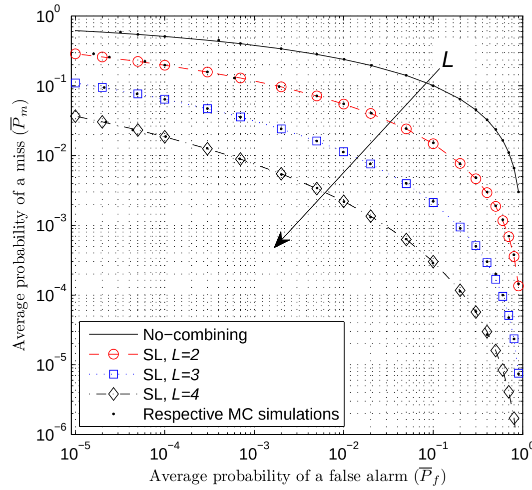

Fig. 4. Complementary ROC curves ($\overline{P}_m$ vs. $\overline{P}_f$) in Nakagami-*m* fading ($\bar{\gamma} = 10\text{dB}, L = 1, 2, 3, 4, m = 2, u = 1$)

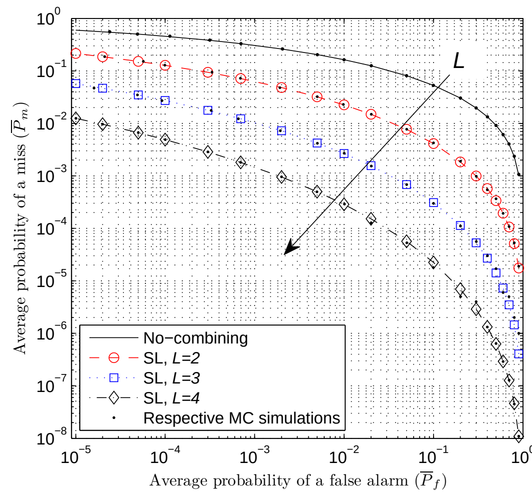

Fig. 5. Complementary ROC curves ($\overline{P}_m$ vs. $\overline{P}_f$) in Rician fading ($\bar{\gamma} = 10\text{dB}, L = 1, 2, 3, 4, K = 7, u = 1$)

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