# Performance Analysis of SDMA with Inter-tier Interference Nulling in HetNets

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***Abstract*—The downlink performance of two-tier (macro/pico) multi-antenna cellular heterogeneous networks (HetNets) employing space division multiple access (SDMA) technique is analyzed in this paper. The number of users simultaneously served with SDMA by each BS in a resource block depends on user distribution, unlike previous studies which assume the number to be any arbitrary value. By exploiting the feasibility of deploying larger number of antennas at macro BS, we propose to utilize the excess spatial degrees of freedom for interference nulling to pico users from their corresponding nearest (dominant) macro BSs. Biased-nearest-distance based user association scheme is proposed as those introduced in previous studies are unsuitable for analyzing the proposed multi-antenna scheme. Coverage probability and average data rate of a typical user are then evaluated. Our results demonstrate that the proposed interference nulling scheme has strong potential to improve performance. However, the system parameters such as association bias, and number of dedicated antennas at each macro BS for serving its own users must be carefully tuned.**

## I. Introduction

Network densification and multi-antenna techniques are well-known for their tremendous potential to increase spectral efficiency of wireless systems. The cost effective approach to network densification is co-channel deployment of a diverse set of low power base-stations (BSs) within the areas covered by macro cellular infrastructure, thus forming a heterogeneous network (HetNet) [1]. BSs equipped with multiple antennas can utilize the additional spatial degrees of freedom for multiplexing, signal power boosting through beamforming, interference suppression or a combination of these. While multi-antenna techniques in general have been very well studied, attempts to study their coexistence with HetNets has started only recently [2]–[5], and choosing the right technique is not quite understood yet.

In this paper, we develop an analytical framework to evaluate the performance of space division multiple access (SDMA) in cellular HetNets. In [3], the performance of zero-forcing (ZF) precoding based closed-loop SDMA technique is compared against single-user-beamforming (SU-BF) and single-antenna communication for multi-tier HetNets through stochastic ordering approach. However, each cell of a tier is assumed to be serving the same number of users with SDMA, say $L$, and it can be any arbitrary integer in the interval $[1, K_i]$, where $K_i$ is the number of antennas in a BS of $i$th tier. This assumption is not suitable for cellular networks since the number of users in a cell, which depends on user distribution, is different from another cell, in general. Similar analysis with emphasis on user association rules is done in [4]. An open-loop SDMA with each antenna serving an independent data stream to its user is analyzed in [5] for two-tier cellular HetNets under the limiting requirement that the number of users in each cell to be at least equal to the number of antennas. In this paper, rather than fixing the number of users served with SDMA to an arbitrary value, we only set the limit on maximum number of users served, say $L_{\max}$. If the total number of users in a cell is below the limit, all the users are served; otherwise only $L_{\max}$ randomly chosen users are served.

One of the key challenges in cellular HetNets is inter-cell interference coordination (ICIC). Due to the huge disparities in transmit power between macro and small-cell nodes such as picos and femtos, ICIC between macro and pico/femto tier is very important as the performance of the small-cell users on the cell edge could be severely degraded, otherwise. While there can be a number of approaches to ICIC such as resource allocation and user scheduling [6], [7], we analyze precoding based *interference nulling* method in this paper. Due to the physical size of macro BSs, it is more feasible to have larger number of antennas at macro BS than at low-power BSs. Thus, the idea is to utilize the extra degrees of freedom at macro BSs to suppress the interference from macro tier to small-cell users through nulling. While SU-BF with interference nulling in single-tier cellular networks is studied in [8], [9], to the best of our knowledge, this is the first work to study user-distribution dependent SDMA scheme with interference nulling in cellular HetNets. The user association rule proposed in [4] is based on the assumption that a deterministic fixed number of users are served with SDMA in each cell. Hence, it does not apply to our proposed SDMA scheme, and a different association rule is introduced in this paper.

***Notations:*** We now present integer-partition notations used in this paper. The set of all possible partitions of a positive integer $n$ is represented by $\Omega_n$. For example: $\Omega_3 = \{\{3\}, \{2, 1\}, \{1, 1, 1\}\}$, where each partition represents a way to express $n = 3$ as a sum of positive integers. The number of all possible partitions of $n$ is represented by $\mathcal{P}(n)$, thus, $\mathcal{P}(3) = 3$. For the $i$th partition $p_i^n$, $\omega_i^n$ represents the number of elements, $\mu_{ij}^n$ is the number of positive integer $j \in \{1, 2, \ldots, n\}$ and $a_{ik}^n$ is the $k$th element ($k \in \{1, 2, \ldots, \omega_i^n\}$).

We have, $\sum_{j=1}^{n} j\mu_{ij}^n = n$ and $\sum_{j=1}^{n} \mu_{ij}^n = \omega_i^n$. Example: for $p_2^3 = \{2,1\}$ in $\Omega_3$, $\omega_2^3 = 2$, $\mu_{21}^3 = 1$, $\mu_{22}^3 = 1$, $\mu_{23}^3 = 0$, $a_{21}^3 = 2$, $a_{22}^3 = 1$.

## II. System Model

We consider the downlink of a two-tier multi-antenna HetNet comprising macro and pico BSs spatially distributed in $\mathbb{R}^2$ plane as independent homogeneous PPPs $\mathbf{\Phi}_m$ with density $\lambda_m$ and $\mathbf{\Phi}_p$ with density $\lambda_p$, respectively. The macro BSs are equipped with $K_m$ transmit antennas and pico BSs with $K_p$ antennas. Similarly, users are assumed to be distributed according to an independent PPP $\mathbf{\Phi}_u$ with density $\lambda_u$, and each has a single receive antenna. The two network tiers share the same spectrum with universal frequency reuse.

