Outage Probability of Underlay Cognitive Relay Networks with Spatially Random Nodes

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***Abstract*—We consider an underlay cognitive relay network coexisting with a primary multicast network (e.g. digital television (TV) broadcasting network), in which secondary user (SU) transmissions are power constrained to limit the interference on any primary receiver in the network. The primary receivers and SU relays are randomly located due to irregular deployments and/or mobility and thus, their spatial distributions are modeled by two independent Poisson point processes (PPPs). In this paper, we analyze an opportunistic relaying scenario and develop a relay-selection scheme by considering the interference constraints on all the primary receivers in the network. We then analytically evaluate the relaying performance in terms of outage probability by using tools from stochastic geometry and point process theory, and finally compare the performance against that of direct communication. Closed-form expressions are derived for the outage probabilities of both the relay and direct links, along with their high-signal-to-noise ratio (SNR) asymptotics.**

I. Introduction

Cognitive radio (CR) is an emerging technology for efficient spectrum utilization [1], [2]. It allows unlicensed (secondary) users to access licensed (primary) users' spectrum with minimal or no impact on primary user (PU) communications by using *interweave*, *overlay* and *underlay* approaches [2]. While the interweave method permits the secondary user (SU) transmissions only over the currently unused primary spectrum, the other two support PU-SU concurrent transmissions. In overlay systems, the SUs must mitigate the interference imposed on the primary receivers by applying sophisticated signal processing [2]. The underlay method, on the other hand, constrains the transmit powers of SUs to ensure that the resultant interference on PUs is below a predefined threshold [2]. Unlike the overlay system, where the SUs require the knowledge of PUs' codebooks and their messages as well, the underlay SUs only require the knowledge of the interference channel gains to PUs. Thus, underlay method is appealing for its low implementation complexity. However, the underlay SUs operating under stringent interference constraints have limited coverage range. Cooperative relaying, in this case, is a natural choice to achieve the adequate radio reception quality at distant SUs. In this paper, we evaluate the gain in outage probability (the commonly used measure for quality of reception in wireless communications) due to cooperative relaying in underlay CR networks with interference constraints.

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In particular, we consider opportunistic relaying by idle SUs in underlay networks. Generally, in practical networks, the users are expected to be highly mobile and their locations vary with time. The SU relays are thus assumed to be randomly and independently located in the network area, and are spatially modeled by a Poisson point process (PPP). The PPP is a widely used spatial model for networks with possibly infinite number of nodes randomly and independently distributed in a finite or infinite area [3]. The PPP based modeling of the locations of PUs and SUs is adopted in [4] to evaluate the aggregate interference caused by SUs on PUs in underlay CR networks, and in [5] to analyze the performance of different CR medium access control (MAC) protocols.

The performance of underlay CR relay network is significantly affected by the transmit power constraints imposed by the coexisting primary network on the SUs. In this paper, we consider the coexisting primary system to be a multicast network such as digital television (TV) broadcasting network. The VHF/UHF TV spectrum is the most promising candidate for cognitive secondary access due to its lower propagation loss and the availability of large amount of spectrum [6]. Different standards have been developed for digital TV broadcasting such as European Digital Video Broadcasting (DVB-T for terrestrial and DVB-H for handhelds), the North American Advanced Television Systems Committee (ATSC) and Integrated Services Digital Broadcasting-Terrestrial (ISDB-T), most of which are based on orthogonal frequency division multiplexing (OFDM) for robustness against multipath distortion [7]. Due to the multicast nature, an underlay secondary transmission must satisfy the interference constraints on all the primary receivers in the network. We assume that the primary receivers (e.g. TV receivers), either fixed or mobile take unplanned/unknown positions and their spatial distribution is modeled by a PPP.

Previous Work and contributions of the paper: Substantial research work on underlay CR relay networks have been reported in the literature [8]–[11]. The outage probability of CR single-relay network, in which the transmit power is constrained according to the interference threshold at the primary receiver is analyzed in [8] under Nakagami-m fading. Optimal power allocation schemes which maximize the overall rate of CR single-relay network are investigated in [9], while adhering to the interference power constraint on the primary receiver. The conventional relay selection schemes are redefined in [10] and [11] for CR multiple-relay network to incorporate the interference constraint in terms of the required outage probability of primary transmission and the maximum tolerable interference power at the primary receiver, respectively. Joint relay selection and power allocation to maximize the system throughput under interference constraint is investigated in [12]. However, these results consider a fixed number of relay nodes, and either ignore the effect of path loss and thus the spatial configuration of the relays or assume the relay locations to be deterministic (known a priori), which are thus not applicable to CR networks with inherent mobility of the SUs. Further, in these work, no two primary receivers have the same transmitter and thus, any secondary transmission needs to satisfy the interference constraint at only one primary receiver. In contrast to these work, we focus on the following aspects in this paper.

