# Cognitive Transmission and Performance Analysis for Amplify-and-Forward Two-Way Relay Networks

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***Abstract*—In this paper, we propose a cognitive transmission scheme for Amplify-and-Forward (AF) two-way relay networks (TWRNs) and investigate its joint sensing and transmission performance. Specifically, we derive the overall false alarm probability, the overall detection probability, the outage probability of the cognitive TWRN over Rayleigh fading channels. Furthermore, based on these probabilities, the spectrum hole utilization efficiency of the cognitive TWRN is defined and evaluated. It is shown that smaller individual or overall false alarm probability can result in less outage probability and thus larger spectrum hole utilization efficiency for cognitive TWRN, and also produce more interference to the primary users. Interestingly, it is found that given data rate, more transmission power for the cognitive TWRN does not necessarily obtain higher spectrum hole utilization efficiency. Moreover, our results show that a maximum spectrum hole utilization efficiency can be achieved through an optimal allocation of the time slots between the spectrum sensing and data transmission phases. Finally, simulation results are provided to corroborate our proposed studies.**

## I. Introduction

The past two decades witness a rapid proliferation of wireless network technology and nowadays the increasing demand for high data rate wireless access and services brings about the problem of spectrum scarcity [1]. Cognitive radio (CR) [2] has been recognized as a promising technology to improve spectrum utilization efficiency and solve the spectrum scarcity problem.

Typically, a cognitive transmission process consists of two essential phases: spectrum sensing phase and data transmission phase. In the spectrum sensing phase, cognitive users attempt to find the spectrum hole, a frequency band assigned to the primary users but is not being utilized by the users at a particular time and specific geographic location. In the data transmission phase, cognitive users transmit data to each other through the detected spectrum hole.

However, the spectrum sensing result by one cognitive user can be fairly inaccurate, which will result in mistakenly transmission and thus cause severe interference to the primary users. To improve the performance of cognitive radio networks, cooperative relay technology can be utilized for spectrum sensing and cognitive transmission [3].

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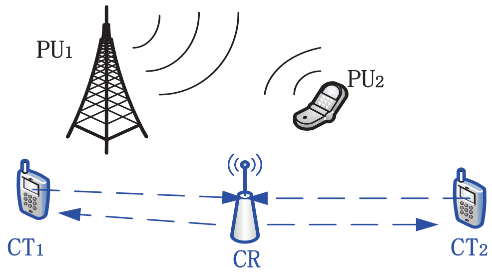


Fig. 1. Cognitive Two-Way Relay Network.

The relay networks applied into cognitive radio can be classified into two types: (1) one-way relay network (OWRN), where data flows unidirectionally from the source via the relay to the destination; (2) two-way relay network (TWRN) where two source nodes simultaneously send their signals to the relay node, which retransmits its received signal to both source nodes. Reference [4] has reported that the overall communication rate between two source terminals in TWRN is approximately twice that achieved in OWRN, making TWRN particularly attractive to bidirectional systems.

Cognitive TWRN has been studied in [6]–[9]. Beamforming methods for cognitive TWRN with several relays have been suggested in [6] while power allocation and relay selection was discussed in [7]. Optimal linear transceiver is designed in [8] for cognitive TWRN with one overlay relay. The reference [9] derives the outage probability for cognitive TWRN with relay selection under interference constraint. However, it focus on three phase Decode-and-Forward TWRN, instead of the two-phase Amplify-and-Forward (AF) TWRN.

To our best knowledge, the cognitive transmission of AF TWRN has not been studied, which motivates our present work. In this paper, we propose a cognitive transmission protocol for AF TWRN and investigate its performance by jointly considering the spectrum sensing phase and data transmission phase over Rayleigh fading channels. Simulation results are then provided to corroborate our proposed studies.

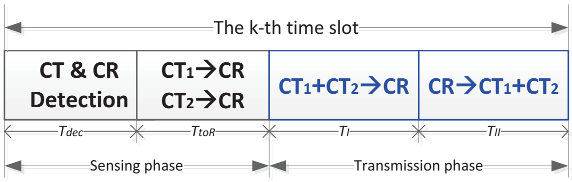


Fig. 2. The $k$-th transmission time slot of cognitive TWRN.

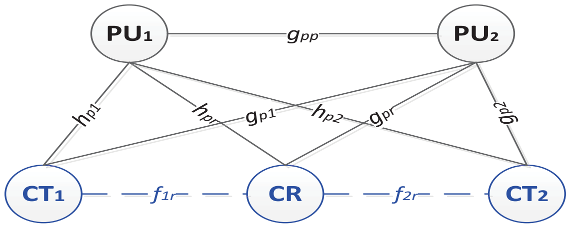


Fig. 3. The channels in cognitive TWRN.

## II. System Model

Consider a joint existence of the primary network and a secondary network as shown in Fig. 1. The primary network consists two primary users $\text{PU}_1$ and $\text{PU}_2$ that are communicating over certain licensed bands. Without loss of generality, we assume $\text{PU}_1$ is the transmitter and $\text{PU}_2$ is the receiver. The secondary network is a cognitive two-way relay network with two destination node $\text{CT}_1$ and $\text{CT}_2$, and a relay node CR.

