On the Performance of Cognitive Underlay Multihop Networks with Imperfect Channel State Information

Vo Nguyen Quoc Bao, *Member, IEEE*, Trung Q. Duong, *Senior Member, IEEE*, and Chintha Tellambura, *Fellow, IEEE*

***Abstract*—This paper proposes and analyzes cognitive multihop decode-and-forward networks in the presence of interference due to channel estimation errors. To reduce interference on the primary network, a simple yet effective back-off control power method is applied for secondary multihop networks. For a given threshold of interference probability at the primary network, we derive the maximum back-off control power coefficient, which provides the best performance for secondary multihop networks. Moreover, it is shown that the number of hops for secondary network is upper-bounded under the fixed settings of the primary network. For secondary multihop networks, new exact and asymptotic expressions for outage probability (OP), bit error rate (BER) and ergodic capacity over Rayleigh fading channels are derived. Based on the asymptotic OP and BEP, a pivotal conclusion is reached that the secondary multihop network offers the same diversity order as compared with the network without back off. Finally, we verify the performance analysis through various numerical examples which confirm the correctness of our analysis for many channel and system settings and provide new insight into the design and optimization of cognitive multihop networks.**

***Index Terms*—Decode-and-forward, dual-hop cognitive relay network, spectrum sharing, multihop, imperfect channel state information, Rayleigh fading, outage probability, bit error rate, ergodic capacity.**

I. Introduction

To alleviate the wireless spectrum scarcity problem cognitive radio has been proposed where an unlicensed user (also known as a cognitive user) is allowed to opportunistically utilize the white space of a licensed spectrum band (called a spectrum hole) for data transmissions [1]. Among existing cognitive protocols, the underlay approach is of particular interest to both the academia and industry due to its advantage in providing concurrent cognitive and non-cognitive communication [2].

In designing spectrum sharing underlay systems, one of the major challenges is to fulfill the two conflicting objectives: i) protecting the primary (licensed) user (PU) from interference and ii) satisfying the quality of service (QoS) requirement of secondary (non-licensed) users (SUs). Between these objectives, the former is of higher priority, making strict regulation of secondary transmit powers necessary. Since the allowable interference level on primary receivers (PU-Rxs) is small, secondary network coverage is limited. In order to extend it, an efficient secondary transmission mechanism is required.

Such a mechanism is multihop relay technology, (see, e.g., [3]–[10]). In [3] and [4], the outage probability (OP) of a multi-hop decode-and-forward (DF) relaying system over Rayleigh and Nakagami-m fading channels under the interference temperature constraint was respectively presented to capture the impact of fading parameters at the interfering links as well as the interference temperature constraint on OP. Thanks to low complexity and easy deployment, amplify-and-forward relaying could be a promising candidate with its outage performance exceeding that of the conventional cognitive radio direct transmission [5]. Over Nakagami-m channels, it was shown in [6] that for the same system model as in [5], the diversity order is strictly defined by the minimum fading severity between the two hops of the secondary network; and the secondary network achieves the full diversity order regardless of the transmit power constraint. For two different types of interference power constraints at the PU-Rx including fixed and proportional interference power constraints, the authors in [7] studied cooperative diversity gain of secondary networks with multiple relays. It is found that the diversity order of the secondary relay network is lost under a fixed interference power constraint and increasing transmit power does not improve the outage performance. In [9], the optimization problem of secondary relay positions is considered.

To the best of our knowledge, most existing works consider perfect channel state information of interference links between the secondary transmitters and the PU-Rx. However, secondary networks may not acquire perfect channel state information (CSI) due to, for example, channel estimation errors and/or the slack cooperation between SUs and the PU. Recently, the performance of cognitive radio networks under imperfect CSI has been considered for single-hop and dual-hop in [11] and [12], respectively.

Motivated by these considerations, we investigate the performance of cognitive multihop networks with imperfect knowledge of interference channels. In particular, we consider the effect of imperfect CSI on the PU-Rx under primary interference probability constraints. Our main contributions are as follows:

1) We analyze the interference probability for primary networks over similar and dissimilar Rayleigh channels. The interference probability is shown to increase with the number of secondary hops.

Manuscript received March 1, 2013; revised August 21, 2013. The editor coordinating the review of this paper and approving it for publication was C. Ling.

V. N. Q. Bao is with the Posts and Telecommunications Institute of Technology, Vietnam (e-mail: baovnq@ptithcm.edu.vn).

T. Q. Duong is with the School of Electronics, Electrical Engineering and Computer Science, The Queens University of Belfast, Belfast BT3 9DT, UK. He was with Blekinge Institute of Technology, Sweden (e-mail: tduong@ieee.org).

C. Tellambura is with the University of Alberta, Edmonton AB, Canada T6G 2V4 (e-mail: chintha@ece.ualberta.ca).

Digital Object Identifier 10.1109/TCOMM.2013.110413.130167

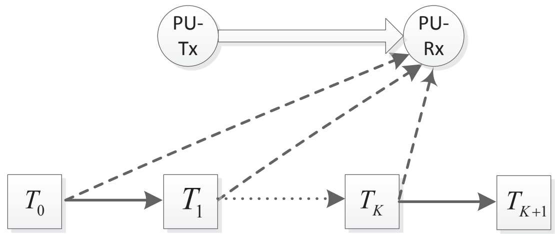

Fig. 1. Secondary underlay multihop transmission.

2) We characterize the effect of imperfect CSI on the performance of secondary networks, which is given by the number of maximum hops and the maximum back-off control powers for a given interference probability.
3) We show that the primary interference probability achieves its minimum if and only if the back-off control powers are set the same.
4) We develop closed-form expressions for secondary OP, bit error rate (BER), and ergodic capacity and their asymptotic bounds at high and low signal-to-noise ratios (SNRs). Moreover, we also compute the secondary performance loss as compared to that of the perfect CSI.

The remainder of this paper is organized as follows. In Section II, the system and channel model are introduced. In Section III, the unified framework to derive the performance measures of the primary and secondary network including interference probability, OP, BER and ergodic capacity is developed in detail. In Section IV, numerical results are provided to illustrate the characteristics of the primary and secondary network. Finally, the paper is concluded in Section V.

II. System Model

We consider a cognitive multihop network over the same spectrum of a primary network with an underlay peak interference power constraint. As illustrated in Fig. 1, the communication between the source ($\mathcal{T}_0$) and the destination ($\mathcal{T}_{K+1}$) takes place in K orthogonal timeslots via serial immediate relays $\mathcal{T}_1, \ldots, \mathcal{T}_K$. In the k-th timeslot, the received signal at relay k is fully decoded and then re-encoded before being forwarded to the next node (or the destination) in the subsequent timeslot. All secondary and primary nodes are equipped with a single-antenna.

We denote h_k and f_k, respectively, as the channel coefficients of the data link and interference link of hop k. Throughout this paper, independent Rayleigh frequency-flat fading links are assumed. As a result, the channel gains, i.e., $|h_k|^2$ and $|f_k|^2$, follow the exponential distribution with parameters $\lambda_{D,k}$ and $\lambda_{I,k}$, respectively.

