An Asymptotically Capacity-Achieving Scheme for the Gaussian Multiple-Access Relay Channel

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***Abstract*—We study the multiple access relay channel (MARC) with relay-destination cooperation (MARC-RDC). This channel resembles the uplink transmission in heterogeneous network where two user equipments (UEs) communicate with macro-cell base station (BS) through small-cell BS. We propose a coding scheme where the transmission is carried over B blocks and each UE performs superposition block Markov encoding. The relay (small-cell BS) first jointly decodes both UEs information using sliding window decoding over two transmission blocks and then forwards these information to the destination (macro-cell BS) coherently with UEs. The destination quantizes its received signal in each block and forwards the quantization index to the relay. The destination then decodes both UEs information using backward decoding. For this scheme, we derive the achievable rate region and compare it with existing schemes and the cut-set bound. Results show that relay-destination cooperation enlarges the rate region as the destination power increases. We further show that the proposed scheme asymptotically achieves the capacity by reaching the cut-set bound when the destination power approaches infinity and the ratio of one UE-destination to UE-relay link amplitudes is equal to that of the other UE. These results make the proposed scheme appealing for deployment in 5G cellular networks.**

I. Introduction

Multiple-access relay channel (MARC) is currently receiving a heightened research interests because of its applications in ad hoc network and the uplink transmission in heterogeneous network (HetNet) for both 4G and 5G cellular standards. In HetNet, small cells coexist with macro cells to provide better coverage and high-speed mobility. In LTE-A release 10 [1], wireless backhaul between small and macro-cells has been standardized and it will be widely used in 5G network that has large number of small cells [2]. In the uplink transmission, MARC resembles the transmission scenario where two UEs communicate with the macro base station (BS) through a small cell BS as shown in Figure 1.

Using basic relaying techniques in [3], several works propose different coding schemes for MARC including full and partial decode-forward (DF) relaying [4]–[6] and compress and quantize-forward relaying [7]–[11]. These works derive the achievable rate region [4]–[6], the outage performances [8] and the optimal quantization [10], [11] of the proposed schemes. Reference [12] derives the sum rate capacity for the degraded Gaussian MARC.

The previous works, however, consider no feedback link from the destination $(\mathcal{D})$ to the relay $(\mathcal{R})$ although in HetNet, the macro BS has more capabilities and much more transmission power $((20-40)\text{W})$ than the small cell BS $((0.02-2)\text{W})$ and UEs $((16-50)\text{mW})$ [1]. Moreover, when $\mathcal{D}$ cooperates with $\mathcal{R}$ or the sources, it can improve the achievable rate region for cooperative multiple access channel (MAC) [13], interference channel [14] and relay channel [15]. It can even achieve the capacity asymptotically for the relay channel with relay-destination cooperation as $\mathcal{D}$ power approaches infinity [15]. Therefore, it is of interest to investigate the impact of RDC in improving the achievable rate region for MARC.

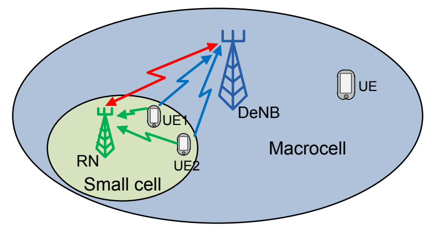

Fig. 1. MARC in heterogeneous network.

In this paper, we show how RDC helps achieve the capacity asymptotically for MARC-RDC shown in Figure 1. We propose a coding scheme in which the transmission is carried over B dependent blocks, each source performs superposition block Markov encoding; $\mathcal{D}$ employs quantize-forward (QF) relaying and backward decoding; and $\mathcal{R}$ employs DF relaying and joint sliding window decoding over two blocks. While $\mathcal{R}$ in [4], [5] decodes both sources information directly at the end of each block, $\mathcal{R}$ in the proposed scheme waits one more block before decoding the sources information in order to receive the quantized signal form $\mathcal{D}$. Hence, transmission in block $k \in \{1, 2, ..., B\}$ in the proposed scheme depends on the signals transmitted in block $k-2$.

We further analyze the asymptotic performance of the proposed scheme when $\mathcal{D}$ power approaches infinity. We prove that the proposed scheme achieves the capacity by reaching the cut-set bound when the ratio of one source-destination to source-relay link amplitudes is equal to that of the other source. As for relay channel with RDC [15], this result agrees with the intuition that when $\mathcal{D}$ power approaches infinity, $\mathcal{D}$ virtually joins $\mathcal{R}$ in one entity and the channel becomes similar to MAC that has a known capacity [16]. For non-asymptotic regimes, we provide results that compare between the proposed and NNC [17] schemes for MARC-RDC, the DF relaying scheme for MARC without RDC [4], [5] and the cut-set bound. Results show that the proposed scheme outperforms all existing schemes as $\mathcal{D}$ power increases.

II. Channel Model

The MARC-RDC consists of two sources $(\mathcal{S}_1, \mathcal{S}_2)$ communicate with a common destination $\mathcal{D}$ with the help from a relay

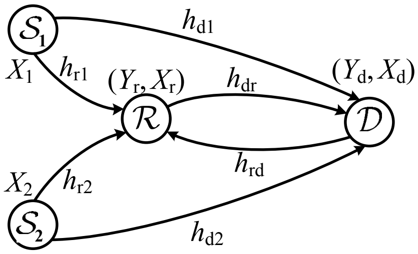

Fig. 2. The channel model of MARC-RDC.

