Outage and Decoding Delay Analysis of Full-Duplex DF Relaying: Backward or Sliding Window Decoding

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***Abstract*—We investigate the outage and decoding-delay performances for full-duplex (FD) decode-forward (DF) relaying with backward and sliding window decoding. In our analysis, we consider a block fading channel with full channel state information (CSI) availability at receivers and with limited CSI at transmitters. For backward decoding where the destination starts decoding from the last transmission block, we derive the average block decoding delay. For sliding window decoding, we analyze both joint and sequential decoding where the destination utilizes two received blocks either simultaneously or sequentially to decode the source data. We then derive their average block decoding delays and outage performances by considering channel variation over two transmission blocks and outage events at both the relay and the destination. When comparing FD relaying with backward decoding and half-duplex (HD) transmission, numerical results show that joint sliding-window decoding is the preferred choice in terms of complexity, decoding delay and outage performances.**

I. Introduction

Wireless relays remain an active research area in order to improve the throughput and reliability of modern and future cellular networks. For example, relays are standardized in LTE-A release 10 [1] and will be used in fifth generation (5G) wireless backhaul provision and device-to-device communication [2]. The performance of any relaying scheme is determined by its decoding delay, achievable rate, and reliability. These characteristics have been analyzed extensively for HD relaying. However, HD relaying achieves lower spectral efficiency than FD relaying. Hence, FD relaying is receiving wide attention, but its reliability and decoding delay have not been analyzed extensively. In 5G networks, FD will be useful for applications such as machine-to-machine communications that require high reliability and low latency [3]. Hence, it is essential to characterize these measures for FD decode-forward (DF) relaying with sliding window decoding (SWD).

Although decode, compress and amplify-forward relaying were developed since [4], [5], their applications in modern and future cellular networks are becoming clear now [2], [3]. As fundamental measures, outage performance and decoding delay have been analyzed for several HD schemes [6]–[10].

However, full-duplexing potentially achieves 100% spectral efficiency improvement, and its use in 5G cellular networks becomes possible by millimeter waves [2] and self-interference cancellation techniques [11], [12]. Therefore, its outage performance has been analyzed for full or partial DF relaying with coherent and/or independent transmission considering backward decoding (BD) [6], [8], [13]. In BD, the destination initializes decoding from the last transmission block, which leads to along decoding delay [14]. However, low latency (≤ 1 ms) is critical for delay-sensitive machine-type communications in 5G networks [3]. Therefore, SWD is a valuable solution since it has lower delay than BD as the destination starts decoding after waiting one extra block only [14].

SWD can be joint or sequential [4], [14]. In joint decoding, the destination simultaneously utilizes two received blocks to decode the source information. In sequential decoding, however, the destination first decodes a bin index about source information in one block and then the source information in an earlier block. Since [14] shows that both BD and SWD achieve the same rate, the same outage analysis for both techniques is performed [8]. However, in a block fading channel, these two methods have different outage probabilities.

In this paper, we first derive the average block decoding delay of SWD and BD. Second, we analyze the outages of joint and sequential SWD and compare them with the outage of BD in [6], [8]. We assume that CSI is fully available at receivers and partially at transmitters. Thus, source ($\mathcal{S}$) and relay ($\mathcal{R}$) can perform coherent transmission and also switch between direct transmission (DT) and DF modes as in [6]. The outage events at both $\mathcal{R}$ and destination ($\mathcal{D}$) and the channel variation over two transmission blocks are considered. Moreover, in sequential decoding, outage events are considered for the bin index and $\mathcal{S}$ information. Results show that while the decoding delay of BD increases with the number of transmission blocks, that of SWD approaches one. Considering the outage measure, BD outperforms SWD while joint decoding outperforms sequential decoding and they both outperform HD DF relaying [6] for a wide range of SNR.

II. Channel Model

In Fig. 1, $\mathcal{S}$ communicates with $\mathcal{D}$ via $\mathcal{R}$. For any transmission over B blocks, the channel model at block $k \in \{1, 2, ..., B\}$ is given as follows:

$$\begin{aligned} Y_{r,k} &= h_{rs,k} X_{s,k} + Z_{r,k}, \\ Y_{d,k} &= h_{ds,k} X_{s,k} + h_{dr,k} X_{r,k} + Z_{d,k}, \end{aligned} \tag{1}$$

where $Y_{r,k}$ $(Y_{d,k})$ is the $k-$th received signal block at $\mathcal{R}$ $(\mathcal{D})$ and $Z_{r,k}$ and $Z_{d,k}$ are i.i.d complex Gaussian noises $(\mathcal{CN}(0,1))$. $X_{s,k}$ $(X_{r,k})$ is the $k-$th signal block transmitted by $\mathcal{S}$ $(\mathcal{R})$. We consider block fading channel where the links remain constant in each block and change independently in the next block. Hence, in block k, each link gain is modeled by Rayleigh fading and pathloss as follows:

$$\begin{aligned} h_{ij,k} &= \tilde{h}_{ij,k}/(d_{ij}^{\alpha_{ij}/2}), \\ &= g_{ij,k} e^{\sqrt{-1}\theta_{ij}}, \ i \in \{r,d\}, \ j \in \{s,r\} \end{aligned} \tag{2}$$

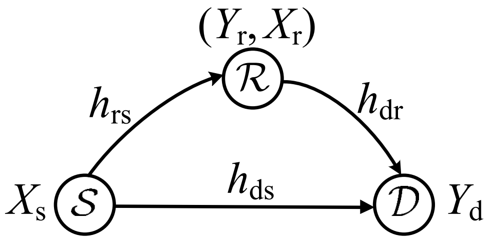

Fig. 1. Basic Relay channel model.

