Clipping Noise-based Tone Injection for PAPR Reduction in OFDM Systems

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***Abstract*—Tone injection (TI) mitigates the high peak-to-average power ratio problem without incurring data rate loss or extra side information. However, optimal TI requires an exhaustive search over all possible constellations, which is a hard optimization problem. In this paper, a novel TI scheme that uses the clipping noise to find the optimal equivalent constellations is proposed. By minimizing the mean error of the clipping noise and possible constellation points, the proposed scheme easily determines the size and position of the optimal equivalent constellations. The proposed scheme achieves significant PAPR reduction while maintaining low complexity.**

***Index Terms*—Multicarrier modulation, orthogonal frequency division multiplexing, tone injection (TI), peak-to-average power ratio.**

I. Introduction

Multicarrier modulation, especially orthogonal frequency division multiplexing (OFDM), has drawn explosive attention in a number of current and future OFDM standard systems including the IEEE 802.11 a/g wireless standard, the IEEE 802.16 (WiMAX), 802.11n standard, and 3GPP LTE owing to the advantages of high spectral efficiency, easy implementation with fast Fourier transform (FFT), and robustness to frequency selective fading channel [1]. However, its suffers from high peak-to-average power ratio (PAPR), which leads to in-band distortion and out-of-band radiation. Therefore, various PAPR reduction techniques have been proposed, including clipping and filtering [2]-[4], probabilistic techniques [5]-[6], and tone injection [7]-[13].

Clipping and filtering (CF) [2] deliberately clips the time-domain OFDM signal to a predefined level and subsequently filters the out-of-band radiation. And yet, this filtering operation leads to the peak regrowth. Therefore, an iterative clipping and filtering (ICF) [3] algorithm has been proposed to both remove the out-of-band radiation and suppress the peak regrowth, although the in-band clipping noise cannot be eliminated via filtering, leading to the increase of bit error rate (BER).

Probabilistic techniques such as partial transmit sequences (PTS) [5], and selected mapping (SLM) [6] statistically improve the PAPR distribution characteristic of the original signals without signal distortion, which are suitable for OFDM systems with a large number of subcarriers. Nevertheless, the complexity of these techniques increases exponentially with the number of subblocks; moreover, side information may be required at the receiver to decode the input symbols. Incorrectly received side information results in burst errors.

To overcome these problems, one class of PAPR reduction techniques uses nonbijective constellations. That is, a data symbol can be mapped to a one of many constellation points. By appropriately choosing the right constellation points among the allowable set of points, the PAPR can be significantly reduced without a data rate loss or extra side information. One of effective method of this type is tone injection (TI) [7]-[13], which uses a cyclic extension of quadrature amplitude modulation (QAM) constellation to offer alternative encoding with a lower PAPR. However, implementation of the TI technique requires solving a hard integer-programming problem, whose complexity grows exponentially with the number of subcarriers. Therefore, suboptimal solutions are sought.

In this paper, a novel TI scheme that uses the clipping noise to find the optimal equivalent constellation points is introduced. We first take the entire samples of original signal larger than threshold as the clipping noise, and then minimize the mean error of these noise and possible equivalent constellation points to determine the optimal size and position of subcarriers with modified constellation points that generate the peak cancellation signal (PCS). The proposed scheme reduces the computational complexity dramatically while maintaining a good PAPR performance.

The outline of the paper is organized as follows. Section II describes the tone injection technique for OFDM. Section III introduces the proposed clipping noise based TI scheme for PAPR reduction. Simulation results are presented in Section V and conclusions in Section VI.

II. Tone Injection for PAPR Reduction

A. PAPR Problem

In a practical OFDM system, the L-times oversampled OFDM signal can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/LN} \,, n{=}0,1,\cdots,LN-1, \quad (1)$$

where X_k are typically chosen from an M-ary signal constellation, e.g., phase shift keying (PSK), or QAM. The PAPR computed from the L-times oversampled time domain signal samples can be formulated as

$$PAPR = \frac{\max\limits_{0 \le n < LN-1} |x_n|^2}{E\left[|x_n|^2\right]}, \quad (2)$$

where $E[\cdot]$ denotes the expectation operation. It has been shown in [14] that for $L \ge 4$, the model in (2) is accurate to approximate the continuous-time PAPR.

