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Uniform Approximations for Wireless Performance in Fading Channels

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***Abstract*—We derive uniform approximations (UAs) for typical performance measures such as error probability, outage probability and capacity of wireless transmissions over flat fading channels impaired by noise. Uniform refers to the fact that these approximations are accurate over the whole range (low to high) of signal-to-noise ratio (SNR) values. First, the high-SNR results of Wang and Giannakis [2] are generalized and unified for an arbitrary performance measure. Second, we develop a Mellin-transform-based procedure to construct low- and high-SNR asymptotics of error probability or outage. Specifically, these asymptotics are related to the left- and right-sided poles of a Mellin product with respect to its fundamental strip. Third, by using multiple low-SNR terms and a single high-SNR term, UAs for the error probability of coherent modulation are constructed for Rayleigh fading, maximal-ratio-combining (MRC), selection-combining (SC), dual hop relaying, and co-channel interference. UAs are also developed for the error probability of single- and multi-channel differential modulation, the product of two Q functions, and the miss probability of energy detection. By using a single low-SNR term and multiple high-SNR terms, the outage probability UA is also developed. Finally, since the capacity measure is not an exponentially decaying function, we derive a UA for an intermediate function which is based on the moment-generating function (MGF) and ultimately, the resulting approximation for the capacity.**

***Index Terms*—Capacity, error probability, fading channels, outage probability, signal-to-noise ratio (SNR).**

I. Introduction

THE performance of wireless transmissions impaired by fading and noise has been extensively analyzed [3]–[5]. This analysis, typically requiring the averaging of the performance metric over the statistical distributions, yields closed-form expressions for the bit error rate (BER), outage, and ergodic capacity, for example. Although such results have been widely reported [3]–[8], analytical tractability and complexity can still be an issue. As an alternative to exact closed-form solutions, various high-signal-to-noise-ratio (SNR) approximations [2], [9], [10] and general approximations [11]–[14] have thus been developed.

A popular high-SNR approximation for the error probability P_E of wireless transmissions over flat fading channels impaired by Gaussian noise is [2], [3]

$$P_E \approx (G_c\rho)^{-G_d}, \tag{1}$$

where ρ is the unfaded SNR (or simply SNR), and G_c and G_d are referred to as the SNR gain and the diversity gain (diversity order), respectively. Equation (1) holds for the error rate of non-Gaussian noise [10] and for the outage probability as well. The diversity order G_d determines the absolute value of the slope of the error rate versus the SNR curve in a log-log scale at high SNR. For a specific error rate, the SNR gain G_c (referred to as coding gain in [2]) determines the reduction in the SNR as compared to the benchmark error rate curve of ρ^{-G_d}. These two parameters may even be evaluated for problems that are otherwise analytically intractable.

While G_c and G_d (eq. (1)) provide a useful high-SNR approximation, evaluation of the exact P_E requires averaging over the distribution of the instantaneous SNR $\gamma = \rho\beta$, where β is a non-negative random variable that depends on the channel gains and diversity-combining methods and has the probability density function (PDF) $f(\beta)$. In a seminal paper [2], Wang and Giannakis derived the critical parameters G_c and G_d by approximating $f(\beta)$ by a monomial for $\beta \to 0^+$. With this approach, they unified the high-SNR analysis of various wireless systems (coded, uncoded, coherent, non-coherent, differential) over different fading channels (e.g., Rayleigh, Nakagami-m). Although this approach has been widely used in recent research, the accuracy of eq. (1) predictably falls as the SNR decreases. In some cases, SNRs over 20 dB are needed for eq. (1) to be accurate. However, low-SNR operation is typical due to low power specifications or high energy-efficiency requirements, and the operating SNRs in many practical systems are often below 20 dB [15]. Thus, approximations that are accurate over the range of low to high values of SNR are highly desirable. Is it possible to develop approximations that are valid over the entire range of SNR (e.g., $0 \le \rho < \infty$)?

In this paper, we give an affirmative answer to this question by developing **uniform approximations** (UAs) for performance measures such as the BER, symbol error rate (SER), outage and capacity. The UAs simultaneously match both the low- and high-SNR series expansions of the performance measures. Not surprisingly, the UA and the conventional approximation [2] coincide in the high SNR regime.

The UA approach differs from that proposed in [13], [14], which also consider rational function approximations. These works consider the Padé approximant for the moment-generating function (MGF) of the SNR, which thus requires further treatment and analysis for computing the error and outage probabilities. In contrast, the UA is a rational approximant for the performance measure itself. Moreover, references [13], [14] perform only a one-sided match such that the rational MGF approximant is obtained by matching only the Taylor series expansion of the MGF at $s = 0$. In contrast, the UAs perform a two-sided match.

The main contributions of the paper are outlined below.

1) Initially, we generalize the high-SNR results in [2] in order to replace its case-by-case approach with a new approach applicable to any arbitrary performance measure.

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The generalized result offers an improved level of accuracy. Various BER and SER approximations of [2] can be derived under the special case of this approach. This generalization also shows a clear relationship between the SNR gain and the modulation format through the Mellin transform of the performance measure, similar to the relation in [10].

2) To compute UAs, we derive both low- and high-SNR expansions of a generalized performance measure that is a weighted sum of either a finite or an infinite number of terms. Specifically, these expansions are related to the poles in the left and right sides of the fundamental strip of a Mellin product.
3) By using multiple low-SNR terms and a single high-SNR term, UAs for the error probability of coherent modulation are constructed for Rayleigh fading, maximal-ratio-combining (MRC), selection-combining (SC), dual hop relaying, and co-channel interference. UAs are also developed for the error probability of single- and multiple-channel differential modulation, the product of two Q functions, and the miss probability of energy detection.
4) By using a single low-SNR term and multiple high-SNR terms, UA for the outage probability is also developed.
5) Since the capacity measure is not an exponentially decaying function, we derive a UA for an intermediate function which is based on the MGF and ultimately, the resulting approximation for the capacity.
6) Although we focus on the use of the Mellin transform to derive the low- and high-SNR expansions, they can also be developed via the PDF or the MGF of the channel gain. Some details and connections are briefly mentioned.

II. Preliminaries

A. Notations

$\mathbb{E}[\cdot]$, $F(s)=\int x^{s-1}f(x)dx$, and $M_\beta(s)=\mathbb{E}[e^{-s\beta}]$ denote the statistical expectation, Mellin transform and MGF, respectively. Further, $\Gamma(\cdot)$, $I_0(\cdot)$, ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$, and $M_{\kappa,\mu}(\cdot)$ denote the gamma function [16, Eq. (6.1.1)], zeroth-order modified Bessel function of the first kind [16, Eq. (9.616)], the Gauss hypergeometric function [16, Eq. (15.1.1)], and the Whittaker's function [16, Eq. (13.1.32)], respectively.

If $f(x)=\sum_{n=0}^{\infty}a_nx^n$ as $x\to 0^+$, we write $f(x)=S_N(x)+O(x^{N+1})$ as $x\to 0^+$, where $S_N(x)=\sum_{n=0}^{N}a_nx^n$, to express that the difference $|f(x)-S_N(x)|$ is smaller than $C|x^{N+1}|$ for some constant C as $x\to 0^+$. Thus, the partial sum $S_N(x)$ is an **asymptotic** (approximation) of $f(x)$ with an error term $O(x^{N+1})$ as $x\to 0^+$. Similar series of x^{-n} forms an asymptotic expansion as $x\to\infty$.

B. Basics of Mellin-transform

Let $f(x)$ be defined on the positive real axis $0\le x<\infty$. The Mellin transform of $f(x)$ on the complex plane is

$$\mathscr{M}[f(x);s]=F(s)=\int_0^\infty x^{s-1}f(x)dx, \qquad (2)$$

for some complex s [17]. $F(s)$ is holomorphic (a function that is complex differentiable in a neighborhood of every point in its domain) in a vertical strip called the fundamental strip. For example, for $\mathscr{M}[\exp(-x);s]=\Gamma(s)$, the fundamental strip is $0\le\Re(s)<\infty$. More generally, if $f(x)=O(x^{-u})$ as $x\to 0^+$ and $O(x^{-v})$ as $x\to\infty$ with $u<v$, the fundamental strip is $u\le\Re(s)<v$.

Crucially, the Mellin transform maps the asymptotic expansions of $f(x)$ at $x=0$ and ∞ to the poles of $F(s)$. In order to understand this result, suppose $f(x)$ decays rapidly as $x\to\infty$, and $f(x)=\sum_{n=0}^{N-1}a_nx^n+O(x^N)$ as $x\to 0^+$. The Mellin transform is then given by

$$\begin{aligned}F(s)&=\int_0^1 x^{s-1}\left(f(x)-\sum_{n=0}^{N-1}a_nx^n\right)dx+\int_0^1 x^{s-1}\sum_{n=0}^{N-1}a_nx^ndx\\&\quad+\int_1^\infty x^{s-1}f(x)dx\\&=\int_0^1 x^{s-1}\left(f(x)-\sum_{n=0}^{N-1}a_nx^n\right)dx+\sum_{n=0}^{N-1}\frac{a_n}{s+n}\\&\quad+\int_1^\infty x^{s-1}f(x)dx.\end{aligned}$$

The first integral converges in the larger half-plane $\Re(s)>-N$ and the second for all complex s. Thus, we see that $F(s)$ is singularity-free for all $\Re(s)>0$ with simple poles of residue a_n at $s=-n$ $(n=0,\ldots,N-1)$ and no other singularities. Thus, the monomial assumption (AS2) made by [2] is equivalent to a simple pole of residue a at $s=-t$, which also reveals the polynomial growth rate of $f(x)$ at 0.

