Approximations for Performance of Energy Detector and p-norm Detector

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***Abstract*—Although the approximations based on the central limit theorem (CLT) for the detection probability (P_d) of the energy detector and of its more generalized cousin, the p-norm detector, are accurate for large sample sizes, their accuracy is poor otherwise. A recent work has addressed this problem by developing a cube-of-Gaussian approximation (CGA). However, CGA may not be the only option and thus this letter investigates five other classical approximations for deriving P_d. They tightly match the exact values even for few samples and thus are more accurate than CLT. These approximations have been unnoticed in the signal detection research community and this letter aims at making them known to a wider audience. Because the range of their potential applications could be diverse, to demonstrate their utility, we derive a novel expression for the area under the receiver operating characteristic curve of the p-norm detector.**

***Index Terms*—Detection probability, energy detector, false alarm probability, p-norm detector.**

I. Introduction

As the IEEE 802.22 based cognitive radios must detect potential spectrum opportunities rapidly, sensing devices such as the energy detector (ED) or the more general p-norm detector [1] must operate with fewest possible samples while offering high detection reliability. However, their probability of detection, P_d, is widely approximated by using the central limit theorem (CLT) [2] which yields P_d in terms of the well-known Gaussian-Q function. The CLT-based approximation has found widespread applications in solving practical problems such as sensing-throughput tradeoff [3], multiple-band spectrum sensing [4], low signal-to-noise ratio (SNR) spectrum sensing [5], [6], and numerous others. However, it is not accurate enough for small sample sizes [7]. Small sample size based analysis is particularly important for highly delay-sensitive applications such as mission critical machine-type communications, say, in future 5G networks [8].

Exact P_d (without approximations) has also been analyzed extensively. For example, works in [9]–[13] treat ED with fading, shadowing, multiple antennas, cooperative diversity, and other factors. For the more general p-norm detector, since exact closed-form P_d appears intractable, several computational methods have been developed [14]. However, these analyses often lead to complicated expressions (residues, infinite series, and so on) rather than closed-forms, which may hinder their rapid use in optimization and low-SNR design [3]–[6].

Thus, simple and accurate P_d approximations valid for arbitrary sample size are necessary. To the best of our knowledge, only the work in [7] has attacked this problem with a cube-of-Gaussian approximation (CGA), originally proposed by Abdel-Aty [15] for approximating non-central chi-square distributions. However, the CGA is not the only option, and other more robust approximations may exist. Thus, in this paper, we investigate five other classical approximations to derive P_d. These approximations (i) yield accurate P_d in closed form; (ii) apply to an arbitrary number of samples; (iii) need only the first few moments/cumulants, and thus are applicable to deterministic or random signal model; and (iv) may be extended to other applications. For example, the p-norm detector is a generalization of the ED [1], [6], [14] for which we will derive a novel approximation for the area under the receiver operating characteristic (ROC) curve (AUC), a single figure of merit useful for characterizing the detection performance.

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Notations

$f_{\mathcal{Z}}(\cdot)$, $\mathbb{P}(\cdot)$, $\mathbb{E}(\cdot)$, $\mathrm{Var}(\cdot)$ denote the probability density function (PDF) of $\mathcal{Z}$, probability measure, expectation, and variance, respectively. χ_k^2 and $\chi_k^2(\epsilon)$ denote the central and non-central chi-square distributions, respectively, with k being the degree of freedom and ϵ being the non-centrality parameter. $\mathcal{CN}(m,\sigma^2)$ denotes complex Gaussian variable with mean m and variance σ^2. $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$, and $\Gamma(z,x) = \int_x^\infty t^{z-1}e^{-t}dt$ are the Gamma function and upper-incomplete Gamma function, respectively. $Q_l(a,b) = \int_b^\infty \frac{x^l}{a^{l-1}} e^{-(x^2+a^2)/2} I_{l-1}(ax)dx$ is the l-th order generalized Marcum-Q function with $I_\nu(z) = \sum_{k=0}^\infty \frac{(z/2)^{2k+\nu}}{k!\Gamma(\nu+k+1)}$ denoting the modified Bessel function of first kind of order ν. $Q(x) = \int_x^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}}dt$ is the Gaussian-Q function. ${}_1\tilde{F}_1(a;b;z) = \sum_{k=0}^\infty \frac{(a)_k z^k}{\Gamma(b+k)k!}$ is the regularized hypergeometric function and ${}_2F_1(a,b;c;z) = \sum_{k=0}^\infty \frac{(a)_k(b)_k z^k}{(c)_k k!}$ is the Gauss hypergeometric function, with $(x)_y$ denoting the Pochhamer symbol.