The cognitive TWRN work in a slotted structure that is shown in Fig. 2. The three nodes in the cognitive TWRN sense the bands simultaneously at beginning of each slot and will start communication when sensing the spectrum hole and stop at the end of the slot. Clearly, each cognitive transmission process is divided into two phases: sensing phase and transmission phase.

Let $H_p(k)$ represent whether or not there is a spectrum hole for the $k$-th time slot. Specifically, $H_p(k) = H_0$ represents that a spectrum hole is available, i.e., the channel is unoccupied by the primary users; otherwise, $H_p(k) = H_1$. As did in [5], we model $H_p(k)$ as a Bernoulli random variable with parameter $P_\emptyset$, i.e., $\Pr(H_p(k) = H_0) = P_\emptyset$ and $\Pr(H_p(k) = H_1) = 1 - P_\emptyset$. For notational convenience, the channels and their distributions are given in Fig. 3 and Table I respectively. For clarity and ease of understanding this paper, the notations used throughout this paper are illustrated in Table II.

### A. Sensing phase

During the $T_{dec}$ sub-slot, the node $\text{CT}_1$ or $\text{CT}_2$ will receive the signal

$$y_i(k,1) = \sqrt{P_p} h_{pi} \theta(k,1) + n_i(k,1), \quad i = 1,2 \tag{1}$$

where $P_p$ is the transmission power of the primary user $\text{PU}_1$, $n_i(k,1)$ is the additive white Gaussian noise (AWGN) with variance $N_0$ at the $\text{CT}_i$ node, and $\theta(k,1)$ is the signal transmitted from the PU

$$\theta(k,1) = \begin{cases} 0, & H_p(k) = H_0 \\ x_p(k,1), & H_p(k) = H_1. \end{cases} \tag{2}$$

TABLE I
List of Distributions of All Channels

| Notation | Distribution | Notation | Distribution |
|---|---|---|---|
| $h_{p1}$ | $\mathcal{CN}(0, \varsigma_{h1}^2)$ | $g_{p1}$ | $\mathcal{CN}(0, \varrho_{g1}^2)$ |
| $h_{p2}$ | $\mathcal{CN}(0, \varsigma_{h2}^2)$ | $g_{p2}$ | $\mathcal{CN}(0, \varrho_{g2}^2)$ |
| $h_{pr}$ | $\mathcal{CN}(0, \varsigma_{hr}^2)$ | $g_{pr}$ | $\mathcal{CN}(0, \varrho_{gr}^2)$ |
| $f_{1r}$ | $\mathcal{CN}(0, \sigma_{f1}^2)$ | $f_{2r}$ | $\mathcal{CN}(0, \sigma_{f2}^2)$ |

TABLE II
List of the Notations Used Throughout This Paper

| | |
|---|---|
| $\beta_i(k,n)$ | Signals transmitted by $\text{CT}_i$ in the $n$-th sub-slot of the $k$-th time slot, where $i = 1,2$ and $n = 1,2,3,4$. |
| $\theta(k,n)$ | Signals transmitted by $\text{PU}_1$ in the $n$-th sub-slot of the $k$-th time slot, where $n = 1,2,3,4$. |
| $y_i(k,n)$ | Signals received by $\text{CT}_i$ in the $n$-th sub-slot of the $k$-th time slot, where $n = 1,2,3,4$. |
| $y_r(k,n)$ | Signals received by CR in the $n$-th sub-slot of the $k$-th time slot, where $n = 1,2,3,4$. |
| $n_i(k,n)$ | Noise at $\text{CT}_i$ in the $n$-th sub-slot of the $k$-th time slot, where $i = 1,2,r$ and $n = 1,2,3,4$. |
| $H_0, H_1$ | Events representing the licensed channel being unoccupied and occupied by a primary user, respectively. |
| $H_p(k)$ | The status of the licensed primary channel at the $k$-th time slot. $H_p(k) = H_0$ or $H_p(k) = H_1$. |
| $\hat{H}_{s,i}(k)$ | The initial spectrum sensing result detected by $\text{CT}_i$ at the $k$-th time slot. $i = 1,2,r$ and $\hat{H}_{s,i}(k) = H_0$ or $\hat{H}_{s,i}(k) = H_1$. |
| $\hat{H}_s(k)$ | The final decision of the spectrum sensing results for the cognitive TWRN. $\hat{H}_s(k) = H_0$ or $\hat{H}_s(k) = H_1$. |
| $P_{d,i}, P_{f,i}$ | Individual detection probability and individual false alarm probability at $\text{CT}_i$ respectively, where $i = 1,2,r$. |
| $P_d, P_f$ | Overall detection probability and overall false alarm probability for the cognitive TWRN respectively, i.e., $P_d = \Pr\{H_s(k) = H_1 \mid H_p(k) = H_1\}$ and $P_f = \Pr\{H_s(k) = H_1 \mid H_p(k) = H_0\}$. |

The node CR will receive the signal

$$y_r(k,1) = \sqrt{P_p} h_{pr} \theta(k,1) + n_r(k,1), \quad i = 1,2. \tag{3}$$

Based on the received signal $y_i(k,1)$ or $y_r(k,1)$, the $\text{CT}_i$ node or the CR node will decide whether the PU is transmitting signals or not by utilizing a detection approach. In this paper, energy detector is chosen to evaluate the spectrum sensing performance due to its simplicity and popularity.