C. Assumptions

The following assumptions where AS1 and AS2 are consistent with those in [2] are stated here for completeness.

AS1) The instantaneous SNR at the receiver is given by $\gamma=\rho\beta$, where ρ is the unfaded link SNR (aka the transmit SNR), or simply the SNR, and a nonnegative random variable, β depends on the channel gains and the diversity-combining techniques.

AS2) Unless otherwise stated, as in [2], the PDF of β can be approximated with a monomial term as $f(\beta)=a\beta^t+O(\beta^{t+1})$ as $\beta\to 0^+$, where $a>0$. For example, if $f(\beta)=e^{-\beta},\quad \beta\ge 0$, then $t=0$ and $a=1$. As developed in Proposition 2, the parameters a and t are also given by the first left pole of the Mellin transform $F(s)$ of the PDF $f(\beta)$. Thus, when $f(\beta)$ is not completely known, the MGF of β or the Mellin transform $F(s)$ furnishes an alternative route. The details of the former option are given in [2] and [11]. The latter option is developed in Proposition 2.

AS3) The system performance metric is denoted by $h(x)$, which is a conditional error probability, outage probability or capacity. Unless dealing with capacity, we can safely assume that $h(x)$ decays exponentially as $x\to\infty$ and admits a polynomial expansion as $x\to 0^+$. From Section II-B, this assumption implies that the Mellin transform $H(s)$ has poles in the left half-plane only. This phenomenon shows up in common $h(x)$ including $Q(\sqrt{\kappa x})=\int_{\sqrt{\kappa x}}^{\infty}\frac{1}{2\pi}e^{-t^2/2}dt$, which represents the BER or SER of various coherent digital modulation and demodulation schemes. Similarly, $h(x)=pe^{-qx}$ or the exponential sum or integral represents the BER or SER of noncoherent demodulation schemes [3], [5].

III. Approximations for Arbitrary Performance Measure

In this section, we lay the groundwork necessary to develop the UAs. First, a general error probability $P_e(\rho)$ and its low- and high-SNR asymptotics are introduced. Second, in Proposition 1, the result of [2] is generalized for an arbitrary conditional error expression. Third, in Proposition 2, the low- and high-SNR asymptotics of the error probability are derived from the poles of $H(s)$ and $F(s)$. These poles also yield diversity and SNR gains immediately.

A. Error Probability

We first consider a general expression for the error probability expressed as

$$P_e(\rho) = \int_0^\infty h(\rho\beta) f(\beta) d\beta, \tag{3}$$

where $h(x)$ represents a conditional error expression that requires averaging over noise, fading and other effects. Extensions to outage and capacity will be provided in Sections V and VI.

We need low-SNR ($\rho \to 0^+$) and high-SNR ($\rho \to \infty$) asymptotics of $P_e(\rho)$. At high SNR, generically $P_e(\rho) = \sum a_n \rho^{-b_n}$, where a_n and b_n are real number sequences, with b_n positive and increasing. The first term of this series dominates and is sufficient to develop UAs. Of course, [2] relates this first term to the monomial expansion (AS2). At low SNR, generically $P_e(\rho) = \sum a'_n \rho^{c_n}$, where a'_n and c_n are real number sequences, with c_n positive and increasing. For coherent modulations, c_n is not an integer sequence because the conditional error is a function of $\sqrt{\rho}$, whereas for non-coherent and differential modulations, c_n is an integer sequence. To treat all such cases in a unified way, we must use the following definition.

Definition 1. The low- and high-SNR asymptotics of $P_e(\rho)$ (eq. (3)) may be given as

$$P_e(\rho) = \begin{cases} \sum_{l=0}^{M_1} c(l) x^l + O(x^{M_1+1}) & \text{as} \quad x \to 0^+ \\ \sum_{l=0}^{M_2} \frac{b(l)}{x^{\delta+l}} + O(x^{-(\delta+M_2+1)}) & \text{as} \quad x \to \infty, \end{cases} \tag{4}$$

where $\gamma = \rho\beta$ as per AS1, and $x = \rho^\tau$ for a positive real number τ such that both expansions have integer powers of x, whereas the series in terms of ρ may not necessarily have integer powers.

All error-probability cases involve either $x = \rho^{1/2}$ or $x = \rho$. The former is needed for coherent schemes and the latter for non-coherent (differential) schemes. Thus, the rationale behind the substitution $x = \rho^\tau$ is the need for the unified treatment of all such cases. We will show under Proposition 2 that δ is related to the diversity order of the system.

B. A general high-SNR approximation

Before presenting our generalization, we list the high-SNR approximation of eq. (3) derived in [2] when $h(x) = Q(\sqrt{\kappa x})$, where $\kappa > 0$ for quick reference:

$$\begin{aligned} P_e(\rho) &= \mathbb{E}[Q(\sqrt{\kappa\gamma})] \\ &= \frac{2^t a \Gamma(t+\frac{3}{2})}{\sqrt{\pi}(t+1)} \frac{1}{(\kappa\rho)^{t+1}} + O\left(\rho^{-(t+2)}\right) \text{ as } \rho \to \infty. \end{aligned} \tag{5}$$

Equation (5) reveals that the diversity order G_d is $t+1$, which directly relates to the growth rate of the PDF $f(\beta)$ (AS2). This result is observed because the exponential decay rate of $Q\left(\sqrt{\kappa\rho\beta}\right)$ as $\rho \to \infty$ ensures that the main contribution to the integral eq. (3) comes from the neighborhood of $\beta = 0$. Thus, the monomial $a\beta^t$ (AS2) determines both the diversity and SNR gains.

Note that eq. (5) holds for coherent modulations such as binary phase shift keying (BPSK) and frequency shift keying (FSK). Other conditional error expressions $h(x)$ involve linear combinations and/or powers of the Q function, and weighted exponentials like $x^p \exp(-qx)$, where $p, q > 0$. Since a case-by-case treatment of all those is tedious, a single, unified expression applicable to any modulation format is highly useful. Such a generalized result is developed next.

Proposition 1. *If the PDF of the channel gain is given by* $f(\beta) \approx \beta^t g(\beta)$ *as* $\beta \to 0^+$ *with* $g(0) \neq 0$, *the error probability* $P_e(\rho)$ *eq.* (3) *can be approximated as*

$$P_e(\rho) = \frac{H(t+1)}{\rho^{t+1}} g\left(\frac{H(t+2)}{\rho H(t+1)}\right) + O(\rho^{-(t+3)}) \quad \text{as} \quad \rho \to \infty, \tag{6}$$

where $H(s)$ *is the Mellin transform of* $h(x)$.

The proof is given in the Appendix.

Remark: $f(\beta) \approx \beta^t g(\beta)$ as $\beta \to 0^+$ implies that $g(\beta)$ is a function with the asymptotic expansion $\sum_{k=0}^{M} a(k)\beta^k + O(\beta^{M+1})$ as $\beta \to 0^+$ such that the first $M+1$ coefficients match with those of the expansion of $f(\beta)$ as $\beta \to 0^+$. The error term associated with this PDF approximation is $O(\beta^{t+M+1})$. For example, the exact PDF of the dual-branch SC system in independent and identically distributed (iid) Rayleigh fading, given by $f(\beta) = 2(1-e^{-\beta})e^{-\beta}$, $\beta > 0$, can be approximated as (i) $f(\beta) = 2\beta e^{-1.5\beta} + O(\beta^3)$ as $\beta \to 0^+$, (ii) $f(\beta) = \beta\frac{18-13\beta}{9+7\beta} + O(\beta^3)$ as $\beta \to 0^+$.

As an application of eq. (6), we consider an N_r-branch MRC system in iid Rayleigh fading. The PDF of β takes the form $f(\beta) = \beta^{N_r-1} g(\beta)$, where $g(x) = \frac{e^{-x}}{(N_r-1)!}$. By using $H(s)$ from Table I, the BER of the coherent BPSK may be obtained as

$$P_e(\rho) = \frac{\Gamma(N_r+\frac{1}{2})}{2\sqrt{\pi}N_r!\rho^{N_r}} e^{-\frac{N_r(N_r+1/2)}{(N_r+1)\rho}} + O(\rho^{-(t+3)}). \tag{7}$$

The exact BER of this system is given by [5, eq. (9.6)]. Figure 1 compares the accuracy of the BER approximation obtained in eq. (7) with that of the classical high-SNR result, eq. (5), where $\kappa = 2$, $a = 1/(N_r-1)!$ and $t = N_r - 1$. The relative errors computed as $|exact - approximate|/exact$ for $N_r = 4$ are plotted. We can observe that eq. (7) is at least an order of magnitude better than eq. (5) in terms of the relative error.

