Secondary User Interference Characterization for Underlay Networks

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***Abstract*—In an underlay cognitive radio network, the secondary (underlay) transmitters generate interference to a primary receiver, while an underlay receiver is subject to interference from both the primary transmitters and other underlay transmitters not associated with it. Although guard regions, maximum allowable underlay transmit powers, and contention distances help guarantee a minimum performance to the primary network, no such safeguard exists for the underlay network. To this end, this paper characterizes the aggregate interference on an underlay receiver while considering power control and receiver association schemes for both networks. Transmitters and receivers of both networks are assumed to be distributed as independent Poisson fields in the 2-D plane, and all links undergo exponential path loss and Rayleigh fading. We derive the moment generating function of the aggregate interference on an underlay receiver and its outage probability. We show that the interference from the primary network does not depend on any node density, and that it dominates the aggregate interference. Furthermore, it is shown that increasing primary and underlay receiver densities reduce the outage probability under lower required power thresholds for the primary receivers.**

I. Introduction

Cognitive radio (CR) networks allow opportunistic access to pre-licensed frequency blocks as a solution to spectrum scarcity. One variant, namely underlay networks allow concurrent spectrum access for secondary users (underlay users) with the licensed primary users (primary users) [1], and have a wide range of applications. Although concurrent primary-secondary transmissions make underlay networks appealing, the mutual interference this generates is also its major drawback. Exclusion regions around the primary system's receivers where underlay transmissions are not allowed, maximum transmit power thresholds for the underlay transmitters, and interference temperature based channel access can be used to limit the interference caused to the primary system. However, no such safeguards exist for the underlay system. An underlay receiver will get interference from both primary transmissions and other underlay transmissions.

Power control schemes, receiver association schemes, random spatial locations of nodes, and channel impairments significantly affect the level of interference. Modern wireless systems employ power control schemes which can be divided into as fixed, distance based, and measurement based schemes [2], and receiver association schemes based on the transmitter-receiver distance and the received signal to noise ratio (SNR) [3]. As such, interference to the underlay system needs to be characterized considering all these factors in order to assess its performance and to optimize it. Therefore, this paper analyzes the interference on an underlay receiver from both primary and other underlay transmissions for a distance based power control scheme and two underlay receiver association schemes while considering channel conditions using a stochastic geometry based approach.

Interference characterization and modeling of underlay networks has received a lot of research interest in academia. Reference [4] develops interference models for CR networks employing power control, contention control, and hybrid power-contention control schemes. Performance metrics such as the success probability, spatial average rate, and area spectral efficiency are derived for cellular and underlay d2d users under Rician fading in [5], and an analytical framework to characterize the area spectral efficiency of a large Poisson underlay CR network is proposed in [6]. Furthermore, [7] proposes an adaptive power control scheme for CR systems in a Rayleigh fading channel, while [8] proposes a limited feedback based underlay spectrum sharing scheme for Poisson cognitive networks. Interference experienced by a primary receiver is characterized in [9] considering constraints on a secondary (underlay) transmitter from both primary and secondary systems.

Although several previous works consider interference issues in underlay networks, most consider only the interference to the primary network. Meanwhile, some works do not consider power control and receiver association schemes for the two networks. However, in practical systems, power control and receiver association schemes take place. Furthermore, there are other works which don't consider the locations and numbers of transmitter/receiver nodes as random, while others restrict the system model to a single primary transmitter receiver pair. Reference [10] has done significant work in characterizing the interference and outage for both primary and secondary users in Poisson CR networks. But, it does not consider power control and receiver association schemes.