$(\mathcal{R})$ where $\mathcal{D}$ and $\mathcal{R}$ cooperate to improve the transmission rates of both sources. Figure 2 shows the channel model for full-duplex Gaussian MARC-RDC. For any transmission over B blocks, the discrete-time channel model at block $k \in \{1, 2, ..., B\}$ is given as follows:

$$\begin{aligned} Y_{r,k} &= h_{r1}X_{1,k} + h_{r2}X_{2,k} + h_{rd}X_{d,k} + Z_{r,k}, \\ Y_{d,k} &= h_{d1}X_{1,k} + h_{d2}X_{2,k} + h_{dr}X_{r,k} + Z_{d,k}, \end{aligned} \tag{1}$$

where for $i \in \{r, d\}$ and $j \in \{1, 2, r, d\}$, h_{ij} is the complex link coefficient from node j to i; $X_{j,k}$ is the transmitted signal form node j in block k; $Y_{i,k}$ is the received signal at node i in block k; $Z_{r,k}$ and $Z_{d,k} \sim CN(0, 1)$ are independent complex AWGN. Each link coefficient is a complex value $h_{ij} = g_{ij}e^{\sqrt{-1}\theta_{ij}}$ where g_{ij} is the real amplitude gain and θ_{ij} is the phase. Using standard assumption of coherent relaying [3], [16], we assume that $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{R}$ know the phases of their respective links to $\mathcal{D}$ and they can transmit coherently. We further assume that the full channel coefficients are known at the respective receivers $(\mathcal{R}, \mathcal{D})$.

III. Transmission Scheme for MARC-RDC

The proposed scheme aims to utilize the feedback signal from $\mathcal{D}$ to $\mathcal{R}$ such that $\mathcal{R}$ can decode both information of $\mathcal{S}_1$ and $\mathcal{S}_2$ at higher rates. Then, $\mathcal{R}$ can forward more information from both sources to $\mathcal{D}$ which enlarges the achievable rate region. Similar to the scheme of relay channel with RDC [15], the transmission scheme is carried over B independent blocks. It is based on superposition block Markov encoding at $\mathcal{S}_1$ and $\mathcal{S}_2$; DF relaying and sliding window decoding at $\mathcal{R}$; and QF relaying and backward decoding at $\mathcal{D}$.

A. Transmission Scheme

In a transmission block $k \in \{1, 2, ..., B\}$, $\mathcal{S}_1$ transmits its new and old information $(w_{1,k}, w_{1,k-2})$ using superposition block Markov encoding. It first generates a codeword U_1 for $w_{1,k-2}$ and then superposes $w_{1,k}$ over U_1 and generates the codeword X_1. $\mathcal{S}_2$ performs similar encoding and generates the codewords U_2 and X_2. $\mathcal{R}$ forwards U_1 and U_2 to $\mathcal{D}$ since it already decoded $w_{1,k-2}$ and $w_{2,k-2}$ using the received signals in blocks $k-2$ and $k-1$. $\mathcal{D}$ has already quantized the received signal in block $k-1$ and determined the quantization index (l_{k-1}). $\mathcal{D}$ then generates a codeword X_d for (l_{k-1}) and transmits X_d to $\mathcal{R}$ in block k.

1) Transmit Signals: In block k, $\mathcal{S}_1$, $\mathcal{S}_2$ $\mathcal{R}$ and $\mathcal{D}$ respectively transmit $X_1(w_{1,k}, w_{1,k-2})$, $X_2(w_{2,k}, w_{2,k-2})$, $X_r(w_{1,k-2}, w_{2,k-2})$ and $X_d(l_{k-1})$. They construct their transmit signals as follows.

$$\begin{aligned} X_1 &= \sqrt{\rho_{n1}}V_1(w_{1,k}) + \sqrt{\rho_{o1}}U_1(w_{1,k-2}) \\ X_2 &= \sqrt{\rho_{n2}}V_2(w_{2,k}) + \sqrt{\rho_{o2}}U_2(w_{2,k-2}) \\ X_r &= \sqrt{\rho_{r1}}U_1(w_{1,k-2}) + \sqrt{\rho_{r2}}U_2(w_{2,k-2}), \\ X_d &= \sqrt{P_d}V_d(l_{k-1}), \;\; \hat{Y}_d = Y_d + \hat{Z}_d, \end{aligned} \tag{2}$$

where $\hat{Y}_d$ is the quantized version of Y_d and $\hat{Z}_d \sim \mathcal{CN}(0, Q)$. The signals V_1, U_1, V_2, U_2, and V_d are all i.i.d Gaussian signals $\sim \mathcal{CN}(0, 1)$ that convey the codewords of the messages $w_{1,k}$, $w_{1,k-2}$, $w_{2,k}$, $w_{2,k-2}$ and the bin index l_{k-1}, respectively. The power allocation parameters ρ_{n1}, ρ_{o1}, ρ_{n2}, ρ_{o2}, ρ_{r1} and ρ_{r2} satisfy the following constrains:

$$\rho_{n1} + \rho_{o1} = P_1, \;\; \rho_{n2} + \rho_{o2} = P_2, \;\; \rho_{r1} + \rho_{r2} = P_r. \tag{3}$$

2) Decoding: $\mathcal{R}$ and $\mathcal{D}$ can decode both sources information using joint typicality (JT) [16] or maximum likelihood (ML) [18] decoding. Here, we briefly describe the decoding techniques and give the full analysis in Appendix A.

At the relay: $\mathcal{R}$ performs sliding window decoding to decode both sources information and the quantization index from $\mathcal{D}$. At the end of block $k+1$, $\mathcal{R}$ has already estimated $\hat{w}_{1,k-2}$, $\hat{w}_{2,k-2}$ and l_{k-2} ($\hat{w}_{1,k-1}$, $\hat{w}_{2,k-1}$ and l_{k-1}) from the decoding in blocks $k-2$ and $k-1$ ($k-1$ and k). $\mathcal{R}$ then simultaneously utilizes the received signals in blocks k and $k+1$ $(Y_{r,k}, Y_{r,k+1})$ to jointly decode both sources information $(w_{1,k}, w_{2,k})$ and the quantization index (l_k).

