A Novel Base Stations-Mobile Stations Association Policy for Cellular Networks

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***Abstract*—We propose a novel base stations (BSs) - mobile stations (MSs) association policy for cellular networks. In this policy, the BS which provides the highest signal-to-interference ratio (SIR) among those located within a predetermined maximum association distance of the MS is selected as the serving BS. This policy encompasses the conventional highest-SIR association as a special case. Application of the new policy in 2- and 3-dimensional single-tier (homogeneous) and 2-dimensional two-tier (heterogeneous) networks is discussed. Coverage probability expressions are derived assuming BSs in each tier are distributed according to an independent homogeneous Poisson point process (PPP). Rayleigh fading and exponential path-loss radio channels are assumed. Analysis is validated by Monte-Carlo simulations. For single-tier networks, two methods are proposed for the selection of the maximum association distance. With such selection, the proposed association policy performs similarly to the highest-SIR association. It is shown that this policy can also be used to manage user offloading to small cells in two-tier heterogeneous networks.**

I. Introduction

In modern cellular networks, the base station (BS) - mobile station (MS) association policy plays an important role in providing the best user-perceived rate [1]. However, because the locations of nodes and users are subject to considerable uncertainty, their spatial distribution needs to be modeled via spatial stochastic processes [1]–[3]. Thus many association policies have been studied under different spatial models for single-tier (homogeneous) and multi-tier (heterogeneous) networks; see [4]–[15] and references therein. They can be divided into three categories: closest-BS [9]–[12], highest-signal-to-interference-plus-noise ratio (SINR) (equivalently, highest-signal-to-interference ratio (SIR) in interference-limited networks) [4], [13]–[15], and biased association [5]–[7]. These policies mainly consider three metrics for the selection of serving BS: (i) BS-MS distances, (ii) received signal-to-interference-plus-noise ratio (SINR), and (iii) biased-SINR or biased-location. In this paper, we propose a new association policy, which uses both SIR and BS-MS distances, and investigate its performance by modelling the distribution of BSs as a homogeneous Poisson point process (PPP). The PPP [16] has extensively been used to model distributions of BSs and MSs in many recent research works [3]–[7], [9]–[15].

A. Prior related research

In closest-BS policy, the serving BS is the geographically closest one. Downlink performance of single-tier networks under this association policy is investigated in [9]. In particular, coverage probability, normalized per user mean rate and coverage gain (and mean rate loss) from static frequency re-use are investigated. Uplink performance of a single-tier network with closest-BS policy and per-mobile power control is investigated in [11]. [12] investigates the downlink performance of multi-tier networks under the same association policy.

In highest-SINR policy, the serving BS is the one offering the highest SINR. Performance of multi-tier networks under highest-SINR policy is investigated in [4], [14], [15]. Reference [13] investigates the downlink coverage probability of a single-tier network under highest-SINR policy.

In biased association policies, a bias is introduced to the original association policy to account for the load of each type of BS. Performance of SINR-based biased association policies is investigated in [5], [6]. In [7], Mukherjee investigates the downlink performance of both SINR-based and location-based biased association policies.

B. Motivation and our contribution

All those aforementioned BS-MS association policies require the information of BS locations or SINR/SIR of the entire network or at least a large part of the network. However, in practical networks, available SINR/SIR and/or location information may be limited to BSs residing within a limited distance from each MS. We propose a new BS-MS association policy suitable for such cases, named 'hybrid BS-MS association policy'. In this policy, for each MS, the BS which provides the highest signal-to-interference ratio (SIR) among those located within a predetermined maximum association distance R is selected as the serving BS. Limiting the maximum association distance will also avoid frequent handovers. The highest-SIR association [4] can be considered as a special case of the new policy, i.e., when $R \rightarrow \infty$. Application of this scheme in 2- and 3-dimensional single-tier homogeneous and 2-dimensional two-tier heterogeneous networks is investigated. For single-tier networks, two methods for the selection of R are proposed. For both single- and two-tier networks, expressions are derived for the coverage probability assuming BSs in each tier are distributed according to an independent homogeneous PPP.