The transmission scheme is SDMA with ZF precoding applied at each BS to serve multiple users simultaneously in each resource block (RB). We assume only one RB per time slot. As BSs and users are independently distributed in $\mathbb{R}^2$ plane, the number of users in different cells are different. Thus, in our proposed SDMA scheme, a typical active macro cell with $N_m \geq 1$ users serves $M_m = \min(N_m, L_{\max}^M)$ users simultaneously in a given time slot, where $L_{\max}^M$ is the maximum number of users it can serve. When $N_m > L_{\max}^M$, the BS choses $L_{\max}^M$ users for service randomly, else, all $N_m$ users are served. Similarly, $M_p = \min(N_p, L_{\max}^P)$ users are simultaneously served by a typical active pico cell in a given time slot, which has $N_p \geq 1$ users and $L_{\max}^P$ is the maximum number it can serve. The macro and pico BSs transmit to each of their users with power $P_m$ and $P_p$, respectively.

### A. User Association

According to the user association rule introduced in [4], a typical user at the origin is associated with the nearest pico BS if $P_p\sqrt{\Delta_p\tau_p}X_p^{-\alpha} \geq P_m\sqrt{\Delta_m\tau_m}X_m^{-\alpha}$, otherwise associated with the nearest macro BS, where $X_m = \min_{x_m \in \mathbf{\Phi}_m} \|x_m\|$ and $X_p = \min_{x_m \in \mathbf{\Phi}_p} \|x_p\|$ are the distances from the origin to the nearest macro and pico BSs, respectively. If associated with the macro tier, $\Delta_m$ is the average desired channel gain from the nearest macro BS and $\tau_m$ is the average interference channel gain from the nearest pico BS. Similarly, $\Delta_p$ and $\tau_p$ are the corresponding parameters, if associated with the pico tier. The average gains of the desired and interference channels depend on the number of users served with SDMA. This association rule is thus not suitable for our proposed SDMA scheme, where the number of users served with SDMA in each cell is a function of number of users in that cell. The number of users, on the other hand, is determined by the association rule. The above rule however can be equivalently expressed as: a user is associated with the pico tier only if

$$X_m \geq \left(\frac{P_m}{P_p}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\eta}\right)^{\frac{1}{\alpha}} X_p \Rightarrow X_m \geq \rho X_p, \tag{1}$$

where $\rho = (\frac{P_m}{P_p}\frac{1}{\eta})^{\frac{1}{\alpha}}$, $\eta = \sqrt{\frac{\Delta_p\tau_p}{\Delta_m\tau_m}}$. This rule can be perceived as biased nearest distance association, where the biasing is due to the difference in transmit power and average channel gains between macro and pico tier, and for load balancing as well. We will investigate the optimal value of $\eta$ for average data rate in Section V, which in turn determine the optimal $\rho$.

Using the fact that $X_m$ and $X_p$ are Rayleigh RVs with mean $1/(2\sqrt{\lambda_m})$ and $1/(2\sqrt{\lambda_p})$, respectively [10], the probability that a typical user at the origin is associated with pico tier is

$$A_p = \mathbb{P}(X_m \geq \rho X_p) = \frac{\lambda_p}{\lambda_p + \lambda_m\rho^2}, \tag{2}$$

and the probability that it is associated with macro tier is $A_m = 1 - A_p$.

These tier association probabilities are also valid for any randomly selected user. As per the given association scheme, the total set of users in the network, $\mathbf{\Phi}_u$ can be divided into two disjoint subsets: $\mathbf{\Phi}_u^m$ and $\mathbf{\Phi}_u^p$, the set of macro and pico users, respectively. $A_m$ and $A_p$ can be interpreted as average number of users belonging to $\mathbf{\Phi}_u^m$ and $\mathbf{\Phi}_u^p$, respectively. As we are interested in the number of users in a typical cell, rather than the actual locations of users, $\mathbf{\Phi}_u^m$ and $\mathbf{\Phi}_u^p$ can be equivalently modeled as independent PPPs with density $A_m\lambda_m$ and $A_p\lambda_p$ respectively. Since each macro user is always associated with the nearest macro BS and each pico-user with the nearest pico BS, the network can be viewed as a superposition of two independent Voronoi tessellations of macro and pico tier. The distribution of the number of users associated with a typical macro and pico BS is derived next.

**Lemma 1.** *Let the number of users in a randomly chosen macro and pico cell be denoted by $U_m$ and $U_p$, respectively. Their probability mass functions (PMFs) are given by*

$$\mathbb{P}(U_l = n) = \frac{3.5^{3.5}\Gamma(3.5+n)(A_l\lambda_u/\lambda_l)^n}{\Gamma(3.5)n!(A_l\lambda_u/\lambda_l + 3.5)^{n+3.5}}, n \geq 0, \quad \forall l \in \{m,p\}, \tag{3}$$

*Proof:* The proof is similar to that of Lemma 2 in [7]. ∎

A BS without any user associated does not transmit at all and is inactive. Given in the following corollary are the PMFs of the number of users in a typical cell of macro and pico tier, under the condition that it is active.

**Corollary 1.** *The PMFs of the number of users in a randomly chosen active cell of macro and pico tier are given by*

$$\mathbb{P}(N_l = n) = \frac{\mathbb{P}(U_l = n)\boldsymbol{1}(n \geq 1)}{p_l}, \forall l \in \{m,p\}, \tag{4}$$

*where $p_m$ and $p_p$ are the probabilities that a typical BS of macro and pico tier, respectively, is active, and are given by*

$$p_l = 1 - \mathbb{P}(U_l = 0) = 1 - \left(1 + 3.5^{-1}\frac{A_l\lambda_u}{\lambda_l}\right)^{-3.5}, \quad \forall l \in \{m,p\}. \tag{5}$$

Let the sets of active macro and active pico BSs be denoted by $\mathbf{\Psi}_m$ and $\mathbf{\Psi}_p$ respectively. $\mathbf{\Psi}_m$ and $\mathbf{\Psi}_p$ are thinned versions of the original PPPs $\mathbf{\Phi}_m$ and $\mathbf{\Phi}_p$, respectively, and hence are independent PPPs with densities $p_m\lambda_m$ and $p_p\lambda_p$, respectively.