II. Problem Statement

In an ED, the decision variable is given as $T = \frac{1}{n}\sum_{i=1}^n |y_i|^2$, where y_i is the i-th sample of the received signal and n is the number of samples. Then, T is compared against a detection threshold λ to decide on the absence (H_0) or presence (H_1) of the primary user (PU) signal. For deterministic signals, false alarm probability, P_f, is $P_f = \Gamma(n,\lambda/2)/\Gamma(n)$, while detection probability, P_d, is [9]

$$P_d = Q_n(\sqrt{2\gamma}, \sqrt{\lambda}), \tag{1}$$

where γ is the SNR. The conditional decision variables $T|H_0$ and $T|H_1$ are distributed as χ_{2n}^2 and $\chi_{2n}^2(2\gamma)$, respectively. Thus, the problem at hand is to find accurate approximations to (1) for arbitrary sample size, unlike CLT, which is limited to large sample size ($n \gg 1$).

III. New Approximations

As expression (1) results from the complementary cumulative distribution function (CCDF) of $T|H_1$ which is $\chi_{2n}^2(2\gamma)$ distributed, we seek accurate approximations for this CCDF by approximating the distribution of $T|H_1$. Let Y denote $T|H_1$. Among the numerous approximations available in the statistical literature, we next consider five important ones to find accurate approximations to the distribution of Y [16]–[20]. Note that for some of our derivations, the cumulant generating function of Y is needed which is obtained as [21] $g(t) = \ln \mathbb{E}\left(e^{tY}\right) = t/(1-2\gamma t) - n\ln(1-2\gamma t), \quad t \le 1/(2\gamma)$. Then, the Taylor expansion of $g(t)$ yields the first three cumulants of Y to be $\kappa_1 = 2(n+\gamma)$, $\kappa_2 = 4(n+2\gamma)$, $\kappa_3 = 16(n+3\gamma)$. The r-th moment of Y is $m_r = 2^r e^{-\gamma}\Gamma(r+n)\,{}_1\tilde{F}_1(r+n;n;\gamma)$. Next, we present the five approximations for deriving P_d.

A. Patnaik's approximation

Patnaik [16] proposed approximating the non-central chi-square random variable (RV) Y by a scaled central one denoted $Z_1 \sim \chi^2_v$, as

$$Y \approx \rho Z_1. \tag{2}$$

The problem then is to find the scaling factor ρ and the new degree of freedom v. By matching the first two moments of both sides of (2), these parameters are found to be $\rho = 2(n+2\gamma)/(n+\gamma)$ and $v = 2(n+\gamma)^2/(n+2\gamma)$. Then, the approximate detection probability, denoted P_d^{pn}, can be expressed as

$$P_d^{\text{pn}} = \mathbb{P}(\rho Z_1 > \lambda) = \int_{\lambda/\rho}^{\infty} f_{Z_1}(z)dz = \frac{1}{\Gamma(v/2)}\Gamma\left(\frac{v}{2}, \frac{\lambda}{2\rho}\right). \tag{3}$$

Clearly, P_d^{pn} is in terms of the upper-incomplete Gamma function (as opposed to the more complicated Marcum-Q function (1)) and thus may further lead to tractable analysis (e.g., when averaging P_d over fading, shadowing, antenna/cooperative diversity, or in other applications such as sensing-throughput tradeoff in cognitive radios) for arbitrary sample size unlike the CLT which is valid only for large sample sizes. The accuracy of (3) will be investigated in Section IV.

