Massive MIMO Two-Way Relay Networks with Channel Imperfections

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***Abstract*—This paper investigates the impact of co-channel interference (CCI), imperfect channel state information (CSI) and pilot contamination for multi-pair massive multiple-input multiple-output (MIMO) two-way relay networks (TWRNs). We consider a multi-cell TWRN system consisting of single-antenna user nodes and amplify-and-forward (AF) relay nodes having very large antenna arrays. Under the aforementioned channel imperfections, the asymptotic signal-to-interference-noise ratio and asymptotic sum rate expressions are derived in closed-form whenever the number of relay antennas grows unbounded with respect to the number of user nodes. For perfect CSI case, the transmit power at the user nodes and the relay can be scaled down inversely proportional to the number of antennas at the relay. Moreover, for the imperfect CSI case, these transmit powers can only be scaled down inversely proportional to the square-root of the relay antenna count. Thus, even with imperfect CSI, the benefits of employing a massive MIMO-enabled relay on transmit power savings are significant. Moreover, our analysis shows that although the detrimental effect of CCI can be asymptotically negated completely, the residual interference due to pilot contamination cannot be mitigated even in the limit of infinitely many relay antennas.**

I. Introduction

Massive MIMO or large scale antenna systems are considered as one of the main enabling technologies for future 5G implementations [1], [2]. Research on Massive MIMO have shown very high spectral efficiencies, low transmit powers per bit and high energy efficiencies [3]. Pilot contamination is the residual interference caused by the reuse of non-orthogonal pilot sequences in adjacent cells. It has been identified as the main performance limiting factor in massive MIMO systems [3], [4].

Two-way relay networks (TWRNs) are one of the important research directions in cooperative wireless communications. TWRNs offer two fold increase in the achievable data rate compared to the one-way relay networks (OWRNs) [5]. Multi-pair TWRNs offer mutual data transmissions between pairs of nodes with the aid of intermediate relays [6]. Beamforming techniques based on zero forcing (ZF), minimum mean square error (MMSE) and maximum ratio combining (MRC) can be used at the relay to mitigate the interferences among the user node pairs [7]. Having a massive MIMO relay in a multi-pair TWRN will combine the performance gains provided by both massive MIMO and TWRNs.

Previous research on single-hop massive MIMO systems: In [3], the performance of multi-user massive MIMO in non-cooperative cellular networks is investigated by deriving asymptotic performance metrics. Specifically, [3] concludes that whenever the number of base-station antennas grow unbounded, the simple linear precoders and decoders become asymptotically optimal. Pilot contamination in multi-cell multi-user massive MIMO systems is investigated by deriving rigorous asymptotic signal-to-interference-plus-noise ratio (SINR) expressions [4]. The relative energy efficiency versus spectral efficiency trade-off is investigated for massive MIMO cellular systems with linear precoding and detection [8].

Previous research on dual-hop massive MIMO systems: In [9], the asymptotic performance of multi-pair OWRNs with very large relay antenna arrays is investigated for the case of perfect CSI and co-channel interference (CCI) free scenario. To this end, the asymptotic SINR and sum rate expressions are derived by considering three transmit power scaling laws. In [10], the multi-pair TWRNs with massive MIMO is investigated by employing linear precoders and detectors. Moreover, in [11], the asymptotic performance of AF TWRNs with single-antenna user-nodes and infinitely many relays by employing distributed beamforming is analyzed. It is worth noting that again in [10] and [11], the genie-aided perfect CSI and CCI/pilot contamination free scenario is assumed.

Motivation and our contribution: With the emergence of dense deployment of wireless systems, the effect of CCI and pilot contamination are becoming the primary performance limiting factors. Further, in practice, CSI must be estimated for constructing precoders/detectors, and hence, the system performance degrades due to the detrimental impact of the channel estimation errors (imperfect CSI). Nevertheless, none of the impacts of CCI or the imperfect CSI or pilot contamination has been investigated for multi-pair TWRNs with massive MIMO. Thus, this paper fills this gap by investigating the performance of such systems under the effects of CCI, imperfect CSI and pilot contamination.

Specifically, the asymptotic SINR and sum-rate expressions are derived for three transmit power scaling laws namely (i) power scaling at the user nodes, (ii) power scaling at the relay, and (iii) power scaling at both the relay and user nodes. Our analysis reveals that the transmit power at the user nodes and the relay can be scaled down inversely proportional to the number of relay antennas for CCI case with perfect CSI. Nevertheless, for the imperfect CSI case, the transmit power at the user nodes can only be scaled down inversely proportional to the square-root of the relay antenna count. For the CCI case, the asymptotic SINR expressions asymptotically become independent of the number of co-channel interferers (L), and consequently, the corresponding detrimental impact can be cancelled asymptotically, whenever the relay antenna count grows unbounded. Nevertheless, the asymptotic performance is limited by the residual interference incurred due to pilot contamination, and the corresponding adverse performance impact cannot be completely mitigated even in the limit of infinitely many relay antennas. Notably, the asymptotic performance metrics are independent of the fast fading component of the wireless channel, and hence, the cross-layer resource scheduling becomes simple. Our analysis and Monte-Carlo simulations reveal that substantial sum rate performance gains can be achieved by using very large relay antenna arrays in TWRNs, while allowing the transmit power to become infinitesimal as well.

It is important to identify the practical use of this system model. The discussed TWRN can be used as a heterogeneous wireless entity for the existing cellular network architecture for reducing the workload of the base-stations. If two user groups within a cell are communicating with each other, system designers can bypass the base-station and use the two-way relaying instead. Service providers can use TWRNs to improve the data throughput without making drastic changes to their existing infrastructure.

Moreover, with the emergence of internet of things (IoT), most of the items in our day-to-day life will become connected devices and there will be hundreds of sensors in our environment [12]. Thus the direct communications between these nodes will be important. As an example in a home IoT network, the cooling system will require data from temperature sensors while security system may require data from the motion sensors. In this case, a multi-pair relay network with a central relay node can accommodate the required communications. Thus the proposed system model becomes useful in future wireless communication scenarios. Since there will be several IoT networks within a certain area, it is interesting to identify the effect of co-channel interferences on multi-pair TWRNs.

Notation: $\mathbf{Z}^H$ and $[\mathbf{Z}]_k$ denote the Hermitian-transpose and the kth diagonal element of the matrix, $\mathbf{Z}$, respectively. $\mathbf{I}_M$ and $\mathbf{O}_{M\times N}$ are the $M\times M$ Identity matrix and $M\times N$ matrix of all zeros, respectively. A complex Gaussian random variable X with mean μ and standard deviation σ is denoted as $X\sim\mathcal{CN}\left(\mu,\sigma^2\right)$.

