Outage analysis of ZFB-MRT/MRC Underlay Two-Way Relay Systems

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***Abstract*—We analyse the end-to-end (E2E) outage probabilities (OPs) of zero-forcing beamforming and maximal-ratio-transmission/combining (ZFB-MRT/MRC) for an underlay network, which consists of a single-antenna fixed-gain amplify-and-forward (AF) two-way relay and two multi-antenna terminals. Assuming both path loss effects and small-scale fading and considering both secondary-to-primary (S2P) and primary-to-secondary (P2S) interferences, the exact and asymptotic E2E OPs are derived.**

***Index Terms*—amplify-and-forward relaying, cognitive radio, outage probability, underlay, ZFB-MRT/MRC**

I. Introduction

The cognitive underlay concept yields improved spectral efficiency and spectrum utilization. However, these improvements may be limited due to secondary transmitters having to reduce their transmit power to comply with the S2P interference constraint and secondary receivers are being subject to P2S interferences. To mitigate the first effect, beamforming and one-way relays were used and their OPs were analysed [1]–[3]. However, the performance of beamforming for underlay two-way AF relay networks considering both S2P and P2S interferences has not been analysed.

Therefore, we study an underlay two-way AF relay network, which consisting of two terminals (SU$_1$ and SU$_2$) with M_1 and M_2 antennas, respectively, and a single-antenna half-duplex relay (R) co-exists with a primary transmitter (A) and receiver (B) (Fig. 1). A similar system configuration but with one-way relay was studied in [3]. We consider AF relaying due to its advantages of low complexity and short processing time [4]. In our study, however, the relay is limited to a single-antenna device for two reasons: (1) multi-antenna relays may face the size and cost constraints [5], and (2) the analysis of the multi-antenna relay case is beyond the scope of this letter.

In this configuration, the SU$_j$ →B and A→SU$_j$ interference signals are mitigated via ZFB-MRT and ZFB-MRC, respectively. However, since mutual information exchange between SU$_1$ and SU$_2$ requires two time slots, the A→R interference in time slot one will propagate to SU$_1$ and SU$_2$ in time slot two. To the best of our knowledge, a comprehensive analysis considering both P2S and S2P interference links (Fig. 1) has not yet been furnished. Therefore, considering all those effects, we derive both the exact and asymptotic E2E OPs and important insights are: (1) The location of the relay significantly impacts the outage; (2) As the secondary transmit power $P_s \to \infty$, the diversity order is zero if the interference threshold I_{th} is finite; (3) As $P_s \to \infty$, if $\frac{I_{th}}{P_s}$ is constant, a diversity of order $(\min(M_1, M_2) - 1)$ is achieved.

Notations: $(\bullet)^*$, $(\bullet)^T$, $\| \bullet \|_F$, and $\mathbb{E}\{\bullet\}$ represent complex conjugation, transpose, Frobenius norm, and expectation, respectively. $\boldsymbol{I}$ is the identity matrix. Regularized incomplete Gamma function $P(a,\, b) = \frac{\int_0^b t^{a-1} e^{-t} dt}{\int_0^\infty t^{a-1} e^{-t} dt}$, incomplete Beta function $B(x;\, \alpha,\, \beta)$ [6, Eq. (8.391)] and Beta function $B(\alpha,\, \beta) = B(1, \alpha, \beta)$. The probability density function (PDF) and cumulative distribution function (CDF) of X are $f_X(x)$ and $F_X(x)$. If $f_X(x) = \lambda e^{-\lambda x}, x \geq 0$, we write $X \sim \text{Exp}(\lambda)$. $X \sim \text{Gamma}(k, \theta)\,(k > 0, \theta > 0)$, if $F_X(x) = P(k, \frac{x}{\theta}), x > 0$. $X \sim \text{Beta}(\alpha, \beta)\,(\alpha > 0,\, \beta > 0)$, if $f_X(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\, \beta)}, 0 \leq x \leq 1$. $\mathcal{CN}(\mu,\, \sigma^2)$ denotes complex Gaussian with mean μ and variance σ^2. $\mathcal{K}_\nu(x)$ is the modified Bessel function of the second type of order ν. $\mathbb{C}$ is the set of complex numbers.

II. Underlay Relay Systems and Signal Flow

A. System Configuration and Signal Flow

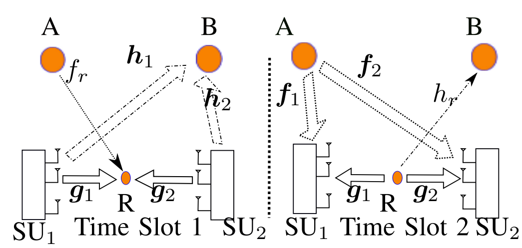

Fig. 1. Underlay Two-way Relay Model.