where $\tilde{h}_{ij,k} \sim \mathcal{CN}(0,1)$ represents the small scale fading. The large scale fading is captured by pathloss where d_{ij} is the distance between nodes i and j and α_{ij} is the attenuation factor. Let $g_{ij,k}$ and $\theta_{ij,k}$ be the amplitude and the phase of a link coefficient in block k, then $g_{ij,k} = |\tilde{h}_{ij,k}|/d_{ij}^{\alpha_{ij}/2}$ has a Rayleigh distribution while $\theta_{ij,k}$ has a uniform distribution between $[0, 2\pi]$. We assume full CSI at receivers and partial CSI at transmitters (obtained via feedback from $\mathcal{D}$ [15]). As receivers, $\mathcal{R}$ knows $h_{rs,k}$ and $\mathcal{D}$ knows $h_{ds,k}$ and $h_{dr,k}$. As transmitters, since the outage probability depends on the statistics of the channel links, statistical (not exact instantaneous) knowledge of the links is required at $\mathcal{S}$ such that it can optimize its transmit power to minimize the outage probability. Moreover, to perform coherent transmission, $\mathcal{S}$ and $\mathcal{R}$ each knows the phase of its respective link to $\mathcal{D}$. Furthermore, they both know the order between $g_{rs,k}^2$ and $g_{ds,k}^2$ to determine the optimal transmission scheme as specified in Section III.

Although the FD relay suffers from the self-interference, it is alleviated with millimeter waves [2] and analog and digital cancellation techniques that reduce it by 110 dB in WiFi radios [16]. However, hardware imperfections leave a small interference, which is seen as additive noise by $\mathcal{R}$ [8], [17].

III. DF Relaying scheme

We consider the FD DF scheme in [4], [5], [14] where the transmission is carried over B independent blocks ($B >> 1$, $B \in \mathbb{N}$) and $\mathcal{S}$ aims to send $B-1$ messages through B blocks[1]. However, instead of always performing DF relaying at any transmission block $k \in \{1, 2, ..., B\}$, our scheme switches between direct transmission (DT) and DF relaying based on amplitude ordering between $\mathcal{S}-\mathcal{R}$ ($g_{rs,k}$) and $\mathcal{S}-\mathcal{D}$ ($g_{ds,k}$) links as in [6]. This is because when $g_{rs,k} < g_{ds,k}$, decoding at $\mathcal{R}$ may constrain the achievable rate. To determine the operating mode for block $k \in \{1, 2, ..., B\}$, $\mathcal{S}$ must know the relative order of $g_{rs,k}$ and $g_{ds,k}$, which is obtained through one bit feedback from $\mathcal{D}$ to $\mathcal{S}$ and $\mathcal{R}$.

A. *Direct Transmission Mode:* ($g_{rs,k} \leq g_{ds,k}$)

This mode for block k occurs when the $\mathcal{S}-\mathcal{R}$ link is weaker than $\mathcal{S}-\mathcal{D}$ link ($g_{rs,k} \leq g_{ds,k}$). Clearly, it is identical to the classical point-to-point communication without $\mathcal{R}$, where $\mathcal{S}$ sends its information directly to $\mathcal{D}$ at rate $R \leq \log\left(1 + g_{ds,k}^2 P_s\right)$.

[1]In the last transmission block, $\mathcal{S}$ sends only the old information (no new information). This may reduce the average achievable rate. However, this rate reduction becomes negligible as $B \to \infty$ [14].

B. *DF Relaying Mode:* ($g_{rs,k} > g_{ds,k}$)

This mode for block k occurs when the $\mathcal{S}-\mathcal{R}$ link is stronger than $\mathcal{S}-\mathcal{D}$ link ($g_{rs,k} > g_{ds,k}$). This scheme uses superposition block Markov encoding[2] where $\mathcal{R}$ decodes $\mathcal{S}$ information in block k (w_k) and then forwards it *coherently* with $\mathcal{S}$ to $\mathcal{D}$ in block $m > k$ in which the $\mathcal{S}-\mathcal{R}$ link is also stronger than $\mathcal{S}-\mathcal{D}$ link ($g_{rs,m} > g_{ds,m}$). $\mathcal{D}$ then utilizes the signals received from $\mathcal{S}$ and $\mathcal{R}$ and performs either BD or SWD for w_k. Note that independent (*non-coherent*) transmission from $\mathcal{S}$ and $\mathcal{R}$ is also possible but leads to lower rate and outage performance [6].

1) Transmission Scheme: Assume that $\mathcal{S}-\mathcal{R}$ link is stronger than $\mathcal{S}-\mathcal{D}$ in blocks v, k and m where $v < k < m$ and $(v, k, m) \in \{1, 2, ..., B\}$. In block k, $\mathcal{S}$ encodes its new information (w_k) by superposing it on the information (w_v) and transmitting the signal that conveys this superposed codeword (for w_k and w_v). Then, in block k, $\mathcal{R}$ decodes w_k and transmits the signal of its codeword (for w_k) in block m.

2) Transmit Signals: $\mathcal{S}$ and $\mathcal{R}$ construct their Gaussian transmit signals as follows:

$$X_{s,k} = \sqrt{\rho_n} U_1(w_k) + \sqrt{\rho_o} U_2(w_v),\ X_{r,k} = \sqrt{P_r} U_2(w_v) \quad (3)$$

where $U_j \sim N(0,1)$ for $j \in \{1, 2\}$. Codeword U_1 conveys new information w_k while U_2 conveys old information w_v. Here, $\mathcal{R}$ allocates its power P_r for the signal U_2 while $\mathcal{S}$ allocates the transmit power ρ_n (ρ_o) for the signal U_1 (U_2). The power parameters (ρ_n, ρ_o) satisfy the constraint $\rho_n + \rho_o = P_s$ where P_s is the transmit power of $\mathcal{S}$.