At the destination: $\mathcal{D}$ performs backward decoding for both sources information. In block k, $\mathcal{D}$ has already estimated the new information $\tilde{w}_{1,k}$ and $\tilde{w}_{2,k}$ from the decoding in block $k+2$. $\mathcal{D}$ then utilizes the received signal in block k $(Y_{d,k})$ to jointly decode both sources old information $(w_{1,k-2}, w_{2,k-2})$.

B. Achievable Rate

The error analyses for the decoding rules at $\mathcal{R}$ and $\mathcal{D}$ leads to some rate constraints that ensure reliable decoding at $\mathcal{R}$ and $\mathcal{D}$. These rate constraints determine the achievable rate region for MARC-RDC as in the following Theorem:

Theorem 1. *For the MARC-RDC, the achievable rate region consists of all rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 &\le \min\{I_1, I_2, I_3\}, \quad R_2 \le \min\{I_4, I_5, I_6\}, \\ R_1 + R_2 &\le \min\{I_7, I_8, I_9\}, \quad \textit{where} \end{aligned} \tag{4}$$

$$\begin{aligned} I_1 &= \mathcal{C}\left(g_{r1}^2\rho_{n1} + \frac{g_{d1}^2\rho_{n1}}{1+Q}\right), \; I_4 = \mathcal{C}\left(g_{r2}^2\rho_{n2} + \frac{g_{d2}^2\rho_{n2}}{1+Q}\right), \\ I_2 &= \mathcal{C}\left(\frac{g_{rd}^2 P_d}{1 + g_{r1}^2\rho_{n1} + g_{r2}^2\rho_{n2}}\right) + \mathcal{C}(g_{r1}^2\rho_{n1}) - \mathcal{C}(\frac{1}{Q}), \\ I_3 &= \mathcal{C}\left(g_{d1}^2 P_1 + g_{dr}^2\rho_{r1} + 2g_{d1}g_{dr}\sqrt{\rho_{o1}\rho_{r1}}\right), \\ I_5 &= \mathcal{C}\left(\frac{g_{rd}^2 P_d}{1 + g_{r1}^2\rho_{n1} + g_{r2}^2\rho_{n2}}\right) + \mathcal{C}(g_{r2}^2\rho_{n2}) - \mathcal{C}(\frac{1}{Q}), \\ I_6 &= \mathcal{C}\left(g_{d2}^2 P_2 + g_{dr}^2\rho_{r2} + 2g_{d2}g_{dr}\sqrt{\rho_{o2}\rho_{r2}}\right), \\ I_7 &= \mathcal{C}\left(g_{r1}^2\rho_{n1} + g_{r2}^2\rho_{n2} + \frac{g_{d1}^2\rho_{n1} + g_{d2}^2\rho_{n2}}{1+Q}\right), \\ I_8 &= \mathcal{C}\left(g_{r1}^2\rho_{n1} + g_{r2}^2\rho_{n2} + g_{rd}^2 P_d\right) - \mathcal{C}(\frac{1}{Q}), \\ I_9 &= \mathcal{C}\Big(g_{d1}^2 P_1 + g_{d2}^2 P_2 + g_{dr}^2 P_r \\ &\quad + 2g_{d1}g_{dr}\sqrt{\rho_{o1}\rho_{r1}} + 2g_{d2}g_{dr}\sqrt{\rho_{o2}\rho_{r2}}\Big), \end{aligned} \tag{5}$$

where $\mathcal{C}(x) = \log(1 + x)$. The power allocation parameters satisfies (3) and the rate constraints are subjected to

$$Q \geq Q_c \;\; Q_c = \frac{1 + g_{r1}^2 \rho_{n1} + g_{r2}^2 \rho_{n2}}{g_{rd}^2 P_d}, \tag{6}$$

Proof. The constraints I_1, I_2, I_4, I_5, I_7 and I_8 are obtained from the decoding at $\mathcal{R}$ while I_3, I_6 and I_9 are obtained from the decoding at $\mathcal{D}$. The transmission rate for the quantization index sent by $\mathcal{D}$ is bounded by the link quality from $\mathcal{D}$ to $\mathcal{R}$ which bounds the quantization noise variance as shown in (6). For detailed proof, see Appendix A. □

1) Optimal Q^*: This section derives the optimal values of Q^* that maximize the individual rates (R_1, R_2) and the sum rate (R_1+R_2) in Theorem 1. Since I_1 is a decreasing function with Q while I_2 is a an increasing function, the optimal Q_1^* that maximizes R_1 is obtained form the intersection between I_1 and I_2. Similarly holds for the optimal Q_2^* of R_2 and Q_s^* of $R_1 + R_2$. Q_1^*, Q_2^* and Q_s^* are given as follows:

Corollary 1. *The optimal Q_1^*, Q_2^* and Q_s^* that respectively maximize R_1, R_2 and $R_1 + R_2$ in Theorem 1 are given as*

$$\begin{aligned} Q_1^* &= \frac{\left(1 + \frac{g_{r2}^2 \rho_{n2}}{1+g_{r1}^2 \rho_{n1}}\right)\left(1 + (g_{r1}^2 + g_{d1}^2)\rho_{n1}\right)}{g_{rd}^2 P_d}, \\ Q_2^* &= \frac{\left(1 + \frac{g_{r1}^2 \rho_{n1}}{1+g_{r2}^2 \rho_{n2}}\right)\left(1 + (g_{r2}^2 + g_{d2}^2)\rho_{n2}\right)}{g_{rd}^2 P_d}, \\ Q_s^* &= \frac{1 + (g_{r1}^2 + g_{d1}^2)\rho_{n1} + (g_{r2}^2 + g_{d2}^2)\rho_{n2}}{g_{rd}^2 P_d}. \end{aligned} \tag{7}$$

Proof. Q_1^*, Q_2^* and Q_s^* are obtained from the solutions of $I_1 = I_2$, $I_4 = I_5$ and $I_7 = I_8$ in (5), respectively. □

Remark 1. Q_1^*, Q_2^* and Q_s^* in (7) satisfy the condition in (6) since they are greater than Q_c.