Suppose the sensing results given by the $\text{CT}_i$ node is $\hat{H}_{s,i}(k) = H_0$ (implying the existence of the spectrum hole) or $\hat{H}_{s,i}(k) = H_1$ (indicating no spectrum holes). In the following $T_{toR}$ sub-slot, CTs will transmit the sensing results to the CR node over one *orthogonal* common control channel. The received signals at CR from CTs can thus be expressed as

$$y_r(k,2) = \sqrt{P_{si}} f_{ir} x_i(k,2) + n_r(k,2), \quad i = 1,2 \tag{4}$$

where $n_r(k,2)$ is the AWGN with variance $N_0$ at the CR node.

Next CR will decode the received signal $y_r(k,2)$ in order to find out the sensing results of the CTs. Let $\mathcal{C}_{R,SP}$ represent the CTs whose initial sensing results are received and decoded successfully at CR. Clearly, the sample space of such a set is given by $\mathcal{C}_{R,SP} \in \{\emptyset, \text{CT}_1, \text{CT}_2, \text{CT}_1 \bigcup \text{CT}_2\}$.

Suppose the bandwidth of one common control channel is $B_c$ and one symbol period is $T_s$. Note that an initial sensing result $\hat{H}_{s,i}$ carries only one bit information. To successfully decode this bit, the common control channel capacity should be larger than the data rate. Therefore, the case $\mathcal{C}_{R,SP} = \text{CT}_1 \bigcup \text{CT}_2$ can be described as

$$\frac{T_{toR}B_c}{2}\log_2(1+\frac{P_{si}|f_{ir}|^2}{N_0}) > \frac{1}{T_s}, \quad i=1,2 \tag{5}$$

where the factor $1/2$ is due to the fact that $\text{CT}_1$ (or $\text{CT}_2$) is assigned with half of the common control channel.

After decoding the signal $y_r(k,2)$, the CR node will get information about $\mathcal{C}_{R,SP}$ as well as $\hat{H}_{s,1}$ and $\hat{H}_{s,2}$. Then combining the CRs' detection results $\hat{H}_{s,1}$ and $\hat{H}_{s,2}$ with its own detection result $\hat{H}_{s,r}$ through a given fusion rule. Several choices for the fusion rule can be available and their performance analysis can be treated in the same way. An "AND" fusion rule is used throughout this paper. That is, the decision $\hat{H}_s(k) = H_0$ is made only and if only when $\mathcal{C}_{R,SP} = \text{CT}_1 \bigcup \text{CT}_2$, $\hat{H}_{s,1}(k) = H_0$, $\hat{H}_{s,2}(k) = H_0$ and $\hat{H}_{s,r}(k) = H_0$.

Next the cognitive TWRN will move to transmission phase depending on the final decision $\hat{H}_s$. In the case of $\hat{H}_s(k) = H_1$, the CR will transmit a signal to both CTs through a common control channel to inform them the existence of the PU. Therefore, no transmission will begin and both CTs and CR will wait for the next slot. In the case of $\hat{H}_s(k) = H_0$, the transmission phase will begin.

### B. Transmission phase

During the sub-slot $T_I$, the relay node CR will receive the signal

$$\begin{aligned} y_r(k,3) =& \sqrt{P_{s1}}f_{1r}\beta_1(k,3) + \sqrt{P_{s2}}f_{2r}\beta_2(k,3) \\ &+ \sqrt{P_p}h_{pr}\theta(k,3) + n_r(k,3), \end{aligned} \tag{6}$$

where

$$\beta_i(k,3) = \begin{cases} x_{ti}(k,3), & \hat{H}_s(k) = H_0 \\ 0, & \hat{H}_s(k) = H_1 \end{cases} \quad i=1,2 \tag{7}$$

$$\theta(k,3) = \begin{cases} 0, & H_p(k) = H_0 \\ x_p(k,3), & H_p(k) = H_1. \end{cases} \tag{8}$$

In the next sub-slot $T_{II}$, the CR node combines the received signal $y_r(k,3)$ from CTs, amplifies it with a constant factor $\alpha$, and forward it to both CTs. Therefore the signal received at $\text{CT}_1$ is

$$y_1(k,4) = \alpha\sqrt{P_r}f_{1r}y_r(k,3) + \sqrt{P_p}h_{p1}\theta(k,4) + n_1(k,4), \tag{9}$$

where $\alpha$ is the constant amplifying factor and

$$\theta(k,4) = \begin{cases} 0, & H_p(k) = H_0 \\ x_p(k,4), & H_p(k) = H_1. \end{cases} \tag{10}$$

Substituting (6) into (9) will give

$$\begin{aligned} y_1(k,4) =& \alpha\sqrt{P_rP_{s1}}f_{1r}^2\beta_1(k,3) + \alpha\sqrt{P_rP_{s2}}f_{1r}f_{2r}\beta_2(k,3) \\ &+ \alpha\sqrt{P_rP_p}f_{1r}h_{pr}\theta(k,3) + \sqrt{P_p}h_{p1}\theta(k,4) + w_1(k,4), \end{aligned} \tag{11}$$

where $w_1(k)$ is the combined noise defined as

$$w_1(k,4) = \alpha\sqrt{P_r}f_{1r}n_r(k,3) + n_1(k,4). \tag{12}$$

Clearly, $E(w_1(k,4)) = 0$ and the variance of $w_1(k,4)$ is

$$N_{w1} = \alpha^2 P_r\sigma_{f1}^2 N_0 + N_0. \tag{13}$$

## III. Sensing Performance

In this section, we will find out the expression for the overall false alarm probability $P_f$ and overall detection probability $P_d$ to evaluate the sensing performance of the cognitive TWRN.