3) Decoding Techniques: The decoding at $\mathcal{R}$ is straightforward where in block k, $\mathcal{R}$ already knows w_v from the decoding in block v and it can reliably decode w_k at the following rate:

$$R \leq \log\left(1 + g_{rs,k}^2 \rho_n\right) = C_1. \quad (4)$$

$\mathcal{D}$ decodes $\mathcal{S}$ information using backward, joint or sequential SWD techniques that are explained in details in [[14], Chapters 16 and 18]. We explain them briefly in the following sections for the ease of reference.

Backward Decoding (BD): In this decoding, $\mathcal{D}$ decodes w_v in block k given that it knows w_k from the decoding in block m. $\mathcal{D}$ then can reliably decode w_v at the following rate [14]:

$$R \leq \log\left(1 + g_{ds,k}^2 P_s + g_{dr,k}^2 P_r + 2 g_{ds,k} g_{dr,k} \sqrt{\rho_o P_r}\right) = C_2. \quad (5)$$

Remark 1. The term $2 g_{ds,k} g_{dr,k} \sqrt{\rho_o P_r}$ shows the advantage of beamforming obtained by coherent transmission of w_v from both $\mathcal{S}$ and $\mathcal{R}$. With independent (non-coherent) transmission, this beamforming term is zero and the rate becomes similar to that of the maximum ratio combining (MRC) [7].

From (4) and (5), the achievable rate is given as follows:

$$R \leq \min\{C_1, C_2\}, \quad (6)$$

[2]In block Markov encoding, the codeword sent in each block depends on not only on the new information but also old information from an earlier block. Sending two or more information parts is performed through superposition coding as explained in [[14], Chapter 16].

Joint SWD: In joint SWD, $\mathcal{D}$ utilizes the received signals in blocks k and m $(Y_{d,k}, Y_{d,m})$ to decode w_k given that it knows w_v from the previous decoding step (from $Y_{d,v}$ and $Y_{d,k}$). $\mathcal{D}$ then can reliably decode w_k at the following rate [4], [5]:

$$R \leq \log(1 + g_{ds,k}^2 \rho_n) + \log\left(1 + \frac{g_{ds,m}^2 \rho_o + g_{dr,m}^2 P_r + 2 g_{ds,m} g_{dr,m} \sqrt{\rho_o P_r}}{1 + g_{ds,m}^2 \rho_n}\right) = C_3. \quad (7)$$

From (4) and (7), the achievable rate is given as follows:

$$R \leq \min\{C_1, C_3\}, \quad (8)$$

Remark 2. Formulas (6) and (8) show that SWD achieves the same rate as BD in AWGN channels since $C_2 = C_3$ as $g_{ds,k} = g_{ds,m}$. However, the two decoding schemes are not equivalent in fading channels since $C_2 \neq C_3$ as $g_{ds,k} \neq g_{ds,m}$ which can lead to different outages.

Sequential SWD: Another scheme called DF via binning was proposed in [14]. In this scheme, $\mathcal{S}$ messages are sorted into equal size groups and a bin index (l) is given for each group. Then, in each block, $\mathcal{S}$ sends the new message and the bin index of the previous message (l_v). $\mathcal{R}$ decodes the new message in one block and forwards the bin index of that message in the next block. The transmit signals for this scheme are similar to (3) but replacing each w_v by l_v.
$\mathcal{D}$ performs sequential decoding as it first utilizes its received signal in block m to decode l_k at the following rate [14]:

$$R_b \leq \log\left(1 + \frac{g_{ds,m}^2 \rho_o + g_{dr,m}^2 P_r + 2 g_{ds,m} g_{dr,m} \sqrt{\rho_o P_r}}{1 + g_{ds,m}^2 \rho_n}\right) = C_4, \quad (9)$$

where R_b is the transmission rate for the bin index l_k where $R_b < R$. $\mathcal{D}$ next goes back to block k and decodes w_k, given that it knows l_v and l_k where $w_k \in l_k$, at the following rate [14]:

$$R - R_b \leq \log(1 + g_{ds,k}^2 \rho_n) = C_5. \quad (10)$$

Then, from (4), (9) and (10), the achievable rate with sequential decoding is given as in (8).

Remark 3. Unlike joint decoding, sequential decoding simplifies the decoding at $\mathcal{D}$ but complicates the encoding at $\mathcal{S}$ and $\mathcal{R}$ since they need to group the information into bins and determine the optimal binning index rate.

Remark 4. Although joint and sequential SWD achieve the same rate, their outages differ because of the different decoding processes (see Section V). In joint decoding, $\mathcal{D}$ decodes $\mathcal{S}$ information (w_k) using the signals received in blocks k and m simultaneously. However, in sequential decoding, $\mathcal{D}$ first decodes the bin index l_k in block m and then w_k in block k.

IV. Average Block Decoding Delay

Decoding delay is an important criterion for some applications in 5G networks [3] such as the traffic safety that requires a very low latency ($\leq$ 1 ms). We first define the decoding delay as the number of transmissions blocks $\mathcal{D}$ waits before decoding $\mathcal{S}$ information. Hence, from now on, *the decoding delay refers to block decoding delay*. We derive it for BD and SWD. For these schemes, the delay occurs only for DF mode since in DT mode, $\mathcal{D}$ decodes $\mathcal{S}$ information at the end of a transmission block. Hence, the average decoding delay depends on the decoding technique and channel statistics that determine the ratio between DT and DF modes.