Note that a special case of Proposition 1 occurs when $g(\beta) = a$, that is, when the PDF model that appears in AS2 is used. (This case was treated in [2]). The approximation of the error probability $P_e(\rho)$ (eq. (3)) as $\rho \to \infty$ can then be expressed as

$$\begin{aligned} P_e(\rho) &= \frac{aH(t+1)}{\rho^{t+1}} + O(\rho^{-(t+2)}) \\ &= \left((aH(t+1))^{-1/(t+1)}\rho\right)^{-(t+1)} + O(\rho^{-(t+2)}). \end{aligned} \tag{8}$$

TABLE I
MELLIN TRANSFORMS OF $h(x)$

Application	$h(x)$	$H(s)$
Coherent BPSK	$Q(\sqrt{2x})$	$\frac{\Gamma(s+1/2)}{2s\sqrt{\pi}}$
Non-coherent FSK	$\frac{1}{2}e^{-\frac{x}{2}}$	$2^{s-1}\Gamma(s)$
DPSK	$\frac{1}{2}e^{-x}$	$\frac{1}{2}\Gamma(s)$
Coherent FSK	$Q(\sqrt{x})$	$\frac{2^{s-1}\Gamma(s+1/2)}{s\sqrt{\pi}}$
Energy detection	$1-Q_u(\sqrt{2x},\sqrt{\lambda})$	$\frac{e^{-\lambda/2}\Gamma(s)}{2\pi}\oint_{\triangle}\frac{e^{\lambda z/2}}{z^{u-s-1}(z-1)^{s+1}}dz$
Outage probability	$u(\gamma_T-x)$	$\frac{\gamma_T^s}{s}$
Non-coherent M-FSK	$\frac{1}{2(M-1)}\sum_{m=1}^{M-1}(-1)^{m+1}\binom{M}{m+1}e^{-\frac{mx}{m+1}}$	$\frac{\Gamma(s)}{2(M-1)}\sum_{m=1}^{M-1}(-1)^{m+1}\binom{M}{m+1}(1+\frac{1}{m})^s$

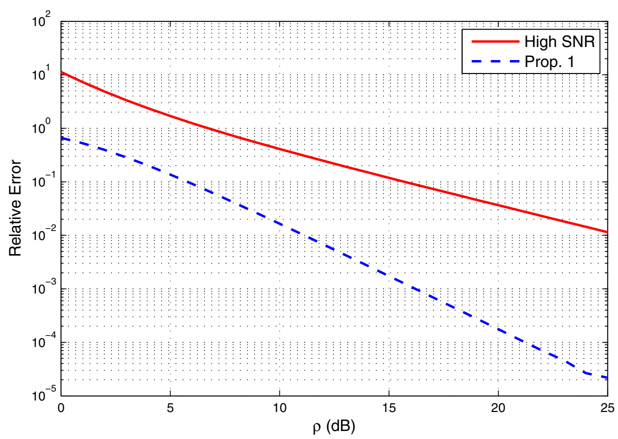

Fig. 1. The relative errors eq. (5) and eq. (7) as a function of the SNR. An MRC system with $N_r = 4$ is considered.

Comparing eq. (8) with the traditional high-SNR approximation eq. (1) shows that the SNR gain depends on $H(s)$, whereas the diversity gain strictly depends on the order of the monomial $a\beta^t$.

C. Low- and high-SNR asymptotics from Mellin Transforms

Mellin transforms are a natural tool for the study of asymptotics. To this end, we consider the Mellin transform of a general $h(x)$ expressed as a sum of either a finite or an infinite number of terms derived from a common basis function. For example, with basis function $Q(\sqrt{x})$, we consider $h(x) = \sum a_k Q(\sqrt{b_k x})$, which is powerful enough to cover all coherent linear modulations, union bound on block and convolutional coded systems and others. With suitable basis functions, differential and non-coherent modulation can also be treated. With this generalized $h(x)$, we next show how the low- and high-SNR asymptotics of $P_e(\rho)$ can be obtained from the left- and right-sided poles of a Mellin transform product with respect to its fundamental strip.

Proposition 2. *Consider a generalized conditional error probability given by the sum*

$$h(x) = \sum_k \lambda_k g(\mu_k x), \tag{9}$$

where $g(x)$ is a general base function. If $G(s)$ and $F(s)$ are the Mellin transforms of $g(x)$ and the PDF $f(\beta)$, respectively, let $\Lambda(s) = G(s)F(1-s)\sum_k \lambda_k \mu_k^{-s}$ has an increasing sequence of right-sided poles $\tilde{p}_0, \tilde{p}_1, \ldots$ and a decreasing sequence of left-sided poles $\tilde{q}_0, \tilde{q}_1, \ldots$ with respect to the fundamental strip. All the poles are assumed to be first-order ones. Then, the low- and high-SNR asymptotics of $P_e(\rho)$ can be expressed as eq. (4), where $\delta = \tilde{p}_0/\tau$ and the non-zero coefficients are given by

$$\left\{\begin{array}{c} c\left(\frac{|\tilde{q}_l|}{\tau}\right) \\ b\left(\frac{(\tilde{p}_l-\tilde{p}_0)}{\tau}\right)\end{array}\right\} = \left\{\begin{array}{l} \lim\limits_{s\to\tilde{q}_l}\left[(s-\tilde{q}_l)G(s)F(1-s)\sum_k \lambda_k\mu_k^{-s}\right] \\ \lim\limits_{s\to\tilde{p}_l}\left[(\tilde{p}_l-s)G(s)F(1-s)\sum_k \lambda_k\mu_k^{-s}\right]\end{array}\right\}. \tag{10}$$

Otherwise, the coefficients are zero.

See Appendix for the proof. Several examples of eq. (10) will be furnished later.

Remarks:

1) Since $H(s) = G(s)\sum\lambda_k\mu_k^{-s}$, the term $\sum\lambda_k\mu_k^{-s}$ does not have any poles, and the poles of $H(s)$ are simply those of the base function. For example, consider an error bound for a digital modulation given by $h(x) = \sum a_k Q(\sqrt{b_k x})$, where a_k and b_k represent the number of the nearest neighbors and their distances. The poles of $H(s)$ are then from the Mellin transform of $Q(\sqrt{x})$ and are not dependent on the a_k and b_k values. The poles are thus contributed by $G(s)$ and $F(1-s)$. These in general yield left-sided and right-sided poles, respectively, which in turn determine the low- and high-SNR performance.
2) The first right-sided pole $\tilde{p}_0$ makes the dominant contribution to high-SNR performance. Thus, the high-SNR error probability may be described by

$$G_d = \tilde{p}_0,$$
$$G_c = \lim_{s\to\tilde{p}_0}\left[(\tilde{p}_0-s)G(s)F(1-s)\sum_k\lambda_k\mu_k^{-s}\right]^{-1/\tilde{p}_o}. \tag{11}$$

As per AS3, $H(s)$ contributes left-sided poles only. The diversity order is thus given by the first right-sided pole of $F(1-s)$.
3) The result in eq. (11) encompasses eq. (5) (the problem treated in [2]). In order to understand this point, note that the monomial $f(\beta) \approx a\beta^t$ corresponds to $F(1-s)$ having a pole at $s = 1+t$. Thus, [2] is derived from $f(\beta)$, while our result is based on $F(s)$.

To sum up, Proposition 2 states that the positive poles of $F(1-s)$ describe the high-SNR expansion whereas the negative poles of $F(1-s)$ together with those of $H(s)$ describe

TABLE II
MELLIN TRANSFORMS OF β

Fading	$f(\beta)$	$F(s)$
Rayleigh	$e^{-\beta}$	$\Gamma(s)$
Nakagami-m	$\frac{m^m \beta^{m-1} e^{-m\beta}}{\Gamma(m)}$	$\frac{m^{1-s}\Gamma(s+m-1)}{\Gamma(m)}$
Rician	$(1+K)e^{-(K+(1+K)\beta)} I_0\left(2\sqrt{K(1+K)\beta}\right)$	$\frac{\Gamma(s)e^{K/2}}{\sqrt{K}(1+K)^s} M_{\frac{1}{2}-s,0}(K)$
Hoyt	$ae^{-a^2\beta} I_0(b\beta)$	$\frac{2^{s-1}\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{s+1}{2}\right)}{\sqrt{a^{2s-1}\pi}}\ {}_2F_1\left(\frac{s+1}{2}, \frac{s}{2}; 1; \frac{b^2}{a^4}\right)$
Weibull	$\frac{b\beta^{\frac{b}{2}-1}}{2a^{\frac{b}{2}}} e^{-\left(\frac{\beta}{a}\right)^{\frac{b}{2}}}$	$a^{s-1}\Gamma\left(\frac{2s+b-2}{b}\right)$
MRC(Rayleigh)	$\frac{\beta^{N_r-1}e^{-\beta}}{(N_r-1)!}$	$\frac{\Gamma(N_r+s-1)}{(N_r-1)!}$
SC(Rayleigh)	$N_r(1-e^{-\beta})^{N_r-1}e^{-\beta}$	$N_r \sum_{l=0}^{N_r-1} \frac{(-1)^l \binom{N_r-1}{l}\Gamma(s)}{(l+1)^s}$

the low-SNR expansion. The first positive pole of $F(1-s)$ gives the diversity order. Tables I and II give $H(s)$ and $F(s)$ for several common cases.