As mentioned before, in our underlay cognitive setup (Fig. 1), the two secondary multi-antenna transceivers SU$_j$ have $M_j \geq 2$ antennas ($j = 1,\, 2$), and the half-duplex single-antenna AF relay is R. No direct link is assumed between SU$_1$ and SU$_2$. As in [2], [3], perfect time synchronization is assumed between the primary and secondary networks. Thus, our results characterize the worst-case interference scenario.

This work considers both path loss and small-scale fading. Consider two nodes x and y with n_x and n_y antennas at a distance $d_{\text{x,y}}$ in Fig. 1. The channel from x to y is thus denoted by a $\mathbb{C}^{n_y \times n_x}$ matrix with independent and identical distributed $\mathcal{CN}(0,\, \lambda_{\text{x,y}})$ entries, where $\lambda_{\text{x,y}}$ accounts for the path loss and satisfies $\lambda_{\text{x,y}} \propto d_{\text{x,y}}^{-\omega}$, where ω is the path loss exponent. Therefore, vectors $\boldsymbol{g}_j \in \mathbb{C}^{M_j \times 1}$, $\boldsymbol{h}_j \in \mathbb{C}^{M_j \times 1}$ and $\boldsymbol{f}_j \in \mathbb{C}^{M_j \times 1}$ ($j = 1,\, 2$) are the reciprocal SU$_j$ ↔R channel, the SU$_j$ →B and A→SU$_j$ interference channels with $g_{j_i} \sim$

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$\mathcal{CN}(\mathbf{0},\,\lambda_j)$, $h_{j_i} \sim \mathcal{CN}(\mathbf{0},\,\lambda_{\text{SU}_j,\text{B}})$ and $f_{j_i} \sim \mathcal{CN}(\mathbf{0},\,\lambda_{\text{A,SU}_j})$ $(i = 1, 2, ..., M_j)$, respectively. $h_r \sim \mathcal{CN}(0,\,\lambda_3)$ and $f_r \sim \mathcal{CN}(0,\,\lambda_0)$ are the R→B and A→R interference channels, respectively. Here, $\lambda_0 = \lambda_{\text{A,R}}$, $\lambda_j = \lambda_{\text{SU}_j,\text{R}}$ $(j = 1,\,2)$, and $\lambda_3 = \lambda_{\text{R,B}}$. $\boldsymbol{m}_j$, $\boldsymbol{d}_j \in \mathbb{C}^{M_j\times 1}$ are the normalized transmit and receive beamforming vectors at SU_j $(j = 1,\,2)$, respectively.

In [1]–[3], channel gains of the whole network, e.g. all the secondary-to-secondary (S2S), P2S and S2P channels, are assumed known at secondary nodes. However because such full CSI requirement necessitates a large overhead, we assume that every secondary node x∈{SU$_1$, SU$_2$, and R} obtains channel gains only of channels involving itself, e.g. x→B, A→x, x→R/SU$_j$ (if x=SU$_j$/R, $j = 1, 2$), via a suitable channel estimation process [7]. With this assumption, SU$_j$ can calculate its own beamforming vectors $\boldsymbol{m}_j$ and $\boldsymbol{d}_j$ $(j = 1, 2)$, and the relay (R) could adapt its relay gain $G \in \mathbb{C}$ if SU$_j$ communicates $\boldsymbol{m}_j$ to R. However, to reduce this overhead, we assume SU$_j$ sends only the average $(\mathbb{E}\{|\boldsymbol{g}_j^T\boldsymbol{m}_j|^2\})$ to R, then R calculates G and feeds it back to SU$_j$ $(j = 1, 2)$. Such averages are more static than the actual instant channel gains themselves, and thus we obtain a fixed-gain relay. However, our numerical comparisons with a relay using a channel-assisted gain are also provided.

Without loss of generality, zero-mean complex additive white Gaussian noise (AWGN) with N_0 variance and unit symbol power are assumed. I_{th} denotes the interference temperature limit. P_s and P_p denote the transmit power at SU$_j$ $(j = 1,\,2)$ and A, respectively.

The two-way relay requires two time slots. In time slot one, SU$_j$ $(j = 1, 2)$ and A transmit symbols s_j and $x^{(1)}$, respectively. Given n_r as the noise at R, the signal y_r and interference signal $x_{int}^{(1)}$ received at R and B are then given as,

$$y_r = \sqrt{P_s}\boldsymbol{g}_1^T\boldsymbol{m}_1 s_1 + \sqrt{P_s}\boldsymbol{g}_2^T\boldsymbol{m}_2 s_2 + \sqrt{P_p} f_r x^{(1)} + n_r, \quad (1)$$

$$x_{int}^{(1)} = \sqrt{P_s}\boldsymbol{h}_1^T\boldsymbol{m}_1 s_1 + \sqrt{P_s}\boldsymbol{h}_2^T\boldsymbol{m}_2 s_2. \quad (2)$$