By using corollary 1, the PMFs of the number of users simultaneously served by a typical active macro/pico BS in a given time slot for $L^l_{\max} > 1$ can be obtained as

$$\mathbb{P}(M_l = n) = \begin{cases} \mathbb{P}(N_l = n), & 1 \le n < L^l_{\max} \\ 1 - \sum_{k=1}^{L^l_{\max}-1} \mathbb{P}(N_l = k), & n = L^l_{\max}, \end{cases} \quad \forall l \in \{m, p\}. \quad (6)$$

For $L^l_{\max} = 1$, $\mathbb{P}(M_l = 1) = 1$, $\forall l \in \{m, p\}$.

### *B. Interference Nulling*

We assume that $K_m$ is typically much larger than $K_p$. The additional spatial degrees of freedom that macro BSs have can be utilized to suppress the strong interference they impose on pico users through nulling strategy. To accomplish this, we propose that each served pico user requests its nearest active macro BS to perform interference nulling. As interference nulling costs macro BSs their available degrees of freedom for their own users, we assume that each macro BS can handle at most $K_m - T_{\min}$ requests only, which ensures that a macro BS has at least $T_{\min} \ge L^M_{\max}$ antennas dedicated for serving its own users. Hence, if $Q_m$ requests are received by a typical active macro BS, it will perform interference nulling to $B = \min(Q_m, K_m - T_{\min})$ pico users. For $Q_m > (K_m - T_{\min})$, the BS will randomly choose $K_m - T_{\min}$ pico users. Hence, not all interference nulling requests are satisfied.

The number of interference nulling requests $Q_m$ received by a typical active macro BS is equal to the number of served pico users within a typical Voronoi cell $\Upsilon$ of the tessellation formed by $\mathbf{\Psi}_m$. Although, the number of pico users served by a typical active pico BS cannot exceed $L^p_{\max}$, $Q_m$ is unbounded because the number of active pico BSs within $\Upsilon$ is Poisson distributed with mean $p_p\lambda_p/(p_m\lambda_m)$. In order to derive the PMF of $Q_m$, we first derive $\mathbb{E}[M_p] = A_p\vartheta_p\lambda_u/(p_p\lambda_p)$, where

$$\vartheta_p = \frac{L^p_{\max} p_p \lambda_p}{A_p \lambda_u} - \frac{3.5^{3.5}}{\Gamma(3.5)} \sum_{k=1}^{L^p_{\max}-1} \left[ \frac{\Gamma(3.5+n)}{n!} \times \frac{(A_p\lambda_u/\lambda_p)^{n-1}(L^p_{\max} - k)}{(A_p\lambda_u/\lambda_p + 3.5)^{n+3.5}} \right], \quad (7)$$

then approximate the set of pico users requesting interference coordination $\mathbf{\Psi}^p_u$ as a PPP with density $A_p\vartheta_p\lambda_u$ so that $\mathbb{E}[Q_m] = A_p\vartheta_p\lambda_u/(p_m\lambda_m)$. By using this approximation, the PMF of $Q_m$ is derived as follows.

**Lemma 2.** *The PMF of the number of interference nulling requests received by a typical active macro BS is given by*

$$\mathbb{P}(Q_m = n) = \frac{3.5^{3.5}\Gamma(3.5+n)\left(\frac{A_p\vartheta_p\lambda_u}{p_m\lambda_m}\right)^n}{\Gamma(3.5)n!\left(\frac{A_p\vartheta_p\lambda_u}{p_m\lambda_m} + 3.5\right)^{n+3.5}}, n \ge 0. \quad (8)$$

### *C. Channel Model and Precoding Matrices*

For independent unit-mean Rayleigh multipath fading between any transmit-receive antenna pair and standard power law path-loss with exponent $\alpha$, the received signal $z_m$ at a typical user $u$ located at the origin if $u \in \mathbf{\Phi}^m_u$ is given by

$$z_m = \sqrt{P_m} D_m^{-\frac{\alpha}{2}} \mathbf{h}^*_{b_m,1} \mathbf{W}_{b_m} \mathbf{s}_{b_m} + \sum_{q\in\{m,p\}} \sqrt{P_q} \sum_{x_q \in \mathbf{\Psi}_q \setminus b_m} ||x_q||^{-\frac{\alpha}{2}} \mathbf{g}^*_{x_q,1} \mathbf{W}_{x_q} \mathbf{s}_{x_q} + n_m, \quad (9)$$

where $b_m$ is the serving BS at a distance $D_m$, which is serving $M'_m$ other users simultaneously, $\mathbf{h}_{b_m,1} \sim \mathcal{CN}(\mathbf{0}_{K_m\times 1}, \mathbf{I}_{K_m})$ and $\mathbf{g}_{x_q} \sim \mathcal{CN}(\mathbf{0}_{K_q\times 1}, \mathbf{I}_{K_q})$ are the desired and interference channel vectors from the tagged BS $b_m$ and the interfering BS at $x_q$, respectively, $n_m \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN), $\mathbf{s}_{b_m} = [s_{b_m,i}]_{1\le i\le M'_m+1} \in \mathbb{C}^{(M'_m+1)\times 1}$ is the signal vector transmitted from $b_m$ to its $M'_m + 1$ served users with the symbol $s_{b_m,1}$ intended for $u$, and $\mathbf{W}_{b_m} = [\mathbf{w}_{b_m,i}]_{1\le i\le(M'_m+1)} \in \mathbb{C}^{K_m\times(M'_m+1)}$ is the corresponding precoding matrix.