B. Tone Injection

In TI [7], the original constellation is extended to several equivalent points so that the same information can be carried by any of these points. Thus, these extra freedom degrees can be exploited to reduce the PAPR. Mathematically, the modified TI signal is given by

$$\tilde{x}_n = x_n + c_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (X_k + C_k) e^{j2\pi nk/LN}, \quad (3)$$

where C_k is the extra freedom constellation, and c_n is known as the PCS. However, finding the optimal constellation of C_k to obtain the lowest PAPR for $\tilde{x}_n$ is an nondetermined-polynomial hard problem [8]. Therefore, good suboptimal solutions are required.

However, extra constellation points will increase the transmit signal power. Thus, to prevent a greater power increase, only constellation points located on the outer ring could be shifted, and the corresponding equivalent constellation is nearly symmetrical about the origin. This extended constellation for 16-QAM is illustrated in Fig. 1. As shown, the original constellation points in the outer ring are duplicated cyclically onto the surrounding region. Therefore, the modified constellation includes 12 alternative constellation points. In this case, the corresponding equivalent point can be rewritten as [10], [13]

$$S(X_k) = \begin{cases} -\frac{d}{2}p_k - j\frac{d}{2}M'' & (p_k > -M', q_k = M') \\ -\frac{d}{2}p_k + j\frac{d}{2}M'' & (p_k < -M', q_k = -M') \\ -\frac{d}{2}M'' - j\frac{d}{2}q_k & (p_k = M', q_k < M') \\ \frac{d}{2}M'' - j\frac{d}{2}q_k & (p_k = -M', q_k > M') \\ X_k & otherwise \end{cases}, \quad (4)$$

where $M' = \sqrt{M} - 1$, $M'' = \sqrt{M} + 1$, p_k and q_k are integers, $\sqrt{M}$ and d represent the number of levels per dimension and the minimum distance between constellation points, respectively.

III. Proposed TI Scheme and Its Complexity

In this section, we first determine the index set of subcarriers (denote as $\mathcal{S}$) which contribute to the PCS, and then introduce the proposed TI scheme for the reduction of PAPR.

A. Index Set of Subcarriers Contribute to the PCS

Size of $\mathcal{S}$: in order to obtain the size of $\mathcal{S}$, according to Fig. 1, the transmit signal in (3) may rewritten as

$$\hat{x}_n = x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_k + \Delta_k) e^{j2\pi nk/LN}, \quad (5)$$

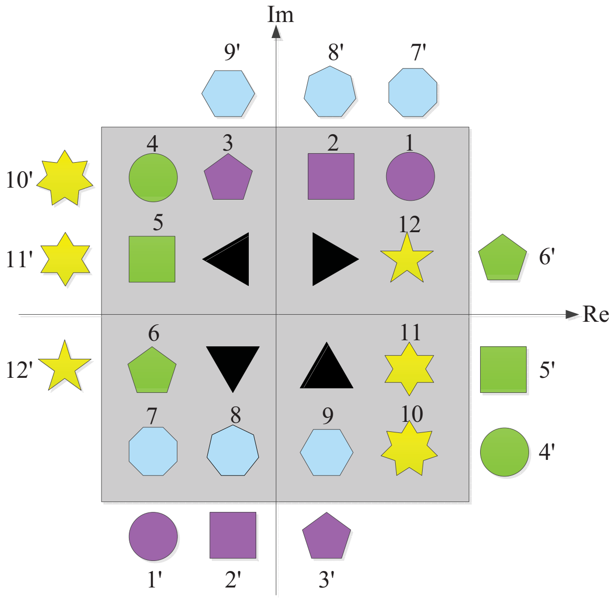

Fig. 1. The cyclically extended 16-QAM constellation diagram.

where $|\Delta_k|$[1] and α_k are defined as

$$\begin{cases} |\Delta_k| = d\sqrt{M}, \quad \alpha_k = 2 & k \in \mathcal{S} \\ |\Delta_k| = 0, \quad \alpha_k = 0 & otherwise \end{cases}.$$

Thus, we have

$$\begin{aligned} |\hat{x}_n| &\ge |x_n| - \left| \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_k + \Delta_k)\, e^{j2\pi(nk/LN)} \right| \\ &= |x_n| - \frac{1}{\sqrt{N}} \cdot \sum_{k \in \mathcal{S}} |2X_k + \Delta_k|. \end{aligned} \quad (6)$$