In this paper, we consider independent homogeneous Poisson point processes (PPPs) in $\mathbb{R}^2$ for primary and underlay devices, and an exclusion region is enforced around all primary receivers. Both the primary and underlay transmitters employ a distance based transmit power control scheme, while the underlay devices have the added constraint of having to have receivers within a maximum allowable transmit distance. For the primary system, receivers would associate with the nearest transmitter. For the underlay system, we consider two association schemes where 1) a transmitter initiates a connection with the closest receiver, and 2) a receiver initiates an association with the closest transmitter. Under this system, we characterize the interference on an associated underlay receiver by deriving the moment generating function (MGF). The MGF allows the sum rate and moments to be obtained easily. Moreover, the performance of the underlay receiver is analyzed using the outage probability.

Notations: $\Gamma(x,a) = \int_a^{\infty} t^{x-1}e^{-t}dt$ and $\Gamma(x) = \Gamma(x,0)$ [11]. $\Pr[A]$ is the probability of event A, $f_X(\cdot)$ is the probability density function (PDF), $F_X(\cdot)$ is the cumulative distribution function (CDF), $M_X(\cdot)$ is the MGF, and $E_X[\cdot]$ denotes the expectation over random variable X.

II. System model

This section describes the spatial distribution of the network as well as the signal model.

A. Spatial model

The system consists of four different types of devices; namely primary transmitters, primary receivers, underlay transmitters, and underlay receivers. We consider all types of devices to be distributed in $\mathbb{R}^2$. Because the locations of the nodes are random, a stochastic geometry based approach is needed for modeling [12]. Although the assumption of transmitter-only and receiver-only devices hold true for primary networks, underlay networks can have devices which may switch from being a transmitter to a receiver and vice versa. However, using a stochastic model helps to incorporate this switching. To this end, we will use the PPP to model each type [13]. Modeling random node distributions with PPPs has received significant popularity in the fields of cognitive networks, d2d networks, and heterogeneous networks [4], [14]. In order to be provide a general analysis and to avoid special cases, we would assume distributions of each device type to be independent of each other and homogeneous, where a homogeneous PPP is one such that the intensity measure doesn't change with location. Let the processes of primary transmitters, primary receivers, underlay transmitters, and underlay receivers respectively be $\Phi_{p,t}, \Phi_{p,r}, \Phi_{s,t}$, and $\Phi_{s,r}$ with intensities $\lambda_{p,t}, \lambda_{p,r}, \lambda_{s,t}$, and $\lambda_{s,r}$. We also assume that the primary transmitter and underlay transmitter densities are significantly lower than the primary and underlay receiver densities. This is a valid assumption because practically, primary networks are terrestrial television networks or cellular networks where a base station provides coverage to a multitude of receivers.

One significant aspect of underlay networks is the guard region [10], [14]. This is a physical region around either the primary transmitters or the primary receivers where no underlay transmitters are active. This region helps to ensure that the interference experienced by primary receivers is limited. Underlay nodes can be aware of the guard region either through a centralized process where dynamic location information is made available through the network itself, or via a distributed mechanism where each underlay node senses the spectrum for pilot signals transmitted from the primary receivers. We will assume that a guard region exists around each primary receiver having a constant radius of R_G, and that the underlay nodes are fully aware of it.

B. Signal model

We assume that all signals undergo Rayleigh fading and path loss. The path loss model of [15] is used where the received power P_R at a distance r from the transmitter is $P_R = Pr^{-\alpha}$, where P is the transmit power level and α is the path loss exponent. The fading between any transmitter receiver pair is assumed to be independent and identically distributed. Moreover, the fading is independent of the underlying PPP. Under Rayleigh fading, the channel power gain $|h|^2$ is distributed as $f_{|h|^2}(x) = e^{-x}, 0 < x < \infty$.

C. Power control and transmitter-receiver association

1) Primary network: In the primary network, the receiver (user) would be initiating the transmission attempt. It would select the primary transmitter (base station) which is closest to it. Associating with the closest transmitter provides the best average received power (after averaging the effects of fading), and are valid for primary networks such as digital terrestrial television. However, within a certain resource block (time-frequency), there can only be one receiver associated with the transmitter. Other receivers who initiate transmission would be assigned separate resource blocks if available. A distance based power control scheme would be used where the path loss effects would be inverted in order to provide a constant average receiver power level $P_{c,p}$ [4]. Furthermore, we assume that there is no cut-off transmit power level (A cut-off transmit power level would either be the maximum allowable transmit power level of the network or the maximum possible transmit power of the device depending on its power source).