C. Discussion

We have some remarks on the proposed scheme.

Remark 2. As in [15], this scheme uses a second order block Markov encoding where the transmitted codeword in block k depends on the codeword transmitted in block $k - 2$. This is because $\mathcal{R}$ does not decode $\mathcal{S}_1$ and $\mathcal{S}_2$ information in block k directly. However, $\mathcal{R}$ waits another block $k + 1$ to receive the quantized signal from $\mathcal{D}$ and then decodes $\mathcal{S}_1$ and $\mathcal{S}_2$ information sent in block k using its received signals in blocks k and $k+1$. $\mathcal{R}$ next forwards these information in block $k+2$.

Remark 3. The proposed scheme can be generalized by using partial DF relaying at $\mathcal{R}$. However, we only use full DF relaying as we focus on understanding the impact of $\mathcal{R}$-$\mathcal{D}$ cooperation in improving the achievable rate region.

Remark 4. The scheme includes the following existing schemes as special cases:

- The DF scheme for MARC without RDC [4]. This can be verified by setting $X_d = \hat{Y}_d = \emptyset$ and $P_d = 0$.
- The DF scheme for relay channel [3]. This can be verified by setting $X_2 = X_d = \hat{Y}_d = \emptyset$ and $P_2 = P_d = 0$.

Moreover, if we generalize the scheme to partial DF relaying as in Remark 3, it will include the partial DF relaying with RDC for relay channel in [15] by setting $X_2 = \emptyset$ and $P_2 = 0$.

Remark 5. Noisy network coding (NNC) scheme [17] can be applied on the channel model in Figure 2. This scheme is based on message repetition at the sources in all transmission blocks, QF relaying at $\mathcal{R}$ and $\mathcal{D}$, and simultaneous joint decoding over all transmission blocks at $\mathcal{D}$. By applying Theorem 1 in [17] into the MARC-RDC, we obtain the achievable rate region that consists of all rate pairs (R_1, R_2) satisfying

$$\begin{aligned} &R_1 \leq \min\{J_1, J_2\}, \; R_2 \leq \min\{J_3, J_4\} \\ &R_1 + R_2 \leq \min\{J_5, J_6\} \text{ where} \\ &J_1 = \mathcal{C}\left(g_{d1}^2 P_1 + \frac{g_{r1}^2 P_1}{1 + Q_r}\right), \; J_3 = \mathcal{C}\left(g_{d2}^2 P_2 + \frac{g_{r2}^2 P_2}{1 + Q_r}\right), \\ &J_2 = \mathcal{C}\left(g_{d1}^2 P_1 + g_{dr}^2 P_r\right) - \mathcal{C}(\frac{1}{Q_r}), \\ &J_4 = \mathcal{C}\left(g_{d2}^2 P_2 + g_{dr}^2 P_r\right) - \mathcal{C}(\frac{1}{Q_r}), \\ &J_5 = \mathcal{C}\Big(g_{d1}^2 P_1 + g_{d2}^2 P_2 \\ &\qquad + \frac{g_{r1}^2 P_1 + g_{r2}^2 P_2 + (g_{r1} g_{d2} - g_{r2} g_{d1})^2 P_1 P_2}{1 + Q_r}\Big), \\ &J_6 = \mathcal{C}\left(g_{d1}^2 P_1 + g_{d2}^2 P_2 + g_{dr}^2 P_r\right) - \mathcal{C}(\frac{1}{Q_r}), \end{aligned} \tag{8}$$

where Q_r is the quantization noise variance obtained from the quantization at $\mathcal{R}$. The optimal Q_r^* for individual and sum rates are obtained in similar way to Q_1^*, Q_2^* and Q_s^* in Corollary 1. Unlike the proposed MARC-RDC scheme, the feedback from $\mathcal{D}$ to $\mathcal{R}$ in NNC scheme does not improve the rate region. This is because in the proposed scheme, $\mathcal{R}$ decodes the users' information which is improved with the received signal from $\mathcal{D}$. However, $\mathcal{R}$ quantizes its received signal in NNC scheme and $\mathcal{D}$ won't benefit from receiving a quantized version of a signal it knows already. Hence, NNC schemes for classical MARC and MARC-RDC achieve the same rate region.

IV. Capacity Achieving at High $\mathcal{D}$ Power

In HetNet, as specified in Section I, the transmission power of macro BS is about 200x of the small cell BS and 1000x of the UEs. Therefore, for MARC-RDC which represents the uplink transmission in HetNet, $\mathcal{D}$ can cooperate with $\mathcal{R}$ at high power to enlarge the rate region. As a theoretical limit, this section shows that the rate region of the proposed scheme can asymptotically achieve the capacity as $P_d \to \infty$ by reaching the cut-set bound when the amplitude link ratios $\frac{g_{d1}}{d_{r1}} = \frac{g_{d2}}{g_{r2}}$.

A. Achievable Rate Region at $P_d \to \infty$

When $\mathcal{D}$ has enough power $(P_d \to \infty)$, it can reduce the quantization noise by increasing the number of bin indices. This allows $\mathcal{D}$ to send a very clear version of its received signal to $\mathcal{R}$ which improves the decoding at $\mathcal{R}$ and leads to the following rate region:

Corollary 2. *The proposed MARC-RDC scheme with $P_d \to \infty$ achieves a similar rate region to Theorem 1 with $I_1 = I_2$, $I_4 = I_5$ and $I_7 = I_8$ where*

$$I_1 \to \mathcal{C}\left((g_{r1}^2+g_{d1}^2)\rho_{n1}\right),\ I_4 \to\ \mathcal{C}\left((g_{r2}^2+g_{d2}^2)\rho_{n2}\right),$$
$$I_7 \to \mathcal{C}\left((g_{r1}^2+g_{d1}^2)\rho_{n1}+(g_{r2}^2+g_{d2}^2)\rho_{n2}\right), \tag{9}$$

and I_3, I_6, and I_9 are given as in Theorem 1.