A. Backward Decoding (BD)

Here, the decoding delay in block k is either 0 during DT mode or $B-k$ during DF mode. Therefore, the decoding delay T_k^{BD} for any transmission block k is given as follows:

$$T_k^{BD} = \mathsf{P}[g_{rs,k} \leq g_{ds,k}] * 0 + \mathsf{P}[g_{rs,k} > g_{ds,k}] * (B-k), \stackrel{(a)}{=} \frac{\mu_{rs}}{\mu_{rs} + \mu_{ds}}(B-k), \quad (11)$$

where (a) is obtained by using the Rayleigh distribution of $g_{ds,k}$, and $g_{rs,k}$ and μ_i is the mean of g_i^2 for $i \in \{ds, dr, rs\}$. The average delay $\overline{T}^{BD}$ can then be derived as follows:

Theorem 1. *The average decoding delay for FD DF relaying with BD* ($\overline{T}^{BD}$) *is given by:*

$$\overline{T}^{BD} = 0.5B\left(\mu_{rs}/(\mu_{rs} + \mu_{ds})\right). \quad (12)$$

Proof. Obtained by summing T_k^{BD} in (11) over all $k \in \{1, 2, ..., B-1\}$, dividing this summation over $B-1$ and then using the summation identity $\sum_{k=1}^{n} k = 0.5n(n+1)$. □

Theorem 1 shows that the delay increases with B, thus limiting the usefulness of BD for delay sensitive applications.

B. Sliding Window Decoding

In SWD, the average decoding delays for both joint and sequential decoding are identical as $\mathcal{D}$ utilizes the received signals in two blocks to decode $\mathcal{S}$ information. The decoding delay for block k is 0 during DT mode and varies between $1, 2, ..., B-k$ during DF relaying mode depending on channel realizations in the following blocks after block k that determine the DT and DF modes. The decoding delay in block k is $i \in 1, 2, ..., B-(k+1)$ if $g_{rs,k} > g_{ds,k}$, $g_{rs,k+l} < g_{ds,k+l}$ for all $l \in \{1, 2, ..., i-1\}$ and $g_{rs,k+i} > g_{ds,k+i}$ since $\mathcal{D}$ will wait till block $k+i$ to decode w_k. However, if $g_{rs,k} > g_{ds,k}$ and $g_{rs,n} \leq g_{ds,n}$ for all $k < n < B-k-1$, the decoding delay then is $B-k$ as $\mathcal{R}$ will forward w_k in the last block B regardless of the link order. Then, the decoding delay T_k^{SWD} for any transmission block k is given as follows:

$$T_k^{SWD} = \mathsf{P}[g_{rs,k} \leq g_{ds,k}] * 0 + \mathsf{P}[g_{rs,k} > g_{ds,k}] * P^*,$$
$$P^* = \sum_{i=1}^{B-(k+1)} i\mathsf{P}[g_{rs,k+i} > g_{ds,k+i}] \prod_{l=1}^{i-1} \mathsf{P}[g_{rs,k+l} < g_{ds,k+l}] + (B-k) \prod_{l=1}^{B-k-1} \mathsf{P}[g_{rs,k+l} < g_{ds,k+l}], \quad (13)$$

by using Rayleigh distributions, T_k^{SWD} becomes as follows:

$$T_k^{SWD} = \frac{\mu_{rs}}{\mu_{rs} + \mu_{ds}} * \Big[\sum_{i=1}^{B-(k+1)} i * \frac{\mu_{ds}^{i-1}\mu_{rs}}{(\mu_{ds} + \mu_{rs})^i} (B-k)(\mu_{ds}/(\mu_{ds} + \mu_{rs}))^{B-k-1}\Big]. \quad (14)$$

The average delay $\overline{T}^{SWD}$ can then be obtained as follows:

Theorem 2. *The average decoding delay for FD DF relaying with joint or sequential SWD ($\overline{T}^{SWD}$) is given as follows:*

$$\overline{T}^{SWD}= 1 - \frac{\mu_{ds}}{\mu_{rs}(B-1)} \left[1 - (\mu_{ds}/(\mu_{ds}+\mu_{rs}))^{B-1}\right]. \quad (15)$$

Proof. Obtained by summing T_k^{SWD} in (13) over all $k \in \{1, 2, ..., B-1\}$, dividing over $B-1$ and then using the summation identity $\sum_{k=1}^{n} x^k = x(x^n - 1)(x-1)^{-1}$. □

Remark 5. Theorem 2 implies that the average decoding delay approaches 1 as $B \to \infty$. This is because if $\mathcal{S}$ sends a message in block k through DF mode and $\mathcal{D}$ decodes it in block m, the delay for this message is $m-k$ while the delays for other messages sent in blocks $k+1, k+2, ..., m$ are zero which makes the average equal to 1. Moreover, for a message sent in block k, (13) implies that the probability of decoding this message in block m decreases as m increases. Hence, SWD is more attractive than BD for delay sensitive applications.

V. Outage for Sliding Window Decoding

High reliability is required by many wireless applications. For example, VoIP service in LTE release 8 can tolerate an outage of 2% [18]. Hence, we analyze the outage of FD DF relaying with joint and sequential SWD since the outage of BD is already derived in [6], [13], [19]. For both SWD techniques, outage is derived by considering both DT and DF modes over two blocks and the outage events at both $\mathcal{R}$ and $\mathcal{D}$.

A. Joint Sliding Window Decoding

The outage probability is related to the achievable rates for DT and DF modes that are given in (4) and (7), respectively. Define $\mathbb{P}_1$ and $\mathbb{P}_2$ as outage probabilities in DT and DF modes, respectively. We then obtain the following theorem:

Theorem 3. *For a given target rate (R) with specific power allocation (ρ_n, ρ_o), the average outage probability $(\bar{\mathbb{P}}_o^{JSWD})$ of the FD DF scheme with joint SWD is given as follows:*

$$\begin{aligned} \bar{\mathbb{P}}_o^{JSWD} &= \mathbb{P}_1 + \mathbb{P}_2 = \mathbb{P}_1 + \mathbb{P}_r + \mathbb{P}_d, \quad (16)\\ \textit{with}\ \ \mathbb{P}_1 &= \mathsf{P}[R > \log\left(1 + g_{ds,i}^2 \rho_n\right),\ g_{rs,i} \le g_{ds,i}],\\ \mathbb{P}_r &= \mathsf{P}\left[R > C_1,\ g_{rs,i} > g_{ds,i}\right],\\ \mathbb{P}_d^{JSWD} &= \mathsf{P}\big[R > C_3,\ R \le C_1,\ g_{rs,i} > g_{ds,i}\big],\ \textit{where} \end{aligned}$$