2) Underlay network: We will consider 2 different association policies: 1) the receiver initiates the communication, and selects the closest available transmitter, and 2) the transmitter initiates the communication and selects the closest available receiver. These policies are valid for networks such as ad-hoc and d2d networks. For both aforementioned schemes, the selection of transmitters and receivers would only occur whenever they are within a distance of D from the initiating receiver or transmitter. A distance based power control scheme would again be assumed where the required receiver power level is denoted as $P_{c,s}$. The maximum allowable distance D ensures that the transmit power level would not increase arbitrarily, and is analogous to having a maximum cut-off power level.

III. Outage analysis

Within this section, we derive the outage probability of an underlay receiver which is already conducting a transmission. For the receiver in question, we assume that an association has already been made successfully.

Let r be the distance between the underlay receiver and the associated transmitter. The received power from the associated transmitter P_R is written as $P_R = P_{c,s}r^{\alpha}r^{-\alpha}|h|^2 = P_{c,s}|h|^2$. Thus, given that an association has occurred, the association policy for the underlay network nor r play any role in the received power from the associated transmitter. Signals from other underlay transmitters and all primary transmitters generate interference. Let I_p, I_s, and σ^2 be the interference from the primary and underlay networks, and the noise variance respectively. The signal to interference and noise ratio (SINR) at the underlay receiver (γ) is written as $\gamma = \frac{P_{c,s}|h|^2}{I_p+I_s+\sigma^2}$. The CDF of the SINR ($F_\gamma(x)$) can be written as

$$F_\gamma(x) = 1 - e^{(-\frac{x\sigma^2}{P_{c,s}})} M_{I_p}\left(\frac{x}{P_{c,s}}\right) M_{I_s}\left(\frac{x}{P_{c,s}}\right). \quad (1)$$

Substituting the required SINR threshold (T) instead of x gives us the outage probability. However, in order to evaluate this, the MGFs of the interference from primary transmitters and other underlay transmitters are needed. Those would be derived in the next subsections.

A. Interference from the primary network

The total interference from the primary network I_p can be written as $I_p = \sum_{i \in \Phi_{p,t}} I_{p,i}$, where $I_{p,i}$ is the interference from the i-th primary transmitter. We assume that there is always a receiver connected to the transmitter within the resource block in question because $\lambda_{p,t} << \lambda_{p,r}$. When the resource block is vacant for certain transmitters, our assumption provides the worst case scenario for the interference.

We can write $I_{p,i}$ as $I_{p,i} = P_p|h|^2 r_p^{-\alpha}$, where P_p is the transmit power of a primary transmitter and r_p is the distance from a primary transmitter to the underlay receiver in question. Because an infinite network is considered, there is no loss of generality in taking the underlay receiver for which the performance is investigated to be at the center. As such, the 1-D intensity of the primary transmitters with respect to the underlay receiver ($\tilde{\lambda}_{p,t}$) can be written as $\tilde{\lambda}_{p,t} = 2\pi\lambda_{p,t} r_p$.

The MGF of I_p is defined as $M_{I_p}(s) = E[e^{-sI_p}]$. By the Campbell's theorem [13], $M_{I_p}(s)$ is written as

$$M_{I_p}(s) = e^{\left(\int_0^\infty E\left[e^{-sP_p|h|^2 r_p^{-\alpha}} - 1\right] 2\pi\lambda_{p,t} r_p dr\right)}, \quad (2)$$

where the expectation is with respect to $|h|^2$ and P_p. In order to evaluate (2), the distribution of P_p needs to be derived.