Proof. By substituting $P_d \to \infty$ into Theorem 1 where all optimal quantization noises Q_1, Q_2, Q_s and $Q_c \to 0$. □

B. Cut-Set Bound

The cut-set outer bound [16] for the Gaussian MARC-RDC in (1) is given as follows.

Corollary 3. *The capacity region of the Gaussian MARC-RDC is upper bounded by the rate pairs (R_1, R_2) satisfying*

$$R_1 \le \min\{C_1, C_2\}, \quad R_2 \le \min\{C_3, C_4\},$$
$$R_1+R_2 \le \min\{C_5, C_6\},\ \textit{where} \tag{10}$$

$$C_1 = \mathcal{C}\left((g_{r1}^2+g_{d1}^2)\beta_1 P_1\right),\ C_3 = \mathcal{C}\left((g_{r2}^2+g_{d2}^2)\gamma_1 P_2\right),$$
$$C_2 = \mathcal{C}\left(g_{d1}^2\beta_2 P_1 + g_{dr}^2\beta_3 P_2 + 2g_{d1}g_{dr}\beta_4\sqrt{P_1P_2}\right),$$
$$C_4 = \mathcal{C}\left(g_{d2}^2\gamma_2 P_2 + g_{dr}^2\gamma_3 P_2 + 2g_{d2}g_{dr}\gamma_4\sqrt{P_1P_2}\right),$$
$$\begin{aligned}C_5 = \mathcal{C}\Big(&(g_{r1}^2+g_{d1}^2)\mu_1 P_1 + (g_{r2}^2+g_{d2}^2)\mu_2 P_2\\ &- 2(g_{d1}g_{d2}+g_{r1}g_{r2})\mu_3\sqrt{P_1P_2}\\ &+ (g_{r1}g_{d2}-g_{r2}g_{d1})^2\mu_4 P_1P_2\Big),\end{aligned}$$
$$\begin{aligned}C_6 = \mathcal{C}\Big(&g_{d1}^2P_1(1-\delta_{1d}^2) + g_{d2}^2P_2(1-\delta_{2d}^2)\\ &+ g_{dr}^2P_r(1-\delta_{rd}^2) - 2g_{d1}g_{d2}\delta_{1d}\delta_{2d}\sqrt{P_1P_2}\\ &+ 2g_{d1}g_{dr}\sqrt{P_1P_r}(\delta_{1r}-\delta_{1d}\delta_{rd})\\ &+ 2g_{d2}g_{dr}\sqrt{P_2P_r}(\delta_{2r}-\delta_{2d}\delta_{rd})\Big),\ \textit{and}\end{aligned} \tag{11}$$

$$\beta_1 = \frac{\eta}{1-\delta_{2r}^2-\delta_{2d}^2-\delta_{rd}^2+2\delta_{2r}\delta_{2d}\delta_{rd}},$$
$$\begin{aligned}\eta = 1 &- \delta_{1r}^2-\delta_{1d}^2-\delta_{2r}^2-\delta_{2d}^2-\delta_{rd}^2+\delta_{1d}^2\delta_{2r}^2+\delta_{2d}^2\delta_{1r}^2\\ &+2\delta_{1r}\delta_{1d}\delta_{rd}+2\delta_{2r}\delta_{2d}\delta_{rd}-2\delta_{1r}\delta_{1d}\delta_{2r}\delta_{2d},\end{aligned}$$
$$\beta_2 = 1-\frac{\delta_{1d}^2}{1-\delta_{2d}^2},\ \beta_3 = 1-\frac{\delta_{2r}^2+\delta_{rd}^2-2\delta_{2r}\delta_{2d}\delta_{rd}}{1-\delta_{2d}^2}$$
$$\beta_4 = \delta_{1r}-\frac{\delta_{1d}\delta_{rd}-\delta_{1d}\delta_{2d}\delta_{2r}}{1-\delta_{2d}^2}$$
$$\mu_1 = 1-\frac{\delta_{1d}^2+\delta_{1r}^2}{1-\delta_{rd}^2},\ \mu_2 = 1-\frac{\delta_{2d}^2+\delta_{2r}^2}{1-\delta_{rd}^2},\ \mu_4 = \frac{\eta}{1-\delta_{rd}^2}$$
$$\mu_3 = \frac{\delta_{1d}\delta_{2d}+\delta_{1r}\delta_{2r}-\delta_{rd}(\delta_{1d}\delta_{2r}+\delta_{2d}\delta_{1r})}{1-\delta_{rd}^2}, \tag{12}$$

where γ_i, for $i \in \{1,2,3,4\}$ is similar to β_i except switching all indices from $1 \to 2$ and $2 \to 1$. $\delta_{mn} \in [-1,+1]$ for $m \in \{1,2,r\}$ and $n \in \{r,d\}$ is the correlation factor between X_m and X_n and

$$\eta \ge 0. \tag{13}$$

Proof. By determining the mutual information of the 6 cutsets for the discrete memoryless MARC-RDC; applying them into the Gaussian channel in (1); and showing that the optimal input distribution (X_1, X_2, X_r, X_d) is jointly Gaussian $\sim \mathcal{N}(0,\Sigma)$ where (13) insures that Σ is a positive semi-definite covariance matrix. For detailed proof, see Appendix B. □

Remark 6. The cut-set bound for classical MARC (without RDC) is given as in Corollary 3 but with $\delta_{1d} = \delta_{2d} = \delta_{rd} = 0$. In both bounds for classical MARC and MARC-RDC, $0 \le (\beta_i, \gamma_i, \mu_i) \le 1$ for $i \in \{1,2,3,4\}$. Therefore, the two bounds have the same region.