In multi-tier networks, BSs with smaller coverage areas, such as pico-BSs, will have fewer active users and hence will be lightly loaded compared to macro-BSs [1]. Therefore, an intelligent cell association policy designed for multi-tier networks should be capable of offloading users from highly loaded BSs to lightly loaded ones so that BSs offer them the best user-perceived rate [1]. We show that the proposed policy can also be used to handle user offloading in two-tier networks.

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II. System Model: Single-Tier Network

We consider a network with BSs distributed in $\mathbb{R}^d, d \in \{2,3\}$ according to a homogeneous PPP Φ with intensity λ. Without loss of generality, we consider that all the BSs transmit with unity power. Rayleigh-fading (with envelope power normalized to one) is assumed along with exponential path-loss model, where path-loss exponent $\alpha > d$. The condition $\alpha > d$ is to maintain a finite received power at each MS. For simplicity, we ignore the background thermal noise since its impact is negligible in an interference-limited network. In the new BS-MS association policy, serving BS is the one, which provides the highest SIR among all BSs located within distance R. We refer to this new BS-MS association policy as 'hybrid BS-MS association policy' as this is a hybrid version of the highest-SIR and the location-based BS association policies. The highest-SIR association [4] is a special case of the new policy when $R \to \infty$.

III. Performance Analysis: Single-Tier Network

This section presents the SIR based coverage probability analysis. Without loss of generality, the analysis is focused on an MS located at the center of $\mathbb{R}^d$. In the new association policy, maximum association distance R divides the original space $\mathbb{R}^d$, over which the BSs are distributed, into two disjoint sub-spaces: $S_1 = \{x \in \mathbb{R}^d ||x|| < R\}$ and $S_2 = \{x \in \mathbb{R}^d : ||x|| > R\}$. Here $||x||$ represents the Euclidean distance between point $x \in \Phi$ and the origin. According to the restriction theorem [17], BSs residing in S_1 and S_2 form two independent PPPs: Φ_1 and Φ_2, respectively. Assuming the MS is connected to BS $y \in \Phi_1$, its SIR can be written as

$$SIR(y) = \frac{||y||^{-\alpha} h_y}{\sum_{w \in \Phi_1 \backslash y} ||w||^{-\alpha} h_w + \sum_{z \in \Phi_2} ||z||^{-\alpha} h_z}, \quad (1)$$

where h_y, h_w, and h_z represent fading gains. $\Phi_1 \backslash y$ denotes Φ_1 excluding BS located at y. The resulting SIR at the MS is given by $SIR = \max_{y \in \Phi_1} \{SIR(y)\}$. An MS is considered to be in coverage when its SIR equals or exceeds a given threshold T. By using a similar approach to that in [4], the coverage probability P_c under hybrid BS-MS association policy can be derived as follows. P_c can be written as

$$P_c = \Pr\left(\bigcup_{y \in \Phi_1} SIR(y) \geq T\right) \leq \mathbb{E}\left[\sum_{y \in \Phi_1} \mathbf{1}\left(SIR(y) \geq T\right)\right], \quad (2)$$

where the indicator function $\mathbf{1}(\cdot) = 1$, if the condition in the argument holds, or 0 if it fails. The bound in (2) is the union bound on P_c. Using Lemma 1 of [4], it can be easily shown that the union bound in (2) is exact for threshold SIRs $T > 0$ dB and is an upper bound for $T < 0$ dB. Now (2) can be written as

$$P_c \overset{a}{\leq} \lambda \int_{S_1} \Pr\left(h_y \geq \frac{T(I_{1\backslash y} + I_2)}{||y||^{-\alpha}}\right) dy$$

$$\overset{b}{\leq} \lambda \int_{S_1} \mathcal{L}_{I_{\sum \backslash y}}\left(\frac{T}{||y||^{-\alpha}}\right) dy, \quad (3)$$

where (a) follows from the Campbell-Mecke theorem [16] and (b) follows from the Rayleigh fading assumption. $I_{1\backslash y}$ is the total interference power from all the BSs in Φ_1 when MS is connected to BS y. I_2 is the total interference power from Φ_2. $I_{\sum \backslash y} = I_{1\backslash y} + I_2$. $\mathcal{L}_{I_{\sum \backslash y}}(s)$ is the Laplace transform of $I_{\sum \backslash y}$, which can be derived as shown in the following.
Since Φ_1 and Φ_2 are independent PPPs over S_1 and S_2 respectively, $I_{1\backslash y}$ and I_2 are statistically independent. Therefore,

$$\mathcal{L}_{I_{\sum \backslash y}}(s) = \mathbb{E}_{\Phi_1, h_x}\left[\exp\left(-s \sum_{x \in \Phi_1 \backslash y} ||x||^{-\alpha} h_x\right)\right] \times \mathbb{E}_{\Phi_2, h_w}\left[\exp\left(-s \sum_{w \in \Phi_2} ||w||^{-\alpha} h_w\right)\right]. \quad (4)$$