In the second time slot, R transmits Gy_r. The interference signal $x_{int}^{(2)}$ received at primary receiver B is given as

$$x_{int}^{(2)} = h_r G y_r. \quad (3)$$

Because the underlay mode requires the interference power at B below the interference temperature limit (I_{th}), we choose G such that $\mathbb{E}\{|x_{int}^{(2)}|^2\} \le I_{th}$. Therefore, the fixed relay gain G must satisfy,

$$G^2 = \frac{I_{th}}{\mathbb{E}\{|h_r|^2\}[P_s\mathbb{E}\{|\boldsymbol{g}_1^T\boldsymbol{m}_1|^2\} + P_s\mathbb{E}\{|\boldsymbol{g}_2^T\boldsymbol{m}_2|^2\} + P_p\mathbb{E}\{|f_r|^2\} + N_0]}. \quad (4)$$

Also in time slot two, A transmits $x^{(2)}$. Knowing G and $\boldsymbol{g}_j$ $(j = 1, 2)$, SU$_j$ can eliminate the self-interference part $\boldsymbol{g}_j G \boldsymbol{g}_j^T \boldsymbol{m}_j s_j$ in its received signal perfectly. After that and receive beamforming, the resulting signal $\hat{y}_j$ is represented as

$$\hat{y}_j = \underbrace{\sqrt{P_s}G\boldsymbol{d}_j^T\boldsymbol{g}_j\boldsymbol{g}_{\bar{j}}^T\boldsymbol{m}_{\bar{j}} s_{\bar{j}}}_{\text{Signal}} + \underbrace{G\boldsymbol{d}_j^T\boldsymbol{g}_j n_r + \boldsymbol{d}_j^T\boldsymbol{n}_j}_{\text{Noise}} + \underbrace{\sqrt{P_p}G\boldsymbol{d}_j^T\boldsymbol{g}_j f_r x^{(1)} + \sqrt{P_p}\boldsymbol{d}_j^T\boldsymbol{f}_j x^{(2)}}_{\text{P2S Interference}} \quad (5)$$

where $\boldsymbol{n}_j \in \mathbb{C}^{M_j\times 1}$ is the AWGN at SU$_j$. And $\bar{j} = 1$, if $j = 2$, and vise versa.

B. ZFB-MRT/MRC Beamforming at Transmision/Reception

With ZFB-MRT, $\boldsymbol{m}_j$ $(j = 1,\,2)$ is determined to nullify the SU$_j$ →B interference while maximizing the SU$_j$ →R signal power. Consequently, $\boldsymbol{m}_j$ is computed as the projection of the SU$_j$ →R channel $\boldsymbol{g}_j$ onto the sub-space $\Phi_j = \boldsymbol{I} - \frac{\boldsymbol{h}_j^*\boldsymbol{h}_j^T}{\|\boldsymbol{h}_j\|_F^2}$, which is orthogonal to the SU$_j$ →B interference channel $\boldsymbol{h}_j$. Thus, $\boldsymbol{m}_j = \frac{\Phi_j\boldsymbol{g}_j^*}{\sqrt{\boldsymbol{g}_j^T\Phi_j\boldsymbol{g}_j^*}}$, where the denominator is the normalizing factor. Similarly, ZFB-MRC is employed to compute the receive beamforming vector $\boldsymbol{d}_j$ $(j = 1,\,2)$ as $\boldsymbol{d}_j = \frac{\Psi_j\boldsymbol{g}_j^*}{\sqrt{\boldsymbol{g}_j^T\Psi_j\boldsymbol{g}_j^*}}$, where $\Psi_j = \boldsymbol{I} - \frac{\boldsymbol{f}_j^*\boldsymbol{f}_j^T}{\|\boldsymbol{f}_j\|_F^2}$.

After receive beamforming, the processed signal has the residual accumulated interference of $x^{(1)}$, which must be considered in performance analysis.

III. End-to-End Outage Probability

In this section, the exact and asymptotic E2E OPs and the fixed relay gain G are derived.

A. E2E Outage Probability

After substituting the beamforming vectors $\boldsymbol{m}_j$ and $\boldsymbol{d}_j$ into (5), the SINR S_j at SU$_j$ $(j = 1,\,2)$ may be represented as

$$S_j = \frac{\gamma_{\bar{j}R}\gamma_{Rj}}{\gamma_{Rj}(\gamma_3+1)+C}, \quad (6)$$

where $\gamma_{jR} = \bar{\gamma}\|\boldsymbol{g}_j\|_F^2\rho_{\boldsymbol{h}_j}$, $\rho_{\boldsymbol{h}_j} = 1 - \frac{|\boldsymbol{h}_j^T\boldsymbol{g}_j^*|^2}{\|\boldsymbol{h}_j\|_F^2\|\boldsymbol{g}_j\|_F^2}$, $\gamma_{Rj} = \bar{\gamma}\|\boldsymbol{g}_j\|_F^2\rho_{\boldsymbol{f}_j}$, $\rho_{\boldsymbol{f}_j} = 1 - \frac{|\boldsymbol{f}_j^T\boldsymbol{g}_j^*|^2}{\|\boldsymbol{f}_j\|_F^2\|\boldsymbol{g}_j\|_F^2}$, $\gamma_3 = \gamma_0|f_r|^2$, $C = \frac{\bar{\gamma}}{G^2}$, $\bar{\gamma} = \frac{P_s}{N_0}$, and $\gamma_0 = \frac{P_p}{N_0}$.