From Corollaries 2 and 3, we obtain the following theorem:

Theorem 2. *The proposed scheme for MARC-RDC achieves the capacity as $P_d \to \infty$ by reaching the cut-set bound when the ratio of one $\mathcal{S}-\mathcal{D}$ to $\mathcal{S}-\mathcal{R}$ link amplitudes is equal to that of the other source $g_{d1}/g_{r1} = g_{d2}/g_{r2}$.*

Proof. By comparing formulas (9) with (3) and (10) with (13) and after some mathematical manipulations, we show that they almost have identical constraints ($I_1 \triangleq C_1$, $I_3 \triangleq C_2$, $I_4 \triangleq C_3$, $I_6 \triangleq C_4$, $I_9 = C_6$) except for $I_7 \ne C_5$ because of the term $(g_{r1}g_{d2}-g_{r2}g_{d1})^2\mu_4P_1P_2$ in C_5 in (10). However, this term is 0 when $g_{d1}/g_{r1} = g_{d2}/g_{r2}$. □

Remark 7. Theorem 2 is interesting for practical designers of 5G cellular systems. The theorem implies that two UEs in a small cell can guarantee to have the maximum possible throughput when macro and small cell base stations cooperate. However, the two UEs need to pay for such a service since the macro base station will cooperate with high power.

V. Numerical Results

We now provide numerical results for the achievable rate regions of MARC obtained by the proposed scheme with different values of the destination power. We also compare our scheme with existing DF relaying scheme without RDC [4], [5] and NNC for MARC-RDC [17]. The channel parameters and transmit powers are given in each figure.

Figure 3 shows how the proposed MARC-RDC scheme enlarges the rate region of the MARC compared with DF relaying [4], [5] and NNC [17] schemes. RDC improves the rate region as $\mathcal{D}$ power increases until it reaches the cut-set bound. This result implies that $\mathcal{S}_1$ and $\mathcal{S}_2$ can improve their transmission rates by not only increasing their transmission powers but also by increasing the transmission power from $\mathcal{D}$. Hence, in the uplink transmission for HetNet where $\mathcal{D}$ (e.g. macro BS) is more powerful than UEs, $\mathcal{D}$ can change its transmission power to adapt the transmission rate of both UEs based on a specific service requirement.

Figure 4 compares between the cut-set bound and the asymptotic achievable rate region at $P_d \to \infty$ for different sets of channel configurations. Results confirm Theorem 2 where the proposed scheme achieves the capacity when $\frac{g_{d1}}{d_{r1}} = \frac{g_{d2}}{g_{r2}}$ as in sets 1 and 2 of channel parameters in Figure 4. However, in set 3, the cut-set bound has a higher sum rate than the proposed scheme. Moreover, for any channel configurations, the individual rates of the proposed scheme achieve the cut-set bound individual rates which is also proved in [15].

Figure 5 shows the sum rate gap between the proposed MARC-RDC scheme and the DF relaying scheme without RDC [4], [5] for the symmetric channel. The rate improvement increases with P_d until it achieves the cut-set bound and the maximum gap occurs when $\mathcal{S}-\mathcal{R}$ and $\mathcal{S}-\mathcal{D}$ have the same strength ($g_{r1} = g_{d1}$). This is because DF relaying scheme

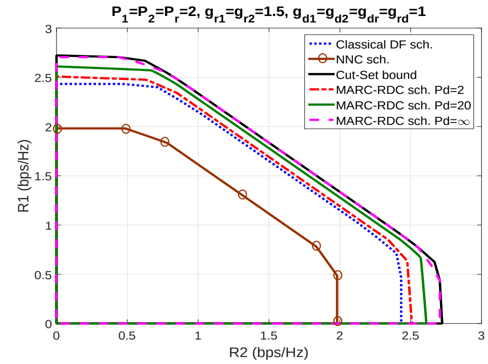

Fig. 3. Achievable rate regions of the MARC for the proposed MARC-RDC, DF relaying [4], [5] and NNC [17] schemes and the cut-set bound.

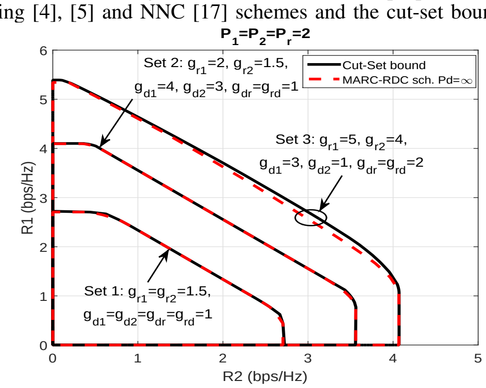

Fig. 4. Comparison between the achievable rate region of the proposed MARC-RDC scheme when $P_d \to \infty$ and the cut-set bound.

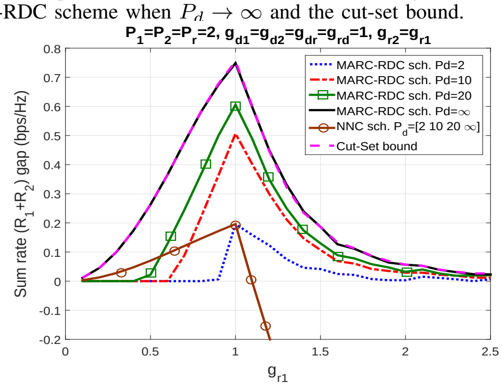

Fig. 5. Sum rate gaps of the MARC between the proposed RDC, NNC [17] and DF relaying [4], [5] schemes versus g_{r1} for the symmetric channel.