- *$\mathbb{P}_1$ is the outage during DT mode,*
- *$\mathbb{P}_r$ and $\mathbb{P}_d^{JSWD}$ are the outages during DF mode,*
  - *$\mathbb{P}_r$ is the outage at $\mathcal{R}$;*
  - *$\mathbb{P}_d^{JSWD}$ is the outage at $\mathcal{D}$ when there is no outage at $\mathcal{R}$.*

Proof. From the outage events in DT and DF modes when R is higher than the achievable rate. □

The analytical expressions for $\mathbb{P}_1$ and $\mathbb{P}_r$ are given in [[19], Theorem 2] and we omit them here because of space limitation. $\mathbb{P}_d^{JSWD}$ is given in the following Lemma:

Lemma 1. *The probability $\mathbb{P}_d^{JSWD}$ in Theorem 3 can be expressed as follows:*

$$\mathbb{P}_d^{JSWD} = e^{-\frac{\eta_1^2}{\mu_{rs}}} \times \begin{cases} \int_0^{\eta_1}\int_0^{\zeta_2} f(\gamma_1,\beta_1,\zeta_1) d\beta_1 d\gamma_1, & \textit{if } \frac{P_s}{\rho_n} > 2^R \\ \int_0^{\eta_0}\int_0^{\infty} f(\gamma_1,\beta_1,\zeta_1) d\beta_1 d\gamma_1 & \\ + \int_{\eta_0}^{\eta_1}\int_0^{\zeta_2} f(\gamma_1,\beta_1,\zeta_1) d\beta_1 d\gamma_1, & \textit{if } \frac{P_s}{\rho_n} \le 2^R \end{cases}, \quad (17)$$

where $\eta_0 = \sqrt{\frac{2^R}{P_s} - \frac{1}{\rho_n}}, \quad \eta_1 = \sqrt{\frac{2^R - 1}{\rho_n}}, \quad (18)$

$$\zeta_1 = P_r^{-0.5}\left(\sqrt{\delta} - \beta_1\sqrt{\rho_o}\right),\ \zeta_2 = \sqrt{\frac{2^R - (1+\gamma_1^2\rho_n)}{P_s(1+\gamma_1^2\rho_n) - 2^R\rho_n}},$$

$$\delta = \left(1+\beta_1^2\rho_n\right)\left(\frac{2^R}{1+\gamma_1^2\rho_n} - 1\right),$$

$$f(\gamma_1,\beta_1,\zeta_1) = (4\gamma_1\beta_1/\mu_{ds}^2) e^{-\frac{\gamma_1^2+\beta_1^2}{\mu_{ds}}}\left(1 - e^{-\frac{\zeta_1^2}{\mu_{dr}}}\right)$$

Proof. $\mathbb{P}_1$ and $\mathbb{P}_r$ are derived in [6], [8] while $\mathbb{P}_d^{JSWD}$ is obtained by using the Rayleigh distribution of $g_{ds,k}$, $g_{ds,m}$, $g_{rs,k}$, $g_{dr,k}$, and $g_{dr,m}$ as shown in Appendix A. □

Remark 6. We obtain numerically the optimal resource allocation (ρ_n, ρ_o) that minimizes $\bar{\mathbb{P}}_o^{JSWD}$ since Theorem 3 specifies the outage for a fixed resource allocation.

B. Sequential Sliding Window Decoding

The outage probability for DF relaying with sequential decoding is similar to joint decoding except for the outage at $\mathcal{D}$ during DF relaying mode. For sequential decoding, an outage at $\mathcal{D}$ can occur not only for $\mathcal{S}$ information (w) but also for the bin index (l). An outage for l leads to an outage for w since each l represents a group of w $(w \in l)$. The average outage probability $(\bar{\mathbb{P}}_o^{SSWD})$ can be obtained as follows:

Theorem 4. *For a given target rate R with specific resource allocation (power (ρ_n, ρ_o) and binning rate (R_b)), the average outage probability $(\bar{\mathbb{P}}_o^{SSWD})$ of the FD DF scheme with sequential SWD is given as in Theorem 3 except replacing $\mathbb{P}_d^{JSWD}$ by $\mathbb{P}_d^{SSWD}$ which is given as follows:*

$$\begin{aligned} \mathbb{P}_d^{SSWD} &= \mathbb{P}_{d1} + \mathbb{P}_{d2}, \quad (19)\\ \mathbb{P}_{d1} &= \big[R_b > C_4,\ R \le C_1,\ g_{rs,k} > g_{ds,k}\big]\\ \mathbb{P}_{d2} &= \big[R - R_b > C_5,\ R_b \le C_4,\ R \le C_1,\ g_{rs,k} > g_{ds,k}\big],. \end{aligned}$$

- *$\mathbb{P}_{d1}$ is the outage probability of the bin index (l) at $\mathcal{D}$ when there is no outage at $\mathcal{R}$,*
- *$\mathbb{P}_{d2}$ is the outage probability of $\mathcal{S}$ information (w) at $\mathcal{D}$ when there is no outage at $\mathcal{R}$ for w or at $\mathcal{D}$ for l.*