In order to satisfy the PAPR restriction ($\hat{x}_n$ smaller than the PAPR threshold A), a necessary condition is that $\mathcal{S}$ must satisfy

$$\frac{1}{\sqrt{N}} \sum_{k \in \mathcal{S}} (2|X_k| + |\Delta_k|) \ge |x_n|_{\max} - A. \quad (7)$$

Let $\bar{d} = E\{|X_k|\}$, then for square M-ary QAM

$$\bar{d} = \frac{4}{M} \sqrt{\frac{6}{M-1}} \sum_{p=1}^{\sqrt{M}/2} \sum_{q=1}^{\sqrt{M}/2} \sqrt{(p-0.5)^2 + (q-0.5)^2}. \quad (8)$$

Therefore, the minimum size $\mathcal{S}$ that satisfy (7) can be calculated as

$$N_S = \left\lceil \frac{\sqrt{N}\,(|x_n|_{\max} - A)}{2\bar{d} + |\Delta_k|} \right\rceil, \quad (9)$$

where $\lceil x \rceil$ represent the smallest integer greater than x. □

[1]In order not to increase BER at the receiver, the value of Δ_k should be larger than the minimum distance between the constellation points (at least $d\sqrt{M}$, see [7]). Since we only use the amplitude of Δ_k in the following derivation, we neglect the phase of Δ_k.

Position of $\mathcal{S}$: clipping method limits the peak envelope of the input signal to a predetermined threshold A, where A is determined by the saturation level of the power amplifier (PA). Thus, the clipping noise can be calculated as

$$f_n = \begin{cases} x_n - Ae^{j\phi} & |x_n| > A \\ 0 & |x_n| \leq A \end{cases}, \tag{10}$$

where ϕ represents the phase of x_n. In order to obtain the PCS, we can project the frequency domain clipping noise F_k to X_k.

$$\hat{f}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P_k X_k e^{j2\pi(nk/LN)}, \tag{11}$$

where

$$P_k = \frac{\Re\mathfrak{e}[F_k X_k^*]}{|X_k|^2}, \tag{12}$$

$\Re\mathfrak{e}[x]$ represents the real part of x and $(\cdot)^*$ is the complex conjugate operation.

To further minimize the transmit signal, $\hat{f}_n$ can be scaled by a factor β,

$$\begin{aligned} \hat{x}'_n &= x_n - \beta \hat{f}_n \\ &= x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \beta \cdot P_k X_k e^{j2\pi(nk/LN)}, \end{aligned} \tag{13}$$

Here, we choose a suboptimal solution [15] by minimizing the out-of-range power to obtain the optimal β,

$$\min_{\beta} \sum_{|\hat{x}'_n| > A} (|\hat{x}'_n| - A)^2, \tag{14}$$

and the optimal solution is given by

$$\beta = \frac{\Re\mathfrak{e}\left[\sum f_n \hat{f}_n^*\right]}{\sum \left|\hat{f}_n\right|^2}. \tag{15}$$

We can minimize the mean squared rounding error of (5) and (13) to achieve the optimal position of $\mathcal{S}$,

$$\begin{aligned} \varepsilon &= E\left\{|\hat{x}_n - \hat{x}'_n|^2\right\} \\ &= \frac{1}{LN} \sum_n \left| \frac{1}{\sqrt{N}} \sum_{k\in S} [(2-\beta P_k)X_k + \Delta_k] e^{j2\pi(nk/LN)} \right. \\ &\quad \left. - \frac{1}{\sqrt{N}} \sum_{k\notin S} \beta \cdot P_k X_k e^{j2\pi(nk/LN)} \right|^2 \\ &\leq \frac{1}{\sqrt{N}} \left[\sum_{k\in S} [|(2-\beta P_k) X_k| + |\Delta_k|] + \sum_{k\notin S} |\beta P_k X_k| \right]^2. \end{aligned} \tag{16}$$

Theorem 1: Define

$$Z_m = |(2-\beta P_m) X_m| + |\beta P_m X_m|. \tag{17}$$

Therefore, ε_{max} is minimized if

$$Z_m < Z_n, \quad \text{for all} \quad \mathrm{m} \in \mathcal{S} \quad \text{and} \quad \mathrm{n} \notin \mathcal{S}. \tag{18}$$