B. Pearson's approximation

Pearson's method is a generalization of Patnaik's idea where Y is approximated with a linear transformation of a central chi-square RV denoted $\mathcal{X} \sim \chi^2_{v'}$, as $Y \approx Z_2 = a\mathcal{X} + b$, where a, b and v' are to be determined via moment matching. By matching the first two moments on both sides, we get [17]

$$Z_2 = \frac{\mathcal{X} - v'}{\sqrt{2v'}}\text{Var}(Y) + \mathbb{E}(Y) = \frac{n+3\gamma}{n+2\gamma}\mathcal{X} - \frac{2\gamma^2}{n+3\gamma}. \tag{4}$$

Parameter v' is obtained by matching the third moments of Z_2 (4) and Y as $v' = 2(n+2\gamma)^3/(n+3\gamma)^2$. Then, the approximate detection probability, $P_d^{\text{ps}} = \mathbb{P}(a\mathcal{X} + b > \lambda)$, is obtained as

$$P_d^{\text{ps}} = \frac{1}{\Gamma(v'/2)}\Gamma\left(\frac{v'}{2}, \left(\lambda + \frac{2\gamma^2}{n+3\gamma}\right)\cdot\frac{n+2\gamma}{2(n+3\gamma)}\right). \tag{5}$$

Despite matching the first three moments, the final P_d^{ps} expression has a form similar to P_d^{pn} (3) and thus has the same computational ease. Intuitively, we expect (5) to be highly accurate as it utilizes the first three exact moments.

C. Three-parameter Gamma

This method approximates the PDF of Y by a three-parameter Gamma PDF [18] as

$$f_Y(y) = \frac{(y-\delta)^{\alpha-1}e^{-(y-\delta)/\beta}}{\beta^\alpha\Gamma(\alpha)}, \tag{6}$$

where $\delta < y < \infty$, $\alpha = 4\kappa_2^3/\kappa_3^2$, $\beta = \kappa_3/(2\kappa_2)$ and $\delta = \kappa_1 - 2\kappa_2^2/\kappa_3$. Thus, the parameters α, β and δ in (6) are readily obtained from the cumulants of Y. The approximate detection probability, P_d^{tg}, can hence be expressed as

$$P_d^{\text{tg}} = \int_{\lambda}^{\infty} f_Y(y)dy = \frac{1}{\Gamma(\alpha)}\Gamma\left(\alpha, \frac{\lambda-\delta}{\beta}\right). \tag{7}$$

Since both (7) and (5) utilize third-order statistics, their accuracy should be similar.

So far, we have presented all the approximations based on moment/cumulant matching. In contrast, the next two approximations are based upon finding a rapid transformation of Y that approaches a Gaussian RV. The basis is as follows. If we denote Y as $Y(n)$ (a function of n), then we know $Y(n)$ converges to a Gaussian RV when $n \to \infty$. However, this condition requires a large number of samples. Alternatively, if a transform $\mathcal{G}(\cdot)$ can be found which makes $\mathcal{G}(Y)$ Gaussian without the large-sample assumption, that forms the basis for approximating the non-central chi-square tail probability. Transforms of type $\mathcal{G}(x) = (ax+b)^\beta$ have been used in the literature as discussed in the following sub-sections.

D. Sankaran's third approximation

Following the basis above, Sankaran [19] proposed the following transformation of Y: $X = \left(\frac{Y}{2(n+\gamma)}\right)^h$ to be Gaussian with mean $\mu_{\text{sk}} = 1 + \frac{h(h-1)(n+2\gamma)}{2(n+\gamma)^2} - \frac{h(h-1)(2-h)(1-3h)(n+2\gamma)^2}{8(n+\gamma)^4}$, and variance $\sigma^2_{\text{sk}} = \frac{h^2(n+2\gamma)}{(n+\gamma)^2}\left[1 - (1-h)(1-3h)\frac{(n+2\gamma)}{2(n+\gamma)^2}\right]$, where $h = 1 - [2(n+\gamma)(n+3\gamma)]/[3(n+2\gamma)^2]$. As X is Gaussian distributed, the desired approximate detection probability, P_d^{sk}, can thus be obtained in terms of the Gaussian-Q function after some algebraic manipulations to be

$$P_d^{\text{sk}} = \mathbb{P}(Y > \lambda) = Q\left(\frac{\lambda^h - \mu_{\text{sk}}2^h(n+\gamma)^h}{\sigma_{\text{sk}}2^h(n+\gamma)^h}\right). \tag{8}$$

Like the CLT approximation, expression (8) avoids occurrence of the Marcum-Q function (in P_d), yet does not invoke the large-sample assumption needed for the CLT. Thus, expression (8) should work for any number of samples (low to high).