$\mathbb{P}_{d1}$ and $\mathbb{P}_{d2}$ can be analytically expressed as follows:

$$\begin{aligned} \mathbb{P}_{d1} &= \left(e^{-\frac{\eta_1^2}{\mu_{rs}}} - \frac{\mu_{ds}}{\mu_{ds}+\mu_{rs}} e^{-\frac{\eta_1^2(\mu_{ds}+\mu_{rs})}{\mu_{ds}\mu_{rs}}}\right)\\ &\quad \times \begin{cases} \int_0^{\zeta_4} f_1(\beta_1,\zeta_3) d\beta_1, & \textit{if } \frac{P_s}{\rho_n} > 2^{R_b} \\ \int_0^{\infty} f_1(\beta_1,\zeta_3) d\beta_1, & \textit{if } \frac{P_s}{\rho_n} \le 2^{R_b} \end{cases} \quad (20)\\ \mathbb{P}_{d2} &= e^{-\frac{\eta_1^2}{\mu_{rs}}}\left(1 - e^{-\frac{\eta_2^2}{\mu_{ds}}}\right)\\ &\quad \times \begin{cases} e^{-\frac{\zeta_4^2}{\mu_{ds}}} + \int_0^{\zeta_4} f_2(\beta_1,\zeta_3) d\beta_1, & \textit{if } \frac{P_s}{\rho_n} > 2^{R_b} \\ \int_0^{\infty} f_2(\beta_1,\zeta_3) d\beta_1, & \textit{if } \frac{P_s}{\rho_n} \le 2^{R_b} \end{cases}, \end{aligned}$$

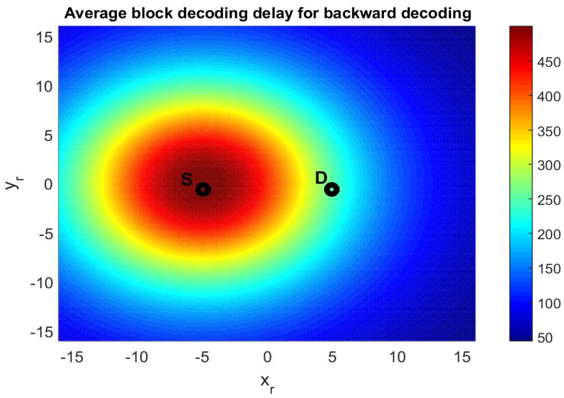

Fig. 2. Average block decoding delay of FD DF relaying with BD for any $\mathcal{R}$ location in $2D$ plan where $B = 1000$.

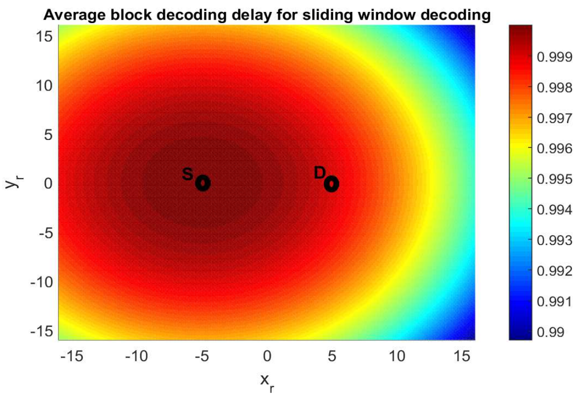

Fig. 3. Average block decoding delay of FD DF relaying with SWD for any $\mathcal{R}$ location in $2D$ plan where $B = 1000$.

$$\text{where } \eta_2 = \sqrt{\frac{2^{R-R_b}-1}{\rho_n}}, \quad \zeta_4 = \sqrt{\frac{2^{R_b}-1}{P_s - 2^{R_b}\rho_n}},$$

$$\zeta_3 = P_r^{-0.5}\left(\sqrt{(2^{R_b}-1)(1+\beta_1^2\rho_n)} - \beta_1\sqrt{\rho_o}\right),$$

$$f_1(\beta_1,\zeta_3) = \frac{2\beta_1}{\mu_{ds}} e^{-\frac{\beta_1^2}{\mu_{ds}}}\left(1 - e^{-\frac{\zeta_3^2}{\mu_{dr}}}\right)$$

$$f_2(\beta_1,\zeta_3) = \frac{2\beta_1}{\mu_{ds}} e^{-\frac{\beta_1^2}{\mu_{ds}}} e^{-\frac{\zeta_3^2}{\mu_{dr}}} \tag{21}$$

Proof. Using the Rayleigh distribution of $g_{ds,k}$, $g_{ds,m}$, $g_{rs,k}$, $g_{dr,k}$, and $g_{dr,m}$ in similar way to that of Lemma 1. □

Remark 7. As in Theorem 3, we obtain numerically the optimal resource allocation (ρ_n, ρ_o, R_b) that minimizes $\bar{\mathbb{P}}_o^{SSWD}$. Compared with joint decoding, sequential decoding simplifies the decoding at $\mathcal{D}$ but needs R_b to be optimized.

VI. Numerical Results

We now provide numerical results for the average delay and outage probabilities of FD DF with different decoding techniques. In these simulations, $\mathcal{S}$ and $\mathcal{R}$ have the same transmit power ($P_s = P_r = P$) in Watt and all links experience Rayleigh fading. Hence, the average channel gain for each link from node j to i is given as $\mu_{ij} = \frac{1}{d_{ij}^\alpha}$ where d_{ij} is the inter-node distance in meters and α is the pathloss factor (we choose $\alpha = 2.4$). The average received SNR in dB at $\mathcal{D}$ for the signal from $\mathcal{S}$ is defined as follows: $\text{SNR} = 10\log\left(P/d_{ds}^\alpha\right)$.

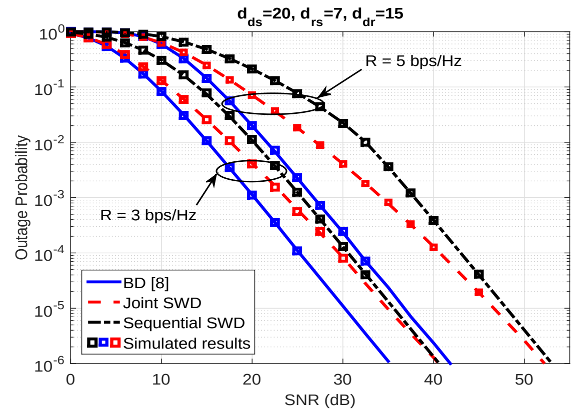

Fig. 4. Outage probabilities of FD DF relaying with BD, joint and sequential SWD at different target rates R.