Theorem 1. *The random variables γ_{jR} and γ_{Rj} $(j = 1,\,2)$ are Gamma$(M_j - 1, \bar{\gamma}\lambda_j)$ distributed.*

Proof. Define $\hat{\boldsymbol{h}}_j = \frac{\boldsymbol{h}_j}{\sqrt{\lambda_{SU_j,B}}}$ and $\hat{\boldsymbol{g}}_j = \frac{\boldsymbol{g}_j}{\sqrt{\lambda_j}}$ $(j = 1,\,2)$. Both $\hat{\boldsymbol{h}}_j$ and $\hat{\boldsymbol{g}}_j$ are thus $\mathcal{CN}(0,\,1)$ distributed and $\rho_{\boldsymbol{h}_j}$ remains unchanged when $\boldsymbol{h}_j$ and $\boldsymbol{g}_j$ are replaced by $\hat{\boldsymbol{h}}_j$ and $\hat{\boldsymbol{g}}_j$, respectively. It has been proven in [8] that the random variable $X = \frac{|\hat{\boldsymbol{h}}_j^T\hat{\boldsymbol{g}}_j^*|^2}{\|\hat{\boldsymbol{h}}_j\|_F^2\|\hat{\boldsymbol{g}}_j\|_F^2}$ is Beta$(1,\,M_j - 1)$ distributed and is independent from both $\hat{\boldsymbol{g}}_j$ and $\hat{\boldsymbol{h}}_j$ because X is the normalized correlation between $\hat{\boldsymbol{g}}_j$ and a uniformly distributed variable $\boldsymbol{v} = \frac{\hat{\boldsymbol{h}}_j}{\|\hat{\boldsymbol{h}}_j\|_F}$. Since the Beta distribution has the property that if $X \sim$ Beta(α, β), then $1 - X \sim$ Beta(β, α), $\rho_{\boldsymbol{h}_j} = 1 - X$ is Beta$(M_j - 1,\,1)$-distributed.

The random variable $Y_j = \|\boldsymbol{g}_j\|_F^2$ is a sum of M_j absolute square $\mathcal{CN}(0,\,\lambda_j)$ terms and as such is Gamma$(M_j,\,\lambda_j)$ distributed. Then by expanding $P(s, x) = 1 - \sum_{i=1}^{s}\frac{x^{s-i}e^{-x}}{(s-i)!}$, given s is a positive integer, the CDF of γ_{jR} is derived as,

$$\begin{aligned} F_{\gamma_{jR}}(\gamma) &= \int_0^{\frac{\gamma}{\bar{\gamma}}} f_{Y_j}(y)\mathrm{d}y + \int_{\frac{\gamma}{\bar{\gamma}}}^{\infty} \left(\frac{\gamma}{\bar{\gamma}y}\right)^{M_j-1} \frac{y^{M_j-1}e^{-\frac{y}{\lambda_j}}}{\int_0^\infty t^{M_j-1}e^{-t}dt\lambda_j^{M_j}}\mathrm{d}y \\ &= 1 - \sum_{i=1}^{M_j}\frac{\gamma^{M_j-i}e^{-\frac{\gamma}{\bar{\gamma}\lambda_j}}}{(\bar{\gamma}\lambda_j)^{M_j-i}(M_j-i)!} + \frac{\gamma^{M_j-1}e^{-\frac{\gamma}{\bar{\gamma}\lambda_j}}}{(\bar{\gamma}\lambda_j)^{M_j-1}\Gamma(M_j)} \\ &= P\left(M_j - 1, \frac{\gamma}{\bar{\gamma}\lambda_j}\right) \end{aligned} \quad (7)$$

The proof for γ_{Rj} is analogous and omitted here. □

Theorem 2. *Given $\gamma_{th} > 0$ as the minimum SINR required at SU_j to decode the received signal, the E2E OP $P_j^{out}(\gamma_{th}) = Pr[S_j \le \gamma_{th}]$ at SU_j ($j = 1, 2$) is given by* (8).