### A. False Alarm Probability

An energy detection is considered in evaluation the performance of the spectrum sensing phase. In the case of $H_p(k) = H_0$, according to (1) and (3), the received signal at node CTs and CR is only noise. Therefore, the individual false alarm probability $P_{f,i}$ at node $\text{CT}_i$ (i=1,2) or CR (i=r) can be obtained as

$$\begin{aligned} P_{f,i} =& \Pr\{\hat{H}_{s,i}(k) = H_1|H_p(k) = H_0\}, \\ =& \Pr\{\sum_{k=1}^{T_{dec}}\frac{|n_i(k,1)|^2}{N_0/2} > \frac{\lambda_{ED}}{N_0/2}\}, \quad i=1,2,r \end{aligned} \tag{14}$$

where $\lambda_{ED}$ is the decision threshold of the energy detector. Note that $\sum_{k=1}^{T_{dec}}\frac{|n_i(k,1)|^2}{N_0/2}$ can be considered as a central chi-squared random variable $\mathcal{X}_{2T_{dec}}^2$ with $2T_{dec}$ degrees of freedom. Therefore, equation (14) can be found as [12]

$$P_{f,i} = \exp(-\frac{\lambda_{ED}}{N_0})\sum_{k=0}^{T_{dec}-1}\frac{(\frac{\lambda_{ED}}{N_0})^k}{k!}. \tag{15}$$

Since the final decision $\hat{H}_s = H_0$ requires $\mathcal{C}_{R,SP} = \text{CT}_1 \bigcup \text{CT}_2$ and all the three nodes, $\text{CT}_1$, $\text{CT}_2$ and CR node, get the same sensing result $H_0$, we can obtain

$$\begin{aligned} &\Pr\{\hat{H}_s(k) = H_0|H_p(k) = H_0\} \\ =& P_{\mathcal{C}}(1-P_{f,1})(1-P_{f,2})(1-P_{f,r}), \end{aligned} \tag{16}$$

where $P_{\mathcal{C}} = \Pr\{\mathcal{C}_{R,SP} = \text{CT}_1 \bigcup \text{CT}_2\}$.

From (5), we can find

$$\begin{aligned} P_{\mathcal{C}} =& \Pr\{\mathcal{C}_{R,SP} = \text{CT}_1 \bigcup \text{CT}_2\} \\ =& \Pr\{\frac{P_{s1}}{N_0}|f_{1r}|^2 > \breve{R}_{sr}\}\Pr\{\frac{P_{s2}}{N_0}|f_{2r}|^2 > \breve{R}_{sr}\} \\ =& e^{-\frac{\breve{R}_{sr}N_0}{P_{s1}\sigma_{f1}^2}}e^{-\frac{\breve{R}_{sr}N_0}{P_{s2}\sigma_{f2}^2}} \end{aligned} \tag{17}$$

where $\breve{R}_{sr} = 2^{\frac{2}{T_sT_{toR}B_c}} - 1$.

Finally, using the property of condition probability and combining (17), (15) and (16), we can obtain the overall false alarm probability as

$$\begin{aligned} P_f =& \Pr\{\hat{H}_s(k) = H_1|H_p(k) = H_0\} \\ =& 1 - \Pr\{\hat{H}_s(k) = H_0|H_p(k) = H_0\} \\ =& 1 - e^{-\frac{\breve{R}_{sr}N_0}{P_{s1}\sigma_{f1}^2}-\frac{\breve{R}_{sr}N_0}{P_{s2}\sigma_{f2}^2}}\left(1 - e^{-\frac{\lambda_{ED}}{N_0}}\sum_{k=0}^{T_{dec}-1}\frac{(\frac{\lambda_{ED}}{N_0})^k}{k!}\right)^3. \end{aligned} \tag{18}$$

### B. Detection Probability

In the case of $H_p(k) = H_1$, according to (1) and (3), the received signal at node CTs and CR is the signal sent from PU plus noise. Therefore, the individual detection probability $P_{d,i}$ at node CT$_i$ (i=1,2) or CR (i=r) can be obtained as

$$\begin{aligned} P_{d,i} &= \Pr\{\hat{H}_{s,i} = H_1 | H_p(k) = H_1\} \qquad i = 1, 2, r \\ &= \Pr\{\sum_{k=1}^{T_{dec}} \frac{|P_p h_{pi} \theta(k,1) + n_i(k,1)|^2}{(P_p \varsigma_{hi}^2 + N_0)/2} > \frac{\lambda_{ED}}{(P_p \varsigma_{hi}^2 + N_0)/2}\}. \end{aligned} \tag{19}$$