Since fading coefficients are independent and identically distributed (i.i.d.) and using the fact that the fading process is independent from PPPs

$$\mathcal{L}_{I_{\sum \backslash y}}(s) = \mathbb{E}_{\Phi_1}\left[\prod_{x \in \Phi_1 \backslash y} \frac{1}{1 + s||x||^{-\alpha}}\right] \times \mathbb{E}_{\Phi_2}\left[\prod_{w \in \Phi_2} \frac{1}{1 + s||w||^{-\alpha}}\right]. \quad (5)$$

Using the Slivnyak-Mecke theorem [16] along with the definition of probability generating functional of PPP [16] we get:

$$\begin{aligned}\mathcal{L}_{I_{\sum \backslash y}}(s) =& \exp\left[-\lambda \int_{S_1} \left(1 - \frac{1}{1 + s||x||^{-\alpha}}\right) dx\right] \\ & \times \exp\left[-\lambda \int_{S_2} \left(1 - \frac{1}{1 + s||w||^{-\alpha}}\right) dw\right] \\ =& \exp\left[-\lambda \int_{\mathbb{R}^d} \left(1 - \frac{1}{1 + s||u||^{-\alpha}}\right) du\right]. \quad (6)\end{aligned}$$

Converting from Cartesian to polar/spherical coordinates, (6) for d-dimensional networks can be written as

$$\begin{aligned}\mathcal{L}_{I_{\sum \backslash y}}(s) &= \exp\left[-\kappa_d \pi \lambda \int_0^\infty r^{d-1}\left(1 - \frac{1}{1 + sr^{-\alpha}}\right) dr\right] \\ &= \exp\left[\frac{-\kappa_d \pi^2 \lambda s^{\frac{d}{\alpha}}}{\alpha \sin\left(\frac{d\pi}{\alpha}\right)}\right], \quad (7)\end{aligned}$$

where $\kappa_2 = 2$, and $\kappa_3 = 4$. Converting (3) from Cartesian to polar/spherical coordinates and substituting (7) in this new expression, P_c for d-dimensional networks can be expressed as

$$P_c \leq \left[1 - \exp\left(\frac{-\kappa_d \pi^2 R^d T^{\frac{d}{\alpha}} \lambda}{\alpha \sin\left(\frac{d\pi}{\alpha}\right)}\right)\right] \frac{\alpha \sin\left(\frac{d\pi}{\alpha}\right)}{d\pi T^{\frac{d}{\alpha}}}. \quad (8)$$

According to (8), the impact of λ on P_c diminishes as $R \to \infty$. A similar observation is made for 2-D networks in Corollary 1 of [4].

Selection of Maximum Association Distance (R)

One way to select the maximum association distance R is to choose it such that the closest n BSs are located within R with a reasonably high probability q. For example, $n=2$ and $q=0.95$ will guarantee that at least two BSs are located within R distance from MS with a probability of 0.95. In a network with a homogeneous PPP distribution of BSs in $\mathbb{R}^d$ with intensity λ, for a given number of BSs n and probability q, R can be computed as [18]

$$R=\left[\frac{\Gamma_{in}^{-1}(n,(1-q)(n-1)!)}{\lambda c_d}\right]^{1/n}, \qquad (9)$$

where $c_2=\pi, c_3=4\pi/3$ and $\Gamma_{in}^{-1}(\cdot,\cdot)$ is the inverse incomplete gamma function.