Let the distance between a primary transmitter, and the associated receiver be $r_{p,tx}$. Then, according to the distance dependent power control strategy, $P_p = P_{c,p} r_{p,tx}^\alpha$. In the primary network, each receiver connects to it's closest transmitter. As such, the primary network can be envisioned as a set of voronoid cells where each receiver within a cell can connect to the corresponding transmitter. Using the distribution of the nearest node from a point within a PPP [16], the distance distribution $r_{p,tx}$ can be found out to have the approximate PDF

$$f_{r_{p,tx}}(x) \approx 2\pi\lambda_{p,t} x e^{-\pi\lambda_{p,t}x^2}. \quad (3)$$

It should be noted that (3) is not the exact PDF of $r_{p,tx}$. If a particular receiver is closer to a transmitter, it would bring forth a dependence.(3) has been derived for any transmitter assuming it was independent from all other transmitters which is not the case in reality. Furthermore, there is the non zero probability that $r_{p,tx}$ would not exist for a particular transmitter. This would occur when there are no receivers associated with a transmitter. However, this probability is trivial when $\lambda_{p,r} >> \lambda_{p,t}$, which is the case in practice. Fig. 1 compares the theoretical CDF of $r_{p,tx}$ (3) with simulated CDFs for two different primary transmitter densities ($\lambda_{p,t}$). It is seen that the discrepancy under $\lambda_{p,t} = 1 \times 10^{-5}$ is lower than that when $\lambda_{p,t} = 1 \times 10^{-6}$.

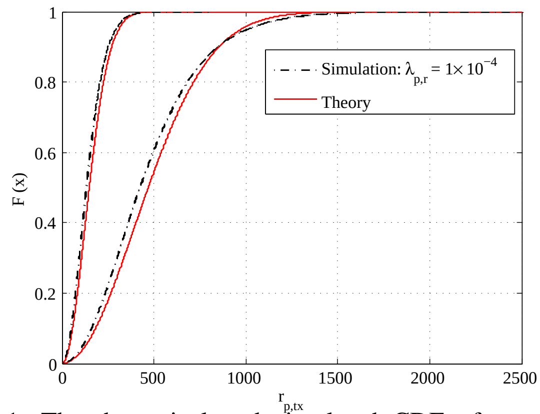

Fig. 1: The theoretical and simulated CDF of $r_{p,tx}$ under different primary transmitter and receiver densities.

Coming back to the original objective of finding $M_{I_p}(s)$, we can perform the expectation on (2) with respect to $|h|^2$ and obtain $M_{I_p}(s) = e^{\left(\int_0^\infty E\left[\frac{1}{1+sP_{c,p}r_{p,tx}^\alpha r_p^{-\alpha}} - 1\right] 2\pi\lambda_{p,t} r_p dr_p\right)}$, where the remaining expectation is with respect to $r_{p,tx}$. When $\alpha > 2$, changing the order of integration and averaging results in

$$M_{I_p}(s) = e^{\left(-\frac{2\pi^2\lambda_{p,t}}{\alpha}\frac{(sP_{c,p})^{\frac{2}{\alpha}}}{\sin\left(\frac{2\pi}{\alpha}\right)} E\left[r_{p,tx}^2\right]\right)} = e^{\left(-\frac{2\pi}{\alpha}\frac{(sP_{c,p})^{\frac{2}{\alpha}}}{\sin\left(\frac{2\pi}{\alpha}\right)}\right)}. \quad (4)$$

The mean interference $E[I_p]$ is another important performance measure. From the Campbell's theorem, we can write $E[I_p]$ as

$$E[I_p] = \int_0^\infty E_{|h|^2, r_p, r_{tx}}\left[P_{c,p}|h|^2 r_{p,tx}^\alpha r_p^{-\alpha}\right] 2\pi\lambda_{p,t} r_p dr_p. \quad (5)$$