Since $\sum_{k=1}^{T_{dec}} \frac{|P_p h_{pi} \theta(k,1)+n_i(k,1)|^2}{(P_p \varsigma_{hi}^2+N_0)/2}$ can be considered as a central chi-squared random variable $\mathcal{X}_{2T_{dec}}^2$ with $2T_{dec}$ degrees of freedom, we can further obtain (19) as

$$P_{d,i} = \exp(-\frac{\lambda_{ED}}{P_p \varsigma_{hi}^2 + N_0}) \sum_{k=0}^{T_{dec}-1} \frac{(\frac{\lambda_{ED}}{P_p \varsigma_{hi}^2 + N_0})^k}{k!}. \tag{20}$$

The overall detection probability can be written as

$$\begin{aligned} P_d &= \Pr\{\hat{H}_s(k) = H_1 | H_p(k) = H_1\} \\ &= 1 - \Pr\{\hat{H}_s(k) = H_0 | H_p(k) = H_1\}. \end{aligned} \tag{21}$$

Similar with the process in finding $P_f$, we can obtain the overall detection probability

$$\begin{aligned} P_d = &1 - e^{-\frac{\breve{R}_{sr} N_0}{P_{s1} \sigma_{f1}^2} - \frac{\breve{R}_{sr} N_0}{P_{s2} \sigma_{f2}^2}} \\ &\times \left(1 - e^{-\frac{\lambda_{ED}}{P_p \varsigma_{h1}^2 + N_0}} \sum_{k=0}^{T_{dec}-1} \frac{(\frac{\lambda_{ED}}{P_p \varsigma_{h1}^2 + N_0})^k}{k!}\right) \\ &\times \left(1 - e^{-\frac{\lambda_{ED}}{P_p \varsigma_{h2}^2 + N_0}} \sum_{k=0}^{T_{dec}-1} \frac{(\frac{\lambda_{ED}}{P_p \varsigma_{h2}^2 + N_0})^k}{k!}\right) \\ &\times \left(1 - e^{-\frac{\lambda_{ED}}{P_p \varsigma_{hr}^2 + N_0}} \sum_{k=0}^{T_{dec}-1} \frac{(\frac{\lambda_{ED}}{P_p \varsigma_{hr}^2 + N_0})^k}{k!}\right). \end{aligned} \tag{22}$$

## IV. Outage Probability

The outage probability for the node CT$_1$ can be expressed as

$$\begin{aligned} P_{\text{out}} = &P_\emptyset (1 - P_f) \Pr\{C_{T1} < R | H_p(k) = 0, \hat{H}_s(k) = 0\} \\ &+ (1 - P_\emptyset)(1 - P_d) \Pr\{C_{T1} < R | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ &+ (1 - P_\emptyset) P_d + P_\emptyset P_f. \end{aligned} \tag{23}$$

Clearly, the expression for $\Pr\{C_{T1} < R | H_p(k) = 0, \hat{H}_s(k) = 0\}$ and $\Pr\{C_{T1} < R | H_p(k) = 1, \hat{H}_s(k) = 0\}$ are required to find out $P_{\text{out}}$, which will be obtained in the following two subsections respectively.

### A. Case: successfully detect the existence of spectrum hole

In the case of $H_p(k) = 0$ and $\hat{H}_s(k) = 0$, the equation (11) can be rewritten as

$$y_1(k, 4) = \alpha \sqrt{P_r P_{s2}} f_{1r} f_{2r} x_{t2}(k, 3) + w_1(k, 4). \tag{24}$$

Therefore, the corresponding channel capacity achievable at the CT$_1$ is

$$C_{T1} = \frac{T_{II}}{T_{slot}} \log_2(1 + \frac{\alpha^2 P_r P_{s2} |f_{1r}|^2 |f_{2r}|^2}{N_{w1}}) \tag{25}$$

Note that $T_{II}/T_{slot} = \eta_t$ and we can further obtain

$$\begin{aligned} &\Pr\{C_{T1} < R | H_p(k) = 0, \hat{H}_s(k) = 0\} \\ &= \Pr\{|f_{1r}|^2 |f_{2r}|^2 < \frac{N_{w1}}{\alpha^2 P_r P_{s2}} (2^{R/\eta_t} - 1)\} \\ &= 1 - \int_0^\infty \frac{1}{\sigma_{f2}^2} e^{(-\frac{y}{\sigma_{f2}^2} - \frac{A_1}{y \sigma_{f1}^2})} dy. \end{aligned} \tag{26}$$

where $A_1 = \frac{\breve{R}}{\alpha^2 P_r P_{s2}/N_{w1}}$ and $\breve{R} = 2^{R/\eta_t} - 1$.