1) Considering the multicasting primary network, we take into account the spatial distribution of primary receivers and derive the outage probability of the direct link between an underlay secondary source node and its destination, while satisfying the interference constraints on all the primary receivers.
2) We next consider the opportunistic relaying between the source-destination pair and derive the outage probability of the relay link, while taking into account the spatial distribution of the relays.
3) The relay-selection scheme is designed by not just considering the source-relay and relay-destination links but also the stringent interference constraints on all the primary receivers.
4) We finally compare the outage probability of the relay link with that of the direct link.

The rest of the paper is organized as follows. The system model and the relaying scheme are presented in Section II. In Section III, the outage probabilities of the direct link and the relay link are derived in closed forms along with their high-signal-to-noise ratio (SNR) asymptotics. The analytical results are verified through Monte Carlo simulations in Section IV. Finally, some concluding remarks are presented in Section V.

Notations: Throughout the paper, the two-dimensional (2-D) Euclidean space is denoted by $\mathbb{R}^2$ and a positive real line by $\mathbb{R}^+$. The Euclidean distance between two points $x, y \in \mathbb{R}^2$ is denoted by $||x - y||$. We use $\mathbb{P}(\cdot)$ and $\mathbb{E}(\cdot)$ to denote the probability measure and the expectation operator, respectively, and Exp[1] to denote exponential distribution with unit mean.

II. System Model

A. System Structure

We consider a CR system consisting of primary and secondary networks. The multicasting primary network consists of a primary transmitter and a number of primary receivers, which are spatially distributed according to a homogeneous PPP $\Phi_p = \{y_1, y_2, y_3 \ldots\}$ on $\mathbb{R}^2$ with density λ_p, where y_i is the location of the ith receiver. The wireless communication between a secondary source node S (located at the origin o without loss of generality) and a secondary destination node D (located at $l_d \in \mathbb{R}^2$) is considered, where the distance between S and D, $||o - l_d||$ is fixed at L. The $S-D$ communication occurs either directly or through opportunistic relaying by a set of idle SUs. The spatial distribution of idle SUs on $\mathbb{R}^2$ is denoted by $\Phi_s = \{x_1, x_2, x_3, \ldots\}$, where x_i is the location of the ith idle SU. Φ_s is assumed to be a homogeneous PPP with density λ_s. A realization of the primary receivers and SU relays spatially distributed according to independent PPPs is shown in Fig. 1. From the definition of PPP [13], [14], the number of primary receivers N_p and the number of SU relays N_s in a given area A are independent Poisson random variables (RVs) with mean $A\lambda_p$ and $A\lambda_s$, respectively. Also, the numbers of primary-receivers in disjoint areas are independent, and so are the numbers of SU relays.

The primary multicast network is assumed to be based on OFDM and each primary receiver thus, occupies a number of frequency channels called subcarriers. A secondary transmission uses a frequency channel from the primary spectra. Each frequency channel undergoes independent flat fading. We assume all the primary and secondary nodes to have single-antenna for transmission and reception. The single antenna model leads to simple analytical results with sufficient insights on important system parameters. Extension to various multiple antenna techniques to analyze their impact on the performance will be considered in future work.

B. Channel Model and Transmission Schemes

Independent Rayleigh multipath fading is assumed between any pair of nodes and across frequency channels. A general power-law path loss model with loss exponent α is also considered in which the signal power decays at the rate of $r^{-\alpha}$ with distance r from the transmitter. Consequently, the channel power gain of the jth frequency channel between a pair of nodes at x and y is given by $h_{xy}^j ||x-y||^{-\alpha}$, where h_{xy}^j is the fading power gain, which is exponentially distributed with unit mean. The value of α is typically in the range of 1.6 to 6 [15], where $\alpha = 2$ is for free space propagation.

1) Direct mode: Let the direct $S-D$ communication occurs over frequency channel n. The source S, while transmitting, must ensure that the interference imposed on each primary receiver is below a predefined threshold $\bar{I}$. If this constraint is satisfied for the primary receiver to which it generates the largest interference power, then the constraint is satisfied for all other receivers. Let $H_{S_n} = \max_{y \in \Phi_p} h_{oy}^n ||o - y||^{-\alpha}$ is the largest interference channel gain associated with S on the nth frequency channel. Then, the transmit power of S on this frequency channel is constrained as $P_{S_n} \leq \bar{I}/H_{S_n}$. The information about the largest interference channel can be acquired through primary receiver detection algorithms [16] or through beacons [17]. If S transmits with the maximum allowable power, the received SNR at the destination D is

$$\mathrm{SNR}_{SD}(o, l_d) = \frac{\bar{I}}{N_0 H_{S_n}} h_{ol_d}^n L^{-\alpha}, \tag{1}$$

where N_0 is the noise variance. The interference signal from the primary transmitter is treated as Gaussian noise [2].