[4], [5] starts improving the rate region over classical MAC (without $\mathcal{R}$) when $(g_{r1} > g_{d1})$ and hence, reducing the gap with MARC-RDC scheme. Although NNC has the smallest achievable rate region for the channel configuration in Figure 3, Figure 5 shows that it achieves a larger sum rate than MARC-RDC when g_{r1} is weaker or slightly stronger than g_{d1}.

VI. Conclusion

We have considered the uplink transmission in HetNet with joint small-macrocell cooperation that is modeled as Gaussian MARC-RDC with relay-destination cooperation. We show that this cooperation can improve the achievable rate region and even achieve the capacity asymptotically with high destination power. We proposed a coding scheme based on superposition block Markov encoding at the sources, DF relaying with joint sliding window decoding at the relay, QF relaying and backward decoding at the destination. We proved that when the ratios of source-destination to source-relay links for both sources are equal, the proposed scheme asymptotically achieves the capacity as the destination power approaches infinity by reaching the cut-set bound. We further provided numerical results that compare between the proposed, NNC and DF schemes for MARC. These results are appealing for the uplink transmission in HetNets and encourage further studying over fading channels and multi small-cell scenario.

Appendix A: Proof of Theorem 1

We proof Theorem 1 using the information theoretic analysis of a discrete memoryless MARC-RDC specified by a collection of pmf $p(y_d, y_r|x_1, x_2, x_r, x_d)$, where $x_m,\ m \in \{1, 2, r, d\}$ is the input signal of node m while $y_l,\ l \in \{r, d\}$ is the output signal of node l. We define a $(2^{nR_1}, 2^{nR_2}, n, Pe)$ code based on standard definitions as in [16].

We consider B independent transmission blocks each of length n. Two sequences of $B-2$ messages $w_{1,k}$ and $w_{2,k}$ for $k \in [1 : B-2]$ are to be sent over the channel in nB transmissions. Therefore, S_1 and S_2 do not send new information in the last two blocks ($B-1$ and B) which reduces the achievable rates in (4) by a factor of $2/B$ (e.g. $R_1(1-2/B) \le \min\{I_1, I_2, I_3\}$). This factor, however, becomes negligible as $B \to \infty$.

A. Codebook generation

The codebook generation of the proposed coding scheme in block $k \in \{1 : B\}$ can be explained as follows. After fixing $P^\dagger = p(u_1)p(x_1|u_1)p(u_2)p(x_2|u_2)p(x_d)p(\hat{y}_d|x_d)$, independently generate 2^{nR_μ} codewords $u_\mu^n(w_{\mu,k-2})$ and 2^{nR_μ} codewords $x_\mu^n(w_{\mu,k}|w_{\mu,k-2})$ that encode $w_{\mu,k-2}$ and $w_{\mu,k}$, respectively where $\mu \in \{1, 2\}$. For each pair $(u_1^n(w_{1,k-2}), u_2^n(w_{2,k-2}))$, generate one sequences $x_r^n(w_{1,k-2}, w_{2,k-2})$. Similarly generate 2^{nR_d} codewords $x_d^n(l_{d,k-1})$ and 2^{nR_d} codewords $\hat{y}_d^n(l_k|l_{k-1})$ that encode the quantization indices l_{k-1} and l_k, respectively.

B. Encoding

Let $(w_{1,k}, w_{2,k})$ be the new messages to be sent in block k. Then, S_1 (S_2) transmits $x_1^n(w_{1,k}, w_{1,k-2})$ ($x_2^n(w_{2,k}, w_{2,k-2})$). $\mathcal{R}$ ($\mathcal{D}$) has estimated $(\hat{w}_{1,k-2}, \hat{w}_{2,k-2})$ ($\hat{l}_{k-1}$) in block $k-1$. Then, $\mathcal{R}$ ($\mathcal{D}$) transmits $x_r^n(\hat{w}_{1,k-2}, \hat{w}_{2,k-2})$ ($x_d^n(\hat{l}_{k-1})$) in block k. Moreover, in block k, $\mathcal{D}$ finds an index l_i such that

$$(\hat{y}_d^n(l_k|L_{k-1}), x_d^n(L_{k-1}), y_d^n(k)) \in A_\epsilon^n \tag{14}$$

By covering lemma [16], such l_i exists if

$$R_d > I(\hat{Y}_d; Y_d|X_d). \tag{15}$$

C. Decoding

Without loss of generality, assume that all transmitted messages and the quantization indices are equal to 1. Then, the decoding can be described as follows.

1) At $\mathcal{R}$: At the end of block $k+1$, $\mathcal{R}$ already knows $l_{k-1} = L_{k-1}$, $(w_{1,k-2}, w_{2,k-2}) = (1, 1)$ and $(w_{1,k-1}, w_{2,k-1}) = (1, 1)$ from the decoding in blocks $k-1$ and k. $\mathcal{R}$ then utilizes the received signals in blocks k and $k+1$ to find a unique triple $(\hat{w}_{1,k}, \hat{w}_{2,k}, \hat{l}_k)$ such that

$$\big(x_1^n(\hat{w}_{1,k},1), u_1^n(1), x_2^n(\hat{w}_{2,k},1), u_2^n(1), x_d^n(L_{k-1}), \hat{y}_d^n(\hat{l}_k|L_{k-1}), y_r^n(k)\big) \in A_\epsilon^n$$
$$\text{and } \big(u_1^n(1), u_2^n(1), x_d^n(\hat{l}_k) y_r^n(k+1)\big) \in A_\epsilon^n$$