Noting that [11, page 337, equation 3.324.1]

$$\int_0^\infty \exp(-\frac{\beta}{4y} - \gamma y) dy = \sqrt{\frac{\beta}{\gamma}} K_1(\sqrt{\beta\gamma}), \quad [\text{Re}\beta \geq 0, \text{Re}\gamma > 0], \tag{27}$$

hence we can further simplify (26) as

$$\begin{aligned} &\Pr\{C_{T1} < R | H_p(k) = 0, \hat{H}_s(k) = 0\} \\ &= 1 - \sqrt{\frac{4A_1}{\sigma_{f1}^2 \sigma_{f2}^2}} K_1 \left(\sqrt{\frac{4A_1}{\sigma_{f1}^2 \sigma_{f2}^2}}\right), \end{aligned} \tag{28}$$

where $K_1(z)$ is the modified Bessel function of the second kind [12]

$$K_1(z) = \frac{z}{4} \int_0^\infty \frac{e^{-t - z^2/4t}}{t^2} dt. \tag{29}$$

### B. Case: no spectrum hole and wrong detection

In the case of $H_p(k) = 1$ and $\hat{H}_s(k) = 0$, the equation (11) can be rewritten as

$$\begin{aligned} y_1(k, 4) = &\alpha \sqrt{P_r P_{s2}} f_{1r} f_{2r} x_{t2}(k, 3) + w_1(k, 4) \\ &+ \underbrace{\alpha \sqrt{P_r P_p} f_{1r} h_{pr} x_p(k, 3) + \sqrt{P_p} h_{p1} x_p(k, 4)}_{I_1(k,4)}. \end{aligned} \tag{30}$$

Hence the corresponding channel capacity achievable at the CT$_1$ is

$$C_{T1} = \eta_t \log_2 \left(1 + \frac{\alpha^2 P_r P_{s2} |f_{1r}|^2 |f_{2r}|^2}{\alpha^2 P_r P_p |f_{1r}|^2 |h_{pr}|^2 + P_p |h_{p1}|^2 + N_{w1}}\right). \tag{31}$$

Therefore, we can have

$$\begin{aligned} &\Pr\{C_{T1} < R | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ &= \Pr\left\{\frac{\alpha^2 P_r P_{s2} |f_{1r}|^2 |f_{2r}|^2}{\alpha^2 P_r P_p |f_{1r}|^2 |h_{pr}|^2 + P_p |h_{p1}|^2 + N_{w1}} < \breve{R}\right\} \\ &= \Pr\left\{|f_{1r}|^2 (|f_{2r}|^2 - \frac{\breve{R} P_p}{P_{s2}} |h_{pr}|^2) < \frac{\breve{R} P_p |h_{p1}|^2}{P_{r,s2}} + \frac{\breve{R} N_{w1}}{P_{r,s2}}\right\}, \end{aligned} \tag{32}$$

where $P_{r,s2} = \alpha^2 P_r P_{s2}$.

Define

$$w = (|f_{2r}|^2 - \frac{\breve{R}P_p}{P_{s2}}|h_{pr}|^2)|f_{1r}|^2. \quad (33)$$

The PDF of the random variable $w$ can be found as [10]

$$p_w(w) = \begin{cases} \frac{1}{\sigma_{f1}^2(\sigma_{f2}^2 + a\varsigma_{hr}^2)} \int_0^\infty \frac{1}{v} e^{-\frac{v}{\sigma_{f1}^2} - \frac{w}{v\sigma_{f2}^2}} dv, & w > 0 \\ \frac{1}{\sigma_{f1}^2(\sigma_{f2}^2 + a\varsigma_{hr}^2)} \int_0^\infty \frac{1}{v} e^{\frac{w}{av\varsigma_{hr}^2} - \frac{v}{\sigma_{f1}^2}} dv, & w \leq 0. \end{cases} \quad (34)$$

Now let us return back to (32). Note that if $u = |f_{2r}|^2 - \frac{\breve{R}P_p}{P_{s2}}|h_{pr}|^2 \leq 0$, then $\Pr\{C_{T1} < R\} = 1$. Thus we can have

$$\begin{aligned} &\Pr\{C_{T1} < R, u \leq 0 | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ = &\Pr\{u \leq 0\} = \frac{a\varsigma_{hr}^2}{\sigma_{f2}^2 + a\varsigma_{hr}^2}. \end{aligned} \quad (35)$$

Next we focus on the case that $u > 0$. We can further simplify (32) as

$$\begin{aligned} &\Pr\{C_{T1} < R, u > 0 | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ = &\Pr\left\{ |f_{1r}|^2 u < \frac{\breve{R}P_p|h_{p1}|^2}{P_{r,s2}} + \frac{\breve{R}N_{w1}}{P_{r,s2}}, u > 0 \right\} \\ = &\int_0^\infty dx \int_0^{bx+c} \frac{e^{-\frac{x}{\varsigma_{h1}^2}} dw}{\varsigma_{h1}^2 \sigma_{f1}^2 (\sigma_{f2}^2 + a\varsigma_{hr}^2)} \int_0^\infty \frac{1}{v} e^{-\frac{v}{\sigma_{f1}^2} - \frac{w}{v\sigma_{f2}^2}} dv \\ = &\rho_\sigma - \frac{\rho_\sigma}{\sigma_{f1}^2} \int_0^\infty \frac{v}{b\varsigma_{h1}^2/\sigma_{f2}^2 + v} e^{-\frac{c}{v\sigma_{f2}^2} - \frac{v}{\sigma_{f1}^2}} dv, \end{aligned} \quad (36)$$

where $b = \breve{R}P_p/P_{r,s2}$, $c = \breve{R}N_{w1}/P_{r,s2}$ and $\rho_\sigma = \sigma_{f2}^2/(\sigma_{f2}^2 + a\varsigma_{hr}^2)$.