Low- and high-SNR expansions form the basis for the UA, which will be developed subsequently. Specifically, to develop the error probability UA, only $b(0)$ and δ from high-SNR series and $c(l)$ for $l = 0, 1, 2, \ldots, M_1$ from low-SNR series are needed. As mentioned above, the first high-SNR term is determined by the first positive pole $\tilde{p}_0$ of $F(1-s)$; thus,

$$b(0) = aH(\tilde{p}_0), \tag{12}$$

where $a = \lim_{s\to\tilde{p}_0}[(\tilde{p}_0 - s)F(1-s)]$ is the residue of $F(1-s)$ at $s = \tilde{p}_0$ and is also the coefficient of the monomial approximation to $f(\beta)$ considered in [2] (AS2). As per this monomial, $\tilde{p}_0$ is equivalently $t+1$. Note that the high-SNR approximation in [2] uses two parameters a and t. As $b(0)$ and δ are closely related to a and t, either set of parameters may thus be used to develop UAs.

For channel fading with $F(1-s)$ having positive poles only (this condition is satisfied for most of the popular fading models and diversity-combining systems), the low-SNR coefficients $c(l), l = 0, 1, 2, \ldots$ given by the first equality in eq. (10) can be shown to depend directly on the moments of β. This point will be verified by the subsequent examples of UAs.

Although we focus on the use of Mellin transform to obtain these low- and high-SNR coefficients, they can also developed via the PDF or the MGF of β. We briefly comment on these alternative approaches below.

1) **PDF**: if one follows [2], a and t are given by monomial expansion of $f(\beta)$ near 0. Similarly, by using the PDF $f(\beta)$, the moments $\mu_n = \int_0^\infty \beta^n f(\beta)d\beta$ can be easily obtained.
2) **MGF**: as MGF is readily available in some problems (e.g., the case of MRC), it is fairly simple to extract a and t by the monomial expansion of $M_\beta(s)$ near $s = \infty$ [2]. Fortunately, the fractional moments are also simple to compute from the MGF [18]. Consider an N_r-branch MRC system in iid Rayleigh fading as an example. The MGF of β is $M_\beta(s) = (1+s)^{-N_r}$, which can be expanded for $s\to\infty$ as $M_\beta(s) = s^{-N_r} + O\left(s^{-(N_r+1)}\right)$, and hence, a and t are obtained to be $1/\Gamma(N_r)$ and $N_r - 1$, respectively [2].

The fractional moments of β, $\mu_{l/2}$, $l = 1, 3, 5, \ldots$ can be computed by using [18] as

$$\mu_{l/2} = \mathbb{E}[\beta^{l/2}] = \Gamma(\lambda)^{-1}\int_0^\infty t^{\lambda-1}\zeta(-t)dt,$$

where λ is chosen to be $1/2$ such that $n = l/2 + \lambda$ is a positive integer while satisfying $0 < \lambda < 1$; $\zeta(s) = \frac{d^n M_\beta(s)}{ds^n}$.

By using the above equation, $\mu_{l/2}$ can be obtained as

$$\begin{aligned}\mu_{l/2} &= \frac{\Gamma(N_r+n)}{\Gamma(1/2)\Gamma(N_r)}\int_0^\infty t^{-1/2}(1+t)^{-(N_r+n)}dt \\ &= \frac{\Gamma(N_r + l/2)}{\Gamma(N_r)},\end{aligned}$$

where the last equality is obtained by using [19, eq. (3.191.3)].

IV. AVERAGE PROBABILITY OF ERROR

Proposition 3 is our main result for the error probability UA. We use the asymptotics of eq. (4) to develop the UA; however, only the first term of the high-SNR series and several low-SNR terms are utilized. These terms feed into a linear equation to yield the coefficients of the numerator and the denominator of the UA. To illustrate this process, UAs for the error probability of coherent modulation for diversity-combining systems, relay channels, and co-channel interference are derived. The UAs for the error probability of non-coherent or differential detection schemes, the product of two Q functions, and the miss probability of energy detection are also derived.

A UA is a rational function that matches with both the low- and high-SNR asymptotics simultaneously [20]. That is, if the UA is expanded into two series of ρ^{-n} and ρ^k, then those expansions will match the appropriate terms of eq. (4).

Definition 2. A rational function $r(x)$ is given by

$$r(x) = \frac{p_0 + p_1 x + p_2 x^2 + \ldots + p_L x^L}{q_0 + q_1 x + q_2 x^+ \ldots + q_K x^K}, \tag{13}$$

where L and K are are the degrees of the numerator and the denominator, respectively. To fit this rational function into $P_e(\rho)$ eq. (3), the coefficients $p_0, p_1, \ldots p_L$ and $q_1, q_2, \ldots, q_K$ are determined from the asymptotics in eq. (4). The values of L and K depend on the number of the low- and high-SNR terms used to construct $r(x)$. If $L < K$, the rational function is called proper, and this is the case in our applications.

Proposition 3. *The UA for the error probability $P_e(\rho)$ eq. (3) is given by*

$$P_e(\rho) = r(x) + \varepsilon(x), \tag{14}$$

where $r(x)$ is a rational function defined in eq. (13), $x = \rho^\tau$, $K = L+\delta$, $L \geq 2$ is an integer, $p_0 = 1$ and $q_0 = 1/c(0)$. The denominator coefficient vector $\mathbf{q} = (q_1, q_2, \ldots, q_K)^T$ is given by

$$\mathbf{q} = -1/c(0)\mathbf{W}^{-1}\left(\tilde{c}(1) \quad \tilde{c}(2) \quad \ldots \quad \tilde{c}(K-1) \quad \tilde{c}(K)\right)^T, \tag{15}$$

where $\tilde{c}(l) = c(l+L-2)$ and $\mathbf{W} = \{w_{ij}\}$, $i = 1,\ldots,K$, $j = 1,\ldots,K$ with

$$w_{ij} = \begin{cases} \tilde{c}(i-j) - b(0) & i = 1,2; j = i+K-2 \\ \tilde{c}(i-j) & \text{otherwise}. \end{cases} \tag{16}$$

The numerator coefficient vector $\mathbf{p} = (p_1, p_2, \ldots, p_L)^T$ is given by

$$p_i = \begin{cases} c(i)/c(0) + \sum_{k=1}^{i} q_k c(i-k) & i = 1,\ldots L-2 \\ b(0)\, q_{K-j} & j = 0,1; i = L-j. \end{cases} \tag{17}$$

The error term $\varepsilon(x)$ of eq. (14) is $O\left(x^{K+L-1}\right)$ as $x \to 0^+$ (low-SNR) and $O\left(x^{-(\delta+1)}\right)$ as $x \to \infty$ (high-SNR).

Proposition 3 is a general result for error probability, applicable to a variety of modulation schemes, fading channels and interference. The basic inputs required are τ, several coefficients $(c(l), l = 0,1,2,\ldots)$ of the low-SNR series and the first coefficient $b(0)$ and δ of the the high-SNR series. The extraction of these inputs for the construction of several UAs is demonstrated next.

A. Coherent modulations

1) Single-branch BPSK in Rayleigh fading: We first show how Proposition 2 helps to get the low- and high-SNR expansions of $P_e(\rho)$ (eq. (3)) for coherent modulations. For the BERs of the coherent BPSK and the coherently detected binary FSK modulations (Table I), we have generic $h(x) = Q\left(\sqrt{\kappa x}\right)$. Table I reveals that $H(s)$ has simple poles at $s = 0, -1/2, -3/2, \ldots$. These poles together with the negative poles of $F(1-s)$ describe the low-SNR expansion of $P_e(\rho)$. However, for common fading models, $F(1-s)$ has a sequence of positive poles $\tilde{p}_0, \tilde{p}_1, \ldots$ only, which describe the high-SNR $P_e(\rho)$. The non-integer poles of $H(s)$ indicate that the low-SNR expansion involves $\sqrt{\rho}$. We thus substitute $x = \sqrt{\rho}$ with $\tau = 1/2$. Then, $\delta = 2\tilde{p}_0$, and the necessary coefficients computed according to proposition 2, are as follows:

$$b(0) = a\frac{2^{\tilde{p}_0-1}\Gamma(\tilde{p}_0+1/2)}{\tilde{p}_0\sqrt{\pi}\kappa^{\tilde{p}_0}}$$

$$c(l) = \begin{cases} \frac{1}{2} & l = 0 \\ \dfrac{(-1)^{(l+1)/2}(\kappa/2)^{l/2}F(l/2+1)}{\sqrt{\pi}\, l\,\Gamma[(l+1)/2]} & l = 1,3,\ldots \\ 0 & \text{otherwise}. \end{cases} \tag{18}$$

From the definition of Mellin transform eq. (2), $F(n+1) = \mathbb{E}[\beta^n] = \mu_n$ is the n-th moment of β. Note that $c(l)$ can also be obtained by using the series expansion of $Q\left(\sqrt{\kappa\rho\beta}\right)$ [19] and averaging over $f(\beta)$.