E. Moschopoulos' approach

This approach [20] exploits the following transformation (abusing the notation X) of Y: $X = \left(\frac{Y+b}{\kappa_1}\right)^h$, where h (abusing previous notations) and b are determined from the first three cumulants of Y as $h = 1 - \kappa_1\kappa_3/(3\kappa_2^2)$ and $b = \kappa_2/(2\kappa_1) - \kappa_3/(4\kappa_2)$. Then, X is Gaussian distributed with mean $\mu_{\text{mo}} = 1 + \frac{h}{\kappa_1}\left[\frac{(h-1)\kappa_2}{2\kappa_1} + b\right]$, and variance $\sigma^2_{\text{mo}} = h^2\kappa_2/\kappa_1^2$. Then, similar to (8), the desired approximate detection probability, P_d^{mo}, can be expressed as

$$P_d^{\text{mo}} = \mathbb{P}(Y > \lambda) = Q\left(\frac{(\lambda+b)^h - \kappa_1^h\mu_{\text{mo}}}{\kappa_1^h\sigma_{\text{mo}}}\right). \tag{9}$$

Since the detection probabilities (8) and (9) both utilize up to third-order statistics of Y and have the same functional complexity, these approximations may possess similar accuracy.

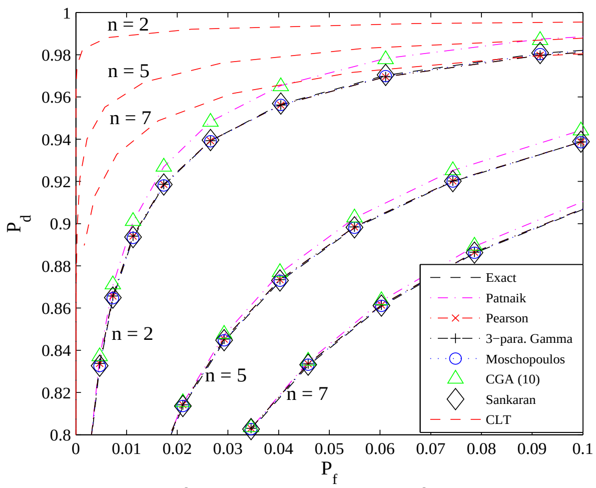

Fig. 1: ROC curves for various sample sizes for $P_f \leq 0.1$, $\gamma = 10$ dB.

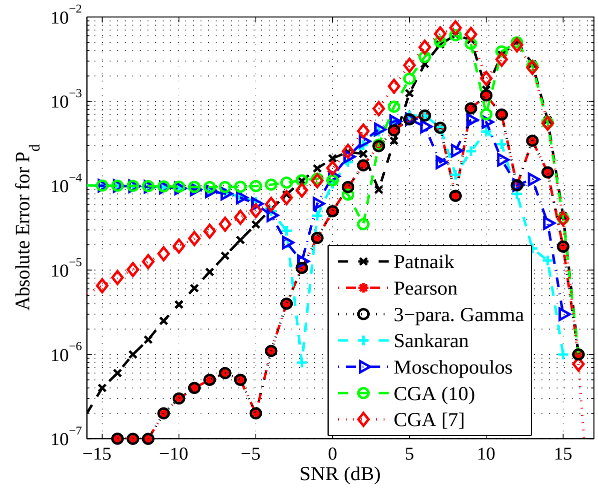

Fig. 2: Absolute error for P_d vs. γ for $P_f = 0.01$, $n = 5$.

IV. Numerical Results and Discussions

Section III presented five approximations for detection probability. In addition, the approximate detection probability using CGA [15], P_d^{cga}, can be derived to be (derivation omitted for brevity)

$$P_d^{\text{cga}}=Q\left(\left[\left(\frac{\lambda}{2(n+\gamma)}\right)^{1/3}+\frac{n+2\gamma}{9(n+\gamma)^2}-1\right]\sqrt{\frac{9(n+\gamma)^2}{n+2\gamma}}\right). \quad (10)$$

Fig. 1 compares the ROC curves for the six approximations (along with the CLT approximation) and the exact P_d (1). Note that we restrict $P_f \leq 0.1$ as per the specifications in IEEE 802.22 standard for cognitive radios [2]. While the CLT is the least accurate, P_d^{cga} (10) and P_d^{pn} (Patnaik) almost match the exact values and become more accurate with increasing n. The other proposed approximations tightly match the exact values.