Suppose $\bar{b} = b\varsigma_{h1}^2/\sigma_{f2}^2$ and using (27), we can further obtain (36) as

$$\begin{aligned} &\Pr\{C_{T1} < R, u > 0 | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ = &\rho_\sigma - \rho_\sigma \sqrt{\frac{4c}{\sigma_{f1}^2 \sigma_{f2}^2}} K_1 \left( \sqrt{\frac{4c}{\sigma_{f1}^2 \sigma_{f2}^2}} \right) \\ &+ \frac{\rho_\sigma}{\sigma_{f1}^2} \int_0^\infty \frac{\bar{b}}{v + \bar{b}} e^{-\frac{c}{v\sigma_{f2}^2} - \frac{v}{\sigma_{f1}^2}} dv. \end{aligned} \quad (37)$$

Combining (35) and (37) will give

$$\begin{aligned} &\Pr\{C_{T1} < R | H_p(k) = 1, \hat{H}_s(k) = 0\} \\ = &\rho_\sigma - \rho_\sigma \sqrt{\frac{4c}{\sigma_{f1}^2 \sigma_{f2}^2}} K_1 \left( \sqrt{\frac{4c}{\sigma_{f1}^2 \sigma_{f2}^2}} \right) + \frac{a\rho_\sigma \varsigma_{hr}^2}{\sigma_{f2}^2} \\ &+ \frac{\rho_\sigma}{\sigma_{f1}^2} \int_0^\infty \frac{\bar{b}}{v + \bar{b}} e^{-\frac{c}{v\sigma_{f2}^2} - \frac{v}{\sigma_{f1}^2}} dv. \end{aligned} \quad (38)$$

### *C. Spectrum hole utilization efficiency*

In the ideal case, the cognitive TWRN can perfect detect the spectrum hole and only transmit signals on the spectrum holes. Thus the final signal received at $\text{CT}_1$ will be

$$y_1(k, 4) = \alpha \sqrt{P_r P_{s2}} f_{1r} f_{2r} \beta_2(k, 3) + w_1(k, 4). \quad (39)$$

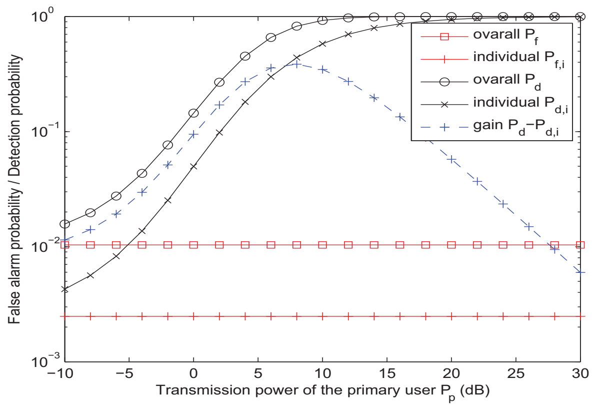


Fig. 4. False alarm probability and detection probability versus transmission power of the primary user with $T_{dec} = 1$, $T_{toR} = 1$, $B_c = 10kHz$, $T_s = 1ms$, $N_0 = 1$ and $P_{s1} = P_{s1} = 100$.

Clearly, the achievable channel capacity for this ideal case is

$$C_{\text{ideal}} = \eta_t \log_2(1 + \frac{\alpha^2 P_r P_{s2} \sigma_{f1}^2 \sigma_{f2}^2}{N_{w1}}), \quad (40)$$

and the average achievable channel capacity for cognitive TWRN in the ideal case is $P_\emptyset C_{\text{ideal}}$.

Based on the derived outage probability, we can define a spectrum hole utilization efficiency as

$$\eta_{CR} = \frac{(1 - P_{\text{out}})R}{P_\emptyset C_{\text{ideal}}}, \quad (41)$$

where $1 - P_{\text{out}}$ indicates the spectrum holes that are utilized by the cognitive TWRN for its successful data transmission and $(1 - P_{\text{out}})R$ implies the average practical data rate. Therefore, $\eta_{CR}$ can be considered as a measure to quantify the utilization efficiency of spectrum holes by the cognitive TWRN.

## V. Simulation Results

In this section, we numerically evaluate the performance of our proposed cognitive transmission scheme. Firstly, we increase the transmission power of the primary users $P_p$ from -10dB to 30dB, and find the individual false alarm probability $P_{f,i}$ and overall false alarm probability $P_f$ as well as individual detection probability $P_{d,i}$ and overall detection probability $P_d$. Fig. 4 shows the sensing performance versus transmission power $P_p$. The gain in joint detection probability defined as $P_d - P_{d,i}$ is also plotted. Clearly, the joint sensing scheme of the cognitive TWRN can improve detection probability at the cost of a bit increase in false alarm probability.

The outage probability and spectrum hole utilization efficiency versus signal-to-noise ratio (SNR) at both destination nodes CTs is also studied. Fig. 5 plots the outage probability versus SNR at CTs in the case of individual false alarm probability $P_{f,i} = 0.01$, 0.02, 0.05 or 0.1 and data rate $R = 0.5$ or 1 respectively. It can be seen that higher transmission power at CTs, as well as smaller individual false alarm probability $P_{f,i}$ and lower data rate $R$, will result in lower outage probability. Fig. 6 plots spectrum hole utilization efficiency $\eta_{CR}$ versus SNR at CTs for the same cases with Fig. 5.