The proof of Theorem 1 is provided in Appendix. Based on this theorem, it follows that the index set $\mathcal{S}$ is made up of the smallest N_s of Z_m. □

B. Proposed TI Scheme

The structure of the proposed TI scheme can now be summarized as follows:

Procedure clipping noise-based TI-PAPR

1. *Initialization: set up the PAPR threshold A and the maximum iteration number T.*
   $t \leftarrow 0$
2. *Calculate x_n according to (1).*
   while *($PAPR_{x_n} > A$)* **do**
   $t \leftarrow t+1$
3. *Set the index set $\mathcal{S} = \emptyset$ and choose f_n from (10).*
4. *Obtain N_s, P_k, β, and Z_m by using (9), (12), (15), and (17), respectively.*
5. *Find the smallest N_s of Z_m to make up the index set $\mathcal{S}$; let the $\alpha_i = 2$ for $i \in \mathcal{S}$, and other to 0.*
6. *Calculate C_k according to set $\mathcal{S}$.*
7. *Update the peak-reduced signal by using (5).*
8. **if** *($t \geq T$ or $PAPR_{\hat{x}_n} \leq A$)* **then**
   Transmit $\hat{x}_n$.
   end if
   end while

C. Analysis of the Computational Complexity

To find the optimal constellation points in (3), conventional TI requires solving an integer programming problem with exponential complexity. Assume there are L candidates per constellation, if K dimensions are to be shifted, we must search over all [8]

$$C_N^K \cdot L^K \approx \frac{N^K}{K!} \approx (NL)^K \tag{19}$$

combinations, and each combination requires an IFFT operation.

The overall complexity of the proposed TI scheme is mainly determined by (5) and (11) (for calculating $\hat{x}_n$, f_n and $\hat{f}_n$), which needs three FFTs per iteration. Assume N_f is the number of nonzero samples in f_n. In [15], it is shown that the mean of N_f is a function of N and can be found as

$$\bar{N}_f = LNe^{-A^2/2\sigma^2}. \tag{20}$$

For example, when $A/\sqrt{2\sigma}$ = 6 dB, $\bar{N}_f$ = $1.87 \times 10^{-2} LN$. In order to simplify the complexity comparison, we loosely upper bound the complexity of the proposed TI scheme as three FFTs per iteration. In Section IV, simulations will show that our TI scheme dramatically reduces the search time to obtain a considerable PAPR reduction.

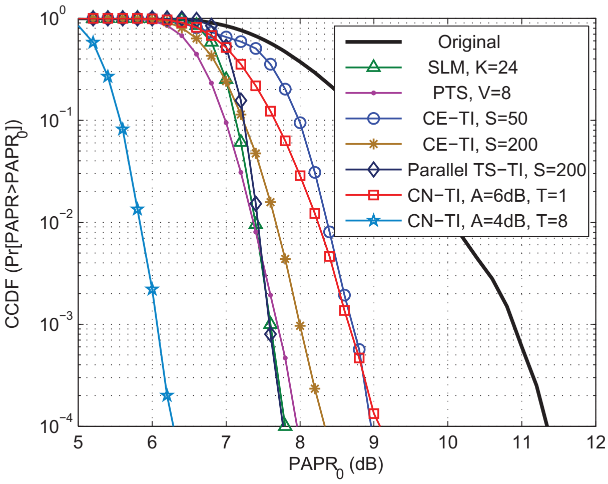

Fig. 2. CCDFs of the PAPR for various schemes with 128 subcarriers.

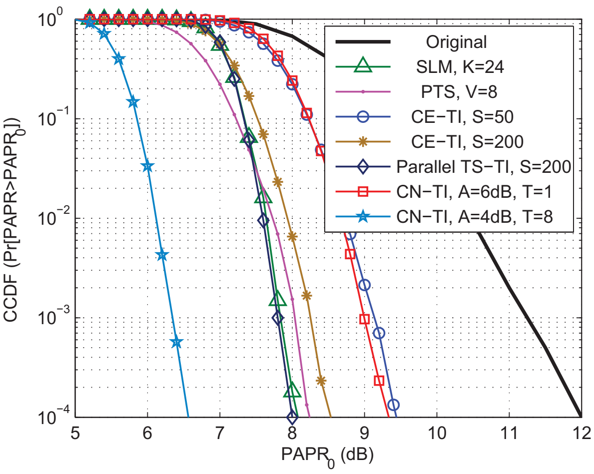

Fig. 3. CCDFs of the PAPR for various schemes with 256 subcarriers.