Improving P_c by adaptive increase of R: In hybrid BS-MS association policy, the MS is considered to be in outage if no BS is found within the maximum association distance R. This can degrade performance of the network, especially when R is small. Also if R is set to a very high value, more BSs will have to be considered. This can increase complexity of the BS-MS association process and reduce power efficiency of the network. However, the problem can be alleviated by initializing the association policy with a small value for R (equivalently, small n and q), and by increasing it iteratively. For example, a system may select initial maximum association distance R with lower values for n and q (e.g., $n=1$ and $q=0.8$, which guarantee at least one BS within R, 80% of the time). Then increase R iteratively until a given number of BSs are found (e.g., new maximum association distance after 1st iteration $R_1=bR,\ b>1.\ R_2=b^2R$).

IV. Application of hybrid BS-MS association policy in two-tier heterogeneous networks

In this section we consider the application of hybrid BS-MS association policy in a two-tier 2-dimensional heterogeneous network consisting of macro- and pico-BSs. According to the association policy, a pico-BS located within distance R of the MS is selected as the serving BS. If more than one pico-BS is available, one providing the highest SIR is selected. When the MS is associated with a pico-BS, the closest macro-BS can choose either to remain silent or continue its transmission (serve another MS) in the channel (time-frequency block) occupied by pico-BS to MS link. If no pico-BS is available within distance R, the closest macro-BS is selected as the serving BS. Thus this association policy has the ability to manage user offloading to small cells by changing the maximum association distance R. For example, 70% of users will be served by pico-BSs, when R is selected with $q=0.7$ and $n=1$ (70% of the time there is at least one pico-BS within R). Further, R can be adaptively selected depending on the load on different types of BSs (two tiers).

A. Network Parameters: Two-tier Network

We characterize the spatial distribution of macro-BSs and pico-BSs by independent homogeneous PPPs Φ_m and Φ_p with intensities λ_m and λ_p respectively. Macro-BSs transmit with power P_m, while pico-BSs transmit with power P_p. All the links are assumed to be subject to exponential path-loss and Rayleigh fading. Path-loss exponents of macro- and pico-BSs are given by α_m and α_p ($\alpha_m,\alpha_p>2$). Target SIR thresholds of macro- and pico-BSs are given by T_m and T_p. Without loss of generality the coverage probability of an MS located at the center of the network is analyzed.

B. Performance Analysis: Two-Tier Network

In this section we derive the coverage probability of the network described previously. Using the theorem on total probability the coverage probability becomes

$$P_c=P_vP_c^{\text{macro}}+(1-P_v)P_c^{\text{pico}}, \qquad (10)$$

where $P_v=\exp(-\lambda_p\pi R^2)$ represents the probability that no pico-BS is found within distance R from the MS. P_c^{macro} is the coverage probability when the user is served by the closest macro-BS provided that there is no pico-BS within distance R. P_c^{pico} gives the coverage probability when the user is served by the highest SIR pico-BS residing within R. In the development of theoretical expressions for P_c^{pico}, it is assumed that all the macro-BSs simultaneously transmit with pico-BSs. P_c^{pico} with closest macro-BS muting is studied using computer simulations.

Following a similar approach to the derivation of (2) and (3), $\hat{P}_{\text{pico}}=(1-P_v)P_c^{\text{pico}}$ can be derived as

$$\hat{P}_{\text{pico}}\le\lambda_p\int_{\mathcal{A}_R}\mathcal{L}_{I_{p\backslash y}+I_m}\left(\frac{T_p}{P_p||y||^{-\alpha_p}}\right)dy$$
$$\le 2\pi\lambda_p\int_0^R w\mathcal{L}_{I_{p\backslash y}+I_m}\left(\frac{T_p}{P_pw^{-\alpha_p}}\right)dw, \qquad (11)$$

where $\mathcal{A}_R$ represents the circular region around the MS with radius R. $\mathcal{L}_{I_{p\backslash y}+I_m}(s)$ represents the Laplace transform of $I_{p\backslash y}+I_m$. $I_{p\backslash y}$ represents the aggregate interference from pico-BSs when MS is served by pico-BS located at $y\in\Phi_p$. I_m represents the aggregate interference from macro-BSs. Similar to (3), the bound in (11) is exact for threshold SIRs $T>0$ dB and is an upper bound for $T<0$ dB. Since fading coefficients are i.i.d. and fading process is independent from PPPs $\mathcal{L}_{I_{p\backslash y}+I_m}(s)=\mathcal{L}_{I_{p\backslash y}}(s)\times\mathcal{L}_{I_m}(s)$, where $\mathcal{L}_{I_{p\backslash y}}(s)$ and $\mathcal{L}_{I_m}(s)$ represent the Laplace transforms of $I_{p\backslash y}$ and I_m. $\mathcal{L}_{I_{p\backslash y}}(s)$ can be obtained by replacing s, λ, and α in (7) with sP_p, λ_p, and α_p, respectively. Therefore

$$\mathcal{L}_{I_{p\backslash y}}(s)=\exp\left[\frac{-2\pi^2\lambda_p(sP_p)^{\frac{2}{\alpha_p}}}{\alpha_p\sin\left(\frac{2\pi}{\alpha_p}\right)}\right]. \qquad (12)$$