II. System, channel and signal model

This section presents the system, channel, and signal models for the multi-pair massive MIMO TWRNs in the presence of CCI.

A. System and channel model

The system model consists of L adjacent TWRNs each having $2K$ number of users. The users in the lth TWRN is denoted as $(U_{l,1},\cdots,U_{l,2K})$, where a particular user node $U_{l,k}$ exchange data signals with its paired-user node $U_{l,k'}$ via the half-duplex AF relay node R_l for $k,k'\in\{1,\cdots,2K\}$ and $l\in\{1,\cdots,L\}$. User nodes are single-antenna terminals, and the relay node is equipped with N antennas. The number of relay antennas can grow unbounded with respect to the total number of user nodes ($N >> 2K$). The channel matrix from $2K$ users in the jth TWRN to the lth relay is defined as $\mathbf{G}_{jl}=\mathbf{F}_{jl}\mathbf{D}_{jl}^{\frac{1}{2}}$, where $\mathbf{F}_{jl}\sim\mathcal{CN}_{N\times 2K}\left(\mathbf{0}_{N\times 2K},\mathbf{I}_N\otimes\mathbf{I}_{2K}\right)$ accounts for the small-scale fading, and $\mathbf{D}_{jl}=\operatorname{diag}\left(\eta_{j,l,1},\cdots,\eta_{j,l,2K}\right)$ represents the large-scale fading. The channel coefficients are assumed to be fixed during two consecutive time-slots, and hence, the relay-to-user channel matrix can be defined by using the channel reciprocity property as $\mathbf{G}_{jl}^T$. The CCI on the lth TWRN occurs due to the data transmissions of the other $L-1$ TWRNs with relay R_j, where $j\in\{1,\cdots,L\}$ and $j\neq l$.

B. Signal model

During two time-slots, $2K$ user nodes in each TWRN exchange their information pair-wisely via their assigned relay node. Specifically, the paired users $(U_{l,2i-1},U_{l,2i})$ exchange their data signals $(x_{l,2i-1},x_{l,2i})$, where $i\in\{1,\cdots,K\}$ and $l\in\{1,\cdots,L\}$. In the first time-slot, user nodes transmit $2K\times 1$ signal vector $\mathbf{x}_l$, which is the combined data signals of $2K$ users in the lth TWRN, towards the relay R_l. The signal vector $\mathbf{x}_l$ satisfies $\mathcal{E}\left[\mathbf{x}_l\mathbf{x}_l^H\right]=\mathbf{I}_{2K}$. The received signal at the relay node R_l can be written as

$$\mathbf{y}_{R_l}=\sqrt{P_S}\sum_{j=1}^{L}\mathbf{G}_{jl}\mathbf{x}_j+\mathbf{n}_{R_l}, \tag{1}$$

where $\mathbf{n}_{R_l}$ is the $N\times 1$ additive white Gaussian noise (AWGN) vector at the relay satisfying $\mathcal{E}\left[\mathbf{n}_{R_l}\mathbf{n}_{R_l}^H\right]=\mathbf{I}_N\sigma_{R_l}^2$. Here, P_S is the transmit power of the user nodes. During the second-time slot, the relay first amplifies and then forwards its received signal towards the end-users. The transmitted signal from the relay can thus be written as $\mathbf{y}'_{R_l}=\beta_l\mathbf{W}_l\mathbf{y}_{R_l}$, where $\mathbf{W}_l$ is the concatenated beamforming-and-amplification matrix at the relay and β_l is the amplification factor to satisfy the relay power constraint which will be presented in the sequel. Here, $\mathbf{W}_l$ is designed to cancel sub-channel interference within a given TWRN, and hence, it can be constructed by using the receive-ZF and transmit-ZF precoding and detection concepts as follows:[1]

$$\mathbf{W}_l=\mathbf{G}_{ll}^*(\mathbf{G}_{ll}^T\mathbf{G}_{ll}^*)^{-1}\mathbf{P}(\mathbf{G}_{ll}^H\mathbf{G}_{ll})^{-1}\mathbf{G}_{ll}^H, \tag{2}$$

where $\mathbf{P}$ is the block diagonal permutation matrix representing the user pairing format. Thus, it is constructed as $\mathbf{P}=\operatorname{diag}(\mathbf{P}_1,\cdots,\mathbf{P}_K)$ and $\mathbf{P}_i=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ for $i\in\{1,\cdots,K\}$. Further, the relay power constraint can be written as

$$P_R=\operatorname{Tr}\left(\mathbf{y}'_{R_l}\mathbf{y}'^H_{R_l}\right). \tag{3}$$

The received signal at the user node $U_{l,k'}$ can next be written as

$$y_{l,k'}=\mathbf{g}_{lk'}^T\mathbf{y}'_{R_l}+n_{l,k'}=\beta_l\sqrt{P_S}\mathbf{g}_{lk'}^T\mathbf{W}_l\sum_{j=1}^{L}\mathbf{G}_{jl}\mathbf{x}_j+\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{n}_{R_l}+n_{l,k'} \tag{4}$$

where $U_{l,k}$ and $U_{l,k'}$ are the paired-users exchanging their data signals for $(k,k')=(2i-1,2i)$ for $i\in\{1,\cdots,K\}$. Further, $\mathbf{g}_{lk'}$ is the k'th row vector of the matrix $\mathbf{G}_{ll}$ for $k\in\{1,\cdots,K\}$ and $n_{l,k'}$ is the AWGN term at the user nodes with variance $\sigma_{n_{l,k'}}^2$. By using the fact that $\mathbf{g}_{lk}^T\mathbf{W}_l\mathbf{g}_{li}=\delta_{k,i}$ where $\delta_{k,i}=1$ when $k=i$ and zero otherwise, (4) can be further simplified to yield

$$y_{l,k'}=\beta_l\sqrt{P_S}x_{l,k}+\beta_l\sqrt{P_S}\mathbf{g}_{lk}^T\mathbf{W}_l\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j+\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{n}_{R_l}+n_{l,k'}. \tag{5}$$

C. Calculation of value β_l

By using (1) and $\mathbf{y'}_{R_l}=\beta_l\mathbf{W}_l\mathbf{y}_{R_l}$, the power constraint (3) can be derived as follows:

$$\begin{aligned}P_R&=\beta_l^2\operatorname{Tr}\left(\mathbf{W}_l\left(P_S\sum_{j=1}^{L}\mathbf{G}_{jl}\mathbf{G}_{jl}^H+\sigma_{R_l}^2\mathbf{I}_N\right)\mathbf{W}_l^H\right)\\&=\beta_l^2P_S\operatorname{Tr}\left(\mathbf{W}_l\sum_{j=1}^{L}\mathbf{G}_{jl}\mathbf{G}_{jl}^H\mathbf{W}_l^H\right)+\beta_l^2\sigma_{R_l}^2\operatorname{Tr}\Big(\mathbf{W}_l\mathbf{W}_l^H\Big).\end{aligned} \tag{6}$$

By substituting $\mathbf{W}_l$ into (6), β_l can be derived as shown in (7) at the top of next page.