IV. Simulation Results

In this Section, the complementary cumulative distribution function (CCDF) is used to evaluate the PAPR performance, which is given by

$$CCDF_x(PAPR_0) = Prob(PAPR > A). \qquad (21)$$

This is the probability that the PAPR of a symbol exceeds the threshold level A. The simulations below are performed for the expanded 16-QAM modulated OFDM symbols, with the subcarriers N under the condition of four times oversampling. Furthermore, we choose four PAPR reduction schemes, PTS and SLM [5], cross entropy (CE)-TI [12], parallel tabu search (TS)-TI [13], as well as the original OFDM, for comparison.

Figs. 2 and 3 depict the CCDF curves of five PAPR reduction schemes with the subcarriers $N = 128$ and $N = 256$. To achieve the same PAPR performance, the complexity of the proposed clipping noise-based (CN) TI scheme is much

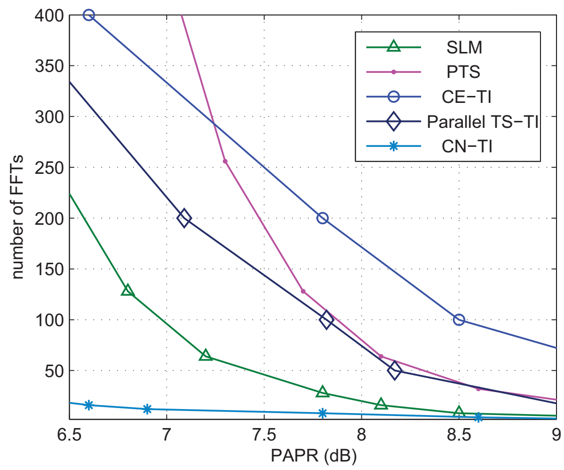

Fig. 4. PAPR vs. computational complexity comparison of the various schemes with 256 subcarriers.

TABLE I
Power Increase, Average Runtime and its Runtime Comparison of the Schemes Based on Cross-Entropy (CE) TI, SLM, PTS and Proposed Clipping Noise-Based (CN) TI Algorithms, with 4 dB PAPR threshold and 128 subcarriers

Scheme	CE-TI	SLM	PTS	CN-TI
PAPR	8.36 dB	7.81 dB	7.98 dB	6.31 dB
Power Increase	1.27 dB	No	No	0.5 dB
Average Runtime	117.4 ms	10.1 ms	58.3 ms	7.1 ms
Number of FFTs	200	24	128	24
Side Information	No	Yes	Yes	No
BER Degradation	No	No	No	No

lower than the other schemes. Specially, to achieve a 9 dB PAPR at 10^{-4} clipping probability, the proposed scheme only needs three FFTs, while the CE-TI needs fifty FFTs[2]. Fig. 3 also shows the similar situations. For 256 subcarriers at 10^{-4} clipping probability, the PAPR reduction of the proposed scheme with eight iterations is about 1.5 dB better than SLM with 24 candidates, 1.7 dB better than PTS with eight random subblock partition. Here, both the proposed scheme and SLM require 24 FFTs per OFDM block, while PTS needs $2^7 = 128$ FFTs. Some comparisons of these four algorithms are listed in Table I.

Fig. 4 illustrates the PAPR and computational complexity comparison of these schemes. It can be observed that, with the same complexity, our TI scheme achieves much smaller PAPR than other schemes. Specially, to achieve a 6.5 dB PAPR, our scheme only needs twenty-four FFTs, while other schemes need hundreds of FFTs.

Fig. 5 shows the CCDF curves of proposed PAPR reduction scheme with eight iterations under different PAPR thresholds. As seen, different threshold offers different PAPR performances. The proposed scheme achieves the best PAPR

[2]According to [12], we choose the parameters $\lambda = 0.8$ and $\rho = 0.1$ for the CE-TI. In addition, S denotes the total number of samples (FFTs).