Fig. 2 shows the absolute errors (AEs) of the six approximations and of the approximation method in [7] which uses CGA for both P_d and P_f. Here, AE is given as $|P_d^{\text{ex}} - P_d^{\text{app}}|$, where P_d^{ex} is the exact (1) and P_d^{app} is the approximate. Clearly, for $4 \leq \gamma\,\text{dB} \leq 14$, all except the Patnaik and the two CGA approximations have AE $\leq 10^{-3}$. The average (over all SNRs) AEs of the Pearson's and the three-parameter Gamma are the lowest. Note that the CGA [7] has lower AE than CGA (10) for $\gamma < -1$ dB. Another interesting observation is that for low SNRs ($\gamma \leq -5$ dB), the Patnaik approximation outperforms Sankaran's, Moschopoulos' and the CGA approximations.

V. AUC of the p-norm Detector

Recently, the p-norm detector, whose decision variable is of the form [1] $T_p = 1/n \sum_{i=1}^{n} |y_i|^p$, has received attention as the more generalized version of the traditional ED ($p = 2$ reverts to the ED) suitable for improving the detection performance in non-Gaussian noise [6], fading [14], and co-operative spectrum sensing [22]. However, the general form of the p-norm detector decision variable makes its exact closed-form performance analysis intractable. To demonstrate the applicability of one of the approximations, we thus consider the p-norm detector in this section and derive its approximate AUC performance, its exact AUC being analytically intractable.

Section IV shows that P_d^{pn} is highly accurate at low SNRs (with AE $< 3\times10^{-6}$ at SNRs below -15 dB). For instance, IEEE 802.22 requires spectrum sensing at SNRs as low as -22 dB [2]. We thus adopt P_d^{pn} to derive the AUC of the p-norm detector. The AUC varies from 1/2 to 1 and serves as a single figure of merit of detection performance [23]. Similar to [1], [6], we consider a random PU signal model [2], [3], [5]. In this model, the PU signal samples and the additive white Gaussian noise (AWGN) samples are assumed to be $s_i \sim \mathcal{CN}(0,\sigma_s^2)$ and $n_i \sim \mathcal{CN}(0,\sigma_w^2)$, respectively, $\forall i \in \{1,2,...,n\}$. Thus, the distribution of T_p appears intractable as $p > 0$ is an arbitrary real number. In contrast, the Patnaik's approximation for $T_p|H_1$ and $T_p|H_0$ yields (see Appendix)

$$P_d = \frac{1}{\Gamma(\theta/2)}\Gamma\left(\frac{\theta}{2},\frac{\lambda}{2\rho_1}\right), \qquad P_f = \frac{1}{\Gamma(\theta/2)}\Gamma\left(\frac{\theta}{2},\frac{\lambda}{2\rho_0}\right), \quad (11)$$

with the parameters $\theta = 2n\Gamma^2(p/2+1)/[\Gamma(p+1)-\Gamma^2(p/2+1)]$ and $\rho_j = [\Gamma(p+1)-\Gamma^2(p/2+1)][2n\Gamma(p/2+1)A_j^{p/2}]$, $j = 0, 1$, with $A_0 \triangleq 1$, $A_1 \triangleq 1/(1+\gamma)$, and $\gamma \triangleq \sigma_s^2/\sigma_w^2$. Then, by substituting (11) into the definition of the AUC [2] AUC $= -\int_0^\infty P_d(\gamma,\lambda)\frac{\partial P_f}{\partial\lambda}d\lambda$, and then using [24, eq. (6.455.1)] to solve the integral, the AUC can be expressed as

$$\text{AUC} = \frac{2\Gamma(\theta)(1+\gamma)^{p\theta/4}{}_2F_1\left(1,\theta;\frac{\theta}{2}+1;\frac{(1+\gamma)^{p/2}}{(1+\gamma)^{p/2}+1}\right)}{\theta\Gamma^2(\theta/2)(1+(1+\gamma)^{p/2})^{\theta}}. \quad (12)$$

This new expression (12) helps to study the dependence of AUC on SNR, n and p as depicted in Fig. 3 and Fig. 4.

In Fig. 3, (12) is compared against the simulation results for $p = 1.5$ and $p = 2.5$. The close match between (12) and the simulation results is evident, thus validating its accuracy.