$$P_j^{out}(\gamma_{th}) = 1 - 2\sum_{i=2}^{M_{\bar{j}}}\sum_{k=0}^{M_{\bar{j}}-i}\sum_{l=0}^{k}\frac{e^{-\frac{\gamma_{th}}{\bar{\gamma}\lambda_{\bar{j}}}}(\frac{\gamma_{th}}{\lambda_{\bar{j}}})^{\frac{M_1+M_2-i+k-1}{2}}}{(M_j-2)!(M_{\bar{j}}-i-k)!(k-l)!} \cdot\frac{(\frac{C}{\lambda_j})^{\frac{M_1+M_2-i-k-1}{2}}(\gamma_0\lambda_0)^l}{\bar{\gamma}^{(M_1+M_2-i-l-2)}(\frac{\gamma_{th}\gamma_0\lambda_0}{\lambda_{\bar{j}}}+\bar{\gamma})^{l+1}}\mathcal{K}_{M_j-M_{\bar{j}}+i+k-1}\left(2\sqrt{\frac{C\gamma_{th}}{\bar{\gamma}^2\lambda_1\lambda_2}}\right) \tag{8}$$

Proof. Since $f_r \sim \mathcal{CN}(0, \lambda_0)$, γ_3 is Exp($\frac{1}{\gamma_0\lambda_0}$)-distributed.

Without loss of generality, due to the symmetric system setup, we derive the CDF $F_{S_1}(\gamma)$ ($\gamma > 0$) here. Since in Theorem 1, we have proved that γ_{jR} and γ_{Rj} ($j = 1, 2$) are identically distributed, both are denoted as γ_j here for simplicity. Then $F_{S_1}(\gamma)$ is derived as follows.

$$F_{S_1}(\gamma) = \int_0^\infty\int_0^\infty F_{\gamma_2}\left(\left(\gamma_3+1+\frac{C}{\gamma_1}\right)\gamma\right)f_{\gamma_3}(\gamma_3)f_{\gamma_1}(\gamma_1)\mathrm{d}\gamma_3\mathrm{d}\gamma_1 \tag{9}$$

By replacing $F_{\gamma_2}\left(\left(\gamma_3+1+\frac{C}{\gamma_1}\right)\gamma\right)$ in (9) with the expansion $P(s,x) = 1 - e^{-x}\sum_{t=0}^{s-1}\frac{x^t}{t!}$, given s is a positive integer, we obtain (10), where the second equality follows the Binomial expansion. Then applying [6, Eq. (3.351.3)] and [6, Eq.(3.471.9)], the end-to-end OP (8) is derived. □

Note that (8) includes only λ_i ($i = 0, 1, 2, 3$). Therefore, only the location of the relay impacts the OP.

B. Calculation of Fixed Relay Gain G

Since (4) shows that the phase of G does not matter, we assume G is positive real valued. Then substituting $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ in Section II-B into (4), the relay gain is calculated as,

$$G = \sqrt{\frac{I_{th}}{\lambda_3[P_s\lambda_1(M_1-1)+P_s\lambda_2(M_2-1)+P_p\lambda_0+N_0]}}, \tag{11}$$

which results from the following two facts. First, because $\rho_{\boldsymbol{h}_j}$ ($j = 1, 2$) is Beta distributed with parameter $M_j - 1$ and 1, and independent from both $\boldsymbol{g}_j$ and $\boldsymbol{h}_j$, it is true that $\mathbb{E}\{\|\boldsymbol{g}_j\|_F^2\rho_{\boldsymbol{h}_j}\} = \mathbb{E}\{\|\boldsymbol{g}_j\|_F^2\}\mathbb{E}\{\rho_{\boldsymbol{h}_j}\}$ and $\mathbb{E}\{\rho_{\boldsymbol{h}_j}\} = \frac{M_j-1}{M_j}$. Secondly, we have $\mathbb{E}\{|h_r|^2\} = \lambda_3$, $\mathbb{E}\{|f_r|^2\} = \lambda_0$ and $\mathbb{E}\{\|\boldsymbol{g}_j\|_F^2\} = M_j\lambda_j$ ($j = 1, 2$).

C. E2E Outage Probability with $I_{th} \to \infty$

When $I_{th} \to \infty$, the underlay network is equivalent to a conventional two-way relay network, where the secondary nodes can transmit freely and A is an interference source. Then, to derive the asymptotic E2E OP, we expand $\mathcal{K}_\nu(z)$ as,

$$\mathcal{K}_\nu(z) = \frac{1}{2}(\frac{1}{2}z)^{-\nu}\sum_{p=0}^{\nu-1}\frac{(\nu-p-1)!}{p!}\left(-\frac{1}{4}z^2\right)^p + (-1)^{\nu+1} \cdot\left(\frac{z}{2}\right)^\nu\sum_{p=0}^{\infty}\frac{\left(\frac{z^2}{4}\right)^p\left[\ln\left(\frac{z}{2}\right)-\frac{1}{2}\psi(p+1)-\frac{1}{2}\psi(\nu+p+1)\right]}{p!(\nu+k)!} \tag{12}$$