Let the channel vectors from the tagged BS $b_m$ to its $M'_m$ users other than $u$ be represented by $[\,\mathbf{h}_{b_m,i}\,]_{2\le i\le M'_m+1}$, and the interference channel vector from the tagged BS to $B = \min(Q_m, K_m - T_{\min})$ pico users chosen for interference nulling by $\mathbf{F} = [\mathbf{f}_i]_{1\le i\le B}$. Under perfect channel state information (CSI) assumption, ZF precoding vectors $\mathbf{W}_{b_m} = [\mathbf{w}_{b_m,i}]_{1\le i\le(M'_m+1)}$ are designed such that $|\mathbf{h}^*_{b_m,j}\mathbf{w}_{b_m,j}|^2$ is maximized for each $j = 1, 2, \ldots, M'_m + 1$, while satisfying the orthogonality conditions $\mathbf{h}^*_{b_m,j}\mathbf{w}_{b_m,i} = 0$ for $\forall i \ne j$ and $\mathbf{f}^*_i\mathbf{w}_{b_m,j} = 0, \forall i = 1, 2, \ldots, B, \forall j = 1, 2, \ldots, M'_m + 1$. This can be achieved by choosing $\mathbf{w}_{b_m,i}$ in the direction of the projection of $\mathbf{h}_{b_m,i}$ on $\text{Null}\big([\,\mathbf{h}_{b_m,j}\,]_{1\le j\le(M'_m+1),j\ne i}, [\,\mathbf{f}_i\,]_{1\le i\le B}\big)$. The nullspace is $K_m - M'_m - B$ dimensional. Under Rayleigh fading, the desired channel power $\beta_{b_m} = |\mathbf{h}^*_{b_m,1}\mathbf{w}_{b_m,1}|^2 \sim \text{Gamma}(\Delta_m, 1)$, where $\Delta_m = K_m - M'_m - B$ [11]. Given that an interfering macro BS at $x_m$ is serving $M_m$ users simultaneously, the interference channel power $\zeta_{x_m} = ||\mathbf{g}^*_{x_m,1}\mathbf{W}_{x_m}||^2 \sim \text{Gamma}(M_m, 1)$ [3].

One possible choice of $\mathbf{W}_{b_m} = [\mathbf{w}_{b_m,i}]_{1\le i\le(M'_m+1)}$ is the normalized columns of the pseudo inverse of $\tilde{\mathbf{H}}^*_{b_m}$, i.e., $\mathbf{W}_{b_m} = \tilde{\mathbf{H}}_{b_m}(\tilde{\mathbf{H}}^*_{b_m}\tilde{\mathbf{H}}_{b_m})^{-1}$, where $\tilde{\mathbf{H}}_{b_m} = [\tilde{\mathbf{h}}_{b_m,i}]_{1\le i\le(M'_m+1)} \in \mathbb{C}^{K_m\times(M'_m+1)}$, $\tilde{\mathbf{h}}_{b_m,i} = (\mathbf{I}_{K_m} - \mathbf{F}(\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^*)\mathbf{h}_{b_m,i}$ being the projection of $\mathbf{h}_{b_m,i}$ on the nullspace of $\mathbf{F} = [\,\mathbf{f}_i\,]_{1\le i\le B}$.

Similarly, the received signal $z_p$ at $u$ when $u \in \mathbf{\Phi}^p_u$ is

$$z_p = \sqrt{P_p} D_p^{-\frac{\alpha}{2}} \mathbf{h}^*_{b_p,1} \mathbf{W}_{b_p} \mathbf{s}_{b_p} + \xi + \sum_{q\in\{m,p\}} \sqrt{P_q} \sum_{x_q\in\mathbf{\Psi}_q\setminus\{v_m,b_p\}} ||x_q||^{-\frac{\alpha}{2}} \mathbf{g}^*_{x_q,1} \mathbf{W}_{x_q} \mathbf{s}_{x_q} + n_p, \quad (10)$$

where

$$\xi = \begin{cases} 0, & \text{if } u \in \chi \\ \sqrt{P_m} V_m^{-\frac{\alpha}{2}} \mathbf{g}^*_{v_m,1} \mathbf{W}_{v_m} \mathbf{s}_{v_m}, & \text{if } u \notin \chi, \end{cases} \quad (11)$$

$b_p$ is the serving BS at a distance $D_p$, which is serving $M'_p$ other users simultaneously, $n_p \sim \mathcal{CN}(0, \sigma^2)$ is AWGN, $v_m$ is the nearest active macro BS to $u$ at a distance $V_m$, $\chi$ is the set of pico users to which interference from their corresponding nearest active macro BSs are nulled. The ZF precoding vectors $\mathbf{W}_{b_p} = [\mathbf{w}_{b_p,i}]_{1\le i\le(M'_p+1)}$ are given by the normalized columns of $\mathbf{H}_{b_p}(\mathbf{H}_{b_p}^*\mathbf{H}_{b_m})^{-1}$, where $\mathbf{H}_{b_p} = [\mathbf{h}_{b_p,i}]_{1\leq i\leq (M_p'+1)} \in \mathbb{C}^{K_p\times(M_p'+1)}$ is the channel matrix from the tagged BS $b_p$ to its $M_p'+1$ served pico users. The desired channel power $\beta_{b_p} = |\mathbf{h}_{b_p,1}^*\mathbf{w}_{b_p,1}|^2 \sim \text{Gamma}(\Delta_p, 1)$, where $\Delta_p = K_m - M_p'$ and the interference channel power $\zeta_{x_p} = ||\mathbf{g}_{x_p,1}^*\mathbf{W}_{x_p}||^2 \sim \text{Gamma}(M_p, 1)$.