To protect the PU network communication as well as to enhance the secondary network performance, all SUs should transmit with maximal allowable power P_k as long as the PU received interference power is below the maximum tolerable interference level, I_{p}. Therefore, the transmit power of node k is considered as

$$P_k = \frac{I_{\mathrm{p}}}{|f_k|^2}, \qquad k = 0, 1, \ldots, K \tag{1}$$

if the channel f_k is perfectly known at the transmitter of hop k. In practice, perfect knowledge of f_k is not available because of various uncertainties such as errors in the feedback transmission and/or the outdated feedback due to the time-varying wireless channels. This uncertain relation between f_k and its estimate $\tilde{f}_k$ can be modeled as [13]

$$\tilde{f}_k = \rho f_k + \sqrt{1-\rho^2}\mu_k, \tag{2}$$

where μ_k is a circular symmetric complex Gaussian random variable with mean zero and variance $\lambda_{I,k}$. Here, the terms μ_k with $k = 1, \ldots, K$ are mutually independent. In addition, ρ denotes the channel correlation factor modeling the channel estimation quality, which is expressed by the pilot symbol assisted modulation (PSAM) parameters such as the rate of pilot symbol insertion and average SNR [14]. The correlation coefficient ρ is assumed to be the same between all the interference channel pairs. Under Rayleigh fading, the joint probability density function (PDF) of $|f_k|^2$ and $|\tilde{f}_k|^2$ are given by [13]

$$f_{|f_k|^2, |\tilde{f}_k|^2}(x, y) = \frac{e^{-\frac{x+y}{(1-\rho^2)\lambda_{I,k}}}}{(1-\rho^2)\lambda_{I,k}{}^2} I_0\left(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\lambda_{I,k}}\right), \tag{3}$$

where $I_0(x) = \frac{1}{\pi}\int_0^\pi e^x \cos\theta \mathrm{d}\theta$ denotes the zeroth-order modified Bessel function of the first kind [15]. As a result, the transmit power at hop k, P_k, is now

$$P_k = \frac{I_{\mathrm{p}}}{\left|\tilde{f}_k\right|^2} \tag{4}$$

resulting in the instantaneous SNR at hop k as

$$\gamma_k = \frac{I_p}{N_0}\frac{|h_k|^2}{\left|\tilde{f}_k\right|^2}. \tag{5}$$

Eq. (4) shows that improper regulation of the transmit powers due to imperfect channel estimation can cause an interference that is higher than the maximum tolerable interference level. In Fig. 1, primary communication is protected if all PU received interference powers in the K hops are lower than I_{p}. Otherwise, the primary communication fails. To analyze this phenomenon, we use primary interference probability concept, which is defined as the probability that the interference power received at the PU-Rx is higher than I_{p}. Note that since the interference probability is computed after the arrival of the source data at the destination, its evaluation reflects the effect of the entire secondary system on the primary system. In the next section, we will derive the interference probability due to an arbitrary number of secondary hops over Rayleigh fading channels.

III. Performance Analysis

In this section, we investigate the performance of the primary and the secondary multihop networks. For the primary network, the interference probability is derived over Rayleigh fading channels. For the secondary network, the performance metrics including OP, BER and ergodic capacity are derived. The secondary network behaviors at high and low SNRs are also provided.

A. Interference Probability of the Primary System

Utilizing the theorem of total probability, the interference probability of the primary system can be evaluated as

$$\begin{aligned} P_I =& \Pr(P_1|f_1|^2 > I_\mathrm{p}) + \Pr(P_2|f_2|^2 > I_\mathrm{p}) \Pr(P_1|f_1|^2 \le I_\mathrm{p}) \\ &+ \cdots + \Pr(P_K|f_K|^2 > I_\mathrm{p}) \Pr(P_{K-1}|f_{K-1}|^2 > I_\mathrm{p}) \ldots \\ &\times \Pr(P_1|f_1|^2 \le I_\mathrm{p}) \\ =& \sum_{k=1}^{K} \Pr(P_k|f_k|^2 > I_\mathrm{p}) \prod_{\ell=1}^{k-1} \Pr(P_\ell|f_\ell|^2 \le I_\mathrm{p}). \end{aligned} \quad (6)$$

To compute (6), we first need to derive $\Pr(P_k|f_k|^2 > I_\mathrm{p})$ and $\Pr(P_k|f_k|^2 \le I_\mathrm{p})$. Before giving continuity to our analysis, the following lemma is of importance in this regard.

Lemma 1: For two given random variables $|f_k|^2$ and $|\tilde{f}_k|^2$, which are correlated with correlation coefficient ρ , $\Pr\left(\frac{|f_k|^2}{|\tilde{f}_k|^2} > z\right)$ is given by

$$\Pr\left(\frac{|f_k|^2}{\left|\tilde{f}_k\right|^2} > z\right) = \frac{1}{2}\left[1 + \frac{1-z}{\sqrt{(1+z)^2 - 4\rho^2 z}}\right]. \quad (7)$$

Proof: The proof can be easily obtained by making use of the definition of the first order Marcum Q-function [16] along with the help of [17, Eq. (6-60)] and [18, Eq. (3)]. ■

It is noted from (7) that $\Pr\left(\frac{|f_k|^2}{|\tilde{f}_k|^2} > z\right)$ depends only on the correlation coefficient (ρ) and the threshold (z), not the average channel power of the interference link ($\lambda_{I,k}$). Such a phenomenon will significantly affect the system design, which is presented in the next part.

Lemma 1 allows us to compute the interference probability of primary systems. In particular, letting $z = 1$ in (7) and combining the resultant with (6) gives

$$\begin{aligned} P_I &= \sum_{k=1}^{K} \Pr\left(\frac{|f_k|^2}{\left|\tilde{f}_k\right|^2} > 1\right) \prod_{\ell=1}^{k-1} \Pr\left(\frac{|f_\ell|^2}{\left|\tilde{f}_\ell\right|^2} < 1\right) \\ &= \sum_{k=1}^{K} \frac{1}{2^k} = 1 - \frac{1}{2^K}. \end{aligned} \quad (8)$$

As a special case, we consider $K = 2$ in (8) leading to $P_I = 0.75$, which agrees with the result reported in [12, Eq. (7)].

The expression in (8) illustrates the relationship between the interference outage (P_I) and the number of hops (K) showing that the minimum of the interference outage is $1/2$ and it increases with the number of hops. Furthermore, it is easy to show that $P_I \to 1$ when K approaches infinity. As such, maintaining acceptable interference for the primary network under imperfect CSI is one of the critical concerns for cognitive networks. To guarantee the interference probability from the secondary network, back-off transmit power control mechanism is a simple and efficient solution. We denote ε_k with $0 < \varepsilon_k \le 1$ as the back-off power control coefficient of hop k, then the reduced transmit power at hop k can be given by

$$P_k' = \varepsilon_k P_k = \frac{\varepsilon_k I_\mathrm{p}}{\left|\tilde{f}_k\right|^2}. \quad (9)$$

1) Identical back-off coefficients: In this case, the back-off coefficient of all links is identical, i.e., $\varepsilon_1 = \cdots = \varepsilon_K = \varepsilon$, and the interference probability of the primary network is given by

$$P_I = \sum_{k=1}^{K} \Pr(P_k'|f_k|^2 > I_p) \prod_{\ell=1}^{k-1} \Pr(P_\ell'|f_\ell|^2 < I_p). \quad (10)$$

From (7), we can rewrite (10) as follows:

$$P_I = \frac{1-\Phi}{2} \sum_{k=1}^{K} \left(\frac{1+\Phi}{2}\right)^{k-1}, \quad (11)$$

where $\Phi = \frac{\varepsilon-1}{\sqrt{(\varepsilon+1)^2-4\rho^2\varepsilon}}$. Similarly, it is easy to see that

$$P_I = 1 - \left(\frac{1+\Phi}{2}\right)^K = 1 - \left(\frac{1}{2} + \frac{1-\varepsilon}{2\sqrt{(1+\varepsilon)^2 - 4\rho^2\varepsilon}}\right)^K. \quad (12)$$

It follows from (12) that in the limited cases of ρ, the interference OP of primary networks becomes

$$P_I = \begin{cases} 1 - \left(\frac{1}{2} + \frac{1-\varepsilon}{2\sqrt{(1+\varepsilon)^2}}\right)^K, & \rho = 0 \\ 1 - \frac{1}{2^K}, & \rho = 1 \end{cases}. \quad (13)$$

2) Non-identical back-off coefficients: While identical back-off coefficients simplifies our analysis, in certain environments, it may be more appropriate to consider the non-identical case, i.e., $\varepsilon_1 \ne \varepsilon_2 \ne \cdots \ne \varepsilon_K$. In practice, energy conditions at each secondary node can be different and the distance between any two nodes is usually not equal, which can cause different back-off power control coefficients among the relaying links. Similar to the previous case, we have

$$\begin{aligned} P_I =& \sum_{k=1}^{K} \Pr(P_k'|f_k|^2 > I_p) \prod_{\ell=1}^{k-1} \Pr(P_\ell'|f_\ell|^2 < I_p) \\ =& \frac{1-\Phi_1}{2} + \frac{(1+\Phi_1)(1-\Phi_2)}{2} + \cdots \\ &+ \frac{(1+\Phi_1)\cdots(1+\Phi_{K-1})(1-\Phi_K)}{2^K} \\ =& \sum_{k=1}^{K} \frac{1-\Phi_k}{2} \prod_{\ell=1}^{k-1} \left(\frac{1+\Phi_\ell}{2}\right), \end{aligned} \quad (14)$$

where $\Phi_k = \frac{\varepsilon_k - 1}{\sqrt{(\varepsilon_k+1)^2-4\rho^2\varepsilon_k}}$. After some manipulations, (14) can be expressed in a mathematically tractable form as

$$P_I = 1 - \prod_{k=1}^{K} \left(\frac{1+\Phi_k}{2}\right). \quad (15)$$

At this point, a natural question arises for a given ρ is what kind of back-off coefficients gives less interference on the primary network. The following theorem will answer such a question.