[1]The results obtained in Sections III, IV, and V can also be obtained for other detection and precoding methods such as matched filter and MMSE. However due to the page restrictions we have only included results for ZF detection and ZF precoding.

$$\beta_l = \sqrt{\frac{P_R}{P_S \sum_{j=1}^{L} \mathrm{Tr}\left(\mathbf{G}_{jl}\mathbf{G}_{jl}^H\mathbf{G}_{ll}\left[\mathbf{G}_{ll}^H\mathbf{G}_{ll}\right]^{-1}\mathbf{P}[\mathbf{G}_{ll}^T\mathbf{G}_{ll}^*]^{-1}\mathbf{P}[\mathbf{G}_{ll}^H\mathbf{G}_{ll}]^{-1}\mathbf{G}_{ll}^H\right) + \sigma_{R_l}^2 \mathrm{Tr}\left(\left[\mathbf{G}_{ll}^H\mathbf{G}_{ll}\right]^{-1}\mathbf{P}\left[\mathbf{G}_{ll}^T\mathbf{G}_{ll}^*\right]^{-1}\mathbf{P}\right)}}, \quad (7)$$

III. Impact of co-channel interference (CCI)

In this section, the asymptotic performance metrics are derived for TWRNs with CCI by considering transmit power scaling at user nodes whenever the relay antenna count grows unbounded.

A. Transmit power scaling at user nodes

In this case, the transmit power at users is scaled inversely proportional to the number of relay antennas. Thus, by substituting $P_S = E_S/N$ into (5), the received signal can be re-written as

$$\frac{y_{l,k'}}{\sqrt{N}} = \frac{\beta_l}{N}\sqrt{E_S}x_{l,k} + \frac{\beta_l\sqrt{E_S}\mathbf{g}_{lk'}^T\mathbf{W}_l}{N}\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \frac{\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{n}_{R_l}}{\sqrt{N}} + \frac{n_{l,k'}}{\sqrt{N}}, \quad (8)$$

where E_S is a constant defined to keep the product $P_S N$ fixed for variable N and P_S. Also we have assumed $P_R = E_R$, where E_R is a constant. The asymptotic value of the normalized relay gain, β_l/N, can be derived as (see appendix A for the proof)

$$\lim_{N\to\infty}\frac{\beta_l}{N} = \sqrt{\frac{E_R}{E_S\sum_{i=1}^{2K}\eta_{l,l,i}^{-1} + 2\sigma_{R_l}^2\sum_{i=1}^{K}\eta_{l,l,2i-1}^{-1}\eta_{l,l,2i}^{-1}}}. \quad (9)$$

By using similar techniques used for the derivation of (9), the asymptotic value of $\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{n}_{R_l}/\sqrt{N}$ can be derived as

$$\frac{\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{n}_{R_l}}{\sqrt{N}} \xrightarrow[N\to\infty]{d} \left(\frac{E_R}{E_S\sum_{i=1}^{2K}\eta_{l,l,i}^{-1} + 2\sigma_{R_l}^2\sum_{i=1}^{K}\eta_{l,l,2i-1}^{-1}\eta_{l,l,2i}^{-1}}\right)^{\frac{1}{2}}\frac{\tilde{n}_l}{\eta_{l,l,k}} \quad (10)$$

where $\tilde{n}_l \sim \mathcal{CN}\left(0, \eta_{l,l,k}\sigma_{R_l}^2\right)$. Next, the asymptotic value of $\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{G}_{jl}\mathbf{x}_j/N$ for $j \neq i$ can be derived as

$$\lim_{N\to\infty}\beta_l\mathbf{g}_{lk'}^T\mathbf{W}_l\mathbf{G}_{jl}\mathbf{x}_j/N = 0. \quad (11)$$

By using (9), (10) and (11) , the asymptotic SINR at the user node $U_{l,k'}$ for case of transmit power scaling at the user nodes under the effects of CCI and perfect CSI can be written as

$$\lim_{N\to\infty}\gamma_{l,k'} = E_S\eta_{l,l,k}/\sigma_{R_l}^2. \quad (12)$$

B. Transmit power scaling at the relay

For transmit power scaling at the relay node, it can be shown that the relay transmit power can be scaled inversely proportional to the number of relay antennas. Thus, we can define $P_R = E_R/N$ and $P_S = E_S$, where E_R and E_S are constants. The asymptotic SINR at the user node $U_{l,k'}$ for relay transmit power scaling case can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{E_R}{\sigma_{n_{l,k'}}^2\sum_{i=1}^{2K}\eta_{l,l,i}^{-1}}. \quad (13)$$

C. Transmit power scaling at both user nodes and relay

The asymptotic SINR at the user node $U_{l,k'}$ for CCI under the transmit power scaling at the relay and user nodes (where $P_S = E_S/N$ and $P_R = E_R/N$) can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{\left(\frac{E_S\eta_{l,l,k}}{\sigma_{R_l}^2}\right)\left(\frac{E_R}{\sigma_{n_{l,k'}}^2\sum_{i=1}^{2K}\eta_{l,l,i}^{-1}}\right)}{\frac{E_S\eta_{l,l,k}}{\sigma_{R_l}^2} + \frac{E_R}{\sigma_{n_{l,k'}}^2\sum_{i=1}^{2K}\eta_{l,l,i}^{-1}} + \frac{2\eta_{l,l,k}\sum_{i=1}^{K}\eta_{l,l,2i-1}^{-1}\eta_{l,l,2i}^{-1}}{\sum_{i=1}^{2K}\eta_{l,l,i}^{-1}}}. \quad (14)$$

Remark III.1: Notably, the asymptotic SINRs at the nodes of lth TWRN (12), (13), and (14) are independent of the parameters of other j TWRNs, where $j \in \{1, \cdots L\}$ and $j \neq l$. Thus, the impact of CCI can be asymptotically cancelled by using a relay with massive MIMO. Moreover, these asymptotic SINRs are independent of the fast fading component of the wireless channels.