Following similar steps to the derivation of (7), $\mathcal{L}_{I_m}(s)$ can be derived as

$$\mathcal{L}_{I_m}(s)=\mathbb{E}_{\Phi_m}\left[\prod_{x\in\Phi_m}\mathbb{E}_{h_x}\left[\exp\left(-sP_m||x||^{-\alpha_m}h_x\right)\right]\right]$$
$$\overset{a}{=}\exp\left[\frac{-2\pi^2\lambda_m(sP_m)^{\frac{2}{\alpha_m}}}{\alpha_m\sin\left(\frac{2\pi}{\alpha_m}\right)}\right]. \qquad (13)$$

P_c^{macro} can be derived as shown in the following. When MS is served by the closest macro-BS $z \in \Phi_m$, SIR is given by

$$SIR(z) = \frac{P_m r_z^{-\alpha_m} h_z}{I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}}, \tag{14}$$

where distance between MS and the closest macro-BS is denoted by $r_z = ||z||$. $I_{m\backslash z} = \sum_{x\in\Phi_m\backslash z} P_m ||x||^{-\alpha_m} h_x$ and $I_{p\backslash \mathcal{A}_\mathcal{R}} = \sum_{y\in\Phi_p, ||y||>R} P_p ||y||^{-\alpha_p} h_y$. Therefore

$$\begin{aligned} P_c^{\text{macro}} &= \int_0^\infty \Pr\left[h_z \geq \frac{T_m \left(I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}\right)}{P_m r_z^{-\alpha_m}} \middle| r_z = t\right] f_{r_z}(t) dt \\ &\stackrel{a}{=} \int_0^\infty \mathcal{L}_{I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}} \left(\frac{T_m}{P_m t^{-\alpha_m}}\right) f_{r_z}(t) dt, \end{aligned} \tag{15}$$

where the Laplace transform of $I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}$ is given by $\mathcal{L}_{I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}}(s)$. Step (a) follows from the Rayleigh fading assumption. The probability distribution function of r_z is given by [18], $f_{r_z}(t) = 2\pi\lambda_m t \exp\left(-\lambda_m \pi t^2\right), 0 < t < \infty$. $\mathcal{L}_{I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}}(s) \stackrel{a}{=} \mathcal{L}_{I_{m\backslash z}}(s) \times \mathcal{L}_{I_{p\backslash \mathcal{A}_\mathcal{R}}}(s)$ can be derived as shown in the following.

$$\begin{aligned} \mathcal{L}_{I_{m\backslash z} + I_{p\backslash \mathcal{A}_\mathcal{R}}}(s) &\stackrel{b}{=} \mathbb{E}_{\Phi_m, h_x}\left[\exp\Big(-s \sum_{x\in\Phi_m\backslash z} P_m ||x||^{-\alpha_m} h_x\Big)\right] \\ &\times \mathbb{E}_{\Phi_p, h_y}\left[\exp\Big(-s \sum_{y\in\Phi_p, ||y||>R} P_p ||y||^{-\alpha_p} h_y\Big)\right] \\ &\stackrel{c}{=} \mathbb{E}_{\Phi_m}\left[\prod_{x\in\Phi_m\backslash z} \mathbb{E}_{h_x}\left[\exp\big(-sP_m ||x||^{-\alpha_m} h_x\big)\right]\right] \\ &\times \mathbb{E}_{\Phi_p}\left[\prod_{y\in\Phi_p, ||y||>R} \mathbb{E}_{h_y}\left[\exp\big(-sP_p ||y||^{-\alpha_p} h_y\big)\right]\right] \\ &\stackrel{d}{=} \exp\left[-2\pi\lambda_m \int_t^\infty u\left(1 - \frac{1}{1 + sP_m u^{-\alpha_m}}\right) du\right] \\ &\times \exp\left[-2\pi\lambda_p \int_R^\infty v\left(1 - \frac{1}{1 + sP_p v^{-\alpha_p}}\right) dv\right] \\ &= \exp\left[\frac{-2\pi\lambda_m P_m t^{2-\alpha_m} s \; {}_2F_1\left[1, \frac{\alpha_m - 2}{\alpha_m}; 2 - \frac{2}{\alpha_m}, \frac{-P_m s}{t^{\alpha_m}}\right]}{\alpha_m - 2}\right] \\ &\times \exp\left[\frac{-2\pi\lambda_p P_p R^{2-\alpha_p} s \; {}_2F_1\left[1, \frac{\alpha_p - 2}{\alpha_p}; 2 - \frac{2}{\alpha_p}, \frac{-P_p s}{R^{\alpha_p}}\right]}{\alpha_p - 2}\right]. \end{aligned} \tag{16}$$