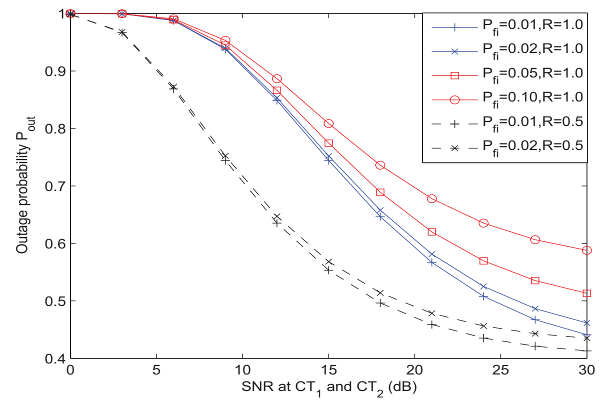


Fig. 5. Outage probability versus transmission power $P_{si}, i = 1, 2$ with $P_{\emptyset} = 0.6$, $\eta_t = 1/4$, $P_p = 10$, $T_{toR} = 1$, $B_c = 10kHz$, $T_s = 1ms$, and $N_0 = 1$.

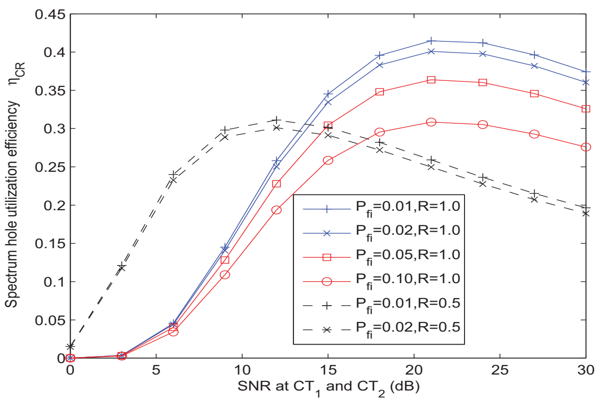


Fig. 6. Spectrum hole utilization efficiency versus transmission power $P_{si}, i = 1, 2$ with $P_{\emptyset} = 0.6$, $\eta_t = 1/4$, $P_p = 10$, $T_{toR} = 1$, $B_c = 10kHz$, $T_s = 1ms$ and $N_0 = 1$.

Interestingly, it can be found that the maximum spectrum hole utilization efficiency $\eta_{CR}$ is not obtained at the largest SNR, which implies larger transmission power by the cognitive users does not necessarily produce higher spectrum hole utilization efficiency. However, we can find from Fig. 6 that larger data rate requires large SNR at CTs, which means the optimal choice of transmission power depends on the requirement of the data rate.

Finally, we evaluate the spectrum hole utilization efficiency $\eta_{CR}$ at different transmission ratio $\eta_t$. Note that $\eta_t = T_{II}/T_{slot} = T_I/T_{slot}$ and thus $0 < \eta_t < 0.5$. The curves of the spectrum hole utilization efficiency $\eta_{CR}$ versus the transmission ratio $\eta_t$ are plotted in Fig. 7. It can be seen that when the data rate $R$ is small, less transmission ratio $\eta_t$ is optimal to obtain largest spectrum hole utilization efficiency $\eta_{CR}$, while when the the data rate $R$ is larger, more transmission part is required to get the highest spectrum hole utilization efficiency $\eta_{CR}$.

## VI. Conclusion

This paper proposed a cognitive transmission scheme for AF TWRNs and investigated its joint sensing and outage performance. The overall false alarm probability, overall detection probability, outage probability and spectrum hole utilization efficiency were derived to evaluate the performance of the cognitive TWRN over flat Rayleigh fading channels. Our numerical results show that more transmission power for the cognitive TWRN does not necessarily produce higher spectrum hole utilization efficiency. It was also shown that a maximum spectrum hole utilization efficiency can be achieved through an allocation of the time slots between the spectrum sensing and data transmission phases.

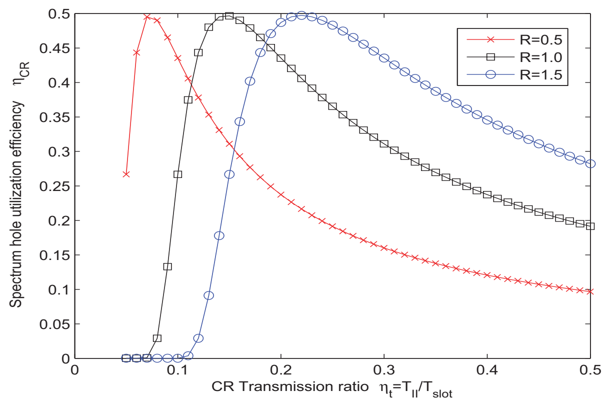


Fig. 7. Spectrum hole utilization efficiency $\eta_{CR}$ versus $\eta_t$ with $P_{\emptyset} = 0.6$, $P_p = 100$, $\lambda_{ED} = 4.6$, $P_{f,i} = 0.01$, $T_{toR} = 1$, $B_c = 10kHz$, $T_s = 1ms$, $N_0 = 1$ and $P_{s1} = P_{s1} = 1000$.

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