Theorem 1: For a given fixed ρ for all interference links, the interference outage of primary networks, P_I, will achieve its minimum at

$$P_I = 1 - \left(\frac{1}{2} + \frac{1-\varepsilon}{2\sqrt{(1+\varepsilon)^2 - 4\rho^2\varepsilon}}\right)^K, \quad (16)$$

if and only if

$$\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_K = \varepsilon. \tag{17}$$

Proof: Applying the inequality of arithmetic and geometric means, we can write

$$\prod_{k=1}^{K}\left(\frac{1+\Phi_k}{2}\right) \le \left(\frac{1}{K}\sum_{k=1}^{K}\frac{1+\Phi_k}{2}\right)^K. \tag{18}$$

The equality happens if and only if

$$\frac{1+\Phi_1}{2} = \cdots = \frac{1+\Phi_K}{2} \tag{19}$$

leading to

$$\varepsilon_1 = \cdots = \varepsilon_K = \varepsilon. \tag{20}$$

This completes the proof. ■

The setting of the back-off coefficients is an additional important issue not only to the primary network but also the secondary network. If these coefficients are chosen too high, the interference probability will increase. On the other hand, if they are chosen too low, reduced transmit powers result in poor performance of secondary networks. Hence, it is important to determine the appropriate value for the back-off coefficient. In particular, the problem is to find the maximum value of the back-off coefficient for the given desired interference value. The corresponding solution is given in the following theorem.

Theorem 2: Given an interference outage (P_I), the interference channel coefficient (ρ), and the number of hops (K), the maximum back-off coefficient which offers the best secondary network performance is determined as

$$\varepsilon_{\max} = \frac{1+\zeta^2-2\zeta^2\rho^2-2\sqrt{\zeta^2-\zeta^2\rho^2-\zeta^4\rho^2+\zeta^4\rho^4}}{1-\zeta^2}, \tag{21}$$

where $\zeta = 2\sqrt[K]{1-P_I}-1$.

Proof: Letting $\zeta = 2\sqrt[K]{1-P_I}-1$ and rephrasing (12), the back-off coefficient is the root of the quadratic equation as follows:

$$(1-\zeta^2)\varepsilon^2 - (2+2\zeta^2-4\rho^2\zeta^2)\varepsilon + 1 - \zeta^2 = 0. \tag{22}$$

Let us denote $\Psi(\zeta) = (1-\zeta^2)\varepsilon^2-(2+2\zeta^2-4\rho^2\zeta^2)\varepsilon+1-\zeta^2$. Note that because $0 < \zeta \le 1$, we have $\Psi(0) = 1-\zeta^2 \ge 0$ and $\Psi(1) = -4\zeta^2(1+\rho^2) < 0$ leading to the fact that this quadratic equation has at least one admissible positive real root in the range of $[0\ 1]$ as presented in (21). ■

Having determined the maximum back-off coefficient for interference outage of primary networks, let us now turn to find the maximum number of secondary multihop networks meeting the desired interference constraint, which is stated in the following theorem.

Theorem 3: For given P_I, ε, and ρ, the maximum number of hops for secondary networks $K_{\max}$ is given by

$$K_{\max} = \left\lfloor \frac{\log(1-P_I)}{\log\left(\frac{1}{2}+\frac{1-\varepsilon}{2\sqrt{(1+\varepsilon)^2-4\rho^2\varepsilon}}\right)} \right\rfloor, \tag{23}$$

where $\log(.)$ and $\lfloor x \rfloor$ denote the natural logarithm and the positive integer closest to x, respectively.

Proof: From (12), we have

$$K^* = \frac{\log(1-P_I)}{\log\left(\frac{1}{2}+\frac{1-\varepsilon}{2\sqrt{(1+\varepsilon)^2-4\rho^2\varepsilon}}\right)}. \tag{24}$$

Since $K_{\max}$ only takes integer values, we choose $K_{\max}$ as the largest integer that is not greater than K to satisfy the interference constraints, which completes the proof. ■

B. Outage probability of secondary networks

The OP is defined as the probability that the end-to-end SNR of the cognitive secondary multihop networks (γ_Σ) is less than a predetermined threshold γ_{th}. Thus, the OP is given by

$$\mathrm{OP} = \Pr(\gamma_\Sigma < \gamma_{th}) \overset{(a)}{=} 1 - \prod_{k=1}^{K}\left[1 - F_{\gamma'_k}(\gamma_{th})\right], \tag{25}$$

where $\gamma_\Sigma = \min_{k=1,\ldots,K}\gamma'_k$ with $\gamma'_k = \varepsilon_k\gamma_k$ and step (a) follows with the assumption that all γ'_k are independent of each other.

In (25), $F_{\gamma'_k}(\gamma)$ is computed as follows:

$$\begin{aligned} F_{\gamma'_k}(\gamma) &= \int_0^\infty\int_0^\infty F_{\frac{\varepsilon_k I_p}{N_0}|h_k|^2}(\gamma x) f_{|f_k|^2,|\tilde{f}_k|^2}(x,y)\,dx\,dy \\ &= \frac{\gamma}{\gamma+\alpha_k}, \end{aligned} \tag{26}$$

where $\alpha_k = \frac{\varepsilon_k I_p \lambda_{D,k}}{N_0\lambda_{I,k}}$. From (26), the end-to-end closed-form expression for the OP is written as

$$\mathrm{OP} = 1 - \prod_{k=1}^{K}\frac{\alpha_k}{\gamma_{th}+\alpha_k}. \tag{27}$$

It is worth noting that the OP of secondary networks involves only finite multiplication of $\frac{\alpha_k}{\gamma_{th}+\alpha_k}$, thus can be calculated in closed-form. Next, we will study the asymptotic form of the OP, which is useful for evaluating the system performance at high SNRs in a more intuitive and concise way. The following theorem is to be utilized.

Theorem 4: At a high SNR regime, the OP can be readily and accurately lower-bounded by

$$\mathrm{OP} \to \sum_{k=1}^{K}\frac{\gamma_{th}}{\varepsilon_k\alpha_k}. \tag{28}$$

Proof: We start the proof by rewriting (25) as

$$\mathrm{OP} = 1 - \prod_{k=1}^{K}\left(1+\frac{\gamma_{\mathrm{th}}}{\alpha_k}\right)^{-1}. \tag{29}$$

By making use of the fact that $\left(1+\frac{\gamma_{\mathrm{th}}}{\alpha_k}\right)^{-1} \approx 1-\frac{\gamma_{\mathrm{th}}}{\alpha_k}$ at high SNR regime and then neglecting the high order terms, i.e., $\prod_{k=1}^{K}\left(1-\frac{\gamma_{th}}{\alpha_k}\right) \approx 1-\sum_{k=1}^{K}\frac{\gamma_{th}}{\alpha_k}$, we have (28). ■

Lemma 2: For predetermined P_I, ρ and ε, the minimum of outage performance loss of the secondary system in dB as compared to the non back-off system is

$$-10\log_{10}\frac{1+\zeta^2-2\zeta^2\rho^2-2\sqrt{\zeta^2-\zeta^2\rho^2-\zeta^4\rho^2+\zeta^4\rho^4}}{1-\zeta^2}. \tag{30}$$