2) Relaying mode: In the relaying mode, the $S-D$ pair communicate through an intermediate node selected from the set of available idle SUs. The relaying protocol used is decode-and-forward, with the assumption that there is no decoding error if the received SNR is greater than the threshold γ_{th}. Although, all the idle SUs distributed over $\mathbb{R}^2$ are considered as the candidate relays for the selection of the best relay in our analysis, this is equivalent to considering only the idle nodes within a circle of radius $R >> L$ as the candidate relays. As the path loss becomes more pronounced than fading when the relays move away from $S-D$ link, the nodes beyond R are less likely to be chosen. The relays operate in half-duplex mode and hence the information transmission from S to D requires two time-slots. We assume that each candidate relay uses a different frequency channel among the primary spectra [11]. Relay selection thus, involves the selection of primary spectrum as well, and the system also gains from multispectrum diversity [18]. If the source S transmits with the maximum allowable power $P_{S_j \max} = \bar{I}/H_{S_j}$ on the jth frequency channel used by the candidate relay R_j at $x \in \Phi_s$, the received SNR at R_j in the first time-slot is given by

$$\mathrm{SNR}_{SR_j}(o,x) = \frac{\bar{I}}{N_0 H_{S_j}} h_{ox}^{j}\,||o-x||^{-\alpha}. \tag{2}$$

The source S can successfully transmit information to any candidate relay at which the received SNR is greater than the threshold γ_{th}. These nodes are the potential relays (represented by triangles in Fig. 1) to retransmit the successfully decoded message to the destination D in the second time-slot. Let $\hat{\Phi}_s$ denotes the set of potential relays, i.e.,

$$\hat{\Phi}_s = \{x \in \Phi_s, \mathrm{SNR}_{SR_j}(o,x) \geq \gamma_{th}\}. \tag{3}$$

The transmit power of a potential relay R_j at $x \in \hat{\Phi}_s$ is constrained as $P_{R_j} \leq \bar{I}/H_{R_j}$, where $H_{R_j} = \max_{y\in\Phi_p} h_{xy}^{j}\,||x-y||^{-\alpha}$ is the largest interference channel gain associated with R_j. Under the proposed relaying scheme, the best node from the set $\hat{\Phi}_s$ is selected. While in a conventional relay network, the relay that has the best channel to the destination would be selected, our selection criterion considers the interference constraint as well. We denote the location of the selected relay by ζ, i.e.,

$$\zeta = \arg\max_{x\in\hat{\Phi}_s} \frac{h_{xl_d}^{j}||x-l_d||^{-\alpha}}{H_{R_j}}. \tag{4}$$

The corresponding received SNR at the destination D from the relay link when the selected relay transmitted with the maximum allowable power is

$$\mathrm{SNR}_{R_J D}(\zeta, l_d) = \frac{\bar{I}}{N_0 H_{R_J}} h_{\zeta l_d}^{J}\,||\zeta - l_d||^{-\alpha}, \tag{5}$$

where J is the index of the selected relay.

III. Performance Analysis

The outage probability is chosen as the performance metric for the given system. To derive the outage probabilities of both the relay and the direct link, we first derive the distribution of $Z = \max_{y\in\Phi} h_{cy}\,||c-y||^{-\alpha}$, where $c \in \mathbb{R}^2$, $\{y \in \Phi\}$ are the points of a homogeneous PPP Φ on $\mathbb{R}^2$ with density λ, and $\{h_{cy}; y \in \Phi\}$ are independent exponentially distributed RVs with unit mean. We can observe that Z does not depend on the exact coordinates of the points $y \in \Phi$, but rather on their distances from c. Lets define $\Phi_l \triangleq \{l = ||c-y||; y \in \Phi\}$ as the transformed points of Φ by mapping the 2-D space into a positive real line with a function f such that $f(b(c,l)) = [0,l]$ (i.e., each point of a closed disc $b(c,l)$ on $\mathbb{R}^2$ of radius l and center c is mapped into the closed interval $[0,l]$ on $\mathbb{R}^+$). From the mapping theorem [13], Φ_l is also a PPP of density $\lambda(l)$ given by $\int_{[0,l]} \lambda(l)\,dl = \int_{b(c,l)} \lambda\,dx$. Thus, $\lambda(l) = 2\pi\lambda l$.