The UA for the error probability of the BPSK ($\kappa = 2$) modulation in Rayleigh fading can now be readily computed by using eq. (14). The basic inputs required are a, $\tilde{p}_0$ and several moments μ_n. For single-branch Rayleigh fading, $F(1-s)$ (Table II) has the first right-sided pole at $\tilde{p}_0 = 1$ of residue $a = 1$. The required moments can be obtained from $F(s)$ as $\mu_n = \Gamma(n+1)$.

We now present a few additional examples of error probability UAs (Proposition 3) for coherent modulations. For this purpose, examples of diversity combining, relay communications and co-channel interference are used.

2) Performance in N_r-branch MRC in iid Rayleigh fading: MRC is an optimal diversity-combining method when no interference is present [5, Chap. 9]. Its performance has been analyzed extensively; for example, the BER of the coherent BPSK modulation with MRC under Rayleigh fading is given in [3, Sec. 14.4].

The Mellin transform of β is given in Table II. We observe that $F(1-s)$ has simple right-sided poles at $s = N_r, N_r+1, \ldots$. As per Proposition 2, these poles describe the high-SNR $P_e(\rho)$. Since the first pole is at $s = N_r$, the diversity order is N_r. The required low- and high-SNR coefficients for the computation of the UA eq. (14) can be obtained by using eq. (18), where $\tilde{p}_0 = N_r$, $a = 1/\Gamma(N_r)$, and the moments $\mu_n = F(n+1) = \Gamma(N_r+n)/\Gamma(N_r)$. For the single-branch ($N_r = 1$) case, the following simple UA can be obtained for the coherent BPSK modulation:

$$P_e(\rho) = \frac{1+x+0.5x^2}{2+4x+5x^2+4x^3+2x^4} + \varepsilon(x), \tag{19}$$

where $x = \sqrt{\rho}$. The exact error rate for this case is well-known [3, Sec. 14.4]. Note that the UA eq. (19) matches the first three non-zero low-SNR terms and the first high-SNR term. Similar UAs for any other N_r can be readily derived and are omitted for brevity. To test their accuracy, the BER UAs for the BPSK modulation when $N_r = 1,2,4$ are plotted along with the exact result [3, Sec. 14.4] and the conventional high-SNR result (eq. (5)) in Figure 2. Notice that the UA coincides with the exact BER for the entire range $-10 \leq \rho < 30$ dB, while the high-SNR result, eq. (5), fails as the SNR decreases. Clearly, the UA provides an excellent approximation over the whole range of the SNR.

3) Performance in N_r-branch SC in iid Rayleigh fading: SC, a classical diversity-combining technique, has less implementation complexity than MRC, but suffers a relative loss in SNR gain. From Table II, the Mellin transform can be written as

$$F(1-s) = \left[N_r \sum_{n=0}^{N_r-1} (-1)^n \binom{N_r-1}{n} \frac{1}{(n+1)^{1-s}}\right]\Gamma(1-s)$$
$$= \nu(1-s)\Gamma(1-s). \tag{20}$$

Given $\Gamma(1-s)$ has poles at $s = 1,2,3,\ldots$, if these are the poles of $F(1-s)$, then the diversity order of the system would be just one, which is incorrect. Surprisingly, the zeros of $\nu(1-s)$ at $s = 1,2,\ldots,N_r-1$, exactly cancel out the first $N_r - 1$ poles of $\Gamma(1-s)$. Therefore, the right-sided simple poles are at $s = N_r, N_r+1, \ldots$ and they describe the high-SNR approximation for $P_e(\rho)$ (eq. (3)). Due to the first pole at $s = N_r$, the diversity order of the system is N_r. The error probability UA eq. (14)

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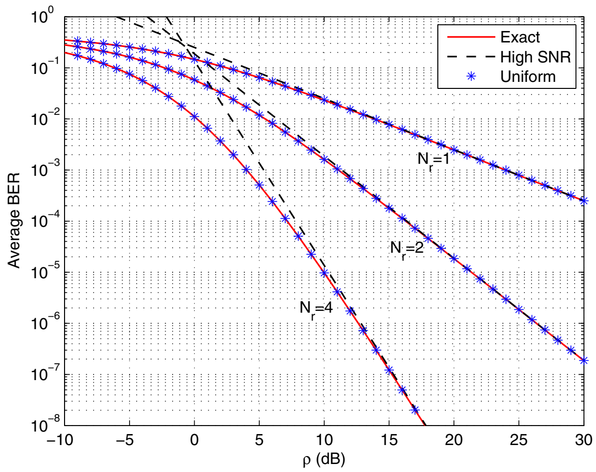

Fig. 2. The exact BER of an N_r-branch MRC system, the high-SNR approximation eq. (5) and the UA eq. (14). In the UA, $L = 2$.

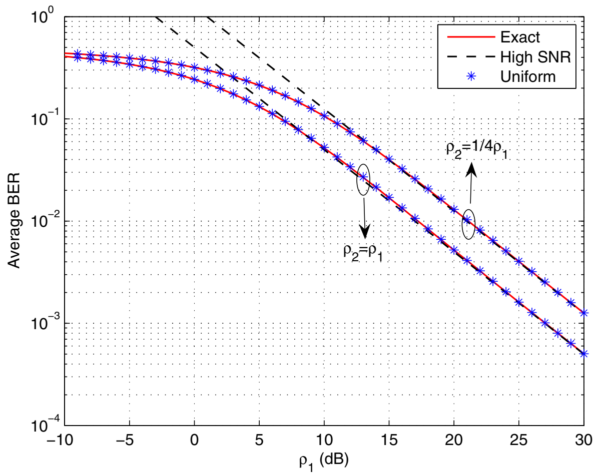

Fig. 3. The BER of a dual-hop relay system in Rayleigh fading.

for general coherent modulation can now be directly computed with $b(0)$ and $c(l)$ given by eq. (18), where $\tilde{p}_0 = N_r$, $a = (-1)^{N_r-1}\nu(-N_r+1)/\Gamma(N_r)$ and the moments $\mu_n = F(n+1) = \nu(n+1)\Gamma(n+1)$.

4) Dual Hop relay Performance: Figure 3 shows the BPSK error performance of a dual hop relay system over Rayleigh fading. The exact curve is plotted by using the results from [21] and the UA by using Proposition 3 with $L = 2$ and the required coefficients of series expansions obtained from eq. (18). The high accuracy of UA in the entire SNR range is visible in the figure. The required parameters a, $\tilde{p}_0$ and moments μ_n in eq. (18) are computed as follows. By using [21, eq. (27)], the first-order expansion of the cumulative distribution function (CDF) of the end-to-end SNR, $F_{\gamma_{eq}}(\gamma)$ can be easily shown to be

$$F_{\gamma_{eq}}(\gamma) = \left(1+\frac{1}{\zeta}\right)\frac{\gamma}{\rho_1} + O\left(\frac{\gamma}{\rho_1}\right), \quad (21)$$

where $\zeta = \rho_2/\rho_1$ ρ_1, and ρ_2 are the average SNRs of the first and second hop, respectively. Let $\gamma = \rho_1\beta$, the coefficient and the order of the monomial $a\beta^t$ (AS2) are thus obtained as $a = (1+\frac{1}{\zeta})$ and $t = 0$, which are also our required parameters a and $\tilde{p}_0$ in eq. (18) ($\tilde{p}_0 = t+1 = 1$). The required moments are then computed as

$$\begin{aligned}\mu_n &= \int_0^\infty n\beta^{n-1}\left(1 - F_{\frac{\gamma_{eq}}{\rho_1}}(\beta)\right)d\beta \\ &= \frac{2n\sqrt{\pi}}{\sqrt{\zeta}}\frac{2B}{(A+B)^{n+2}}\frac{\Gamma(n+2)\Gamma(n)}{\Gamma(n+3/2)} \\ &\times {}_2F_1\left(n+2,\frac{3}{2};n+\frac{3}{2};\frac{A-B}{A+B}\right),\end{aligned} \quad (22)$$

where $A = 1+\frac{1}{\zeta}$, $B = 2\sqrt{\frac{1}{\zeta}}$. The final result for the moment is obtained by substituting [21, eq. (27)] into the integral expression and solving the resultant integral by using [19, eq. (6.621.3)].

5) Performance in co-channel interference: In cellular wireless systems, co-channel interference, rather than noise, limits the performance [22]–[24]. The performance of these systems is analyzed based on the signal-to-interference-and-noise ratio (SINR) which may be defined as

$$\gamma = \frac{\Omega_0 X_0}{\sum_{l=0}^{N_I}\Omega_l X_l + \sigma^2}, \quad (23)$$

where X_0 and $X_l, l = 1,2,\ldots,N_I$ are the channel gains of the desired and N_I interference links, respectively; Ω_0 and $\Omega_l, l = 1,2,\ldots,N_I$ are the average received powers of the desired signal and N_I interfering signals, respectively; and σ^2 is the additive white Gaussian noise power. While eq. (23) can represent a multitude of co-channel scenarios, for the sake of brevity we limit ourselves to the following case: (a) the noise power is negligible and (b) all interferers have identical powers, i.e., $\Omega_l = \Omega_I, (l = 1,2,\ldots,N_I)$. Then γ can be expressed as $\gamma = \rho\beta$, where $\rho = \Omega_0/\Omega_I$, and $\beta = X_0/Z$, $Z = \sum_{l=1}^{N_I} X_l$. The Mellin transforms then relate as

$$F_\beta(s) = F_{X_0}(s)F_Z(2-s).$$

By using $F_\beta(s)$ and $H(s)$, Proposition 2 provides the required asymptotics of $P_e(\rho)$ and finally, Proposition 3 yields the desired UA.