Proof: From (28), it is straightforward to conclude that the performance loss of secondary systems at high SNRs is $-10\log_{10}\varepsilon$ dB. ■

C. Bit error rate of secondary networks

In order to derive the end-to-end BER, we first derive the single-hop average BER, BER_k with $k = 1, \ldots, K$. For square M-QAM, the average BER over Rayleigh fading is given as [19][1]

$$\mathrm{BER}_k = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \int_0^{\infty} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{\upsilon_j} \phi_n^j \,\mathrm{erfc}\left(\sqrt{\omega_k \gamma}\right) f_{\gamma'_k}(\gamma) d\gamma, \quad (31)$$

where $\mathrm{erfc}(.)$ is the complementary error function [20, Eq. (4A.6)]. Here, υ_j, ϕ_n^j, and ω_n are defined respectively as $\upsilon_j = (1-2^{-j})\sqrt{M} - 1$, $\omega_n = \frac{(2n+1)^2 3\log_2 M}{2M-2}$, and $\phi_n^j = (-1)^{\left\lfloor \frac{n2^{j-1}}{\sqrt{M}} \right\rfloor}\left(2^{j-1} - \left\lfloor \frac{n2^{j-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right)$. Using the result in Appendix A, we have the closed-form expression of BER_k as

$$\mathrm{BER} = \sum_{k=1}^{K} \frac{\sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{\upsilon_j} \phi_n^j \left(1 - \sqrt{\omega_n \alpha_k} e^{\omega_n \alpha_k} \sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{\omega_n \alpha_k}\right)\right)}{\sqrt{M}\log_2\sqrt{M}}. \quad (32)$$

For a given set of average BER of K hops, $\mathrm{BER}_1, \ldots, \mathrm{BER}_K$, we are now in a position to derive the end-to-end BER. By taking into account the fact that an even number of wrong single-hop bit transmissions between the source and the destination will make a right bit transmission, we have [21]

$$\mathrm{BER} = \sum_{u=1}^{K} \mathrm{BER}_u \prod_{v=u+1}^{K} (1 - 2\mathrm{BER}_v) \quad (33)$$

After several manipulations, a simplified expression for the end-to-end BER can be written as

$$\mathrm{BER} = \frac{1}{2}\left[1 - \prod_{k=1}^{K} (1 - 2\mathrm{BER}_k)\right]. \quad (34)$$

It is worth noting that the form of (34) is new and has not been reported in the literature. Compared to (33), (34) is more mathematically tractable. Besides, (34) reveals that the BER will achieve its minimum if and only if $\mathrm{BER}_1 = \mathrm{BER}_2 = \cdots = \mathrm{BER}_K$[2].

Theorem 5: At high SNRs, the system BER is approximated as follows:

$$\mathrm{BER} = \frac{\sqrt{M}-1}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{K} \frac{N_0 \lambda_{I,k}}{\varepsilon_k I_p \lambda_{D,k}}. \quad (35)$$

Proof: By utilizing the fact that $\prod_{k=1}^{K}(1-x_k) \approx 1 - \sum_{k=1}^{K} x_k$ for small x_k, from (34), we have

$$\mathrm{BER} \approx \sum_{k=1}^{K} \mathrm{BER}_k \quad (36)$$

In (32), we approximate $\mathrm{erfc}(\sqrt{x})$ by $\frac{e^{-x}}{\sqrt{\pi x}}\left(1 - \frac{1}{2x}\right)$ for large x [22], the system BER is further approximated as

$$\begin{aligned}\mathrm{BER} &\approx \sum_{k=1}^{K} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{\upsilon_j} \frac{\phi_n^j}{\sqrt{M}\log_2\sqrt{M}} \frac{1}{2\omega_n \alpha_k} \\ &\overset{(b)}{\approx} \frac{\sqrt{M}-1}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{K} \frac{N_0 \lambda_{I,k}}{\varepsilon_k I_p \lambda_{D,k}}. \end{aligned} \quad (37)$$

where (b) is based on the fact that the first term, i.e., $j = 1$, is dominant in the inner summation of (37). ■

Moreover, assuming independent and identically distributed (i.i.d.) fading for all links, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_K = \alpha$, the end-to-end BER can be simplified to

$$\mathrm{BER} \approx \frac{(\sqrt{M}-1) K N_0 \lambda_I}{\sqrt{M}\log_2\sqrt{M}\varepsilon I_p \lambda_D}. \quad (38)$$

Lemma 3: Under the constraint of interference outage P_I, $-10\log_{10}\varepsilon$ dB is the performance loss in terms of BER since back-off technique is applied.

Proof: The proof is omitted since it can be done in the same way as for OP. ■

D. Ergodic capacity of secondary networks

In this section, we will derive the ergodic capacity of secondary networks under the interference outage constraint of the primary network. Denoting C as the ergodic capacity of the secondary network, we have

$$\begin{aligned} C &= \frac{1}{K} \int_0^{\infty} \log_2(1+\gamma) f_{\gamma_\Sigma}(\gamma) \mathrm{d}\gamma \\ &\overset{(c)}{=} \frac{1}{K\log 2} \underbrace{\int_0^{\infty} \frac{1}{\gamma+1} \prod_{k=1}^{K} \frac{\alpha_k}{\gamma+\alpha_k} \mathrm{d}\gamma}_{C_1}, \end{aligned} \quad (39)$$

where step (c) follows after the use of integration by parts.

Theorem 6: The ergodic capacities of secondary multihop networks for an arbitrary number of hops are given as

$$C = \frac{1}{K\log 2}\left[\left(\frac{\alpha}{\alpha-1}\right)^{K} \log\alpha - \sum_{k=2}^{K} \left(\frac{\alpha}{\alpha-1}\right)^{K+1-k} \frac{1}{k-1}\right] \quad (40)$$

for i.i.d. channels, or

$$C = \frac{1}{K\log 2} \sum_{k=1}^{K} \frac{\alpha_k}{\alpha_k - 1} \left(\prod_{\ell=1,\ell\neq k}^{K} \frac{\alpha_\ell}{\alpha_k - \alpha_\ell}\right) \log\alpha_k \quad (41)$$

for independent and non-identically distributed (i.n.d.) channels, or

$$C = \frac{1}{K\log 2}\left[\sum_{i=1}^{N} A_{i,1}\log\Theta_i + \sum_{i=1}^{N}\sum_{j=2}^{r_i} \frac{A_{ij}}{(j-1){\Theta_i}^{j-1}}\right] \quad (42)$$

for generalized channels.

Proof: The proof can be found in Appendix B. ■

The exact capacity expression in (40), (41), and (42) involve only elementary mathematical functions and therefore avoids

[1] Although we only consider the BER performance for square M-QAM modulation, the BER derivation can be applied to any general constellation by using the same approach.

[2] The proof follows from the inequality of arithmetic and geometric means.

the need of numerical integration. As a result, it can be applied to many network and channel settings. More importantly, the result can be applied to the case of generalized channels covering the i.i.d. and i.n.d. as special cases. To provide more insights on the network behaviours, theorem 6 can be specialized to two SNR regimes of interest, i.e., low and high SNRs, as follows.

Theorem 7: At high SNRs, the ergodic capacities over i.i.d. and i.n.d. channels are tightly approximated as (43).

Proof: By neglecting small terms in (40) and (41), we obtain (43), which concludes the proof. ■

From Theorem 7, we have two important remarks: i) The secondary network suffers a minimum Shannon capacity loss of $-\log_2 \epsilon_{\max}/K$ to protect the primary networks due to the imperfect CSI of interference links and ii) the Shannon capacity loss at high SNRs, given by

$$\Delta C \triangleq C(K) - C(K+1) = \frac{\log \alpha - \sum_{k=3}^{K} \frac{1}{k-1}}{K(K+1)\log 2}, \quad (44)$$

diminishes as the number of hops increases. We end this section by presenting the following theorem, which describes the approximation form of the ergodic capacity of secondary multihop networks at low regime of SNR.