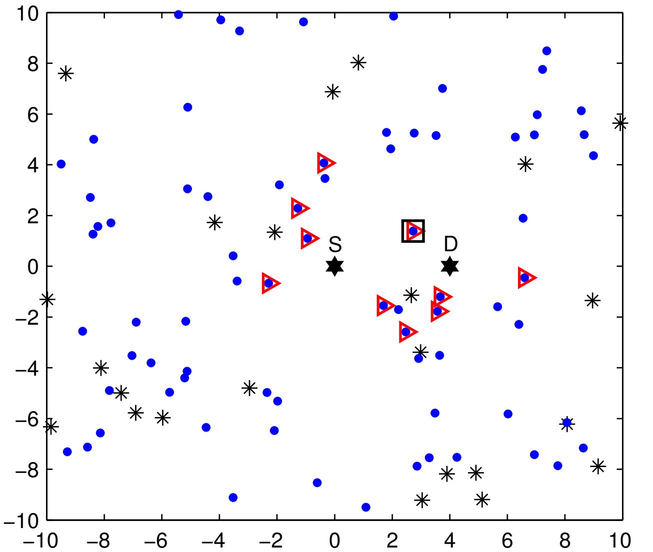

Fig. 1. Spatial distribution of the primary receivers and the SU relays according to independent PPPs. The asterisks are the primary receivers ($\lambda_p = 0.15/\pi$); the dots are the idle SUs which serve as candidate relays ($\lambda_s = 0.5/\pi$); the triangles are the potential relays, i.e., the relays at which the received SNRs are greater than $\gamma_{th} = 5$ dB; and the square is the selected relay as per the proposed scheme. The $S-D$ distance $L = 4$, the path-loss exponent $\alpha = 4$ and the noise normalized interference threshold $\bar{I} = 10$ dB.

Distribution of Z:

$$\begin{aligned} F_Z(z) &= \mathbb{P}\left(\max_{y\in\Phi} h_{cy}\,||c-y||^{-\alpha} < z\right) \\ &= \mathbb{E}_{\Phi}\left[\prod_{y\in\Phi} \mathbb{P}\left(h_{cy}\,||c-y||^{-\alpha} < z|y\right)\right] \\ &= \mathbb{E}_{\Phi_l}\left[\prod_{l\in\Phi_l} \left(1-\exp(-zl^{\alpha})\right)\right], \end{aligned} \tag{6}$$

where the second equality follows from the independence of RVs $\{h_{cy}; y \in \Phi\}$ and (6) from the fact that $h_{cy} \sim \mathrm{Exp}[1]$ and the transformation of Φ to Φ_l. Now, by using the probability generating functional (PGF) of PPP [14], we have

$$\begin{aligned} F_Z(z) &= \exp\left(-2\pi\lambda \int_0^{\infty} \exp(-zl^{\alpha}) l\,dl\right) \\ &= \exp\left(-\frac{\pi\lambda}{z^{2/\alpha}}\Gamma\left(\frac{2}{\alpha}+1\right)\right), \end{aligned} \tag{7}$$

where (7) results from the definition of the Gamma function [19, 8.310]. We can see that the distribution of Z is independent of c. In the following section, we use the notation $\mathcal{Z}(\lambda,\alpha)$ to denote the distribution of $\max_{y\in\Phi} h_{cy}\,||c-y||^{-\alpha}$, $c\in\mathbb{R}^2$.

The transmit power of any secondary node k given by $P_k = \bar{I}/H_k$, where $H_k \sim \mathcal{Z}(\lambda_p,\alpha)$ is thus, independent of the location of the node. The average transmit power of a secondary node can be obtained as

$$\bar{P} = \int_0^\infty \frac{\bar{I}}{z}\,dF_{H_k}(z) = \bar{I}\frac{\Gamma(\alpha/2+1)}{(\pi\lambda_p\Gamma(2/\alpha+1))^{\alpha/2}}. \qquad (8)$$