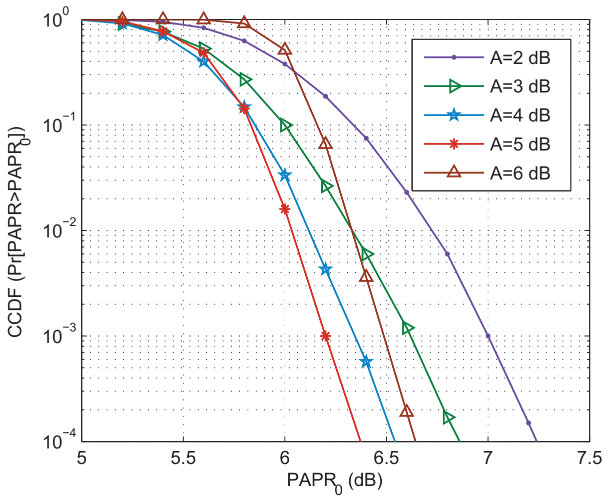

Fig. 5. CCDFs of different PAPR thresholds of CN-TI scheme with eight iterations, 16QAM, and 256 subcarriers.

performance when $A = 5$ dB.

V. Conclusion

This paper presented a novel clipping noise based TI scheme to reduce computational complexity and improve PAPR performance of OFDM signals. By projecting the clipping noise to the nearest equivalent constellation points, the proposed scheme easily determines the peak cancellation signal. Thus, the number of FFTs needed is substantially reduced. Consequently, the proposed scheme reduces the complexity dramatically while achieving a good PAPR reduction.

Appendix

Proof of Theorem 1

Suppose $\mathcal{S}$ satisfies (18). Let $\mathcal{N} = [0, 1, \ldots, N-1]$ is the subcarriers index set of OFDM and

$$\mathcal{N} = \bigcup_{i=1}^{4} \mathcal{S}_i,$$

where $\mathcal{S}_i \cap \mathcal{S}_k = \emptyset$, for $i \neq k$. Given that

$$\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2, \mathcal{S}' = \mathcal{S}_1 \cup \mathcal{S}_3,$$

where the set $\mathcal{S}$ and $\mathcal{S}'$ have the same size. Here, note that $\mathcal{S}_2$ and $\mathcal{S}_3$ are randomly selected. In the following, we show that S has a smaller maximum mean error (i.e., ε in (16)) than S'. The maximum mean error of (16) caused by $\mathcal{S}$ is given by

$$\varepsilon_{\max} = \sum_{k\in\mathcal{S}_1\cup\mathcal{S}_2} [|(2-\beta P_k)\, X_k| + |\Delta_k|] + \sum_{k\in\mathcal{S}_3\cup\mathcal{S}_4} |\beta P_k X_k|,$$

and the maximum mean error of (16) caused by $\mathcal{S}'$ is given by

$$\varepsilon'_{\max} = \sum_{k\in\mathcal{S}_1\cup\mathcal{S}_3} [|(2-\beta P_k)\, X_k| + |\Delta_k|] + \sum_{k\in\mathcal{S}_2\cup\mathcal{S}_4} |\beta P_k X_k|.$$

Note that $\mathcal{S}_2$ and $\mathcal{S}_3$ have the same size because the set $\mathcal{S}$ and $\mathcal{S}'$ have the same size. Therefore,

$$\begin{aligned}\varepsilon_{\max} - \varepsilon'_{\max} &= \sum_{k\in\mathcal{S}_2} [|(2-\beta P_k)\, X_k| + |\Delta_k|] + \sum_{k\in\mathcal{S}_3} |\beta P_k X_k| \\ &\quad - \sum_{k\in\mathcal{S}_3} [|(2-\beta P_k)\, X_k| + |\Delta_k|] - \sum_{k\in\mathcal{S}_2} |\beta P_k X_k| \\ &= \sum_{k\in\mathcal{S}_2} [|(2-\beta P_k)\, X_k| - |\beta P_k X_k|] \\ &\quad - \sum_{k\in\mathcal{S}_3} [|(2-\beta P_k)\, X_k| - |\beta P_k X_k|] \\ &= \sum_{k\in\mathcal{S}_2} Z_k - \sum_{k\in\mathcal{S}_3} Z_k \leq 0.\end{aligned}$$

Since $\mathcal{S}_2$ and $\mathcal{S}_3$ are randomly selected, and $\mathcal{S}_2 \in \mathcal{S}$, $\mathcal{S}_3 \in \mathcal{S}'$, $\mathcal{S}$ minimizes the maximum mean error $\varepsilon_{\max}$. ■

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