The signal-to-interference-and-noise ratio (SINR) of a typical user $u$ when it belongs to $\mathbf{\Phi}_u^l$ can now be expressed as

$$\text{SINR}_l = \frac{P_l\beta_{b_l}D_l^{-\alpha}}{I_{b_l,m}+I_{b_l,p}+\sigma^2}, \forall l \in \{m,p\}, \tag{12}$$

where

$$I_{b_m,m} = P_m\sum_{x_m\in\mathbf{\Psi}_m\backslash b_m}\zeta_{x_m}||x_m||^{-\alpha},\ I_{b_m,p} = P_p\sum_{x_p\in\mathbf{\Psi}_p}\zeta_{x_p}||x_p||^{-\alpha},$$
$$I_{b_p,m} = \begin{cases} P_m\sum\limits_{x_m\in\mathbf{\Psi}_m\backslash v_m}\zeta_{x_m}||x_m||^{-\alpha} & \text{if } u\in\chi \\ P_m\sum\limits_{x_m\in\mathbf{\Psi}_m}\zeta_{x_m}||x_m||^{-\alpha} & \text{if } u\notin\chi, \end{cases}$$
$$I_{b_p,p} = P_p\sum_{x_p\in\mathbf{\Psi}_p\backslash b_p}\zeta_{x_p}||x_p||^{-\alpha}. \tag{13}$$

To compute the desired performance metrics in the following sections, we require $\mathbb{P}(u\in\chi)$ which is derived next.

**Lemma 3.** *The probability that a typical user $u \in \mathbf{\Phi}_u^p$ requesting interference nulling to its nearest active macro BS gets its request fulfilled, i.e., $\varphi = \mathbb{P}(u\in\chi)$ is given by*

$$\varphi = \frac{(K_m - T_{min})p_m\lambda_m}{A_p\vartheta_p\lambda_u}\left(1-\left(1+3.5^{-1}\frac{A_p\vartheta_p\lambda_u}{p_m\lambda_m}\right)^{-3.5}\right)$$
$$-\frac{3.5^{3.5}}{\Gamma(3.5)}\sum_{i=1}^{K_m-T_{min}}\frac{\Gamma(3.5+i)\left(\frac{A_p\vartheta_p\lambda_u}{p_m\lambda_m}\right)^{i-1}(K_m-T_{min}-i)}{i!\left(\frac{A_p\vartheta_p\lambda_u}{p_m\lambda_m}+3.5\right)^{i+3.5}}. \tag{14}$$

*Proof:* Let the BS $v_m$ receives $Q_m'$ other requests along with the one from $u$. Then, conditioned on $Q_m'$, $\varphi = 1$ if $Q_m'+1 \leq K_m - T_{\min}$, otherwise, $\varphi = (K_m - T_{\min})/(Q_m'+1)$. By using the fact that the conditional PDF $f_Y'(y)$ of the area of a Voronoi cell given that a typical user belong to it is equal to $cyf_Y(y)$, where $f_Y(y)$ is the unconditional PDF and $c$ is a constant such that $\int_o^\infty f_Y'(y)dy = 1$ [6], the PMF of $Q_m'$ can be derived as $\mathbb{P}(Q_m' = n) = (n+1)\mathbb{P}(Q_m = n+1)/\mathbb{E}[Q_m]$, $n \geq 0$. Eqn. (14) then follows easily. ∎

## III. Coverage Probability

A user is in coverage if its SINR is greater than a predefined threshold $\gamma$. The coverage probability of a typical user $u$ at the origin is given by

$$\text{P}(\gamma) = \text{P}_m(\gamma)A_m + \text{P}_p(\gamma)A_p. \tag{15}$$

We first evaluate $\text{P}_p(\gamma)$, the coverage probability of a typical pico user as

$$\text{P}_p(\gamma) = \sum_{k=0}^{L_{\max}^p-1}\mathbb{P}(M_p'=k)\underbrace{\mathbb{P}(\text{SINR}_p > \gamma|u\in\mathbf{\Phi}_u^p, M_p'=k)}_{\text{P}_{p|M_p'}(\gamma|k)}.$$

**Theorem 1.** *The coverage probability of a typical pico user $u$ in interference limited scenario, i.e., $\sigma^2 = 0$ is given by*

$$\text{P}_p(\gamma) = \text{T}_1(\gamma)\varphi + \text{T}_2(\gamma)(1-\varphi), \tag{16}$$

*where $\text{T}_1(\gamma) = \mathbb{P}(SINR_p > \gamma|u\in\mathbf{\Phi}_u^p, u\in\chi)$ and $\text{T}_2(\gamma) = \mathbb{P}(SINR_p > \gamma|u\in\mathbf{\Phi}_u^p, u\notin\chi)$ are given by*

$$\text{T}_1(\gamma) = 2p_m\lambda_m\frac{\lambda_p}{A_p}\int_{\theta=0}^{\frac{1}{\rho}}\sum_{k=0}^{L_{max}^p-1}\mathbb{P}(M_p'=k)\sum_{l=0}^{K_p-k-1}\frac{\gamma^l}{l!}\theta^{\alpha l+1}$$
$$\sum_{o=1}^{\mathcal{P}(l)}c_o^l\prod_{q=1}^{l}\Big(p_m\lambda_m\delta^q\Xi_q^m(\delta,\theta,\gamma)+\frac{p_p\lambda_p}{\theta^{\alpha q-2}}\Xi_q^p(1,1,\gamma)\Big)^{\mu_{oq}^l}$$
$$(-1)^{\omega_o^l}\Gamma(\omega_o^l+2)\Big(p_m\lambda_m\Xi_0^m(\delta,\theta,\gamma)+p_p\lambda_p\theta^2\Xi_0^p(1,1,\gamma)$$
$$+(1-p_m)\lambda_m\rho^2\theta^2+(1-p_p)\lambda_p\theta^2\Big)^{-(\omega_o^l+2)}d\theta, \tag{17}$$

$$\text{T}_2(\gamma) = \frac{\lambda_p}{A_p}\sum_{k=0}^{L_{max}^p-1}\mathbb{P}(M_p'=k)\sum_{l=0}^{K_p-k-1}\frac{\gamma^l}{l!}\sum_{o=1}^{P(l)}c_o^l$$
$$\prod_{q=1}^{l}\left(\frac{p_m\lambda_m\delta^q}{\rho^{\alpha q-2}}\Xi_q^m\left(\delta,\frac{1}{\rho},\gamma\right)+p_p\lambda_p\Xi_q^p(1,1,\gamma)\right)^{\mu_{oq}^l}$$
$$(-1)^{\omega_o^l}\Gamma(\omega_o^l+1)\left(p_m\lambda_m\rho^2\Xi_0^m\left(\delta,\frac{1}{\rho},\gamma\right)+p_p\lambda_p\Xi_0^p(1,1,\gamma)\right.$$
$$\left.+(1-p_m)\lambda_m\rho^2+(1-p_p)\lambda_p\right)^{-(\omega_o^l+1)}, \tag{18}$$