The integration in (5) does not necessarily converge because the simplified path loss model doesn't hold when $r_p < 1$. With the simplified path loss model, the received power would up to ∞ for $r_p < 1$. As such, we will take the path loss to be 1 when $r_p < 1$. Moreover, in practical channels $\alpha > 2$, Using these facts and breaking the integration in (5) into two separate parts, we obtain $E[I]$ as

$$E[I_p] = 2P_{c,p}\frac{\Gamma(\frac{\alpha}{2}+1)}{(\pi\lambda_{p,t})^{\frac{\alpha}{2}-1}}\left(\frac{1}{\alpha-2} + \frac{1}{2}\right). \quad (6)$$

B. Interference form the underlay network

We will now look at I_s which is the interference from other underlay nodes.

As mentioned in Section II, underlay transmitters should be inactive whenever they are within the guard region of a primary receiver. Therefore, the active underlay transmitters actually form a Poisson hole process [10]. However, the Poisson hole processes is not mathematically tractable. Instead, we can model the active underlay transmitters as an independently thinned PPP using the coloring theorem [13]. If $\bar{\lambda}_{s,t}$ is the density of the active underlay transmitters, it can be written as $\bar{\lambda}_{s,t} = \mu\lambda_{s,t}$, where μ is the probability that any particular transmitter doesn't fall within the guard region of a primary receiver. Using the void probability, μ is obtained as $\mu = e^{-\pi\lambda_{p,r}R_G^2}$, and $\bar{\lambda}_{s,t} = \lambda_{s,t}e^{-\pi\lambda_{p,r}R_G^2}$. We will now derive the MGF of I_s for two different association schemes.

1) Receiver selects the closest transmitter: In this scheme, a receiver selects the closest transmitter to associate. We assume that all available transmitters (with a density of $\bar{\lambda}_{s,t}$) are associated with a receiver. If not, our analysis would be deriving the worst case performance. Let the process of available underlay transmitters be denoted as $\bar{\Phi}_{s,t}$. If the receiver for which performance is analyzed is connected to the underlay transmitter $z \in \bar{\Phi}_{s,t}$, the total interference from the underlay network is written as $I_s = \sum_{i\in\bar{\Phi}_{s,t}\setminus z} I_{s,i}$, where $I_{s,i}$ is the interference from the i-th underlay transmitter. $I_{s,i}$ is written as $I_{s,i} = \mathcal{B}P_s|h|^2 r_s^{-\alpha}$, where P_s is the transmit power of an underlay transmitter defined as $P_s = P_{c,s}r_{s,tx}^{\alpha}$ and $r_{s,tx}$ is the distance between an underlay transmitter and the associated receiver. r_s is the distance from an interfering underlay transmitter to the underlay receiver in question, and $\mathcal{B}$ is a Bernoulli random variable taking on the value 1 when $r_{s,tx} < D$, and 0 otherwise.

Using the same technique used to obtain the distribution of $r_{p,tx}$, the distribution of $r_{s,tx}$ can be shown to have the approximate PDF $f_{r_{s,tx}}(x) \approx 2\pi\bar{\lambda}_{s,t}xe^{-\pi\bar{\lambda}_{s,t}x^2}$. Let β be the probability that $r_{s,tx} < D$. Then, $\beta = 1 - e^{-\pi\bar{\lambda}_{s,t}D^2}$.

Using the Campbell's theorem, we can write $M_{I_s}(s)$ as

$$M_{I_s}(s) = e^{\left(\int_r^{\infty} E_{r_{s,tx}}\left[1-\beta+\frac{\beta}{1+sP_{c,s}r_{s,tx}^{\alpha}r_s^{-\alpha}}-1\right]2\pi\bar{\lambda}_{s,t}r_s dr_s\right)}, \quad (7)$$

where $r(< D)$ is the distance between the receiver in question and the associated transmitter. For any given associated receiver, r is deterministic. A closed-form solution for (7) is not apparent, and can be solved using numerical techniques. A simplified equation for $M_{I_s}(s)$ obtained after some manipulations and a series expansion when $sP_{c,s}r_{s,tx}^{\alpha}r_s^{-\alpha} < 1$ as (8). This method works when $s << \frac{1}{P_{c,s}}$, and this condition is satisfied for practical system parameters. Furthermore, the mean interference $E[I_s]$ can be derived as (9).