A. Outage probability of the direct mode

The outage probability of the direct $S-D$ link, $P_d = \mathbb{P}(\mathrm{SNR}_{SD}(o,l_d) < \gamma_{th})$ can be obtained by using the SNR expression (1) as follows:

$$\begin{aligned} P_d &= \mathbb{P}\left(\frac{\bar{I}h^n_{ol_d}L^{-\alpha}}{N_0 H_{S_n}} < \gamma_{th}\right) \\ &= \mathbb{E}_{h^n_{ol_d}}\left[\mathbb{P}\left(H_{S_n} > \frac{\bar{I}h^n_{ol_d}}{N_0\gamma_{th}L^\alpha}\middle| h^n_{ol_d}\right)\right], \end{aligned} \qquad (9)$$

where $H_{S_n} = \max_{y\in\Phi_p} h^n_{oy}\,||o-y||^{-\alpha} \sim \mathcal{Z}(\lambda_p,\alpha)$. By applying the cumulative distribution function (CDF) derived in (7), followed by the expectation over $h^n_{ol_d} \sim \mathrm{Exp}[1]$, P_d can be simplified as

$$\begin{aligned} P_d &= 1 - \int_0^\infty \exp\left(-\beta\,L^2u^{-2/\alpha}\right)e^{-u}\,du \\ &= 1 - \Gamma\left(1,0,\beta\,L^2,\frac{2}{\alpha}\right), \end{aligned} \qquad (10)$$

where $\Gamma(\cdot,\cdot,\cdot,\cdot)$ is the extended incomplete Gamma function [20, Eq. 6.2], $\beta = (\Gamma(\alpha/2+1)\gamma_{th}/\rho)^{2/\alpha}$ and $\rho = \bar{P}/N_0$ is the average transmit SNR. The asymptotic performance as SNR $\to\infty$ thus, can be analyzed with the asymptotic $\beta\to 0$. By using $\exp(-x) = 1-x$ as $x\to 0$, the asymptotic outage probability of the direct link is given by

$$P_d \sim b\,L^2\left(\gamma_{th}/\rho\right)^{2/\alpha} + O(\rho^{-4/\alpha}) \quad \text{as} \quad \rho\to\infty, \qquad (11)$$

where $b = \Gamma\left(-2/\alpha+1\right)\left(\Gamma\left(\alpha/2+1\right)\right)^{2/\alpha}$.

B. Outage probability of the relaying mode

The analysis of the outage probability of the relay link given by $P_r = \mathbb{P}(\mathrm{SNR}_{R_JD} < \gamma_{th})$ involves Euclidean distances from the randomly located relays to the source and the destination. As such, it is mathematically convenient to use a polar coordinate system, where $x\in\mathbb{R}^2$ is represented as $x = (r,\theta)$. We take the coordinate axes to be oriented such that $l_d = (L,0)$. The corresponding distances from the relay located at x to the source and the destination are then given by $d_S(x) = ||o-x|| = r$ and $d_D(x) = ||x-l_d|| = \sqrt{r^2+L^2-2rL\cos\theta}$, respectively. According to (4) and (5), the relay link outage probability can be expressed as

$$P_r = \mathbb{P}\left(\max_{x\in\hat{\Phi}_s}\frac{h^j_{xl_d}d_D^{-\alpha}(x)}{H_{R_j}} < \frac{\gamma_{th}N_0}{\bar{I}}\right). \qquad (12)$$

By utilizing the independence of RVs $\{H_{R_j}; x\in\hat{\Phi}_s\}$, we have

$$\begin{aligned} P_r &= \mathbb{E}_{\hat{\Phi}_s}\left[\prod_{x\in\hat{\Phi}_s}\mathbb{E}_{h^j_{xl_d}}\left(\mathbb{P}\left(H_{R_j} > \frac{\bar{I}h^j_{xl_d}}{N_0\gamma_{th}\,d_D^\alpha(x)}\middle| h^j_{xl_d},x\right)\right)\right] \\ &= \mathbb{E}_{\hat{\Phi}_s}\left[\prod_{x\in\hat{\Phi}_s}\left(1-\int_0^\infty \exp\left(-\beta d_D^2(x)v^{-2/\alpha}\right)e^{-v}\,dv\right)\right], \end{aligned} \qquad (13)$$

where (13) follows from the fact that $H_{R_j} \sim \mathcal{Z}(\lambda_p,\alpha)$ and $h^j_{xl_d} \sim \mathrm{Exp}[1]$. In order to compute P_r, we need to first identify the properties of the set of potential relays $\hat{\Phi}_s$. Since the SNRs at candidate relays (2) are independent, the set $\hat{\Phi}_s$ in (3) is formed by independent thinning of the original process Φ_s, i.e., by selecting a point x of the process Φ_s with probability $p = \mathbb{P}(\mathrm{SNR}_{SR_j}(o,x)\geq\gamma_{th})$ independently of the other points in the process. Since Φ_s is a PPP, the thinned process $\hat{\Phi}_s$ is also a PPP [14] with density $\hat{\lambda}_s(x)$ given by