where $\psi(x)$ is the diagamma function. Then, substitute $C = \frac{\bar{\gamma}}{G^2}$ and (11) into (12) and define $\alpha_1 = \frac{\lambda_3}{I_{th}}[\lambda_1(M_1-1)+\lambda_2(M_2-1)]$, $\alpha_2 = \frac{\lambda_3}{I_{th}}(P_p\lambda_0+N_0)$, and $b = \frac{\gamma_{th}N_0}{\lambda_1\lambda_2}(\alpha_1+\frac{\alpha_2}{P_s})$. There are three cases: (1)$(M_j-1)-(M_{\bar{j}}-i-k) < 0$, (2)$(M_j-1)-(M_{\bar{j}}-i-k) = 0$, and (3)$(M_j-1)-(M_{\bar{j}}-i-k) > 0$. Note that $\mathcal{K}_\nu(z)$ has the property that $\mathcal{K}_\nu(z) = \mathcal{K}_{-\nu}(z)$. And with $I_{th} \to \infty$, $\mathcal{K}_{M_j-M_{\bar{j}}+i+k-1}\left(2\sqrt{\frac{b}{I_{th}}}\right) = 0$ for case (1) and (2). For case (3), the second term in (12) converges to zero with $I_{th} \to \infty$, and only the first term left.

Consequently, substituting $C = \frac{\bar{\gamma}}{G^2}$ and (11) into (8), it is found that the non-zero terms are only when $M_{\bar{j}} - i - k = 0$ and $p = 0$. Thus, the asymptotic E2E OP with $I_{th} \to \infty$ is

$$P_j^{out}(\gamma_{th}; I_{th}\to\infty) = 1 - \sum_{i=2}^{M_{\bar{j}}}\sum_{l=0}^{M_{\bar{j}}-i}\frac{e^{-\frac{\gamma_{th}N_0}{P_s\lambda_{\bar{j}}}}(N_0\gamma_{th})^{M_{\bar{j}}-i}(\gamma_0\lambda_0)^l}{\lambda_{\bar{j}}^{M_{\bar{j}}-i}P_s^{M_{\bar{j}}-i}(\frac{P_p\lambda_0\gamma_{th}}{P_s\lambda_{\bar{j}}}+1)^{(l+1)}(M_{\bar{j}}-i-l)!} \tag{13}$$

As $I_{th} \to \infty$, the E2E OP will converge to a constant (13) due to A's interference. Therefore, the diversity order is zero.

D. E2E Outage Probability with $P_s \to \infty$

In this section, we derive the asymptotic E2E OP in high transmit power region ($P_s \to \infty$) for two cases: (1) when I_{th} is fixed and (2) $I_{th} = aP_s$, $a > 0$.

1) Fixed I_{th}: The asymptotic E2E OP $P_j^{out}(\gamma_{th})$ in this case is derived by substituting (11) into (8), and using the fact that with $P_s \to \infty$, $\alpha_1 + \frac{\alpha_2}{P_s} \to \alpha_1$, $e^{-\frac{N_0\gamma_{th}}{\lambda_{\bar{j}}P_s}} \to 1$, $(\frac{P_p\lambda_0\gamma_{th}}{\lambda_{\bar{j}}P_s}+1)^{(l+1)} \to 1$, and the non-zero term in the sum is when $k = 0$ and consequently $l = 0$. Therefore, the E2E OP converges to

$$P_j^{out}(\gamma_{th}; P_s\to\infty) = 1 - 2\sum_{i=2}^{M_{\bar{j}}}\frac{\left(\frac{\alpha_1\gamma_{th}N_0}{I_{th}\lambda_1\lambda_2}\right)^{\frac{M_j+M_{\bar{j}}-i-1}{2}}}{(M_j-2)!(M_{\bar{j}}-i)!} \cdot\mathcal{K}_{M_j-M_{\bar{j}}+i-1}\left(2\sqrt{\frac{\alpha_1N_0\gamma_{th}}{I_{th}\lambda_1\lambda_2}}\right), \tag{14}$$

which is a constant. Therefore, the diversity order is zero in this case.

2) $I_{th} = aP_s$: In this case, the underlay setup is equivalent to conventional two-way relay. We give the following theorem.