For the special case when we don't take r to be deterministic, and an averaged value is needed, it has the distribution $f_r(x) = \frac{2\pi\bar{\lambda}_{s,t}xe^{-\pi\bar{\lambda}_{s,t}x^2}}{1-e^{-\pi\bar{\lambda}_{s,t}D^2}}, 0 < r < D$. Therefore, if (8) is written as $M_{I_s}(s) = e^{\mathcal{W}}$, $M_{I_s}(s)$ for non-deterministic r becomes $M_{I_s}(s) = e^{E_r[\mathcal{W}]}$. However, $E_r[\mathcal{W}]$ does not necessarily converge for $k > 1$. But, when $sP_{c,s} << 1$ and $\alpha < 4$, the summation in (8) can be accurately approximated by the first term. As such, the MGF can be written as (10).

2) Transmitter selects the closest receiver: Now, we look at the scenario where the association attempt is initiated by the underlay transmitter corresponding to a situation where those nodes are the data generators. Within this scheme, an available underlay transmitter ($\in \bar{\Phi}_{s,t}$) selects the nearest underlay receiver to associate with. Again, we assume that all underlay transmitters are associated with a particular receiver, and that occurrences of multiple transmitters associating with a single receiver are permissible.

Let the underlay receiver for which performance is analyzed be connected to the underlay transmitter $z \in \bar{\Phi}_{s,t}$. The total interference from the underlay network is written similar to the previous scheme as $I_s = \sum_{i\in\bar{\Phi}_{s,t}\setminus z} I_{s,i}$. $I_{s,i}$ is written as $I_{s,i} = \mathcal{C}P_s|h|^2 r_s^{-\alpha}$, where P_s is the transmit power of an underlay transmitter defined as $P_s = P_{c,s}r_{s,rx}^{\alpha}$. $r_{s,rx}$ is the distance between an underlay transmitter and the closest receiver and $\mathcal{C}$ is a Bernoulli random variable taking on the value 1 when $r_{s,rx} < D$, and 0 otherwise.

The distance $r_{s,rx}$ has the distribution $f_{r_{s,rx}}(x) = 2\pi\lambda_{s,r}xe^{-\pi\lambda_{s,r}x^2}$. Let μ be the probability that $r_{s,rx} < D$. Then, $\mu = 1 - e^{-\pi\lambda_{s,r}D^2}$.

Unlike the previous scheme, a complication arises when evaluating the MGF of I_s. Although a transmitter selects its closest receiver node, from the point of the underlay receiver for which performance is evaluated, transmitter z is not the closest transmitter in general. As such, we will approximate I_s as $I_s \approx \sum_{i\in\bar{\Phi}_{s,t}} I_{s,i}$. In effect, our approximation gives an upper bound on the interference and thus the outage.

With the Campbell's theorem, the MGF of the interference from underlay nodes $M_{I_s}(s)$ is written as $M_{I_s}(s) = e^{\left(\int_0^{\infty} E_{r_{s,rx}}\left[1-\mu+\frac{\mu}{1+sP_{c,s}r_{s,rx}^{\alpha}r_s^{-\alpha}}-1\right]2\pi\bar{\lambda}_{s,t}r_s dr_s\right)}$. When $\alpha > 2$, the MGF becomes

$$M_{I_s}(s) = e^{\left(-\frac{2\pi\bar{\lambda}_{s,t}e^{-\pi\lambda_{s,r}D^2}}{\alpha\lambda_{s,r}}\frac{(sP_{c,s})^{\frac{2}{\alpha}}}{\sin\left(\frac{2\pi}{\alpha}\right)}\left(e^{\pi\lambda_{s,r}D^2}-\pi\lambda_{s,r}D^2-1\right)\right)}. \quad (11)$$