*where $\delta = \frac{P_m}{P_p}$, $c_o^l = \dfrac{l!}{\prod_{k=1}^{\omega_o^l}a_{ok}^l!\prod_{q=1}^{l}\mu_{oq}^l!}$, and*

$$\Xi_q^l(\varsigma,\kappa,\varepsilon) = \sum_{i=1}^{L_{max}^l}\Bigg[{}_2F_1\left(i+q,-\frac{2}{\alpha}+q,\frac{\alpha-2}{\alpha}+q,-\varsigma\kappa^\alpha\varepsilon\right)$$
$$\times\frac{(i)_q(-\frac{2}{\alpha})_q}{(\frac{\alpha-2}{\alpha})_q}\mathbb{P}(M_l=i)\Bigg], \forall l\in\{m,p\}, \tag{19}$$

*Proof.* The proof is given in the appendix. □

**Remark 1.** *The number of other users served by the BS which is serving the typical user $u \in \mathbf{\Phi}_u^l$ is given by $M_l' = \min(U_l', L_{max}^l - 1)$, where $U_l'$ is the number of other users in the Voronoi cell to which the user $u$ belongs, whose PMF is given by $\mathbb{P}(U_l' = n) = (n+1)\mathbb{P}(U_l = n+1)/\mathbb{E}[U_l]$. The PMF of $M_l'$ for $L_{max}^l > 1$ is thus given by*

$$\mathbb{P}(M_l'=n) = \begin{cases}\mathbb{P}(U_l'=n), & 0\leq n < L_{max}^l - 1 \\ 1-\sum\limits_{k=1}^{L_{max}^l-2}\mathbb{P}(U_l'=k), & n = L_{max}^l-1,\end{cases}$$
$$\forall l\in\{m,p\}. \tag{20}$$

*For $L_{max}^l = 1$, $\mathbb{P}(M_l'=0) = 1$, $\forall l\in\{m,p\}$.*

The coverage probability of a typical macro user

$$\text{P}_m(\gamma) = \sum_{k=0}^{L_{\max}^m-1}\mathbb{P}(M_m'=k)\underbrace{\mathbb{P}(\text{SINR}_m > \gamma|u\in\mathbf{\Phi}_u^m, M_m'=k)}_{\text{P}_{m|M_m'}(\gamma|k)}$$

in interference limited scenario can be similarly derived, where

$$
\begin{aligned}
&\mathrm{P}_{m|M_m'}(\gamma|k) = \frac{\lambda_m}{A_m} \sum_{n=T_{\min}-k}^{K_m-k} \mathbb{P}(\Delta_m = n|M_m' = k) \sum_{l=0}^{n-1} \frac{\gamma^l}{l!} \\
&\sum_{o=1}^{\mathcal{P}(l)} c_o^l \prod_{q=1}^{l} \left(p_m \lambda_m \Xi_q^m\left(1,1,\gamma\right) + p_p \lambda_p \frac{\rho^{\alpha q-2}}{\delta^q} \Xi_q^p\left(\frac{1}{\delta},\rho,\gamma\right)\right)^{\mu_{oq}^l} \\
&(-1)^{\omega_o^l}\Gamma(\omega_o^l+1)\left(p_m\lambda_m\Xi_0^m\left(1,1,\gamma\right) + \frac{p_p\lambda_p}{\rho^2}\Xi_0^p\left(\frac{1}{\delta},\rho,\gamma\right)\right. \\
&\left. +(1-p_m)\lambda_m + (1-p_p)\frac{\lambda_p}{\rho^2}\right)^{-(\omega_o^l+1)}, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbb{P}(\Delta_m = n|M_m' = k) \\
&= \begin{cases} 1 - \displaystyle\sum_{v=0}^{K_m - T_{\min}-1} \mathbb{P}(Q_m = v), & n = T_{\min} - k \\ \mathbb{P}(Q_m = K_m - k - n), & T_{\min} - k + 1 \le n \le K_m - k. \end{cases} \qquad (22)
\end{aligned}
$$

## IV. Average Date Rate

With adaptive modulation so that the Shannon limit can be achieved and interference treated as noise, the data rate of a typical user is given by

$$
R = \sum_{l \in \{m,p\}} S_l W \log_2(1 + \mathrm{SINR}_l)\mathbf{1}(u \in \mathbf{\Phi}_u^l), \qquad (23)
$$

where $S_l = \min(L_{\max}^l/(U_l^{'}+1), 1)$ is the share of resources received by $u$ when $u \in \mathbf{\Phi}_u^l$ and $W$ is the total bandwidth. The average date rate $\bar{R}$ thus can be computed as follows. The proof is omitted for brevity and will be provided in a journal version of the paper.