D. Asymptotic sum rate analysis for CCI case

The average sum rate for the lth $2K$-user TWRN with genie-aided perfect CSI can be defined as [10]

$$\mathcal{R}_l = \frac{1}{2}\sum_{k=1}^{2K}\mathcal{E}\left[\log\left(1+\gamma_{l,k}\right)\right]. \quad (15)$$

Then, the asymptotic sum rate for three power scaling laws can readily be obtained by substituting the corresponding asymptotic SINR expressions in (12), (13), and (14) into (15).

IV. Impact of channel estimation errors

In this section, the impact of channel estimation errors (imperfect CSI) on the performance of multi-pair TWRNs is investigated. The system and channel model is assumed to be similar as in the Section II-A.

A. Channel Estimation

In practice, the users-to-relay channel ($\mathbf{G}_{ll}$) is estimated at the relay node by using the pilot sequences transmitted by the user nodes. To this end, $2K$ user nodes transmit mutually orthogonal pilot sequences of length τ. The corresponding MMSE estimation of the users-to-relay channel can be written as [8]

$$\hat{\mathbf{G}}_{ll} = \left(\mathbf{G}_{ll} + \mathbf{V}_l/\sqrt{P_p}\right)\tilde{\mathbf{D}}_{ll}, \quad (16)$$

where P_p is the transmit power of the pilot sequence that can be represented as $P_p = \tau P_S$. The matrix $\mathbf{V}_l$ consists of i.i.d. $\mathcal{CN}(0,1)$ elements and $\tilde{\mathbf{D}}_{ll} = \left(\frac{1}{P_p}\mathbf{D}_{ll}^{-1} + \mathbf{I}_K\right)^{-1}$. Thus, the estimation error can be written as $\mathbf{E}_l = \hat{\mathbf{G}}_{ll} - \mathbf{G}_{ll}$. The receive-ZF detector and transmit-ZF precoder is assumed to be constructed at the relay by using the CSI with estimation errors. The elements of the columns of $\mathbf{E}_l$ is Gaussian distributed with mean zero and variance $\eta_{l,l,i}/(P_p\eta_{l,l,i}+1)$. Due to the MMSE properties, the matrices $\mathbf{E}_l$ and $\hat{\mathbf{G}}_{ll}$ are statistically independent.

$$\hat{\beta}_l = \sqrt{\frac{P_R}{P_S\sum_{j=1}^{L}\mathrm{Tr}\left(\mathbf{G}_{jl}\mathbf{G}_{jl}^H\hat{\mathbf{G}}_{ll}\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\hat{\mathbf{G}}_{ll}^H\right)+\sigma_{R_l}^2\mathrm{Tr}\left(\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*\right]^{-1}\mathbf{P}\right)}}. \tag{19}$$

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{\tau E_R E_S^2\eta_{l,l,k}^2}{E_R\sigma_{R_l}^2 + \tau E_S^2\eta_{l,l,k}^2\sigma_{n_{l,k'}}^2\sum_{i=1}^{2K}\left(\tau E_S\eta_{l,l,i}^2\right)^{-1} + 2\tau E_S\eta_{l,l,k}^2\sigma_{n_{l,k'}}^2\sigma_{R_l}^2\sum_{i=1}^{K}\left(\tau^2E_S^2\eta_{l,l,2i-1}^2\eta_{l,l,2i}^2\right)^{-1}}. \tag{26}$$

B. Signal model with imperfect CSI

In this subsection, the signal for imperfect CSI is presented, and thereby, the corresponding SINR is derived. In this context, the received signal at the relay R_l can be written as

$$\mathbf{y}_{R_l} = \sqrt{P_S}\mathbf{G}_{ll}\mathbf{x}_l + \sqrt{P_S}\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \mathbf{n}_{R_l}, \tag{17}$$

where $\mathbf{n}_{R_l}$ is the $N\times 1$ AWGN vector at the relay satisfying $\mathcal{E}\left[\mathbf{n}_{R_l}\mathbf{n}_{R_l}^H\right] = \mathbf{I}_N\sigma_{R_l}^2$. Similar to Section II-B, the relay first amplifies and then forwards its received signal to the end users. Thus, the transmitted signal by the relay can be written as $\mathbf{y}'_{R_l} = \hat{\beta}_l\hat{\mathbf{W}}_l\mathbf{y}_{R_l}$, where $\hat{\beta}_l$ is the amplification factor to satisfy the relay power constraint and $\hat{\mathbf{W}}_l$ is the beamforming-and-amplification matrix at the relay constructed by using imperfect CSI and can be defined as

$$\hat{\mathbf{W}}_l = \hat{\mathbf{G}}_{ll}^*(\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*)^{-1}\mathbf{P}(\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll})^{-1}\hat{\mathbf{G}}_{ll}^H, \tag{18}$$

where $\mathbf{P}$ is the permutation matrix defined under (2). The power constraint is designed by using same techniques in (3). By using the similar steps to those in Section II-B, $\hat{\beta}_l$ can be written as shown in (19) on the top of this page.

The received signal at the user node $U_{l,k'}$ can be written as

$$y_{l,k'} = \mathbf{g}_{lk'}^T\mathbf{y}'_{R_l} + n_{l,k'} = \hat{\beta}_l\sqrt{P_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{ll}\mathbf{x}_l + \hat{\beta}_l\sqrt{P_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \hat{\beta}_l\mathbf{g}_{k'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l} + n_{l,k'}, \tag{20}$$

where $\mathbf{g}_{lk'}^T$ and $n_{l,k'}$ are same as in Section II-B. By using the aforementioned signal model, the asymptotic SINRs for estimated CSI are derived for three cases of transmit power scaling at the user nodes and relay in the following subsections.

C. Asymptotic performance analysis for imperfect CSI

In this subsection, the detrimental impact of imperfect CSI is investigated by deriving the asymptotic SINR and sum rate expressions in closed-form for three transmit power scaling laws.