As an example, consider an N_r-branch MRC system in the presence of N_I co-channel interferers. For all signals with Rayleigh fading, X_0 and Z are central chi square with $2N_r$ and $2N_I$ degrees of freedom, respectively [24], and their Mellin transforms are $F_{X_0}(s) = \Gamma(s+N_r-1)/\Gamma(N_r)$ and $F_Z(s) = \Gamma(s+N_I-1)/\Gamma(N_I)$. Thus, we find

$$F_\beta(s) = \frac{\Gamma(s+N_r-1)\Gamma(1+N_I-s)}{\Gamma(N_r)\Gamma(N_I)}.$$

We can observe that $F_\beta(1-s)$ has positive poles at $s = N_r, N_r+1,\ldots$ and negative poles at $s = -N_I, -(N_I+1),\ldots$. Note that $F_\beta(1-s)$ in this case has both positive and negative poles, whereas in the former cases, $F_\beta(1-s)$ has only positive poles, which determine the high-SNR behavior only. As the first positive pole is at $s = N_r$, the diversity order is N_r. The negative poles of $F_\beta(1-s)$ together with those of $H(s)$ thus describe the low-SNR error probability. For coherent modulation ($h(x) = Q(\sqrt{\kappa x})$), the low-SNR expansion includes the powers of $\sqrt{\rho}$ and thus, $\tau = 1/2$. Accordingly, from eq. (10), we obtain the following coefficients:

$$b(0)=\frac{2^{Nr-1}\Gamma(N_I+N_r)\Gamma(N_r+1/2)}{\sqrt{\pi}N_r\kappa^{N_r}\Gamma(N_r)\Gamma(N_I)}$$

$$c(l)=\begin{cases}\frac{1}{2} & l=0\\ \frac{(-1)^{(l+1)/2}(\kappa/2)^{l/2}}{\sqrt{\pi}\,l\,\Gamma[(l+1)/2]}\times\frac{\Gamma(l/2+N_r)\Gamma(N_I-l/2)}{\Gamma(N_r)\Gamma(N_I)} & l=1,3,\ldots\\ \frac{(-1)^{(l+2-2N_I)/2}(\kappa/2)^{l/2}}{\sqrt{\pi}\,l\,\Gamma[(l+2-2N_I)/2]}\times\frac{\Gamma(l/2+N_r)\Gamma((1-l)/2)}{\Gamma(N_r)\Gamma(N_I)} & l=2N_I,2(N_I+1),\ldots\\ 0 & \text{otherwise.}\end{cases} \quad (24)$$

The UA for the average error rate $P_e(\rho)$ can now be readily computed by using Proposition 3.

B. Non-coherent/differential systems

UAs for differential and non-coherent systems with equal energy and equiprobable signaling for single-channel reception and multi-branch receiver with equal gain combining (EGC) are developed.

1) Single channel differential modulations: The conditional BER of such systems can be expressed as [25, eq. (27)]

$$h(\gamma)=\frac{1}{2\pi}\int_0^{\pi}\frac{1-\alpha^2}{1-2\alpha\cos\phi+\alpha^2}\times\exp\left[-\frac{v^2\gamma}{2}(1-2\alpha\cos\phi+\alpha^2)\right]d\phi;\ 0\le\alpha=\frac{u}{v}<1, \quad (25)$$

where u and v are modulation specific parameters such that $v>u$. For example,

- $u=0, v=1$:non-coherent orthogonal binary FSK, $h(\gamma)=\frac{1}{2}e^{-\gamma/2}$
- $u=0, v=\sqrt{2}$: binary differential phase shift keying (DPSK), $h(\gamma)=\frac{1}{2}e^{-\gamma}$
- $u=\sqrt{2-\sqrt{2}}, v=\sqrt{2+\sqrt{2}}$: differential quadrature phase shift keying (DQPSK).

The Mellin transform of $h(x)$ is given by

$$H(s)=\frac{\Gamma(s)2^s(1-\alpha^2)}{2\pi v^{2s}}\int_0^{\pi}\frac{1}{(1-2\alpha\cos\phi+\alpha^2)^{s+1}}d\phi. \quad (26)$$

According to Proposition 2, the simple poles of $H(s)$ at $s=0,-1,-2,\ldots$ describe the low-SNR expansion, a power series of ρ and hence, $\tau=1$. With the first right-sided pole of $F(1-s)$ at $\tilde{p}_0$, the high-SNR term can be found by using eq. (12). The BER UA can finally be expressed as eq. (14) with the required coefficients given by

$$b(0)=a\frac{2^{\tilde{p}_0-1}\Gamma(\tilde{p}_0)}{(v^2-u^2)^{\tilde{p}_0}}P_{\tilde{p}_0}\left(\frac{v^2+u^2}{v^2-u^2}\right)$$

$$c(l)=\begin{cases}\frac{(-1)^lF(l+1)}{2^{l+1}l!}(v^2-u^2)^lP_{l-1}\left(\frac{v^2+u^2}{v^2-u^2}\right) & l=0,1,2,\ldots\\ 0 & \text{otherwise,}\end{cases} \quad (27)$$

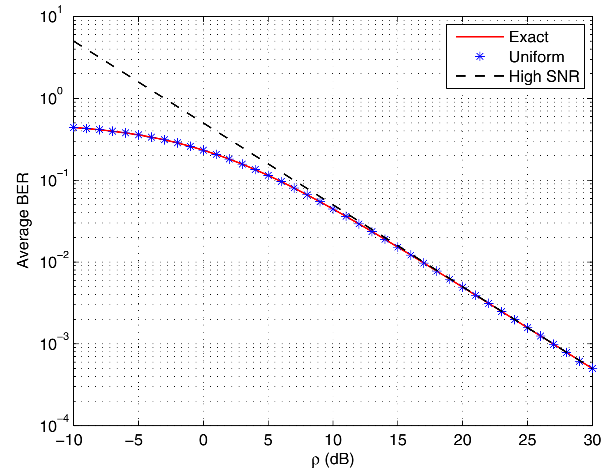

Fig. 4. Average BER of DQPSK over single-channel Rayleigh fading.

where $P_n(\cdot)$ is a Legendre polynomial [19], and $F(l+1)=\mu_l$ are the integer moments of β. Expressions [19, (3.661.3) and (3.661.4)] are used to solve the respective integrals to obtain the above closed-form solutions. For Rayleigh fading, $\tilde{p}_0=1$, $a=1$ and $F(l+1)=\mu_l=\Gamma(l+1)$. For DQPSK over Rayleigh fading, the BER UA ($L=4$), exact BER and high-SNR approximation eq. (8) are compared in Figure 4.

2) Multichannel reception of differential modulations: The conditional BER of N_r-branch non-coherent systems for binary DPSK and orthogonal binary FSK can be expressed as [3, eq. (12.1-13)]

$$h(\gamma)=\sum_{n=0}^{N_r-1}\underbrace{\frac{1}{n!}\sum_{i=0}^{N_r-1-n}\binom{2Nr-1}{i}}_{q_n}\frac{(g\gamma)^le^{-g\gamma}}{2^{2N_r-1}}, \quad (28)$$

where $g=1$ for binary DPSK, and $g=1/2$ for orthogonal binary FSK. The Mellin transform of $h(x)$ is thus given by

$$H(s)=\sum_{n=0}^{N_r-1}q_n\frac{\Gamma(s+n)}{2^{2N_r-1}g^s}, \quad (29)$$

which has simple poles at $s=0,-1,-2,\ldots$. By using Proposition 2, the coefficients of the low- and high-SNR asymptotics can be expressed as

$$b(0)=\frac{a\sum_{n=0}^{N_r-1}q_n\Gamma(n+\tilde{p}_0)}{2^{2N_r-1}\tau^{\tilde{p}_0}}$$

$$c(l)=\begin{cases}\frac{g^lF(l+1)}{2^{2N_r-1}}\sum_{n=0}^{l}\frac{(-1)^{l-n}}{(l-n)!}q_n & 0\le l\le N_r-2\\ \frac{g^lF(l+1)}{2^{2N_r-1}}\sum_{n=0}^{N_r-1}\frac{(-1)^{l-n}}{(l-n)!}q_n & l\ge N_r-1\\ 0 & \text{otherwise.}\end{cases} \quad (30)$$

The UA eq. (14) for the average BER of an N_r-branch DPSK system over Rayleigh fading can be readily obtained with $\tilde{p}_0=N_r$, $a=1/\Gamma(N_r)$, $g=1$ and $F(l+1)=\Gamma(N_r+l)/\Gamma(N_r)$. The UA thus derived by taking $L=2$ is compared against the exact error rate and the high-SNR result eq. (8) in Figure 5 for different values of N_r.