Here, ${}_2F_1\left[a, b; c; z\right]$ is the Gauss's hypergeometric function [19]. Steps (a) and (c) follow from the fact that fading coefficients are i.i.d. and two PPPs Φ_m and Φ_p are independent of each other and independent from the fading process. (b) follows from the definition of the Laplace transform. (d) is due to the Rayleigh fading assumption. (e) follows from the definition of probability generating functional of PPP [16] and by converting from Cartesian to polar coordinates. This concludes the derivation of P_c.

V. Numerical Results

This section first considers a single-tier network with the proposed MS-BS association policy. Its coverage probability is compared with that of closest-BS and highest-SIR policies. Secondly, the coverage probability of a two-tier network, employing the proposed association policy, is investigated.

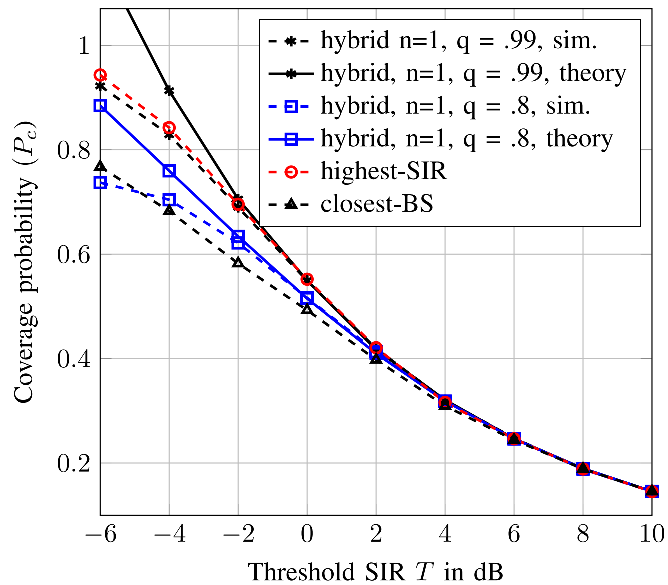

Fig. 1: Variation of P_c in a single-tier network with T. $\alpha = 3.5$, $\lambda = 12 \times 10^{-6}$ m^{-2}.

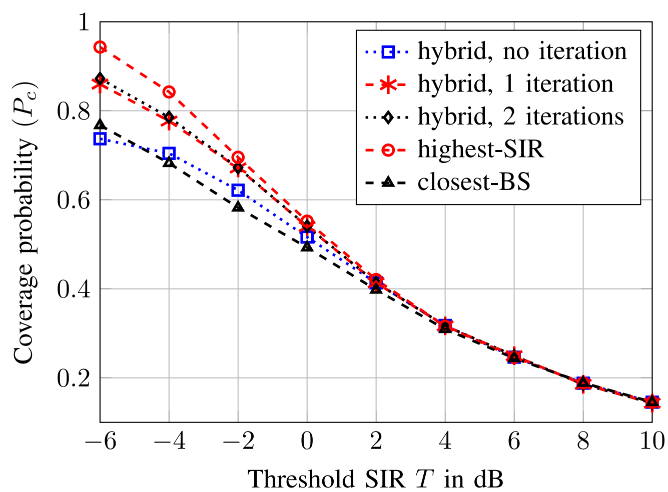

Fig. 2: P_c in a single-tier network with iterative R vs T. $\alpha = 3.5$, $\lambda = 12 \times 10^{-6}$ m^{-2}.