The dependence of AUC on p is depicted in Fig. 4 for a wide range of parameter values: SNRs as low as -20 dB and n in the orders of 10^1 to 10^4. For all the cases, increasing p beyond $p = 2$ has a negative impact on the sensing performance of the p-norm detector since it lowers the AUC. Further, a 15 dB drop in SNR (from -5 dB to -20 dB) requires n to be increased by 3 orders of magnitude (10^1 to 10^4) to maintain a similar performance.

Thus, (12) accurately facilitates the p-norm AUC performance analysis for any SNR and any n. Importantly, the

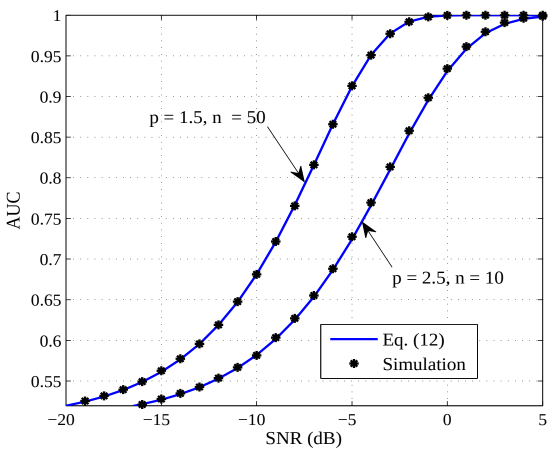

Fig. 3: AUC vs. SNR for different p and n.

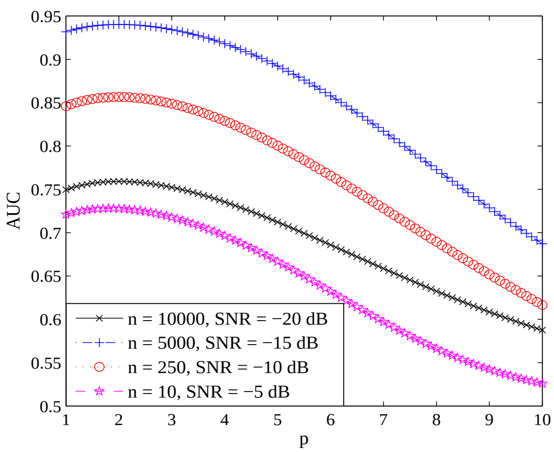

Fig. 4: AUC vs. p for different sample sizes and SNRs

Pearson and three-parameter Gamma approximations (also very accurate for all SNRs) may also be adopted for the AUC analysis. However, we omit similar analyses for brevity.

VI. Conclusion

The CLT approximation for P_d, although offering computational ease, is not accurate for small sample sizes. A previous remedy has been the CGA. This paper investigates five more classical approximations. As a further application, the Patnaik's approximation is utilized to derive a novel AUC expression of the p-norm detector, thus indicating its suitability to problems other than energy detection. This paper thus introduces these important but unnoticed approximations to the research community. Other than detection performance analysis, these approaches may be applicable to more general performance analysis problems involving antenna/co-operative diversity, stochastic interference networks, relaying schemes, etc., which remain as interesting open research problems for further exploration.

Appendix: Derivation of (11)

We have $y_i|H_0 \sim \mathcal{CN}(0, \sigma_w^2)$, and $y_i|H_1 \sim \mathcal{CN}(0, \sigma_w^2(1+\gamma))$. Thus, $|y_i|^2$ normalized with respect to σ_w^2 under hypothesis H_j, $|y_i|^2/\sigma_w^2\,|H_j$, is exponentially distributed with parameter A_j, $j = 0, 1$. Since the samples are independent and identically distributed, the mean of $T_p|H_j$, denoted M_j, can be expressed as (after interchanging the order of integration and summation) $M_j = 1/n \sum_{i=1}^{n} \int_0^\infty x^{p/2} e^{-A_j x} dx = \Gamma(p/2+1)/A_j^{p/2}$. Similarly, the variance of $T_p|H_j$, denoted by σ_j^2, can be obtained to be $\sigma_j^2 = [\Gamma(p+1) - \Gamma^2(p/2+1)]/[nA_j^p]$. Then, use of the transformation (2), matching the corresponding means and variances, and solving for θ and ρ_j yields (11).

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