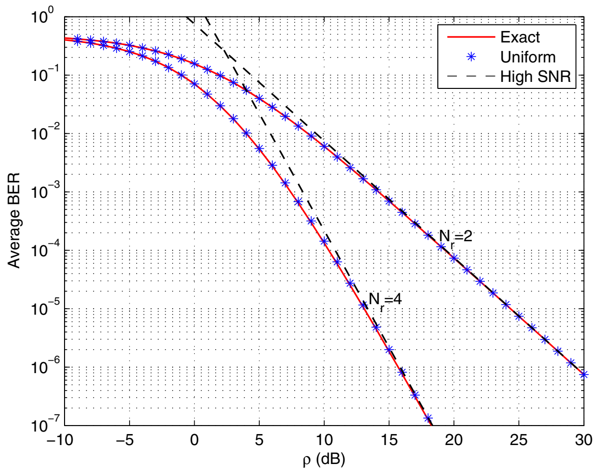

Fig. 5. Average BER of N_r branch system with DPSK modulation over Rayleigh Fading.

C. Other Applications

1) Product of two Gaussian Q functions: The average of the product of two Gaussian Q functions is required for the performance analysis of rectangular quadrature amplitude modulation (QAM) [26], [27], two-user synchronous code-division multiple-access (CDMA) systems [27], and repetition codes [26].

We thus consider $h(x) = Q\left(\sqrt{k_1 x}\right) Q\left(\sqrt{k_2 x}\right)$, whose Mellin transform can be shown to be

$$H(s) = \frac{2^{s-1}\sqrt{k_1 k_2}\Gamma(s+1)}{\pi s(2s+1)(k_1+k_2)^{s+1}} \Bigg({}_2F_1(1, s+1; s+\frac{3}{2}; \frac{k_1}{k_1+k_2}) + {}_2F_1(1, s+1; s+\frac{3}{2}; \frac{k_2}{k_1+k_2}) \Bigg). \quad (31)$$

We can find that $H(s)$ has simple poles at $s = 0, -1/2, -1, -3/2, -2, \ldots$. These negative poles describe the low-SNR expansion of $\mathbb{E}[h(\rho\beta)]$ as a power series of $\sqrt{\rho}$, and thus, $\tau = 1/2$. For Nakagami-m fading, $F(1-s)$ (Table II) has simple positive poles at $s = m, m+1, m+2, \ldots$, which describe the high-SNR $\mathbb{E}[h(\rho\beta)]$. The low- and high-SNR coefficients for the average of the product of two Q functions in Nakagami-m fading can thus be found by using eq. (10) as

$$b(0) = \frac{m^m H(m)}{\Gamma(m)} \quad c(l) = \begin{cases} h_R(l)\dfrac{\Gamma(l/2+m)}{m^{l/2}\Gamma(m)} & l = 0, 1, 2, \ldots \\ 0 & \text{otherwise,} \end{cases} \quad (32)$$

where $h_R(l) = \lim_{s \to -l/2}[(s+l/2)H(s)]$ is the residue of $H(s)$ (eq. (31)) at $s = -l/2$, $l = 0, 1, 2, \ldots$. The details of $h_R(l)$ are omitted here for brevity. Once $h_R(l)$ is determined, the required UA can be obtained by using Proposition 3.

2) Energy detection: The presence or absence of an unknown signal over a noisy channel can be determined from energy detection [28], which has recently been considered for spectrum sensing in cognitive radio [29]–[32]. We next show how the UA can be obtained for the probability of missing detection (P_m) of an energy detector.

For $2u$ samples, with λ threshold and $\gamma = \rho\beta$ instantaneous SNR, [32, Eq. 4]

$$P_m = \mathbb{E}[1 - Q_u(\sqrt{2\rho\beta}, \sqrt{\lambda})], \quad (33)$$

where $Q_M(a,b)$ is the M-th order generalized Marcum-Q function.

Let $h(x) = 1 - Q_u(\sqrt{2x}, \sqrt{\lambda})$. The Mellin transform of $h(x)$ obtained by using the contour integral representation of $Q_M(a,b)$ [33], is given in Table I. We can see that $H(s)$ does not have any positive singular points.

Under Nakagami-m fading, Proposition 2 yields low- and high-SNR coefficients of P_m as

$$b(0) = \frac{m^m H(m)}{\Gamma(m)}$$
$$c(l) = \begin{cases} \displaystyle\sum_{n+k=l} \frac{(-1)^n \Gamma(u+k, \frac{\lambda}{2})\Gamma(m+k+n)}{k!n!\Gamma(u+k)m^{k+n}\Gamma(m)} & l = 1, 2, \ldots \\ 1 - \dfrac{\Gamma(u, \lambda/2)}{\Gamma(u)} & l = 0. \end{cases} \quad (34)$$

The UA now follows from Proposition 3.

V. Outage Probability

This section develops the UA for the outage probability. Recall that previously, error-probability UAs were developed by using a single high-SNR term and multiple low-SNR terms. Similar process must be developed for the outage case.

Outage probability, a common quality-of-service parameter for wireless systems, is the probability that the instantaneous SNR falls below threshold γ_T:

$$P_{out}(\gamma_T, \rho) = \Pr[\rho\beta \le \gamma_T] = \int_0^{\gamma_T/\rho} f(\beta)d\beta. \quad (35)$$

Thus, the performance measure $h(x) = 1$ for $x < \gamma_T$ and $h(x) = 0$ for $x > \gamma_T$. Consequently, such $h(x)$ has only one term in its series expansion as $x \to 0^+$. Thus, the low-SNR coefficients $c(l)$ (eq. (4)) are zero for all l, except for $l = 0$. Previously, an error-probability UA required several low-SNR terms and a single high-SNR term, the latter depending on the first pole of $F(1-s)$. Conversely, matching a single low-SNR term (i.e., $c(0) = 1$) and several high-SNR terms is needed for the outage UA.

Proposition 4. *Generalize AS2 to the following: the fading PDF is* $f(\beta) = \sum_{k=0}^{K-1} a(k)\beta^{(k+t)} + O\left(\beta^{K+t}\right)$ *as* $\beta \to 0^+$ *with* $t \ge 0$. *We assume that* t *is an integer. The outage probability UA is then given by*

$$P_{out}(\gamma_T, \rho) = r(x) + \varepsilon(x), \quad (36)$$

where $r(x)$ *is a rational function defined in eq.* (13), $x = \rho/\gamma_T$, $K = L + (t+1) \ge 2$ *is an integer,* $p_0 = q_0 = 1$ *and* $p_i = q_i$, $i = 1, 2, \ldots, L$. *The coefficient vector* $\boldsymbol{q} = (q_1, q_2, \ldots, q_K)^T$ *is given by* $\boldsymbol{q} = \boldsymbol{W}^{-1}\boldsymbol{e}_{K-L}$, *where* $\boldsymbol{e}_i$ *is a* $K \times 1$ *column vector whose* i^{th} *element is* 1, *all other elements being* 0 *and* $\boldsymbol{W} = \{w_{ij}\}, i = 1, \ldots, K, j = 1, \ldots, K$ *with*

$$w_{ij} = \begin{cases} -1 & i = K-n,\ j = L-n;\ n = 0, \ldots, L-1 \\ b(j-i) & \text{otherwise,} \end{cases} \quad (37)$$

where

$$b(l) = \begin{cases} 0 & l < 0 \\ \frac{a(l)}{l+t+1} & \text{otherwise.} \end{cases} \quad (38)$$

Example 1. The conventional high-SNR outage under AS2 can be readily obtained from eq. (8) and Table I as

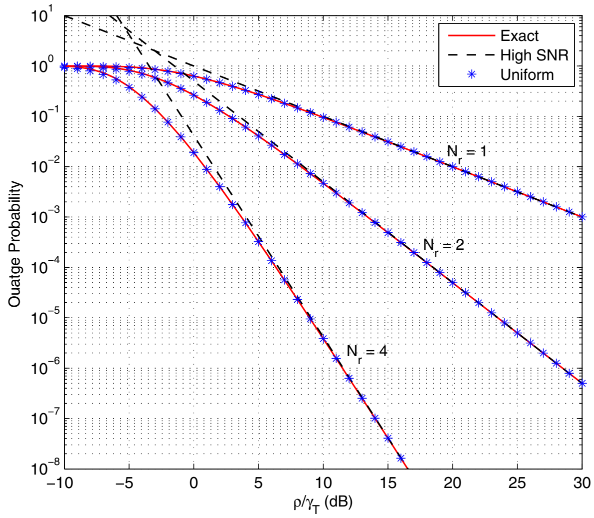

Fig. 6. The UA, exact outage and high-SNR approximation for N_r-branch MRC in iid Rayleigh fading.

$P_{out}(\gamma_T,\rho) \approx \frac{a}{t+1}\left(\frac{\gamma_T}{\rho}\right)^{t+1}$, which is consistent with the result given under Proposition 5 in [2]. Consider a simple case of a single-branch Rayleigh fading channel. Proposition 4 in this case yields the UA:

$$P_{out}(\gamma_T,\rho) = \frac{1+3x+6x^2}{1+3x+6x^2+6x^3} + \varepsilon(x).$$

For an N_r-branch MRC in iid Rayleigh fading, the outage UA ($L = 2$) eq. (36), conventional high-SNR approximation and exact outage are plotted in Figure 6. The excellent accuracy of the UA over the entire range of SNR is clearly evident.