1) Transmit power scaling at the user nodes: Whenever the transmit power at the user nodes is scaled inversely proportional to the relay antenna count, the power of the pilot sequence is also scaled accordingly. Thus, the overall transmit power can only be scaled inversely proportional to the square-root of the relay antenna count ($\sqrt{N}$) for estimated CSI case. By letting $P_S = E_S/\sqrt{N}$, $P_P = \tau E_S/\sqrt{N}$ and $P_R = E_R$ while keeping E_S and E_R fixed, the asymptotic value of the normalized relay gain for unlimited number of relay antennas $\left(\frac{\hat{\beta}_l}{\sqrt{N}}\right)_\infty = \lim_{N\to\infty}\frac{\hat{\beta}_l}{\sqrt{N}}$ can be obtained as:[2]

$$\left(\frac{\hat{\beta}_l}{\sqrt{N}}\right)_\infty = \left(\frac{E_R}{E_S\sum_{i=1}^{2K}\left(\tau E_S\eta_{l,l,i}^2\right)^{-1} + 2\sigma_{R_l}^2\sum_{i=1}^{K}\left(\tau^2E_S^2\eta_{l,l,2i-1}^2\eta_{l,l,2i}^2\right)^{-1}}\right)^{\frac{1}{2}}. \tag{21}$$

By dividing both sides of (20) by $\sqrt[4]{N}$, the expression for the received signal can be rewritten as

$$\frac{y_{l,k'}}{\sqrt[4]{N}} = \frac{\hat{\beta}_l}{\sqrt{N}}\sqrt{E_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{ll}\mathbf{x}_l + \frac{\hat{\beta}_l}{\sqrt{N}}\sqrt{E_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \frac{\hat{\beta}_l}{\sqrt{N}}\sqrt[4]{N}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l} + \frac{1}{\sqrt[4]{N}}n_{l,k'}. \tag{22}$$

The asymptotic value of the third term in (22) can be written as

$$\sqrt[4]{N}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l} \xrightarrow[N\to\infty]{d} \frac{\widetilde{n}_l}{\left(\tau E_S\eta_{l,l,k}^2\right)}, \tag{23}$$

where $\widetilde{n}_l \sim \mathcal{CN}\left(0, \tau E_S\eta_{l,l,k}^2\sigma_{R_l}^2\right)$. All the other terms except the first term and the third term in the equation (22) asymptotically vanish as $N\to\infty$. Thus, the asymptotic SINR for imperfect CSI case under transmit power scaling at the user nodes can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \tau\eta_{l,l,k}^2E_S^2/\sigma_{R_l}^2. \tag{24}$$

2) Transmit power scaling at the relay: In this section, the asymptotic SINR is provided for transmit power scaling at the relay node. Here, it can be shown that the relay transmit power can be scaled inversely proportional to the number of relay antennas. Thus, we can define $P_R = E_R/N$, $P_p = \tau E_S$ and $P_S = E_S$. The asymptotic SINR for relay transmit power scaling case under the imperfect CSI assumption can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{E_R}{\sigma_{n_{l,k'}}^2\sum_{i=1}^{2K}\left(\frac{\tau E_S\eta_{l,l,i}^2}{\tau E_S\eta_{l,l,i}+1}\right)^{-1}}. \tag{25}$$

3) Transmit power scaling at the user nodes and relay: The asymptotic SINR for imperfect CSI under the transmit power scaling at the relay and user nodes (where $P_S = E_S/\sqrt{N}$, $P_P = \tau E_S/\sqrt{N}$ and $P_R = E_R/\sqrt{N}$) can be derived as shown in (26) on the top of this page.

D. Asymptotic sum rate analysis for imperfect CSI case

In this subsection, the asymptotic sum rate analysis is presented for imperfect CSI case. To this end, the average sum rate for the lth $2K$-user TWRN with imperfect CSI can be defined as [3]

$$\mathcal{R}_l = \frac{(T_C-\tau)}{2T_C}\sum_{k=1}^{2K}\mathcal{E}\left[\log\left(1+\gamma_{l,k}\right)\right], \tag{27}$$

where T_C and τ are the coherence time of the wireless channel and length of the pilot sequence used for channel estimation.

[2]The proofs have been omitted in this section due to the page limitations and the similarity to the proofs in Section III.

$$\beta_l = \sqrt{\frac{P_R}{P_S \sum_{j=1}^{L} \mathrm{Tr}\left(\mathbf{G}_{jl}\mathbf{G}_{jl}^H \hat{\mathbf{G}}_{ll}\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\hat{\mathbf{G}}_{ll}^H\right) + \sigma_{R_l}^2 \mathrm{Tr}\left(\left[\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll}\right]^{-1}\mathbf{P}\left[\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*\right]^{-1}\mathbf{P}\right)}}. \tag{30}$$

$$\left(\hat{\beta}_l\right)_{\infty}^2 = \left(\lim_{N\to\infty}\hat{\beta}_l\right)^2 = \frac{E_R}{E_S \sum_{j=1}^{L}\sum_{k=1}^{K}\hat{\eta}_{l,2k-1}^{-1}\hat{\eta}_{l,2k}^{-1}\left(\frac{\eta_{j,l,2k-1}^2}{\hat{\eta}_{l,2k-1}} + \frac{\eta_{j,l,2k}^2}{\hat{\eta}_{l,2k}}\right)}. \tag{35}$$

$$\Lambda_l = \frac{E_R}{E_S \sum_{j=1}^{L}\sum_{k=1}^{K}\hat{\eta}_{l,2k-1}^{-1}\hat{\eta}_{l,2k}^{-1}\left(\frac{\eta_{j,l,2k-1}^2}{\hat{\eta}_{l,2k-1}} + \frac{\eta_{j,l,2k}^2}{\hat{\eta}_{l,2k}}\right) + 2\sigma_{R_l}^2 \sum_{i=1}^{K}\hat{\eta}_{l,2i-1}^{-1}\hat{\eta}_{l,2i}^{-1}}. \tag{38}$$

In particular, the pre-log factor $(T_C - \tau)/T_C$ accounts for the pilot overhead [3]. Next, the asymptotic sum rate for three power scaling laws under imperfect CSI can readily be obtained by substituting the corresponding asymptotic SINR expressions in (24), (25), and (26) into (27).

V. Impact of Pilot Contamination

In this section, the impact of pilot contamination on the performance of multi-pair TWRNs is investigated. The system and channel model is assumed to be similar as in the Section II-A. For the sake of exposition, we assume that the channel estimation at the relay is error-free, nevertheless, it is contaminated with pilot sequences transmitted by L adjacent TWRNs. This assumption allows us to isolate the effect of pilot contamination on the performance of the system.

Let us now revisit the system, channel, and signal models corresponding to the pilot contamination case. Due to reuse of non-orthogonal pilot sequences at the L adjacent co-channel TWRNs, the users-to-relay channel estimation at the relay not only contains the desired channel, but also the undesired channels belonging to $2K$ users in each of the adjacent L TWRNs. Thus, the corresponding channel estimate can be written as

$$\hat{\mathbf{G}}_{ll} = \sum_{j=1}^{L}\mathbf{G}_{jl}, \qquad \text{for} \quad l \in \{1, \cdots L\}, \tag{28}$$

where $\mathbf{G}_{jl}$ is defined in Section II-A.