Theorem 8: At low SNRs, the system capacity is well-approximated as (45), where $\{\beta_1, \ldots, \beta_M\}$ is a set of distinct elements of $\{\alpha_1, \ldots, \alpha_K\}$, $\sum_{i=1}^{M} u_i = K$ and

$$\mathcal{B}_{i,n} = \frac{1}{(u_i - n)!} \left\{ \frac{\partial^{(u_i - n)}}{\partial \gamma^{(u_i - n)}} \left[(\gamma + \beta_i)^{u_i} f_{\gamma_\Sigma}(\gamma) \right] \right\} \Big|_{\gamma = -\beta_i}. \quad (46)$$

Furthermore, I_n is the auxiliary function, which is of the form

$$I_n(\beta_i) = \begin{cases} \frac{\pi}{2\sqrt{\beta_i}}, & n = 2 \\ \frac{\pi}{(n-1)!\beta_k^{\,n-\frac{3}{2}} 2^{n-1}} \prod_{\ell=3}^{n} (2\ell - 5), & n > 2 \end{cases}. \quad (47)$$

Proof: Please refer to Appendix C. ■

Theorem 8 indicates that the system Shannon capacity at low SNRs increases according to the number of hops with respective gain $\frac{C(K+1)}{C(K)} = \frac{2K-1}{2(K+1)}$. It is straightforward to show that the gain becomes one since the number of hops approaches infinity.

Finally, note that our derived approach for the system performance metrics (including OP, BER, and ergodic capacity) is highly precise at high and low SNRs. Additionally, the closed-form expressions contain only elementary functions, and thus its evaluation is instantaneous regardless of network and channel settings.

IV. Numerical Results

In this section, representative numerical examples are provided to highlight the effect of imperfect CSI of interference links on the performance of the primary network and the secondary cognitive multihop network. The network topology is based on the assumption that the secondary network is placed on a straight line connecting the secondary source and the secondary destination. Each cognitive relay node is equidistant from each other, i.e., $d_{\mathcal{T}_k,\mathcal{T}_{k+1}} = 1/K$ for all k. We further assume that the PU-Rx, the cognitive source and the cognitive destination are placed at coordinates $(x_{\rm p},\ x_{\rm p})$, $(0,\ 0)$ and $(1,\ 0)$, respectively. To take into account the effect of pathloss, we set $\lambda_{\mathsf{A},\mathsf{B}} = d_{\mathsf{A},\mathsf{B}}^{-\eta}$, where η is the path loss exponent with $\mathsf{A} \in \{\mathcal{T}_0, \ldots, \mathcal{T}_K\}$ and $\mathsf{B} \in \{\text{PU}, \mathcal{T}_1, \ldots, \mathcal{T}_{K-1}\}$.

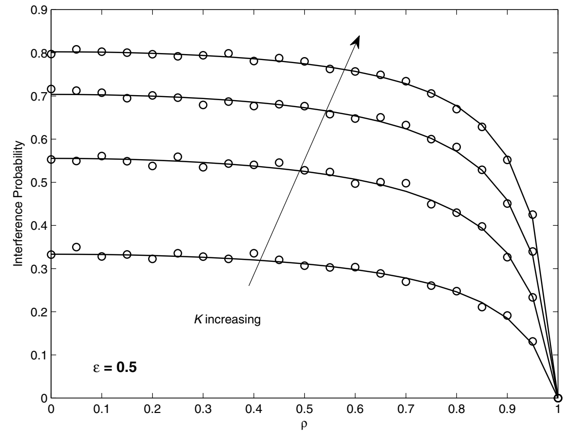

Fig. 2. Interference Probability versus channel coefficient.

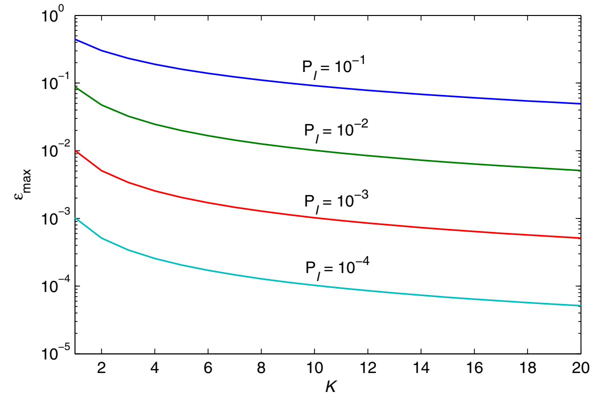

Fig. 3. The maximum back-off coefficient versus number of hops.

Fig. 2 illustrates the effect of the number of hops on the interference probability. As can be clearly seen, the increase of K will increase the interference probability of the primary networks but with diminishing returns. Fig. 2 also shows that the interference probability is a decreasing function of the correlation ratio, which means that the interference probability should decrease as the channel estimation quality increases. Note that P_I is more sensitive to the value of ρ that is close to 1 since we observe a higher dynamic range in the interference probability performance for such values.

In Fig. 3, the maximum back-off power control coefficient is plotted as a function of the number of hops. Clearly, lower target P_I results in smaller $\varepsilon_{\max}$ and this ratio seems to slowly diminish as K increases. The interesting conclusion that one can draw from this figure is that while back-off technique guarantees the minimum interference on primary network in terms of P_I, this gain comes at the expense of certain secondary performance loss on secondary network since the transmit power is reduced by $\varepsilon_{\max}$.

Fig. 4 is a plot of the maximum number of hops, namely $K_{\max}$, as computed from (23), versus the back-off power control coefficient in the constraint of different levels of interference probability. As expected, higher primary interference probability allows a higher number of hops for secondary networks. In addition, since $K_{\max}$ is established for a given

$$C \to \begin{cases} \frac{1}{K\log 2}\left[\log\alpha - \sum_{k=2}^{K}\frac{1}{k-1}\right], & \text{i.i.d. channels} \\ \frac{1}{K}\sum_{k=1}^{K}(-1)^{K-1}\left(\prod_{\ell=1,\ell\neq k}^{K}\frac{\alpha_\ell}{\alpha_k-\alpha_\ell}\right)\log_2\alpha_k, & \text{i.n.d. channels} \end{cases}. \tag{43}$$

$$C \approx \begin{cases} \frac{1}{K}\sum_{k=1}^{K}(-1)^{K-1}\left(\prod_{\ell=1,\ell\neq k}^{K}\frac{\alpha_\ell}{\alpha_k-\alpha_\ell}\right)\frac{\pi\sqrt{\alpha_k}}{2}, & \text{i.n.d. channels} \\ \frac{\pi\sqrt{\alpha}}{K!}\frac{\prod_{\ell=3}^{K+1}(2\ell-5)}{2^K}, & \text{i.i.d. channels} \\ \frac{1}{K}\sum_{i=1}^{M}\sum_{n=2}^{u_i}\mathcal{B}_{i,n}I_n(\beta_i), & \text{generalized channels} \end{cases}, \tag{45}$$

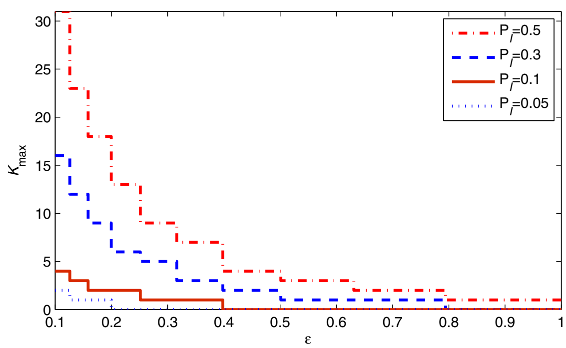

Fig. 4. The maximum number of hops for secondary networks versus back-off coefficient.