JT analysis [16] leads to the following rate constraints:

$$\begin{aligned}
R_d \leq& \zeta_1 + \zeta_2,\ R_1 \leq I(X_1;\hat{Y}_d, Y_r|X_2, U_1, U_2, X_r, X_d),\\
R_1 + R_d \leq& \zeta_1 + \zeta_2 + I(X_1; Y_r|X_2, X_r, X_d, U_1, U_2)\\
R_2 \leq& I(X_2;\hat{Y}_d, Y_r|X_1, U_1, U_2, X_r, X_d),\\
R_2 + R_d \leq& \zeta_1 + \zeta_2 + I(X_2; Y_r|X_1, X_r, X_d, U_1, U_2)\\
R_1 + R_2 \leq& I(X_1, X_2;\hat{Y}_d, Y_r|U_1, U_2, X_r, X_d),\\
R_1 + R_2 + R_d \leq& I(X_1, X_2, X_d; Y_r|U_1, U_2, X_r) + \zeta_2,
\end{aligned} \tag{16}$$

where $\zeta_1 = I(X_d; Y_r|U_1, U_2, X_r)$ and $\zeta_2 = I(\hat{Y}_d; X_1, U_1, X_2, U_2, X_r, Y_r|X_d)$. By combining (15) and (16) and applying them to the Gaussian channel in (1) with the signaling in (2), we obtain the condition in (6) and the rate constraints I_1, I_2, I_4, I_5, I_7 and I_8 in (5).

2) At the Destination: $\mathcal{D}$ employs backward decoding where in block k, $\mathcal{D}$ has already estimated $\tilde{w}_{1,k}$ and $\tilde{w}_{2,k}$ from the decoding in block $k+2$. Then, it looks for a unique message pair $(\tilde{w}_{1,k-2}, \tilde{w}_{2,k-2})$ such that

$$\big(x_1^n(1,\tilde{w}_{1,k-2}), x_2^n(1,\tilde{w}_{2,k-2}), u_1^n(\tilde{w}_{1,k-2}), u_2^n(\tilde{w}_{2,k-2}), x_r^n(\tilde{w}_{1,k-2}, \tilde{w}_{2,k-2}), x_d^n(L_{k-1}), y_d^n(k)\big) \in A_\epsilon^n$$

JT analysis [16] leads to the following rate constraints:

$$\begin{aligned}
&R_1 \leq I(X_1, X_r; Y_d|X_d, X_2, U_2),\ R_2 \leq I(X_2, X_r; Y_d|X_d, X_1, U_1)\\
&R_1 + R_2 \leq I(X_1, X_2, X_r; Y_d|X_d),
\end{aligned} \tag{17}$$

Applying these constraints to the Gaussian in channel in (1), we obtain the rate constraints I_3, I_6, and I_9 in (5).

Appendix B: Proof of Corollary 3

Using the cut-set bound [16], the capacity for MARC-RD is upper-bounded by the maximum information flowing through the 6 cutsets in Figure 6 that are expressed as follows:

$$\begin{aligned}
&R_1 \leq \min\{I(X_1; Y_r, Y_d|X_r, X_d, X_2), I(X_1, X_r; Y_d|X_d, X_2)\},\\
&R_2 \leq \min\{I(X_2; Y_r, Y_d|X_r, X_d, X_1), I(X_2, X_r; Y_d|X_d, X_1)\},\\
&R_1 + R_2 \leq \min\{I(X_1, X_2; Y_r, Y_d|X_r, X_d),\\
&\qquad I(X_1, X_2, X_r; Y_d|X_d)\},
\end{aligned} \tag{18}$$

for some joint distribution $p(x_1, x_2, x_r, x_d)$.

The optimal input distribution that maximizes the rate region in (18) is jointly Gaussian for the Gaussian channel in (1). This is because the distribution of (Z_1, Z_2, Z_r, Z_d) is $\mathcal{CN}(0, I_{4\times4})$ and by using entropy power inequality [16], it is easy to show that the rate region in (18) is maximized if (X_1, X_2, X_r, X_d) is jointly Gaussian, i.e., $(X_1, X_2, X_r, X_d) \sim \mathcal{N}(0, \Sigma)$ where Σ is the covariance matrix given as

$$\Sigma = \mathrm{cov}(X_1, X_2, X_r, X_d) \tag{19}$$
$$= \begin{bmatrix} P_1 & 0 & \delta_{1r}\sqrt{P_1P_r} & \delta_{1d}\sqrt{P_1P_d}\\ 0 & P_2 & \delta_{2r}\sqrt{P_2P_r} & \delta_{2d}\sqrt{P_2P_d}\\ \delta_{1r}\sqrt{P_1P_r} & \delta_{2r}\sqrt{P_2P_r} & P_r & \delta_{rd}\sqrt{P_rP_d}\\ \delta_{1d}\sqrt{P_1P_d} & \delta_{2d}\sqrt{P_2P_d} & \delta_{rd}\sqrt{P_rP_d} & P_d \end{bmatrix}$$

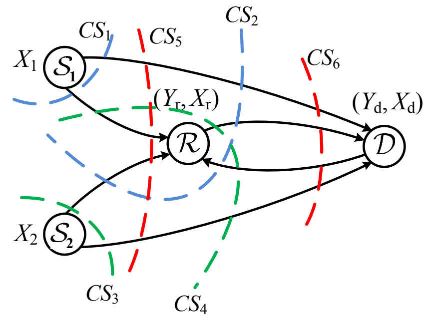

Fig. 6. The 6 cutsets for MARC-RDC.

where δ_{mn} is the correlation factor between X_m and X_n, $(\delta_{1r}, \delta_{1d}, \delta_{2r}, \delta_{2d}, \delta_{rd}) \in [-1, +1]$ and $\delta_{12} = 0$ since X_1 and X_2 are independent. The determinant $\det(\Sigma) \geq 0$ such that Σ is positive semi-definite and a valid covariance matrix. By applying the rate constraints in (18) on the Gaussian channel in (1) with Σ in (19), we obtain the cut set bound in (10).

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