Theorem 3. *If $\frac{I_{th}}{P_s} = a$, $(\min\{M_1, M_2\} - 1)$ diversity can be achieved in high transmit power region ($P_s \to \infty$), and the corresponding asymptotic E2E OP is given by,*

$$P_j^{out}(\gamma_{th}; \frac{I_{th}}{P_s} = a, P_s\to\infty) = \phi(M_1, M_2)\left(\frac{b_1}{P_s}\right)^{\min(M_1,M_2)-1}, \tag{15}$$

where $b_1 = \frac{N_0\gamma_{th}\alpha_1}{a\lambda_1\lambda_2}$ and $\phi(M_1, M_2)$ is given in (16).

$$\phi(M_1,M_2) = \begin{cases} -\sum_{i=1}^{M_{\bar{j}}}\frac{(-1)^{i-1}(M_j-M_{\bar{j}}-1)!}{(M_{\bar{j}}-i)!(M_j-2)!(i-1)!} & \text{if } M_j > M_{\bar{j}} \\ -\sum_{i=1}^{M_{\bar{j}}}\frac{(-1)^i[\ln\left(\frac{b_1}{P_s}\right)-\psi(1)-\psi(i)]}{(M_{\bar{j}}-i)!(M_j-2)!(i-1)!} & \text{if } M_j = M_{\bar{j}} \\ -\sum_{i=1}^{M_{\bar{j}}}\varphi(i) & \text{if } M_j < M_{\bar{j}} \end{cases} \tag{16}$$

$$\varphi(i) = \begin{cases} \frac{(-1)^{M_j-M_{\bar{j}}+i}[\ln\left(\frac{b_1}{P_s}\right)-\psi(1)-\psi(M_j-M_{\bar{j}}+i)]}{(M_j-1)!(M_{\bar{j}}-i)!(M_j-M_{\bar{j}}+i-1)!} & \text{if } M_j-1 > M_{\bar{j}}-i \\ \frac{2\psi(1)-\ln\left(\frac{b_1}{P_s}\right)}{(M_j-1)!(M_{\bar{j}}-i)!} & \text{if } M_j-1 = M_{\bar{j}}-i \\ \frac{(M_{\bar{j}}-M_j-i)!}{(M_j-2)!(M_{\bar{j}}-i)!} & \text{if } M_j-1 < M_{\bar{j}}-i \end{cases}$$

Proof. Without loss of generality, the OP at SU_1 is used here. Substituting $I_{th} = aP_s$ into (14) results in (17).

If $M_1 \ge M_2$, $M_1 - 1 - M_2 + i > 0$ always holds. Then after applying the expansion (12) of $\mathcal{K}_v(\cdot)$, the lowest order of

$$F_{S_1}(\gamma) = 1 - \sum_{i=2}^{M_2} \int_0^\infty \int_0^\infty \frac{e^{-(\gamma_3+1+\frac{C}{\gamma_1})\frac{\gamma}{\lambda_2\bar{\gamma}}} (\frac{\gamma}{\bar{\gamma}\lambda_2})^{M_2-i} \gamma_1^{M_1-2}}{(M_1-2)!(M_2-i)!(\bar{\gamma}\lambda_1)^{M_1-1}\gamma_0\lambda_0} \left(\gamma_3+1+\frac{C}{\gamma_1}\right)^{M_2-i} e^{-\frac{\gamma_3}{\gamma_0\lambda_0}} e^{-\frac{\gamma_1}{\bar{\gamma}\lambda_1}} \mathrm{d}\gamma_3 \mathrm{d}\gamma_1 \tag{10}$$

$$=1-\sum_{i=2}^{M_2}\sum_{k=0}^{M_2-i}\sum_{l=0}^{k}\binom{M_2-i}{k}\binom{k}{l}\frac{(\frac{\gamma}{\bar{\gamma}\lambda_2})^{M_2-i}e^{-\frac{\gamma}{\bar{\gamma}\lambda_2}}C^{M_2-i-k}}{\gamma_0\lambda_0(\bar{\gamma}\lambda_1)^{M_1-1}(M_1-2)!(M_2-i)!}\int_0^\infty \gamma_3^l e^{-\gamma_3(\frac{\gamma}{\bar{\gamma}\lambda_2}+\frac{1}{\gamma_0\lambda_0})}\mathrm{d}\gamma_3\int_0^\infty \gamma_1^{M_1-M_2+i+k-2} e^{-(\frac{\gamma C}{\bar{\gamma}\lambda_2\gamma_1}+\frac{\gamma_1}{\bar{\gamma}\lambda_1})}\mathrm{d}\gamma_1$$