$$\begin{aligned} \hat{\lambda}_s(x) &= \lambda_s\mathbb{P}(\mathrm{SNR}_{SR_j}(o,x)\geq\gamma_{th}) \\ &= \lambda_s\int_0^\infty \exp\left(-\beta\,r^2u^{-2/\alpha}\right)e^{-u}\,du \\ &= \lambda_s\Gamma\left(1,0,\beta\,r^2,\frac{2}{\alpha}\right), \end{aligned} \qquad (14)$$

where (14) is derived by using the fact that $\mathbb{P}(\mathrm{SNR}_{SR_j}(o,x)\geq\gamma_{th}) = 1-P_d$ with the receiver location $l_d = x$. The average number of potential relays can be obtained as

$$\begin{aligned} \hat{\Lambda}_s &= \int_0^{2\pi}\int_0^\infty \hat{\lambda}_s\,rdr\,d\theta \\ &= \lambda_s\int_0^\infty e^{-v}\int_0^{2\pi}\int_0^\infty \exp\left(-\beta r^2u^{-2/\alpha}\right)\,rdr\,d\theta\,du \\ &= \frac{\pi\lambda_s}{\beta}\Gamma\left(\frac{2}{\alpha}+1\right), \end{aligned} \qquad (15)$$

where the second equality is obtained by substituting the integral expression for $\hat{\lambda}_s(x)$, followed by the change in the order of integration. Since $\hat{\Phi}_s$ is a PPP of intensity $\lambda_s(x)$, by using the PGF of a PPP, the P_r in (13) can be simplified as

$$\begin{aligned} P_r &= \exp\left(-\int_{\mathbb{R}^2}\hat{\lambda}_s(x)\int_0^\infty \exp\left(-\beta d_D^2(x)v^{-\frac{2}{\alpha}}\right)e^{-v}\,dv\,dx\right) \\ &= \exp\left(-\lambda_s\Upsilon(\beta)\right), \end{aligned} \qquad (16)$$

where

$$\begin{aligned} \Upsilon(\beta) = &\int_{u=0}^\infty e^{-u}\int_{v=0}^\infty e^{-v}\exp\left(-\beta L^2v^{-\frac{2}{\alpha}}\right) \\ &\times\int_{r=0}^\infty r\exp\left(-\beta(v^{-\frac{2}{\alpha}}+u^{-\frac{2}{\alpha}})r^2\right) \\ &\times\int_{\theta=0}^{2\pi}\exp\left(2\,\beta\,Lv^{-\frac{2}{\alpha}}r\cos\theta\right)\,d\theta\,dr\,dv\,du. \end{aligned} \qquad (17)$$

Eq. (16) is obtained by substituting the integral expression for $\lambda_s(x)$, followed by the conversion of Cartesian to polar coordinates and change in the order of integration. The integral with respect to θ in (17) can be solved by using [19, Eq. 8.431.3] as follows:

$$\Upsilon(\beta) = \pi \int_{u=0}^{\infty} e^{-u} \int_{v=0}^{\infty} e^{-v} \exp\left(-\beta L^2 v^{-\frac{2}{\alpha}}\right) \times \int_{r=0}^{\infty} 2\, r \, \exp\left(-\beta(v^{-\frac{2}{\alpha}} + u^{-\frac{2}{\alpha}}) r^2\right) \times I_0\left(2\beta L v^{-\frac{2}{\alpha}} r\right) dr\, dv\, du, \quad (18)$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind. The integral with respect to r can be reduced to the form $\mathcal{I} = \int_0^\infty \frac{2r}{2\sigma^2} \exp\left(-\frac{r^2+s^2}{2\sigma^2}\right) I_0\left(\frac{s}{\sigma^2} r\right) dr$ by substituting $2\beta L v^{-2/\alpha} = s/\sigma^2$ and $\beta(v^{-2/\alpha} + u^{-2/\alpha}) = 1/(2\sigma^2)$ so that $\mathcal{I}$ integrates to unity. $\Upsilon(\beta)$ can then be simplfied as

$$\Upsilon(\beta) = \frac{\pi}{\beta} \int_{u=0}^{\infty} e^{-u} \int_{v=0}^{\infty} \frac{e^{-v}}{(v^{-2/\alpha} + u^{-2/\alpha})} \times \exp\left(-\frac{\beta L^2}{v^{2/\alpha} + u^{2/\alpha}}\right) dv\, du, \quad (19)$$

which can be accurately approximated by using the Gauss-Laguerre quadrature rule [21, Eq. 25.4.45]. The outage probability of the relay link can finally be expressed as