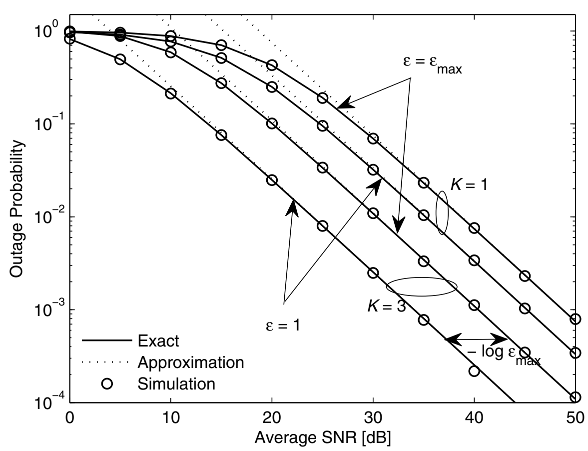

Fig. 5. Outage probability of secondary network versus average SNR.

P_I, only secondary networks with the number of hops equal to or smaller than $K_{\max}$ is valid. For example, only direct transmission is acceptable for secondary networks regardless of secondary channel settings if P_I and ε are chosen at 0.1 and 0.35, respectively.

In Figs. 5 and 6, we plot the OP and BER of secondary networks versus average SNRs. Here we also plot the corresponding no back-off case as a benchmark to evaluate the performance loss. Both figures confirm that a lower OP is achieved by increasing the number of hops in secondary networks. For high average SNRs, the approximations (28) and (35) match the simulation results, confirming the correctness of the analysis approach. Another interesting observation is that a $\log\varepsilon_{\max}$ performance loss between the two systems is found; however, the slope of all curves are the same, that is,

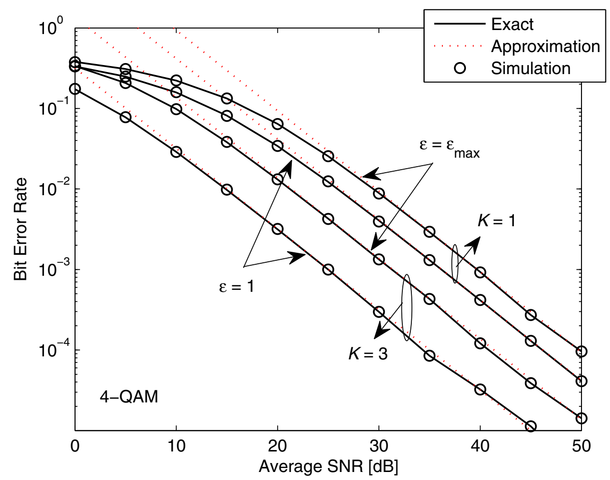

Fig. 6. BER of secondary network for 4-QAM modulation versus average SNR.

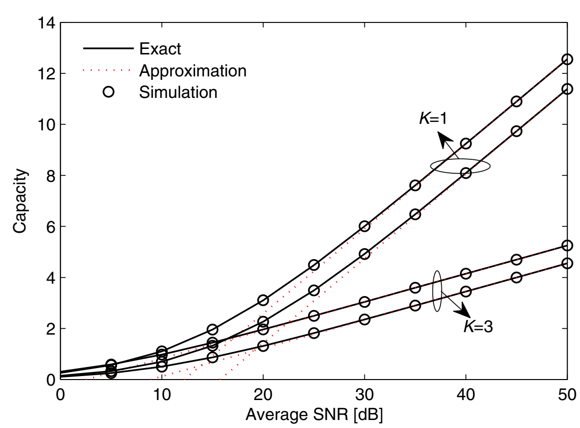

Fig. 7. Ergodic capacity of secondary network versus average SNR.

the back-off technique has no effect on the system diversity except for the system coding gain.

Figure 7 shows the ergodic capacity and its approximation at high SNRs, as given in (43), as a function of average SNRs. Two cases of network topologies, i.e., $K = 1$ and $K = 3$, are considered. It can be observed that for average SNRs higher than 20 dB, the performance loss gap becomes almost steady. We further observe that the secondary network with $K = 3$ has worse capacity than with $K = 1$. This is expected due to the orthogonal channels used in secondary networks.

Figure 8 shows the tightness of proposed approximation for the system ergodic capacity at low SNRs, e.g., ranging from

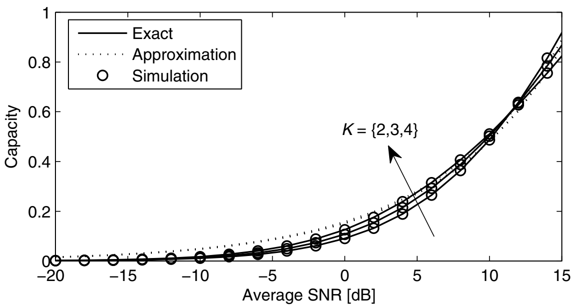

Fig. 8. Ergodic capacity of secondary network versus average SNR.

-20 dB to 15 dB. The asymptotic bounds are quite tight in this case. An important result here is that the capacity at low SNRs increases linearly with K, unlike the high SNR behavior. This effect has also been reported in [23] and it can be explained by using the fact that increasing number of hops in linear networks at low SNRs corresponds to increasing effective SNRs. Finally, along with its simplicity, i.e., requiring only elementary functions, the approximation is found to provide a tight fit for most values of K.

V. Conclusions

This paper has studied the performance of the primary and secondary multihop networks in the presence of imperfect channel knowledge of the primary-secondary links. To protect the primary communication, we proposed to apply the back-off technique so that the interference probability at the primary network is guaranteed. To quantify the loss due to imperfect CSI, three performance metrics of secondary multihop networks including OP, BER and ergodic capacity are derived over Rayleigh fading channels. High-and-low SNR analysis has also been derived to give insights into the system behaviors. The derived closed-form analytical expression is validated by simulations showing an excellent match between the numerical and simulation results.

Acknowledgment

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.04-2012.20.

Appendix A
Proof of Eq. (32)

This appendix is to solve the following integral, i.e.,

$$\Theta = \int_0^\infty \operatorname{erfc}\left(\sqrt{\omega_k \gamma}\right) f_{\gamma_k'}(\gamma) d\gamma, \qquad \text{(A.1)}$$

where $f_{\gamma_k'}(\gamma)$ is obtained from (26) as

$$f_{\gamma_k'}(\gamma) = \frac{d}{d\gamma}\left[\frac{\gamma}{\gamma+\alpha_k}\right] = \frac{\alpha_k}{(\gamma+\alpha_k)^2}. \qquad \text{(A.2)}$$

Using integration by parts, we have

$$\Theta = \underbrace{\frac{\gamma \operatorname{erfc}\left(\sqrt{\omega_k\gamma}\right)}{\gamma+\alpha_p}\Bigg|_{\gamma=0}^{\infty}}_{\Theta_1} + \underbrace{\frac{\omega_k}{\pi}\int_0^\infty \frac{\sqrt{\gamma}e^{-\omega_k\gamma}}{\gamma+\alpha_k} d\gamma}_{\Theta_2}. \qquad \text{(A.3)}$$

It is straightforward to show after using the l'Hospital rule that $\Theta_1 = 0$ resulting in

$$\Theta = \frac{\omega_k}{\pi}\int_0^\infty \frac{\sqrt{\gamma}e^{-\omega_k\gamma}}{\gamma+\alpha_k} d\gamma. \qquad \text{(A.4)}$$

Making a change of integration variables in (A.4), i.e., $u = \sqrt{\gamma}$ and with the help of [24, Eq. (3.32.3) and Eq. (7.1.11)], we have

$$\begin{aligned}\Theta &= 2\int_0^\infty \frac{u^2}{u^2+b} e^{-\omega_p u^2} du \\ &= 2\left[\int_0^\infty e^{-\omega_p u^2} du - \int_0^\infty \frac{b}{u^2+b} e^{-\omega_p u^2} du\right] \\ &= 1 - \sqrt{\omega_n \alpha_k} e^{\omega_n \alpha_k} \sqrt{\pi} \operatorname{erfc}\left(\sqrt{\omega_n \alpha_k}\right). \qquad \text{(A.5)}\end{aligned}$$