We vary the power parameters (ρ_n, ρ_o) and binning rate (R_b) to minimize the outage probability.

Figs. 2 and 3 show the average decoding delay of BD and SWD for a relay network in $2D$ plane. The locations of $\mathcal{S}$ and $\mathcal{R}$ are fixed at $(-5, 0)$ and $(5, 0)$, respectively while the location of $\mathcal{R}$ varies over the entire $2D$ plane, i.e., $\mathcal{R}$ lies at $(x, y) \in \mathbb{R}^2$. These inter-node distances are valid for a small cell in 5G systems. As shown in Theorems 1 and 2, the delay of SWD is less than 1 while it can reach $B/2$ in BD. Moreover, in both decoding techniques, the delay increases as $\mathcal{R}$ gets closer to $\mathcal{S}$ since the DF relaying mode occurs more often than DT mode in this region as the probability of having the $\mathcal{S}-\mathcal{R}$ link stronger than $\mathcal{S}-\mathcal{D}$ link increases (Section IV).

BD is more sensitive to the location of $\mathcal{R}$ than SWD. For any $\mathcal{R}$ locations, the delay varies between 0.99 and 1 for SWD while it varies between 50 and 500 for BD. This is because for any block k with $\mathcal{S}-\mathcal{R}$ link stronger than $\mathcal{S}-\mathcal{D}$, the delay for backward decoding is $B-k$. However, the delay for SWD is $n \in \{1, 2, , B-k\}$ and the probability of decoding in block m where $m = k+n$ decreases as n increases. Therefore, even when $\mathcal{R}$ is close $\mathcal{S}$, the block decoding delay increases slightly compared with other $\mathcal{R}$ locations far from $\mathcal{S}$.

Fig. 4 shows the outage probabilities for the considered decoding techniques of the FD DF relaying scheme with node distances specified in the figure. The simulations are obtained using 10^6 samples for each fading channel, which are sufficient for outage probabilities above 10^{-4}. Results show perfect match between analytical and simulated results. The three decoding techniques achieve a diversity order of 2 at high SNR. However, BD has the best outage performance while sequential decoding has the worst. For delay sensitive services, joint SWD is the best choice as it has shorter decoding delay than BD and lower outage than sequential decoding.

Fig. 5 compares between outage performances of DT, dual-hop transmission [9], FD and HD DF relaying schemes. For the HD scheme, we consider the coherent DF scheme in [6] since it outperforms all other HD schemes in [7], [10][3]. While FD DF relaying with BD outperforms HD DF relaying, FD DF relaying with SWD underperforms HD DF relaying [6] at high

[3]Partial DF relaying outperforms full DF relaying in HD transmission at low SNR [6]. However, it has more parameters to optimize like rate and power allocation for public and private message parts [6].

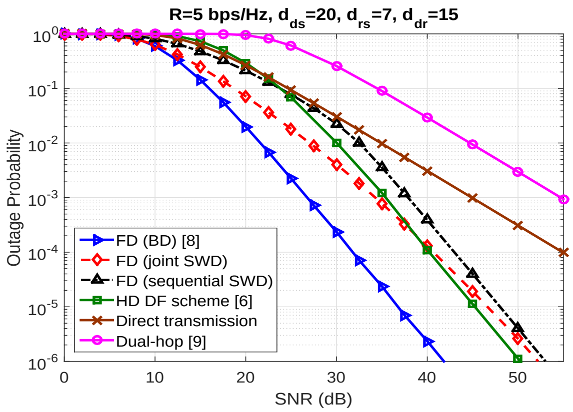

Fig. 5. Comparison between the outage probabilities of FD DF relaying with BD and SWD, HD DF relaying [6], dual-hop [9] and direct transmissions.

SNR. In HD [6], each block is divided into 2 phases: broadcast and coherent relaying phases, which reduces the achievable rate and the outage performance. However, in SWD, $\mathcal{D}$ treats the new information of block m as noise which creates the interference shown in (7) and (9). Hence, results imply that at low SNR, FD with SWD outperforms HD DF because of the rate loss that stems from dividing each block into 2 phases. However, at high SNR, HD outperforms FD since the interference in SWD becomes significant.

VII. Conclusion

We have analytically derived the decoding delay and outage probabilities over Rayleigh fading channels for FD DF relaying with backward and joint and sequential sliding window decoding techniques. Assuming full CSI at the receivers and limited CSI at the transmitters, we take into account outages at both the relay and destination and the channel variation over different blocks obtained with sliding window decoding. Analysis and numerical results show that joint sliding window decoding is a preferred choice for delay sensitive applications since it has much shorter decoding delay than backward decoding and better outage performance than sequential decoding and HD transmission over a wide range of SNR. However, different schemes are optimal for different applications depending on their reliability and latency requirements.