$$P_r \approx \exp\left(-\frac{\lambda_s \pi}{\beta} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \frac{\exp\left(-\frac{\beta L^2}{\vartheta_j^{2/\alpha} + \vartheta_i^{2/\alpha}}\right)}{(\vartheta_j^{-2/\alpha} + \vartheta_i^{-2/\alpha})}\right), \quad (20)$$

where $\vartheta_i (i = 1, 2, \ldots, n)$ are the nodes of Gauss-Laguerre quadrature and $w_i (i = 1, 2, \ldots, n)$ are the corresponding weights. The exponential decrease in the outage probability of the relay link with the increasing density λ_s of the relay nodes can be observed. The asymptotic outage probability of the relay link can be obtained as follows by using $\exp(-x) = 1 - x$ as $x \to 0$ for the last exponential in (19):

$$P_r \sim A \exp\left(-\lambda_s \pi \mathcal{I}_1 \left(\Gamma(\alpha/2+1)\right)^{-2/\alpha} (\rho/\gamma_{th})^{2/\alpha}\right) \times \left(1 + O(\rho^{-2/\alpha})\right) \quad \text{as} \quad \rho \to \infty, \quad (21)$$

where $A = \exp\left(\lambda_s \pi L^2 \mathcal{I}_2\right)$ and $\mathcal{I}_1, \mathcal{I}_2$ are given by

$$\mathcal{I}_1 = \int_{u=0}^{\infty} e^{-u} \int_{v=0}^{\infty} \frac{e^{-v}}{v^{-2/\alpha} + u^{-2/\alpha}} dv\, du,$$

$$\mathcal{I}_2 = \int_{u=0}^{\infty} e^{-u} \int_{v=0}^{\infty} \frac{e^{-v}}{(v^{-2/\alpha} + u^{-2/\alpha})(v^{2/\alpha} + u^{2/\alpha})} dv\, du,$$

which can be readily computed for the given value of α.

IV. Numerical and Simulation Results

In this section, we validate our analytical results through Monte Carlo simulations and assess the impact of various parameters on the performance of the proposed system. The path loss exponent α is assumed to be 4. The interference threshold $\bar{I}$ is normalized by the noise power N_0. We define Λ_p and Λ_s as the average number of primary receivers and idle SUs (candidate relays), respectively within a circle of unit radius, i.e., $\Lambda_p = \pi\lambda_p, \Lambda_s = \pi\lambda_s$. Unless stated otherwise, Λ_p is set to 0.15. To compute the analytical outage probability of the relay link by using (20), we chose $n = 30$.

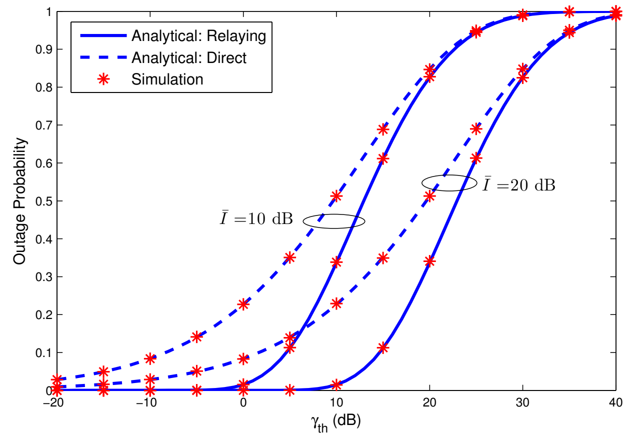

Fig. 2. The outage probability versus threshold γ_{th} for different levels of interference threshold $\bar{I}$ when $\Lambda_s = 0.5$ and $L = 2$.

The outage probability versus the threshold γ_{th} is plotted in Fig. 2 for both the direct and relaying modes. The figure shows an excellent agreement between the analytical and simulation results. Significant outage performance improvement by using relaying mode over direct transmission, is clearly visible. If the primary receivers can tolerate more interference, the secondary nodes can transmit with higher power and the outage performance naturally improves.

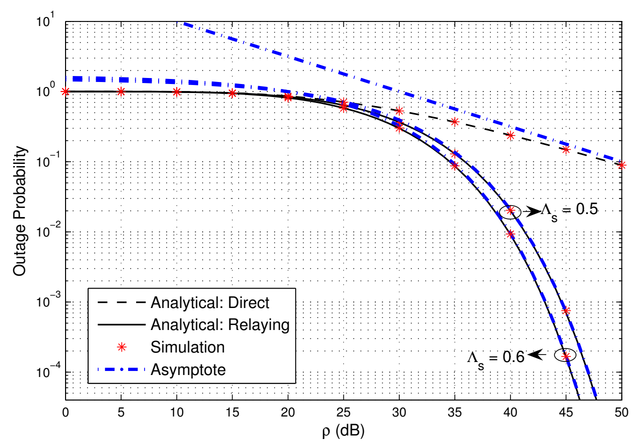

Fig. 3. The outage probability versus average transmit SNR ρ when $\gamma_{th} = 10$ dB and $L = 2$.