Appendix B
Proof of Theorem 6

To obtain C, we first need to derive C_1 in (39). Denoting $\{\Theta_1, \ldots, \Theta_N\}$ as the distinct elements of the set $\{1, \alpha_1, \ldots, \alpha_K\}$, after performing partial fraction expansions, we rewrite C_1 as

$$C_1 = \sum_{i=1}^{N}\sum_{j=1}^{r_i}\int_0^\infty \frac{A_{ij}}{(\gamma+\Theta_i)^{j+1}} \mathrm{d}\gamma, \qquad \text{(B.1)}$$

where the partial-fraction coefficient, A_{ij}, is computed as

$$A_{ij} = \frac{1}{(r_i - j)!}\left\{\frac{\partial^{(r_i-j)}}{\partial\gamma^{(r_i-j)}}\left[(\gamma+\Theta_i)^{r_i} C_i\right]\right\}\Bigg|_{\gamma=-\Theta_i}. \qquad \text{(B.2)}$$

Note that the closed-form expression for the inner integral does not exist when $j = 1$. By separating the summation in (B.1) into two summations and making use of the fact that $\sum_{j=1}^{N} A_{i1} = 0$, the closed-form expression of C_1 is given as follows:

$$C_1 = \sum_{i=1}^{N} A_{i,1}\log\Theta_i + \sum_{i=1}^{N}\sum_{j=2}^{r_i}\frac{A_{ij}}{(j-1)\Theta_i^{j-1}}. \qquad \text{(B.3)}$$

Finally, substituting (B.3) into (39), we obtain the closed-form expression of the ergodic capacity of secondary networks.

For the i.i.d. case, i.e., $\alpha_1 = \cdots = \alpha_N = \alpha \neq 1$, C_1 is rewritten as

$$\begin{aligned}C_1 &= \int_0^\infty \frac{1}{\gamma+1}\left(\frac{\alpha}{\gamma+\alpha}\right)^K \mathrm{d}\gamma \\ &= \left(\frac{\alpha}{\alpha-1}\right)^K \int_0^\infty \left(\frac{1}{\gamma+1} - \frac{1}{\gamma+\alpha}\right) d\gamma \\ &\quad - \sum_{k=2}^{K}\frac{\alpha^K}{(\alpha-1)^{K+1-k}}\int_0^\infty \frac{1}{(\gamma+\alpha)^k} d\gamma \\ &= \left(\frac{\alpha}{\alpha-1}\right)^K \log\alpha - \sum_{k=2}^{K}\left(\frac{\alpha}{\alpha-1}\right)^{K+1-k}\frac{1}{k-1}. \qquad \text{(B.4)}\end{aligned}$$

Since $\alpha_1 = \cdots = \alpha_N = 1$, (B.4) is further reduced to as

$$C_1 = \frac{1}{K\log 2}\int_0^\infty \frac{1}{(\gamma+1)^{K+1}}\mathrm{d}\gamma = \frac{1}{K^2\log 2}. \quad (B.5)$$

For the i.n.d. case, i.e., $\alpha_1 \neq \cdots \neq \alpha_N \neq 1$, we have (B.6).

Appendix C
Proof of Theorem 8

At low SNRs, we can approximate $\log_2(1+\gamma) \approx \sqrt{\gamma}$ resulting in the system capacity being given by

$$C \approx \frac{1}{K}\int_0^\infty \sqrt{\gamma} f_{\gamma_\Sigma}(\gamma)\mathrm{d}\gamma, \quad (C.1)$$

where $f_{\gamma_\Sigma}(\gamma)$ denotes the PDF of γ_Σ, which is derived from (27) as follows:

$$f_{\gamma_\Sigma}(\gamma) = \frac{d}{d\gamma}\left[1 - \prod_{k=1}^{K}\frac{\alpha_k}{\gamma+\alpha_k}\right]. \quad (C.2)$$

Applying again partial fraction technique, (C.2) can be rewritten as follows:

$$f_{\gamma_\Sigma}(\gamma) = \sum_{k=1}^{K}\frac{\prod_{m=1}^{K}\alpha_m}{(\gamma+\alpha_k)\prod_{\ell=1}^{K}(\gamma+\alpha_\ell)} = \sum_{i=1}^{M}\sum_{n=2}^{u_i}\frac{\mathcal{B}_{i,n}}{(\gamma+\beta_k)^n}. \quad (C.3)$$

Substituting (C.3) into (C.1) yields

$$C = \frac{1}{K}\sum_{i=1}^{M}\sum_{n=2}^{u_i}\mathcal{B}_{i,n}\underbrace{\int_0^\infty \frac{\sqrt{\gamma}}{(\gamma+\beta_i)^n}\mathrm{d}\gamma}_{I_n(\beta_i)}. \quad (C.4)$$

In (C.4), $I_n(\beta_i)$ is computed as

$$I_n(\beta_i) = \int_0^\infty \frac{\sqrt{\gamma}}{(\gamma+\beta_i)^n}d\gamma, \ n \geq 2. \quad (C.5)$$

When n is an integer, rewriting (C.5) and then using [24, Eq. (3.194.4)], we immediately get

$$I_n(\beta_i) = \frac{1}{{\beta_i}^n}\int_0^\infty \frac{\sqrt{\gamma}}{(1+\frac{\gamma}{\beta_i})^n}d\gamma = {\beta_i}^{\frac{3}{2}-n}\mathrm{B}\left(\frac{3}{2}, n-\frac{3}{2}\right), \quad (C.6)$$

where $\mathrm{B}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ is the beta function (or the Euler integral of the first kind) [24, Eq. (8.380.1)]. With the help of the identity [24, Eq. (8.384.1)], i.e.,

$$\mathrm{B}\left(\frac{3}{2}, n-\frac{3}{2}\right) = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(n-\frac{3}{2}\right)}{\Gamma(n)}, \quad (C.7)$$

(C.6) is re-expressed as

$$I_n = {\beta_i}^{\frac{3}{2}-n}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(n-\frac{3}{2}\right)}{\Gamma(n)}, \quad (C.8)$$

where $\Gamma(x) = \int_0^\infty e^{-t}t^{z-1}dt$ denotes the Gamma function [24, Eq. (8.310.1)]. Furthermore, from the facts that $\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$ (see [15, Eq. (6.1.9)]), $\Gamma(n) = (n-1)!$ (see [15, Eq. (6.1.6)]), and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (see [15, Eq. (6.1.8)]), a more simplified expression of (C.5) is produced after considerable manipulations as (47) [15, Eq. (6.1.12)].

For the i.i.d. case, from (C.1) and (C.2), we have

$$C = \alpha^K\int_0^\infty \frac{\sqrt{\gamma}}{(\gamma+\alpha)^{K+1}}\mathrm{d}\gamma = \frac{\pi\sqrt{\alpha}}{K!}\frac{\prod_{\ell=3}^{K+1}(2\ell-5)}{2^K}. \quad (C.9)$$

For i.n.d. case, using the same approach, we have

$$\begin{aligned}C &= \frac{1}{K}\int_0^\infty \sqrt{\gamma}\underbrace{\sum_{k=1}^{K}(-1)^{K-1}\left(\prod_{\ell=1,\ell\neq k}^{K}\frac{\alpha_\ell}{\alpha_k-\alpha_\ell}\right)\frac{\alpha_k}{(\alpha+\alpha_k)^2}}_{f_{\gamma_\Sigma}(\gamma)}\mathrm{d}\gamma\\ &= \frac{1}{K}\sum_{k=1}^{K}(-1)^{K-1}\left(\prod_{\ell=1,\ell\neq k}^{K}\frac{\alpha_\ell}{\alpha_k-\alpha_\ell}\right)\alpha_k\underbrace{\int_0^\infty \frac{\sqrt{\gamma}}{(\alpha+\alpha_k)^2}\mathrm{d}\gamma}_{I_2}\\ &= \frac{1}{K}\sum_{k=1}^{K}(-1)^{K-1}\left(\prod_{\ell=1,\ell\neq k}^{K}\frac{\alpha_\ell}{\alpha_k-\alpha_\ell}\right)\frac{\pi\sqrt{\alpha_k}}{2}. \quad (C.10)\end{aligned}$$

References

[1] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun. Mag.*, vol. 6, no. 4, pp. 13–18, Apr. 1999.