$$
\bar{R} = A_m \bar{R}_m + A_p \bar{R}_p, \qquad (24)
$$

where $\bar{R}_m$ and $\bar{R}_p$ are average data rates of a typical macro and pico user, receptively, and are given by

$$
\begin{aligned}
\bar{R}_l = \frac{1}{\ln 2} \int_0^\infty \frac{1}{1+y} \Bigg[ & O_l \, \mathrm{P}_{l|M_l'}(y|L_{\max}^l - 1) \\
& + \sum_{k=0}^{L_{\max}^l - 2} \mathrm{P}_{l|M_l'}(y|k)\mathbb{P}(M_l' = k) \Bigg] dy, \; \forall l \in \{m,p\}, \qquad (25)
\end{aligned}
$$

where

$$
\begin{aligned}
O_l = & \frac{L_{\max}^l \lambda_l}{A_l \lambda_u}\left(1 - \left(1 + 3.5^{-1}A_l\lambda_u/\lambda_l\right)^{-3.5}\right) \\
& - \frac{3.5^{3.5}}{\Gamma(3.5)} \sum_{n=1}^{L_{\max}^l} \frac{\Gamma(3.5+n)\left(\frac{A_l\lambda_u}{\lambda_l}\right)^{n-1} L_{\max}^l}{n!(\frac{A_l\lambda_u}{\lambda_l} + 3.5)^{3.5+n}}. \qquad (26)
\end{aligned}
$$

## V. Numerical Results

In this section, we present some numerical analysis of our analytical results. Unless otherwise stated, we set $\delta = P_m/P_p = 100$, $\lambda_m = 1\mathrm{BS/Km}^2$, and $W = 1$ MHz.

In Figure 1, the average data rate (24) is validated via Monte Carlo simulations for different system configurations. We can observe close match between the analytical and simulation results. The average data rate decrease with increase in user density $\lambda_u$ due to the increase in interference and decrease in user's share of resources.

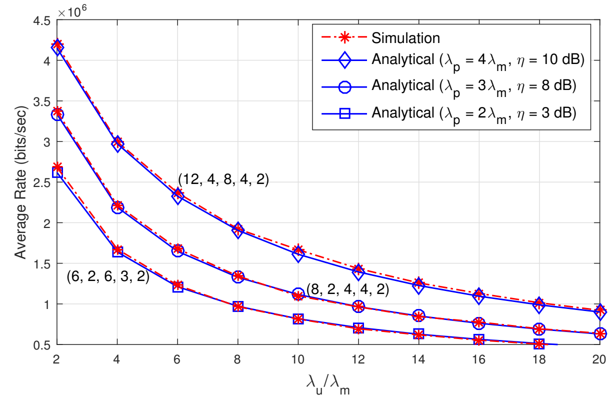


Fig. 1. Validation of the average user data rate (24) via Monte Carlo simulations for different values of $\lambda_p$, $\eta$ and $(K_m, L_{\max}^m, T_{\min}, K_p, L_{\max}^p)$.

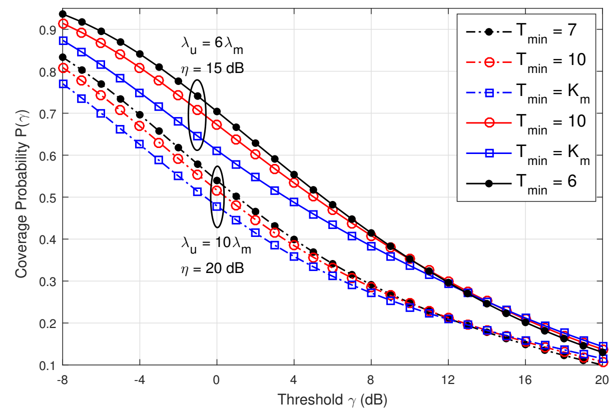


Fig. 2. Impact of $T_{\min}$ on coverage probability: $K_m = 14, L_{\max}^m = 4, K_p = 6, L_{\max}^p = 4, \lambda_p = 6\lambda_m, \alpha = 3.5$

Next, we analyze the impact of interference nulling on coverage probability of a typical user in Figure 2, where $T_{\min} = K_m$ implies no interference nulling employed. We can observe that with properly chosen $T_{\min}$, the coverage probability can be improved with interference nulling. The performance gain is higher for smaller values of thresholds, which indicates that interference nulling significantly improves the SINRs of poor cell edge pico users. If we compare the curves for two different sets of $\lambda_u$ and $\eta$, the performance gain is lower for higher values of $\lambda_u$ and $\eta$, and it is due to insufficient resources for interference nulling.

In Figure 3, the average data rate with and without interference nulling are plotted against $\eta$ for different pico densities. The average rate with an arbitrary $T_{\min}$ and optimum $T_{\min}^*$ are plotted to illustrate the significance of $T_{\min}$ selection on performance. The optimal pair $(T_{\min}, \eta)$ is found to be $(8, 10\,\mathrm{dB})$ and $(6, 11\,\mathrm{dB})$ for pico density $\lambda_p = 4\lambda_m$ and $\lambda_p = 6\lambda_m$, respectively.

## VI. Conclusions

The downlink performance of multi-antenna HetNets with SDMA is analyzed, in which the precoding matrix at macro

BS also considers interference nulling to certain pico users. Further, the number of users served with SDMA in each cell is a function of user distribution. Our results show that the SINRs of victim pico users (those suffering strong interference from macro BS) can be significantly improved with the proposed interference nulling scheme. However, since this costs macro BSs their available degrees of freedom for serving their own users, $T_{\text{min}}$ must be carefully chosen. The optimal choice of $T_{\text{min}}$ is coupled with association bias as it determines the number of users to be served and interference coordination requests to be fulfilled. The optimal $(T_{\text{min}}, \eta, )$ that maximized the average data rate is investigated in this paper.