The mean interference $E[I_s]$ does not converge while using the simplified path loss model. Therefore, as we did when obtaining $E[I_p]$, we will amend the path loss function (denoted as $g(r)$) to $g(r) = \min(r_s^{-\alpha}, 1)$. Using this, $E[I_s]$ can be obtained as

$$E[I_s] = \frac{\pi\alpha\bar{\lambda}_{s,t}P_{c,s}}{(\alpha-2)(\pi\lambda_{s,r})^{\frac{\alpha}{2}}}\left(\Gamma\left(\frac{\alpha}{2}+1\right)-\Gamma\left(\frac{\alpha}{2}+1, \pi\lambda_{s,r}D^2\right)\right). \quad (12)$$

IV. Numerical Results

This section provides numerical results for an underlay receiver's outage probability. We will use the parameters $\lambda_{s,t} = 1 \times 10^{-5}$, $\lambda_{p,t} = 1 \times 10^{-5}$, $r = 50$, $R_G = 20$, $P_{c,s} = 1\times 10^{-8}$, and $\sigma^2 = 0$. The noise variance has been set to 0 in order to highlight the effect of interference. We will denote the underlay association scheme where the transmitter selects the closest receiver as Scheme 1, and the scheme where the receiver selects the closest transmitter as Scheme 2.

Fig. 2 plots the outage probability of an underlay receiver with respect to the required SINR threshold. Although there is a significant difference of the outage probabilities for different α when the threshold (T) is low, the curves converge to 1 as expected when T increases. The outage increase for higher α occurs primarily due to the power control procedures which require an inversion of the path loss. Although the outage of Scheme 2 is higher, the difference is not significant because the main source of interference is the primary network.

The underlay receiver outage is plotted vs. the required primary receiver power level $P_{c,p}$ in Fig. 3. The plots diverge for lower $P_{c,p}$ due to interference from the primary network playing a less dominant role. The outage probabilities drop significantly when the primary and underlay receiver densities are increased. For Scheme 1, this is due to the guard region

$$M_{I_s}(s) = e^{\left(2\pi\bar{\lambda}_{s,t}\sum_{k=1}^{\infty}\frac{(-sP_{c,s})^k}{(\alpha k-2)(\pi\bar{\lambda}_{s,t})^{\frac{\alpha k}{2}}} r^{2-\alpha k}\left(\Gamma\left(\frac{\alpha k}{2}+1\right)-\Gamma\left(\frac{\alpha k}{2}+1,\pi\bar{\lambda}_{s,t}D^2\right)\right)\right)} \tag{8}$$

$$E[I_s] = \frac{(2\pi\bar{\lambda}_{s,t}P_{c,s}}{(\alpha-2)(\pi\bar{\lambda}_{s,t})^{\frac{\alpha}{2}}} r^{2-\alpha}\left(\Gamma\left(\frac{\alpha}{2}+1\right)-\Gamma\left(\frac{\alpha}{2}+1,\pi\bar{\lambda}_{s,t}D^2\right)\right) \tag{9}$$

$$M_{I_s}(s) = e^{\left(\frac{-2sP_{c,s}}{(\alpha-2)(1-e^{-\pi\bar{\lambda}_{s,t}D^2})}\left(\Gamma\left(\frac{\alpha}{2}+1\right)-\Gamma\left(\frac{\alpha}{2}+1,\pi\bar{\lambda}_{s,t}D^2\right)\right)\left(\Gamma\left(2-\frac{\alpha}{2}\right)-\Gamma\left(2-\frac{\alpha}{2},\pi\bar{\lambda}_{s,t}D^2\right)\right)\right)} \tag{10}$$

surrounding each primary receiver. For Scheme 2, in addition to the aforementioned reason, the distance from an interfering underlay transmitter to its associated receiver reduces; causing the transmit power to reduce. Moreover, when the maximum allowable transmit distance D increases, the outage increases because more associations (requiring higher transmit power) are successful. It is also interesting to note that Scheme 1 shows a worse outage performance compared to Scheme 2 when $D = 200$ and $\lambda_{p,r} = \lambda_{s,r} = 1 \times 10^{-3}$.