For the pilot contamination case, the received signal at the relay can also be written as shown in (1). By following steps similar to those in Section II-B, the relay first amplifies and then forwards the signal to the end users. Thus, the transmitted signal by the relay can be written as $\mathbf{y}'_{R_l} = \hat{\beta}_l \hat{\mathbf{W}}_{ll}\mathbf{y}_{R_l}$, where $\hat{\beta}_l$ is the relay amplification factor. Further, $\hat{\mathbf{W}}_{ll}$ is the beamforming-and-amplification matrix at the relay constructed by using the pilot contaminated CSI (28) and can be defined as

$$\hat{\mathbf{W}}_l = \hat{\mathbf{G}}_{ll}^*(\hat{\mathbf{G}}_{ll}^T\hat{\mathbf{G}}_{ll}^*)^{-1}\mathbf{P}(\hat{\mathbf{G}}_{ll}^H\hat{\mathbf{G}}_{ll})^{-1}\hat{\mathbf{G}}_{ll}^H, \tag{29}$$

where $\mathbf{P}$ is the permutation matrix defined under (2). The power constraint is designed by using same techniques in (3). By using (3), a closed-form expression for the relay gain $\hat{\beta}_l$ can be derived as shown in (30) at the top of this page. Under this channel model, the received signal at the $U_{l,k'}$th user is given as

$$y_{l,k'} = \mathbf{g}_{lk'}^T\mathbf{y}'_{R_l} + n_{l,k'} = \hat{\beta}_l\sqrt{P_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{ll}\mathbf{x}_l + \hat{\beta}_l\sqrt{P_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \hat{\beta}_l\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l} + n_{l,k'}. \tag{31}$$

where the $\mathbf{g}_{lk'}^T$ and $n_{l,k'}$ are defined in Section II-A. By using the aforementioned signal model, the asymptotic SINR can be derived for three cases of transmit power scaling at the user nodes and relay in the following subsections.

A. Asymptotic performance analysis for pilot contamination

In this subsection, asymptotic performance metrics are derived for pilot contamination case by considering three power scaling laws at the user nodes and relay.

1) Transmit power scaling at the user nodes: In this case, the transmit power at users is scaled inversely proportional to the number of relay antennas. Thus, by substituting $P_S = E_S/N$ and $P_R = E_R$ into (31), the received signal can be re-written as

$$\frac{y_{l,k'}}{\sqrt{N}} = \frac{\hat{\beta}_l\sqrt{E_S}}{N}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{ll}\mathbf{x}_l + \frac{\hat{\beta}_l\sqrt{E_S}\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l}{N}\sum_{j=1,j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \frac{\hat{\beta}_l\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l}}{\sqrt{N}} + \frac{n_{l,k'}}{\sqrt{N}}. \tag{32}$$

By evaluating the asymptotic values of each term in (32), the asymptotic SINR for transmit power scaling at the users for pilot contamination case can be written as (see appendix B for the proof)

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{\eta_{l,l,k}^2 E_S}{E_S\sum_{j=1,j\neq l}^{L}\eta_{j,l,k}^2 + \frac{\eta_{l,l,k}^2}{\sum_{j=1}^{L}\eta_{j,l,k}}\sigma_{R_l}^2}. \tag{33}$$

2) Transmit power scaling at the relay: In this case, the asymptotic SINR is derived by scaling the transmit power at the relay inversely proportional to the number of relay antennas. Thus, by substituting $P_R = E_R/N$ and $P_S = E_S$ into (31), the asymptotic SINR for relay transmit power scaling under pilot contamination can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{\left(\hat{\beta}_l\right)_{\infty}^2\eta_{l,l,k}^2 E_S}{\left(\hat{\beta}_l\right)_{\infty}^2\sum_{j=1,j\neq l}^{L}\eta_{j,l,k}^2 E_S + \left(\sum_{j=1}^{L}\eta_{j,l,k}\right)^2\sigma_{n_{l,k'}}^2}, \tag{34}$$

where $\left(\hat{\beta}_l\right)_{\infty}$ is the asymptotic value of $\hat{\beta}_l$ as $N\to\infty$ and is written in (35) at the top of this page. For (35) and for the sequel, $\hat{\eta}_{l,k}$ is defined as

$$\hat{\eta}_{l,k} = \sum_{j=1}^{L}\eta_{j,l,k}. \tag{36}$$

3) Transmit power scaling at the user nodes and relay: The asymptotic SINR for pilot contamination case under transmit power scaling at both relay and user nodes (i.e., $P_S = E_S/N$ and $P_R = E_R/N$) can be derived as

$$\lim_{N\to\infty}\gamma_{l,k'} = \frac{\Lambda_l\eta_{l,l,k}^2 E_S}{\Lambda_l\sum_{j=1,j\neq l}^{L}\eta_{j,l,k}^2 E_S + \Lambda_l\frac{\eta_{l,l,k}^2}{\hat{\eta}_{l,k}}\sigma_{R_l}^2 + \sigma_{n_{l,k'}}^2\hat{\eta}_{l,k}^2}, \tag{37}$$

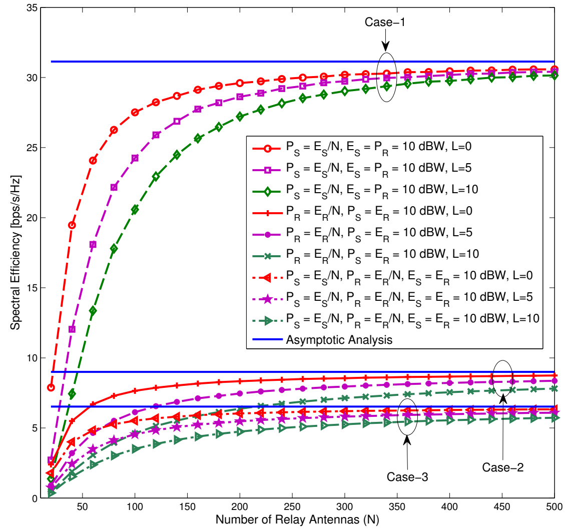

Fig. 1. Spectral efficiency versus the number of relay antennas of an 20-user TWRN with perfect CSI. The channels $\mathbf{G}_{jl}$ is assumed to be independently distributed Rayleigh with $\mathbf{D}_{ll} = \mathbf{I}_{2K}$ and $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$, where $j, l \in \{1, \cdots, L\}$ and $j \neq l$.