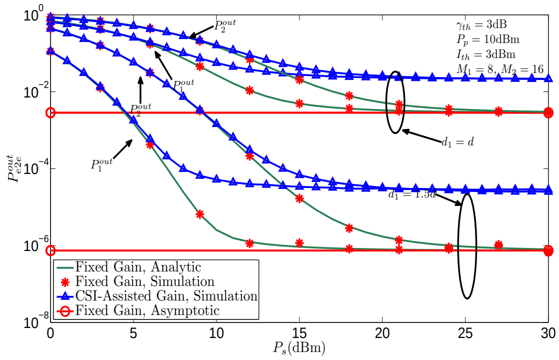

Fig. 2. P_{e2e}^{out} v.s. P_s with $I_{th} = 3$ dBm and $M_1 = 8$, $M_2 = 16$

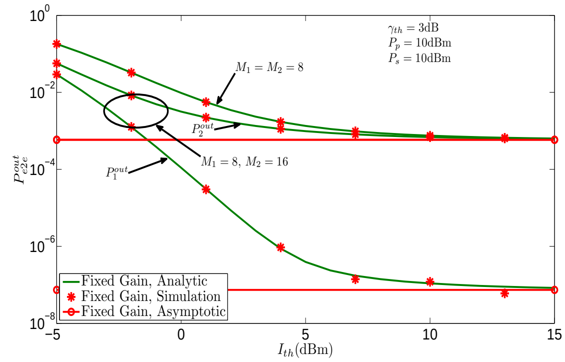

Fig. 3. P_{e2e}^{out} v.s. I_{th} with $d_1 = 1.5d$ and $P_s = 10$ dBm

$\frac{1}{P_s}$ is given by $(M_2 - 1)$ when $M_1 = M_2$ and p in the second term of (12) is zero. If $M_1 > M_2$, the lowest order of $\frac{1}{P_s}$ is given by $(M_2 - 1)$ when p in the first term of (12) takes value of $i - 1$. If $M_1 < M_2$, when $M_1 - 1 - M_2 + i \geq 0$, the lowest order of $\frac{1}{P_s}$ is given by $(M_1 - 1)$ with p in the second term of (12) being zero. When $M_1 - 1 - M_2 + i < 0$, using the $\mathcal{K}_v(\cdot) = \mathcal{K}_{-v}(\cdot)$ property, the lowest order of $\frac{1}{P_s}$ is given by $(M_2 - 1)$ with p in the first term of (12) being $i - 1$.

$$P_1^{out}(\gamma_{th})=1-2\sum_{i=2}^{M_2}\frac{(\frac{b_1}{P_s})^{\frac{M_1+M_2-i-1}{2}}}{(M_1-2)!(M_2-i)!}\mathcal{K}_{M_1-M_2+i-1}\left(2\sqrt{\frac{b_1}{P_s}}\right). \tag{17}$$

□

Comparatively, the underlay setup is equivalent to the conventional two-way relay networks consisting of two multi-antenna terminals and one single-antenna fixed gain AF relay [9]. In [9], the two terminals use MRT/MRC and a diversity of order $\min(M_1, M_2)$ is achieved in high transmit power region. But in our study, ZFB-MRT/MRC applied at the two terminals achieves reduced diversity order (Theorem 3), which is due to orthogonality requirement to enforce ZFB resulting in a small loss of degrees of freedom.

IV. Numerical Results

This section provides numerical results to validate the preceding analysis. The parameters N_0, P_p and γ_{th}, $\omega_{\mathrm{x,y}}$ are set to 0 dB, 10 dBm and 3 dB, 3.5 respectively. We also assume that $d_{\mathrm{SU}_j,\mathrm{R}} = d$ $(j = 1, 2)$ and $d_{\mathrm{A,R}} = d_{\mathrm{R,B}} = d_1$.

Fig. 2 shows the E2E OPs as a function of P_s with $d_1 = d$ and $d_1 = 1.5d$. For comparisons, simulation results of CSI-Assisted gain are provided as well, where R adapts gain as $G = \sqrt{\frac{I_{th}}{|h_r|^2[P_s|\boldsymbol{g}_1^T\boldsymbol{m}_1|^2+P_s|\boldsymbol{g}_2^T\boldsymbol{m}_2|^2+P_p|f_r|^2+N_0]}}$. It is shown that the E2E OPs of CSI-Assisted gain overlap those of fixed gain in lower P_s region, e.g. $P_s \leq 5$ dBm. With increasing P_s, fixed gain outperforms CSI-Assisted gain. This is because with CSI-Assisted gain, G is chosen such that the interference power at B below I_{th} in each transmission, while fixed gain considers the average interference power. It is also shown that when R-A/B distance is larger $(1.5d)$, lower OP is achieved.

Fig. 3 plots a E2E OPs as the function of I_{th} for $M_1 = M_2 = 8$ and $M_1 = 8$, $M_2 = 16$, respectively. With increasing I_{th}, the OPs at SU_2s in both setups converge to the same OP floor since both SU_1s are equipped with 8 antennas.

V. Conclusion

The exact and asymptotic OPs of an underlay multi-antenna network with a two-way relay has been analysed. Both path loss effect and Rayleigh fading were considered, and ZFB-MRT/MRC were used to mitigate interference effects. The location of the relay is shown to significantly impact OP. As $P_s \to \infty$, the diversity order is zero if I_{th} is finite, but if $\frac{I_{th}}{P_s}$ is constant, $(\min(M_1, M_2) - 1)$ diversity is achieved.

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