Fig. 1 shows the variation of P_c in a single-tier network with the threshold T. A closer match between the analytical and simulation results can be observed, especially when $T > -2$ dB. The coverage probability of the new policy is compared with those of the closest-BS and the highest-SIR policies. When the maximum association distance R (or, equivalently, n and q) is selected appropriately, our new policy outperforms the closest-BS association and performs similarly to the highest-SIR policy. For example, the hybrid policy with $n = 1$, and $q = 0.99$ achieves similar performance as the highest-SIR. Due to the exponential path-loss, to achieve a higher SIR an MS needs to associate with a much closer

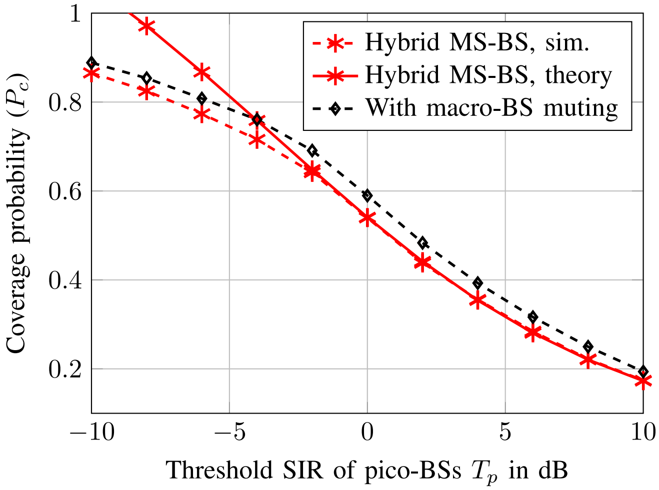

Fig. 3: Variation of P_c in a two-tier network with T_p. $P_m = 20$ W, $P_p = 2$ W, $\alpha_m = 3.5$, $\alpha_p = 3.8$, $n = 1$, $q = 0.7$, $T_m = T_p - 5$ dB, $\lambda_m = 5 \times 10^{-7}$ m^{-2}, $\lambda_p = 20 \times 10^{-6}$ m^{-2}.

BS. Therefore, performances of all the association policies considered in our analysis are equal to that of the closest-BS policy when threshold SIR is higher ($T > 6$ dB).

Coverage probability improvement that can be achieved by iterative selection of R is shown in Fig. 2. In the network considered, R is initially selected with $n = 1$, $q = 0.8$ from (9). In each iteration, R is increased by 50% (b = 1.5). Results show that P_c with initial R is either better or equal to that of the closest-BS policy only when $T > -4$ dB. However, the performance of hybrid policy with one iteration outperforms the closest-BS one. Results also show that only a marginal performance improvement can be achieved by having more than one iteration.

Fig. 3 shows the variation of P_c in a two-tier network with threshold SIR. The maximum association distance for pico-BSs was selected with $n = 1$ and $q = 0.7$. If MSs are uniformly distributed, under this configuration 70% MSs will be served by pico-BSs, while the remaining 30% will be served by macro-BSs. A closer match between analytical and simulation results can be observed, especially when $T_p > -2$ dB. The figure also shows that around 1 dB gain in terms of threshold SIR can be achieved by muting the closest macro-BS when an MS is served by a pico-BS.

VI. Conclusion

This paper has proposed a new MS-BS association policy for cellular networks. In this policy, the serving BS is the one which provides the highest SIR among all BSs located within a preselected maximum association distance R. This policy encompasses the conventional highest-SIR association policy as a special case, i.e., when $R \to \infty$. Application of this policy in both single-tier (homogeneous) and two-tier (heterogeneous) networks has been investigated. For single-tier networks, two methods for the selection of R have been proposed. It has been shown that with proper selection of R, this policy outperforms the closest-BS policy and performs similarly to the highest-SIR association in single-tier networks. It has also been shown that this policy can be used to manage user offloading to small cells in two-tier networks. Coverage probability of 2- and 3-dimensional single-tier and 2-dimensional two-tier networks has been derived and validated by Monte-Carlo simulations.

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