Appendix A: Proof of Lemma 1

Let $\gamma_1 = g_{ds,k}$, $\gamma_2 = g_{dr,k}$, $\gamma_3 = g_{rs,k}$, $\beta_1 = g_{ds,m}$ and $\beta_2 = g_{dr,m}$ be the channel amplitudes in blocks k and m. Using the Rayleigh distribution, we can obtain the outage probability at the destination ($\mathbb{P}_d^{JSWD}$) in (17) as follows:

$$\begin{aligned}\mathbb{P}_d^{JSW} &= \mathsf{P}\big[R > C_3,\ R \le C_1,\ g_{rs,i} > g_{ds,i}\big], \qquad (22)\\ &= \mathsf{P}\big[\eta_0 < \gamma_1 \le \eta_1,\ \gamma_3 > \eta_1,\ \beta_1 \le \zeta_2,\ \beta_2 \le \zeta_1\big],\\ &= \int_{\eta_0}^{\eta_1}\int_{\eta_1}^{\infty}\int_{0}^{\zeta_2}\int_{0}^{\zeta_1} f_0(\gamma_1,\gamma_3,\beta_1,\beta_2)d\gamma_1 d\gamma_3 d\beta_1 d\beta_2\end{aligned}$$

where η_0, η_1, ζ_1, and ζ_2 are given in (18) while

$$f_0(\gamma_1,\gamma_3,\beta_1,\beta_2) = \frac{16\gamma_1\gamma_3\beta_1\beta_2}{\mu_{ds}^2\mu_{rs}\mu_{dr}}e^{-\left(\frac{\gamma_1^2}{\mu_{ds}}+\frac{\gamma_3^2}{\mu_{rs}}+\frac{\beta_1^2}{\mu_{ds}}+\frac{\beta_2^2}{\mu_{dr}}\right)}. \quad (23)$$

After integrating (22), we obtain (17) in Lemma 1.

The lower bound on γ_3 is obtained from (22) as follows:

$$R \le \log\left(1+\gamma_3^2\rho_n\right), \Leftrightarrow \gamma_3 > \eta_1.$$

From the first constraint in (22), we obtain upper bounds on β_1 and β_2 as follows:

$$(1+\gamma_1^2\rho_n)\left(1+\frac{\left(\beta_1\sqrt{\rho_o}+\beta_2\sqrt{P_r}\right)^2}{1+\beta_1^2\rho_n}\right) \le 2^R \Leftrightarrow \beta_2 \le \zeta_1. \quad (24)$$

Similar to β_2, $\beta_1 \in [0,\infty)$. Hence, ζ_1 should also be non-negative ($\zeta_1 > 0$) which adds more constraints on γ_1:

$$\begin{aligned}&\zeta_1 \ge 0,\ \Leftrightarrow\ f(\beta_1) \le 0, \qquad (25)\\ &f(\beta_1) = \beta_1^2\left(P_s(1+\gamma_1^2\rho_n) - 2^R\rho_n\right) - \left(2^R - (1+\gamma_1^2\rho_n)\right)\end{aligned}$$

Then, formula (25) is non-negative in the following cases:

- If $P_s/\rho_n < 2^R$, then $0 \le \beta_1 < \infty$ when $0 < \gamma_1 < \eta_0$ while $0 \le \beta_1 < \zeta_2$ when $\eta_0 < \gamma_1 < \eta_1$.
- If $P_s/\rho_n > 2^R$, then $0 \le \beta_1 < \zeta_2$ when $0 < \gamma_1 < \eta_1$.

From the above cases, we obtain formula (17) in Lemma 1.

References

[1] J. Korhonen, *Introduction to 4G mobile communications*. Artech House, 2014.

[2] Y. Niu, Y. Li, D. Jin, L. Su, and A. V. Vasilakos, "A survey of millimeter wave (mmwave) communications for 5G: opportunities and challenges," *available online: http://arxiv.org/abs/1502.07228.*, Feb. 2015.

[3] Ericsson, "5G radio access: capabilities and technologies," *Ericsson White Paper, Uen 284 23-3204 Rev C, Stockholm, Sweden*, Apr. 2016.

[4] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, pp. 572–584, Sep. 1979.

[5] A. El Gamal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. Inf. Theory.*, vol. 52, no. 4, pp. 1545–1561, Apr. 2006.

[6] A. Abu Al Haija and M. Vu, "Outage analysis for coherent decode-forward relaying over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1162–1177, Apr. 2015.

[7] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[8] L. Pinals, A. Abu Al Haija, and M. Vu, "Link regime and power savings of decode-forward relaying in fading channels," *IEEE Trans. Commun.*, vol. 64, no. 3, pp. 931–946, Mar. 2016.

[9] M. O. Hasna and M. S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh fading channels," in *IEEE VTC*, Apr. 2003.

[10] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[11] A. Sabharwal, P. Schniter, D. Guo, D. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: challenges and opportunities," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1637–1652, Sept. 2014.

[12] Y.-S. Choi and H. Shirani-Mehr, "Simultaneous transmission and reception: algorithm, design and system level performance," *IEEE Trans. Wireless. Commun.*, vol. 12, no. 12, pp. 5992–6010, Dec. 2013.

[13] A. Altieri, L. Rey Vega, P. Piantanida, and C. G. Galarza, "On the outage probability of the full-duplex interference-limited relay channel," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1765–1777, Sept. 2014.

[14] R. El Gamal and Y.-H. Kim, *Network Information Theory*, 1st ed. Cambridge University Press, 2011.

[15] R. Mudumbai, D. Brown, U. Madhow, and H. Poor, "Distributed transmit beamforming: challenges and recent progress," *IEEE Commun. Mag.*, vol. 47, no. 2, pp. 102–110, Feb. 2009.

[16] D. Bharadia, E. Mcmilin, and S. Katti, "Full duplex radios," *ACM, SIGCOMM*, vol. 43, no. 4, pp. 375–386, Aug. 2013.

[17] K. Alexandris, A. Balatsoukas-Stimming, and A. Burg, "Measurement-based characterization of residual self-interference on a full-duplex MIMO testbed," in *IEEE SAM Signal Processing Workshop*, June 2014.

[18] 3GPP, "LTE physical layer framework for performance verification," *TS R1-070674, 3rd Generatin Partnership Project (3GPP)*, Feb. 2007.

[19] A. Abu Al Haija, L. Pinals, and M. Vu, "Outage analysis and power savings for independent and coherent decode-forward relaying," in *IEEE GLOBECOM*, Dec. 2015.