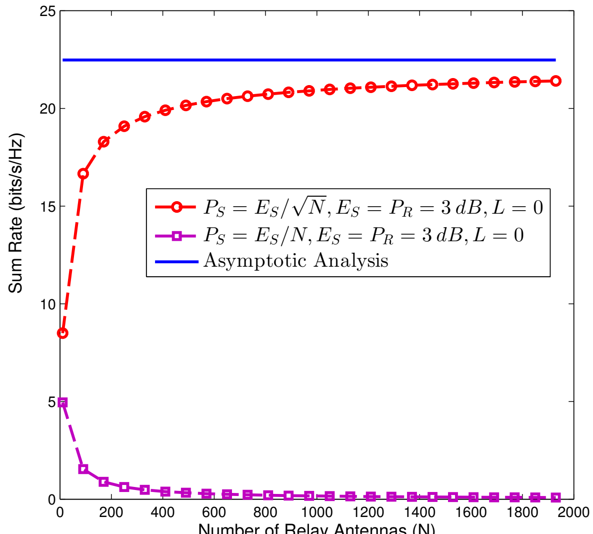

Fig. 2. Spectral efficiency versus the number of relay antennas of an 12-user TWRN with imperfect (estimated) CSI. Simulation parameters are taken as $L = 0$, $K = 6$, $T_C = 196$, and $\tau = K$.

where Λ_l is given in (38) at the top of the previous page.

The sum-rates of the system for the above three cases can be obtained by substituting SINR values (33), (34), and (37) into (15).

Remark V.1: Notably, the asymptotic SINRs (33), (34), and (37) are significantly lower than the asymptotic SINR obtained without the pilot contamination in Section III. Interestingly, whenever $\eta_{j,l,k} = 0$ for $j \neq l$, the asymptotic SINR results in (33), (34), and (37) approaches the same SINRs in Section III.

VI. Simulation Results

In Fig. 1, the impact of CCI is investigated for multi-pair TWRNs with perfect CSI. To this end, sum rate curves for three CCI cases (i.e., $L = 0$ $L = 5$ and $L = 10$) and for three power scaling laws are plotted. Specifically, the sum rate curves corresponding to $L = 0$ represent the CCI free case and plotted for comparison purposes. Fig. 1 reveals the sum rate curves

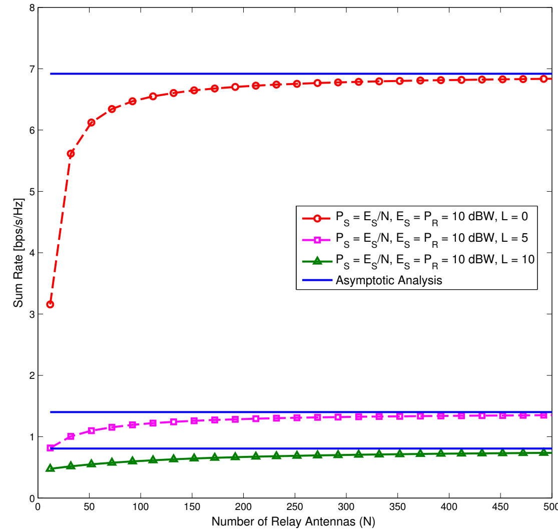

Fig. 3. Spectral efficiency versus the number of relay antennas of an 8-user TWRN with perfect CSI under pilot contamination. The channels $\mathbf{G}_{jl}$ is assumed to be independently distributed Rayleigh with $\mathbf{D}_{ll} = \mathbf{I}_{2K}$ and $\mathbf{D}_{jl} = \frac{1}{2}\mathbf{I}_{2K}$, where $j, l \in \{1, \cdots, L\}$ and $j \neq l$.

approach to a fixed asymptotic value irrespective of L when the relay antenna count grows unbounded. This observation clearly suggests that the impact of CCI can be effectively negated at the asymptotic regime of relay antenna count. Nevertheless, for lower number of relay antennas, the CCI causes substantial sum rate loses (see curves pertinent to $0 < N < 50$ regime). The transmit power scaling at the user nodes (Case-1) provide the highest sum rate, while the transmit power scaling at both user nodes and relay (Case-3) performs the worst. For example, Case-1 provides almost 21 bits/channel-use and 24 bits/channel-use asymptotic sum rate gains over Case-2 and Case-3, respectively.

In Fig. 2, the sum rate curves are plotted for an 6-user TWRN with imperfect CSI under the transmit power scaling at the user nodes. For the imperfect CSI case, the spectral efficiency asymptotically approaches to a non-zero (finite and constant) value only when the transmit powers at the user nodes are scaled down inversely proportional to square-root of the relay antenna count ($\sqrt{N}$). Fig. 2 clearly shows that the spectral efficiency asymptotically decays to zero whenever the transmit power at the user nodes are scaled down inversely proportional to the number of relay antennas (N). Thus, for imperfect CSI case, the fastest rate that the transmit powers at the user nodes can be scaled down is according to $1/\sqrt{N}$. Fig. 2 clearly reveals that two-way relaying with massive MIMO yields a substantial sum rate performance even with imperfect CSI.

In Fig. 3, the impact of pilot contamination is investigated for transmit power scaling at the user nodes by plotting sum rate curves pertinent to three cases; i.e., $L = 0$ $L = 5$ and $L = 10$. The asymptotic sum rate curves are plotted by using (33). In particular, the sum rate curve for $L = 0$ case corresponds to pilot contamination free case and is plotted for comparison purposes. Fig. 3 clearly reveals that the pilot contamination significantly degrades the sum rate performance. For instance, the asymptotic sum rate is degraded by 5.5 bits/s/Hz and 6.1 bits/s/Hz, respectively, when the channel estimation is impaired by pilots transmitted by 5 and 10 adjacent TWRNs. Thus, even in

$$\frac{\beta_l}{N} = \sqrt{\frac{E_R}{E_S \mathrm{Tr}\left(\left[\frac{\mathbf{G}_{ll}^T \mathbf{G}_{ll}^*}{N}\right]^{-1}\right) + \sigma_{R_l}^2 \mathrm{Tr}\left(\left[\frac{\mathbf{G}_{ll}^H \mathbf{G}_{ll}}{N}\right]^{-1} P \left[\frac{\mathbf{G}_{ll}^T \mathbf{G}_{ll}^*}{N}\right]^{-1} P\right) + E_S \sum_{j=1, j\neq l}^{L} \mathrm{Tr}\left(N\Delta_{jl}\right)}}. \tag{42}$$

$$\lim_{N\to\infty} \frac{\hat{\beta}_l}{N} = \left(\frac{\hat{\beta}_l}{N}\right)_\infty = \sqrt{\frac{E_R}{E_S \sum_{j=1}^{L} \sum_{k=1}^{K} \hat{\eta}_{l,2k-1}^{-1} \hat{\eta}_{l,2k}^{-1} \left(\frac{\eta_{j,l,2k-1}^2}{\hat{\eta}_{l,2k-1}} + \frac{\eta_{j,l,2k}^2}{\hat{\eta}_{l,2k}}\right) + 2\sigma_{R_l}^2 \sum_{i=1}^{K} \hat{\eta}_{l,2i-1}^{-1} \hat{\eta}_{l,2i}^{-1}}}. \tag{44}$$

the limit of infinitely many relay antennas, the detrimental impact of pilot contamination cannot be eliminated, and consequently, the asymptotic performance is limited by residual interference incurred due to pilot contamination.