[2] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[3] V. N. Q. Bao and T. Q. Duong, "Outage analysis of cognitive multihop networks under interference constraints," *IEICE Trans. Commun.*, vol. E95-B, no. 03, pp. 1019–1022, Mar. 2012.

[4] C. Zhong, T. Ratnarajah, and K.-K. Wong, "Outage analysis of decode-and-forward cognitive dual-hop systems with the interference constraint in Nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 6, pp. 2875–2879, Jun. 2011.

[5] T. Q. Duong, V. N. Q. Bao, and H.-J. Zepernick, "Exact outage probability of cognitive AF relaying with underlay spectrum sharing," *Electron. Lett.*, vol. 47, no. 17, pp. 1001–1002, Aug. 2011.

[6] T. Q. Duong, D. B. da Costa, M. Elkashlan, and V. N. Q. Bao, "Cognitive amplify-and-forward relay networks over Nakagami-m fading," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2368–2374, May 2012.

[7] H. Jun-pyo, H. Bi, B. Tae Won, and C. Wan, "On the cooperative diversity gain in underlay cognitive radio systems," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 209–219, Jan. 2012.

[8] W. Xu, J. Zhang, P. Zhang, and C. Tellambura, "Outage probability of decode-and-forward cognitive relay in presence of primary user's interference," *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1252–1255, Aug. 2012.

[9] V. N. Q. Bao, T. T. Thanh, N. T. Duc, and V. D. Thanh, "Spectrum sharing-based multihop decode-and-forward relay networks under interference constraints: performance analysis and relay position optimization," *J. Commun. Netw.*, vol. 15, no. 3, pp. 266–275, Jun. 2013.

[10] G. Amarasuriya, C. Tellambura, and M. Ardakani, "Asymptotically-exact performance bounds of AF multi-hop relaying over Nakagami fading," *IEEE Trans. Commun.*, vol. 59, no. 4, pp. 962–967, Apr. 2011.

[11] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 1811–1822, May 2010.

[12] J. Chen, J. Si, Z. Li, and H. Huang, "On the performance of spectrum sharing cognitive relay networks with imperfect CSI," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1002–1005, Jul. 2012.

[13] M. Schwartz, W. Bennett, and S. Stein, *Communication Systems and Techniques*. Wiley-IEEE Press, 1995.

[14] X. Tang, M. Alouini, and A. Goldsmith, "Effect of channel estimation error on M-QAM BER performance in Rayleigh fading," *IEEE Trans. Commun.*, vol. 47, no. 12, pp. 1856–1864, Dec. 1999.

[15] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 10th ed. U.S. Government Printing Office, 1972.

$$
\begin{aligned}
C_1 &= \int_0^\infty \frac{1}{\gamma+1} \prod_{k=1}^{K} \frac{\alpha_k}{\gamma+\alpha_k} \\
&= \int_0^\infty \left[\left(\prod_{k=1}^{K} \frac{\alpha_k}{\alpha_k - 1} \right) \frac{1}{\gamma+1} - \sum_{k=1}^{K} (-1)^{K-1} \frac{\alpha_k}{\alpha_k - 1} \left(\prod_{\ell=1,\ell\neq k}^{K} \frac{\alpha_\ell}{\alpha_k - \alpha_\ell} \right) \frac{1}{\gamma+\alpha_k} \right] d\gamma \\
&= \sum_{k=1}^{K} (-1)^{K-1} \frac{\alpha_k}{\alpha_k - 1} \left(\prod_{\ell=1,\ell\neq k}^{K} \frac{\alpha_\ell}{\alpha_k - \alpha_\ell} \right) \log \alpha_k. \qquad \text{(B.6)}
\end{aligned}
$$

[16] A. H. Nuttall, "Some integrals involving the Q_m function," *IEEE Trans. Inf. Theory*, vol. 21, no. 1, pp. 95–96, Jan. 1975.

[17] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*, 4th ed. McGraw-Hill, 2002.

[18] M. K. Simon and M. S. Alouini, "Some new results for integrals involving the generalized Marcum Q function and their application to performance evaluation over fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 611–615, 2003.

[19] K. Cho and D. Yoon, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1074–1080, Jul. 2002.

[20] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed., ser. Wiley Series in Telecommunications and Signal Processing. John Wiley & Sons, 2005.

[21] K. Dhaka, R. K. Mallik, and R. Schober, "Performance analysis of decode-and-forward multi-hop communication: a difference equation approach," *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 339–345, Feb. 2012.

[22] C. Tellambura and A. Annamalai, "Efficient computation of erfc(x) for large arguments," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 529–532, Apr. 2000.

[23] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, "Bandwidth- and power-efficient routing in linear wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2624–2633, Jun. 2006.

[24] I. S. Gradshteyn, I. M. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of Integrals, Series and Products*, 7th ed. Elsevier, 2007.

Vo Nguyen Quoc Bao (M'10) received the B.E. and M.Eng. degree in electrical engineering from Ho Chi Minh City University of Technology (HCMUT), Vietnam, in 2002 and 2005, respectively, and Ph.D. degree in electrical engineering from University of Ulsan, South Korea, in 2009. In 2002, he joined the Department of Electrical Engineering, Posts and Telecommunications Institute of Technology (PTIT), as a lecturer. Since February 2010, he has been with the Department of Telecommunications, PTIT, where he is currently an Assistant Professor. His major research interests are modulation and coding techniques, MIMO systems, combining techniques, cooperative communications, and cognitive radio. Dr. Bao is a member of Korea Information and Communications Society (KICS), The Institute of Electronics, Information and Communication Engineers (IEICE) and The Institute of Electrical and Electronics Engineers (IEEE). He is also a Guest Editor of *EURASIP Journal on Wireless Communications and Networking*, special issue on "Cooperative Cognitive Networks" and *IET Communications*, special issue on "Secure Physical Layer Communications."

Trung Q. Duong (S'05, M'12, SM'13) received his Ph.D. degree in Telecommunications Systems from Blekinge Institute of Technology (BTH), Sweden in 2012, and then continued working at BTH as a project manager. Since 2013, he has joined Queen's University Belfast, UK as a Lecturer (Assistant Professor). He held a visiting position at Polytechnic Institute of New York University and Singapore University of Technology and Design in 2009 and 2011, respectively. His current research interests include cooperative communications, cognitive radio networks, physical layer security, massive MIMO, cross-layer design, mm-waves communications, localization for radios and networks. Dr. Duong has been a TPC chair for many international conferences and workshops including the most recently IEEE GLOBECOM13 Workshop on Trusted Communications with Physical Layer Security. He currently serves as an Editor for the IEEE COMMUNICATIONS LETTERS, *Wiley Transactions on Emerging Telecommunications Technologies* and the Lead Guest Editor of the special issue on "Secure Physical Layer Communications" of the *IET Communications*, Guest Editor of the special issue on "Green Media: The Future of Wireless Multimedia Networks" of the *IEEE Wireless Communications Magazine*, Guest Editor of the special issue on "Cooperative Cognitive Networks" of the *Eurasip Journal on Wireless Communications and Networking*, Guest Editor of special issue on "Security Challenges and Issues in Cognitive Radio Networks" of the *Eurasip Journal on Advances in Signal Processing*. He is awarded the Best Paper Award at the IEEE Vehicular Technology Conference (VTC-Spring) in 2013 and the Exemplary Reviewer Certificate of the IEEE Communications Letters in 2012.

Chintha Tellambura (F'11) received the B.Sc. degree (with first-class honor) from the University of Moratuwa, Sri Lanka, in 1986, the M.Sc. degree in Electronics from the University of London, U.K., in 1988, and the Ph.D. degree in Electrical Engineering from the University of Victoria, Canada, in 1993. He was a Postdoctoral Research Fellow with the University of Victoria (1993-1994) and the University of Bradford (1995-1996). He was with Monash University, Australia, from 1997 to 2002. Presently, he is a Professor with the Department of Electrical and Computer Engineering, University of Alberta, Canada. His research interests focus on communication theory dealing with the wireless physical layer. Prof. Tellambura was an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the Area Editor for Wireless Communications Systems and Theory in the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He was Chair of the Communication Theory Symposium in Globecom'05 held in St. Louis, MO