## APPENDIX

### A. Proof of Theorem 1

$$\mathbb{P}(\text{SINR}_p > \gamma | u \in \mathbf{\Phi}_u^p, M_p' = k, u \in \chi)$$
$$= \int_{r=0}^{\infty} \int_{r_1=\rho r}^{\infty} \sum_{l=0}^{K_p-k-1} \frac{(-s)^l}{l!} \frac{\mathrm{d}^l}{\mathrm{d}s^l} \mathcal{L}_{I_{b_p}}(s) \Big|_{s=\frac{\gamma r^\alpha}{P_p}} \times f_{V_m|D_p}(r_1|r) f_{D_p}(r)\, dr_1\, dr, \quad (27)$$

where $\mathcal{L}_{I_{b_p}}(s) = \mathcal{L}_{I_{b_p,m}}(s)\mathcal{L}_{I_{b_p,p}}(s)$ is the Laplace transform (LT) of $I_{b_p} = I_{b_p,m} + I_{b_p,p}$. The PDF of the distance $D_p$ from $u$ to the nearest pico BS $b_p$, given that $u \in \mathbf{\Phi}_u^p$ is

$$f_{D_p}(r) = \frac{2\pi\lambda_p}{A_p} r \exp(-\pi(\lambda_m \rho^2 + \lambda_p) r^2). \quad (28)$$

Similarly, the PDF of the distance $V_m$ to the nearest active macro BS $v_m$, given that $u \in \mathbf{\Phi}_u^p$ and its distance to the serving pico BS $b_p$ is $D_p = r$, is

$$f_{V_m|D_p}(r_1|r) = 2\pi p_m \lambda_m r_1 \exp(-\pi p_m \lambda_m (r_1^2 - \rho^2 r^2)), \quad r_1 > \rho r. \quad (29)$$

The proofs for these PDFs are omitted for brevity.

The LT $\mathcal{L}_{I_{b_p,l}}(s) = \mathbb{E}\left[\exp(-s I_{b_p,l})\right]$, $\forall l \in \{m, p\}$ can be derived as

$$\mathcal{L}_{I_{b_p,l}}(s) = \mathbb{E}_{\mathbf{\Psi}_l} \prod_{x_l \in \mathbf{\Psi}_l \setminus b_p} \mathbb{E}_{\zeta_{x_l}} \Big[\exp(-s P_l \zeta_{x_l} ||x_l||^{-\alpha})\Big]. \quad (30)$$

Given $M_l$, $\zeta_{x_l} \sim \text{Gamma}(M_l, 1)$. By performing the expectation over this conditional distribution, followed by the probability generating functional of PPP with density $p_l\lambda_l$, and finally the expectation over the PMF of $M_l$, we have

$$\mathcal{L}_{I_{b_p,l}}(s) = \exp\left(-\pi p_l \lambda_l \varpi_l^2 \left[\Xi_0^l\left(1, 1, \frac{P_l}{\varpi_l^\alpha} s\right) - 1\right]\right), \quad (31)$$

where $\varpi_l$ is the lower bound on distance to the closest interferer from $u$ in the tier $l \in \{m, p\}$. Thus, $\varpi_m = r_1$ and $\varpi_p = r$. Let $y(s) = e^{-\pi s}$, and $t(s) = p_m \lambda_m r_1^2\, \Xi_0^m\left(1, 1, \frac{P_m}{r_1^\alpha} s\right) + p_p \lambda_p r^2\, \Xi_0^p\left(1, 1, \frac{P_p}{r^\alpha} s\right)$. Then $\mathcal{L}_{I_{b_p}}(s) = e^{\pi\left(p_m\lambda_m r_1^2 + p_p \lambda_p r^2\right)} y(t(s))$. The $l$th derivative of a composite function $y(t(s))$ can be evaluated as follows, which is a simplified form of Faà di Bruno's formula.

$$y_s^{(l)}(t(s)) = \sum_{o=1}^{\mathcal{P}(l)} c_o^l\, y_{t(s)}^{(\omega_o^l)}(t(s)) \prod_{q=1}^{l} \left(t_s^{(q)}(s)\right)^{\mu_{oq}^l}, \quad (32)$$

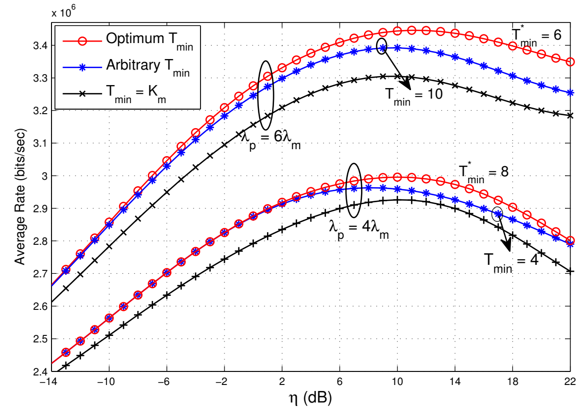


Fig. 3. Effect of pico cell density $\lambda_p$ on the optimal choices of $T_{\text{min}}$ and $\eta$: $\lambda_u = 6\lambda_m$, $K_m = 12$, $L_{\text{max}}^m = 4$, $K_p = 4$, $L_{\text{max}}^p = 4$, $\alpha = 4$.

where $y_{t(s)}^{(k)}(t(s))$ represents $k$th derivative of function $y(t(s))$ with respect to $t(s)$, the summation is over all possible partitions of integer $l$. The notations for integer-partition are presented in Section I. The final result for $\mathrm{T}_1(\gamma)$ in (17) is obtained by applying (32) into (27) where

$$\frac{\mathrm{d}^q}{ds^q} \Xi_0^l\left(1, 1, \frac{P_l}{\varpi_l^\alpha} s\right) = \left(-\frac{P_l}{\varpi_l^\alpha}\right)^q \Xi_q^l\left(1, 1, \frac{P_l}{\varpi_l^\alpha} s\right), \quad (33)$$

followed by changing the order of integration, substituting $\frac{r}{r_1} \to \theta$, $r_1 \to r_1$, then integrating with respect to $r_1$, and finally deconditioning with respect to the PMF of $M_p'$.

$T_2(\gamma)$ is derived in the same way as $T_1(\gamma)$. However, it does not require integration over $f_{V_m}(r_1)$ as $\varpi_m = \rho r$ in this case, given that $D_p = r$.

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