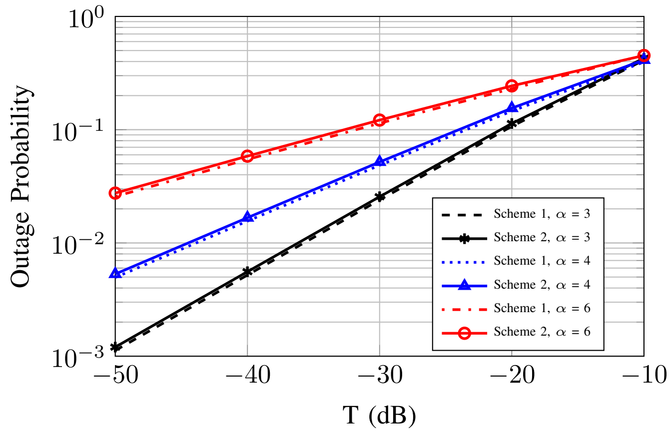

Fig. 2: The underlay receiver outage probability vs. the required SINR threshold T under different path loss exponents α for the two underlay association schemes. $D = 100$, $P_{c,p} = 1 \times 10^{-8}$, $\lambda_{p,r} = 1 \times 10^{-4}$, and $\lambda_{s,r} = 1 \times 10^{-4}$.

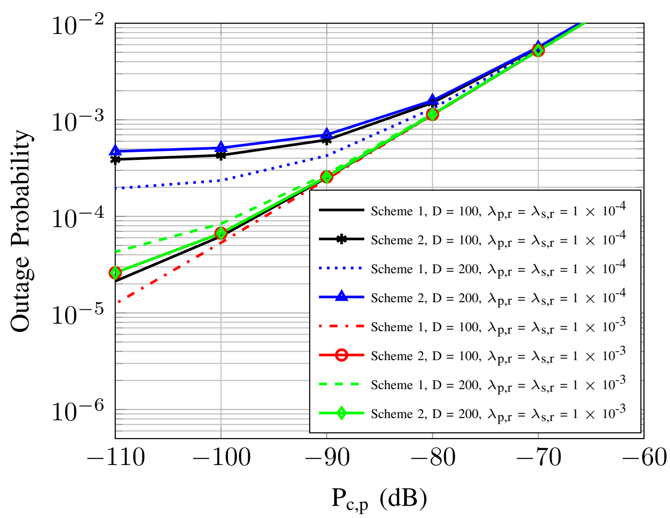

Fig. 3: The underlay receiver outage probability vs. $P_{c,p}$ under different $\lambda_{p,r}$, $\lambda_{s,r}$, and D for the two underlay association schemes. $\alpha = 3$, and $T = 0.0001$.

V. Conclusion

This paper analyzed the aggregate interference on an underlay receiver from primary and other underlay transmitters, where all types of nodes form independent PPPs over the entire 2-D space. We considered a distance based power control scheme based on path loss inversion and a receiver association scheme for the primary network. Moreover, for the underlay network, two association schemes were analyzed along with a distance based power control scheme subject to a maximum allowable transmit distance while incorporating guard regions. The MGF of the aggregate interference on an underlay receiver and its outage probability were derived. The interference from the primary system was shown to be independent of the node densities. Furthermore, when the required power threshold for the primary receiver ($P_{c,p}$) is comparative with the required underlay threshold ($P_{c,s}$), interference from the primary system dominates, and the path loss exponent greatly affects the outage. However, when $P_{c,p} < P_{c,s}$, the outage is significantly affected by the receiver densities and the maximum allowable underlay transmit distance.

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