The outage probability as a function of the average transmit SNR ρ is presented in Fig. 3. The high-SNR asymptotes derived in (11) and (21) for the direct link and the relay link, respectively, are plotted in the figure along with the analytical and simulation curves. At $\rho = 40$ dB, the gain in using relaying mode over the direct mode, $G = P_d/P_r$ is about 10.6 dB for $\Lambda_s = 0.5$ and 14 dB for $\Lambda_s = 0.6$.

Fig. 4 assesses the impact of $S-D$ distance L on the outage performance. As expected, the outage performance of both the direct and relaying modes improves when the $S-D$ distance shrinks. When the density of idle SUs is small and the average transmit SNR is low as well, the source may not have any potential relay node available for retransmission. In this case, outage performance of direct mode is better than the relaying mode. However, if the density of idle SUs is sufficient, the relaying mode outperforms the direct transmission.

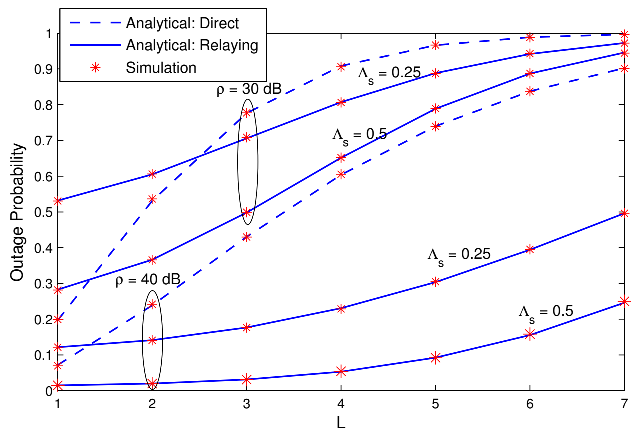

Fig. 4. The outage probability as a function of the $S-D$ distance L for different levels of average transmit SNR ρ and average density of relay nodes Λ_s when γ_{th}=10 dB.

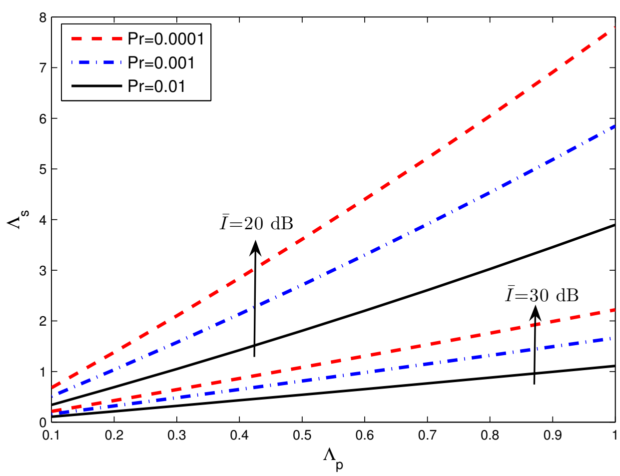

Fig. 5. The required average density of relay nodes Λ_s as a function of average density of primary receivers Λ_p for different levels of outage probability when $\gamma_{th} = 10$ dB and $L = 1$.

Fig. 5 shows the required average density of relay nodes Λ_s as the function of the average density of primary receivers Λ_p to maintain a given outage probability of the relaying mode. From (8), we can see that the average transmit power of the secondary node is inversely proportional to Λ_p. As the transmit power of the secondary node decreases with the increasing Λ_p, the outage probability tends to increase. However, one can maintain the desired outage probability by increasing Λ_s.

V. Conclusion

In this paper, we analyzed the outage in dual-hop underlay secondary network coexisting with the primary multicast network. The spatial distributions of the SU relays and primary receivers are modeled by independent PPPs. An opportunistic relaying scenario with a relay selection scheme that satisfies the interference constraint on any primary receiver is investigated. The impact of various parameters on outage performance is analyzed. We found that the gain in using the relay transmission over direct mode increases with the density of relay nodes. The required density of relay nodes for the desired outage probability increases with the density of primary receivers. The required density however, decreases if the primary receivers can tolerate more interference.

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