VII. Conclusion

The impact of CCI, imperfect CSI, and pilot contamination is investigated for multi-pair massive MIMO TWRNs. The asymptotic SINR and sum rate expressions are derived for three transmit power scaling laws at the user nodes and relay. Importantly, the transmit power at the user nodes and the relay can be scaled down inversely proportional to the number of relay antennas for the perfect CSI case. Notably, even for the imperfect CSI case, the transmit power at the user nodes can be scaled down inversely proportional to the square-root of the relay antenna count without any performance penalty. For the CCI case, the asymptotic performance metrics become independent of the number of co-channel interferers (L) whenever the relay antenna count grows unbounded, and consequently, the corresponding detrimental impact can be asymptotically cancelled. However, the residual interference incurred due to pilot contamination cannot be mitigated completely even by using a massive MIMO enabled relay. Our analytical and simulation results reveal that substantial sum rate performance gains can be achieved by using a very large antenna array at the relay, while allowing the transmit power to become infinitesimal. Massive MIMO helps to mitigate the detrimental impact of channel estimation errors and CCI. Nevertheless, separate methods need to be devised for mitigating the effects of pilot contamination in massive MIMO TWRNs.

Appendix A
Proof of limits for power scaling at the users

In this section, several important limit results are provided. We begin with expressing the following three results [13]. For two independent vectors, $\mathbf{p} \sim \mathcal{CN}_{N\times 1}\left(0, \sigma_p^2\right)$ and $\mathbf{q} \sim \mathcal{CN}_{N\times 1}\left(0, \sigma_q^2\right)$, the following identities are valid.

$$\mathbf{p}^H\mathbf{p}/N \xrightarrow[N\to\infty]{a.s.} \sigma_P^2 \quad \text{and} \quad \mathbf{p}^H\mathbf{q}/N \xrightarrow[N\to\infty]{a.s.} 0, \tag{39}$$

$$\mathbf{p}^H\mathbf{q}/\sqrt{N} \xrightarrow[N\to\infty]{d} \mathcal{CN}\left(0, \sigma_p^2\sigma_q^2\right), \tag{40}$$

where subscripts *a.s.* and d stands for almost sure convergence and the convergence of distributions, respectively. By using the aforementioned identities, it can be shown that

$$\frac{\mathbf{G}_{ll}^H\mathbf{G}_{ll}}{N} = \mathbf{D}_{ll}^{\frac{1}{2}} \left(\frac{\mathbf{F}_{ll}^H\mathbf{F}_{ll}}{N}\right) \mathbf{D}_{ll}^{\frac{1}{2}} \xrightarrow[N\to\infty]{a.s.} \mathbf{D}_{ll}. \tag{41}$$

By first substituting P_S and P_R into (7) and then dividing by N, we obtain (42) shown on top of this page. In (42)

$$\Delta_{jl} = \mathbf{G}_{jl}\mathbf{G}_{jl}^H\mathbf{G}_{ll}\left[\mathbf{G}_{ll}^H\mathbf{G}_{ll}\right]^{-1}\mathbf{P}\left[\mathbf{G}_{ll}^T\mathbf{G}_{ll}^*\right]^{-1}\mathbf{P}\left[\mathbf{G}_{ll}^H\mathbf{G}_{ll}\right]^{-1}\mathbf{G}_{ll}^H. \tag{43}$$

The last term of the denominator of (42), $\mathrm{Tr}\left(N\Delta_{jl}\right) \xrightarrow[N\to\infty]{a.s.} 0$ for $j \neq l$. Thus by substituting values for $\mathbf{W}_l$, the asymptotic value of β_l/N, when $N \to \infty$ can be derived as in (9).

Appendix B
Proof of limits for pilot contamination

This section provides a proof sketch of SINR for pilot contamination under the transmit power scaling at users. The received signal at the users is given in (32). By using techniques similar to those used in appendix A, the value of $\lim_{N\to\infty} \hat{\beta}_l/N$ can be written as in (44) at the top of this page. Next, the asymptotic value of the desired signal component can be derived as

$$\frac{\hat{\beta}_l\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{ll}\mathbf{x}_l}{N} \xrightarrow[N\to\infty]{d} \left(\frac{\hat{\beta}_l}{N}\right)_\infty \frac{\eta_{l,l,k}^2}{\hat{\eta}_{l,k}^2}\widetilde{x}_l, \tag{45}$$

where $\widetilde{x}_l \sim \mathcal{CN}\left(0,1\right)$. Further the asymptotic values of the interference term in (32) can be derived as follows:

$$\frac{\hat{\beta}_l\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{G}_{jl}\mathbf{x}_j}{N} \xrightarrow[N\to\infty]{d} \left(\frac{\hat{\beta}_l}{N}\right)_\infty \frac{\eta_{j,l,k}^2}{\hat{\eta}_{l,k}^2}\widetilde{x}_j, \tag{46}$$

where $\widetilde{x}_j \sim \mathcal{CN}\left(0,1\right)$. The asymptotic values of the noise terms in (32) can be derived as

$$\frac{\hat{\beta}_l\mathbf{g}_{lk'}^T\hat{\mathbf{W}}_l\mathbf{n}_{R_l}}{\sqrt{N}} \xrightarrow[N\to\infty]{d} \left(\frac{\hat{\beta}_l}{N}\right)_\infty \frac{\eta_{l,l,k}}{\hat{\eta}_{l,k}^2}\widetilde{n} \quad \text{and} \quad \frac{n_{l,k'}}{\sqrt{N}} \xrightarrow[N\to\infty]{a.s.} 0, \tag{47}$$

where $\widetilde{n} \sim \mathcal{CN}\left(0, \hat{\eta}_{l,k}\sigma_{R_l}^2\right)$. By using the (45), (46), and (47) the desired asymptotic SINR can be written as shown in (33).

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