VI. Capacity

Capacity refers to the maximum rate of error-free information transfer per unit bandwidth. With the performance metric $h(x) = \log(1+x)$, the average (ergodic) capacity is $\hat{C}(\rho) = \mathbb{E}[\log(1+\gamma)]$. As $h(x)$ does not decay exponentially, our previous UA results (e.g., Propositions 3 and 4) do not apply directly. For this reason, we exploit an MGF-based approach for capacity analysis [34]; i.e.,

$$\hat{C}(\rho) = \int_0^\infty \frac{e^{-x/\rho}}{x}[1-M_\beta(x)]dx = \int_0^\infty e^{-x/\rho}\varphi(x)dx, \quad (39)$$

where $\varphi(x) = \frac{1-M_\beta(x)}{x}$. A UA to $\varphi(x)$ can be computed by using the following asymptotics:

$$\varphi(x) = \begin{cases} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}\mu_{n+1}x^n & \text{as} \quad x \to 0^+ \\ \frac{1}{x} & \text{as} \quad x \to \infty. \end{cases} \quad (40)$$

Again, the moment μ_n follows directly from the Mellin transform $F(s)$. Now, by using these asymptotics, $r(x)$ in eq. (13) can be similarly computed as in Proposition 3. The $x \to \infty$ limit above yields the constraints $K - L = 1$ and $p_L = q_K$. The resulting rational function may be given as

$$r(x) = \sum_{k=1}^{K} \frac{A_k}{x+\alpha_k}. \quad (41)$$

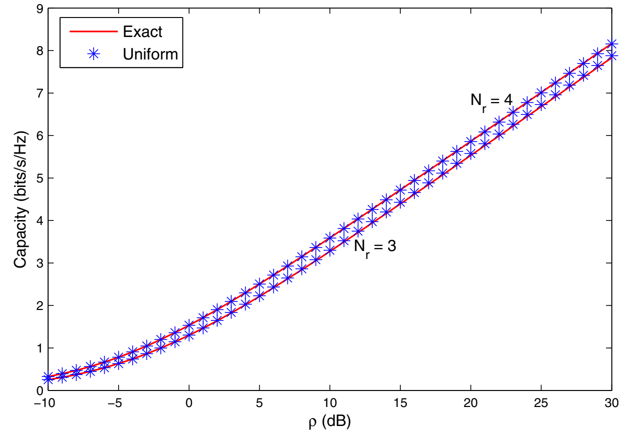

Fig. 7. The UA and exact capacity of an N_r-branch MRC in iid Rayleigh fading.

where α_k and A_k denote partial fraction coefficients. Now, by using eq. (39) and eq. (41), the average capacity can be approximated as

$$\hat{C}(\rho) \approx \sum_{k=1}^{K} -A_k e^{\alpha_k/\rho}\mathrm{Ei}(-\alpha_k/\rho), \quad (42)$$

where $\mathrm{Ei}(x)$ is the exponential integral function [19]. For example, for a 3-branch MRC in iid Rayleigh fading, $\varphi(x)$ can be approximated with the rational function as follows by using $K = 2$ and $L = 1$:

$$\varphi(x) \approx \frac{9+6x}{3+8x+6x^2}. \quad (43)$$

The capacity UA is now obtained by applying eq. (42) after the partial fraction expansion of eq. (43) The UAs for any N_r can be similarly obtained. These UAs are compared against the exact solution in Figure 7.

VII. Conclusion

We have developed UAs for error probability, outage and the ergodic capacity. It is evident from our extensive numerical results that the UAs' main feature is their accuracy over the whole range of SNR (e.g., $-\infty < \rho < \infty$ in a dB scale).

A UA is a rational function that simultaneously matches both the low- and high-SNR expansions of a performance metric. For both error probability and outage, matching the exact performance is feasible, but for capacity, an indirect step is required. Thus, a UA to $\varphi(x) = \frac{1-M_\beta(x)}{x}$ must be developed first.

Although both PDF and MGF can yield the necessary coefficients of the low- and high-SNR expansions, we developed a unified Mellin tranform solution in Proposition 2. In this method, the right-sided poles contributed by $F(1-s)$ determine the high-SNR expansion, the first of which yields the traditional diversity and SNR gains. Similarly, the left-sided poles contributed by $H(s)$ and $F(1-s)$ together, determine the low-SNR expansion. However, for common fading and diversity-combining systems, $F(1-s)$ contributes only right-sided poles.

We also provided a simple generalization (Proposition 1) of the high-SNR results of [2], yielding a unified high-SNR approximation for an arbitrary performance measure. Although not directly related to UAs, this result provides insights on how the SNR gain depends on the modulation format and in general, may be more accurate than that of [2].

This paper appears to be the first one to develop the notion of UA. It provides an alternative to closed-form analysis and also an excellent validation tool for simulations. We presented only a few of the vast number of potential applications, for UAs may also be developed for advanced wireless applications such as relay communications and cognitive radios.

Appendix

A. Proof of Proposition 1

Let $H(s)$ be the Mellin transform of $h(x)$, the performance measure under consideration. It then follows that $\int_0^\infty h(\rho\beta)\beta^q d\beta = H(q+1)/\rho^{q+1}$. Substituting a Taylor series expansion of $g(\beta)$ at $\beta = 0$ in eq. (3), we find

$$P_e(\rho) = \int_0^\infty h(\rho\beta)\beta^t \left(g(0) + g'(0)\beta + \frac{g''(\varepsilon_1)}{2}\beta^2\right) d\beta$$
$$= \frac{H(t+1)}{\rho^{t+1}}\left(g(0) + \frac{H(t+2)}{H(t+1)\rho}g'(0)\right) + \frac{g''(\varepsilon_1)H(t+3)}{2\rho^{t+3}}, \quad (44)$$

where $\varepsilon_1 \in (0,\infty)$. Noting that $g(0) + \frac{H(t+2)}{H(t+1)\rho}g'(0)$ is the first two terms of the Taylor series for $g\left(\frac{H(t+2)}{\rho H(t+1)}\right)$, we get Proposition 1. The error term in eq. (44) is clearly $O\left(\rho^{-(t+3)}\right)$.

B. Proof of Proposition 2

The error probability eq. (3) is a type of convolution and thus can be transformed via the Parseval formula [17] as

$$P_e(\rho) = \frac{1}{2\pi\imath}\int_{c-\imath\infty}^{c+\imath\infty} \frac{1}{\rho^s} H(s)F(1-s)\, ds, \quad (45)$$

where the parameter c is chosen to be in the fundamental strip $\langle u,v\rangle$ where both $H(s)$ and $F(1-s)$ are holomorphic. For $h(x)$ given in eq. (9), the Mellin transform $H(s) = G(s)\sum\lambda_k\mu_k^{-s}$.

To find the asymptotic of eq. (45) as $\rho \to 0^+$, we may consider a large rectangular contour to the left of the fundamental strip with sides $\Re(s) = c$ and $\Re(s) = -M$ for $-M < u$. Given that the functions $G(s)$ and $F(1-s)$ decrease faster than any negative power of $|s|$, and that the series $\sum\lambda_k\mu_k^{-s}$ is of, at most, polynomial growth in the extended strip $\langle -M,u\rangle$ as $|s| \to \infty$, the integrand in eq. (45) when evaluated along the top and bottom lines of the rectangle has a negligible contribution. In contrast, the integral along the vertical line $\Re(s) = -M$ is bounded by $O(\rho^M)$ [17]. By applying the residue theorem,

$$P_e(\rho) = \sum_{s\in H_M} \text{Res}\left\{\frac{1}{\rho^s}G(s)F(1-s)\sum\lambda_k\mu_k^{-s}\right\} + O(\rho^M),$$

where H_M is the set of poles enclosed by the rectangular contour, and M is as large as we want it to be.

One can similarly consider a large rectangular contour to the right of the fundamental strip with sides $\Re(s) = c$ and $\Re(s) = M$ for $M > v$ to get the asymptotic expansion as $\rho \to \infty$. However, there is an additional negative sign due to the contour being clockwise. By assuming first-order poles, we get the formula (10) and complete the proof.

C. Proof of Proposition 3

To construct UA, we require the rational function $r(x)$ in eq. (13) to satisfy the low- and high-SNR asymptotics in eq. (4) simultaneously. Let us use only the first term of the high-SNR expansion. As $x \to \infty$, we find that

$$\frac{p_{L-1}x^{L-1} + p_L x^L}{q_{K-1}x^{K-1} + q_K x^K} = \frac{b(0)}{x^{K-L}}$$

if $p_{L-1} = b(0)\,q_{K-1}$ and $p_L = b(0)\,q_K$, where $K - L = \delta$. These conditions thus determine the values of p_i for $i = L-1, L$ as expressed in eq. (17). To satisfy the low-SNR expansion, we must have

$$\left(\sum_{l=0}^{\infty} c(l)x^l\right)\left(q_0 + \sum_{k=1}^{K} q_k x^k\right) = p_0 + \sum_{l=1}^{L} p_l x^l. \quad (46)$$

By comparing the coefficients of x^l on both sides of eq. (46) for $l = L-1, L, \ldots, K+L-2$, we get

$$c(L-2+i)q_0 + \sum_{j=1}^{K} c(L-2+i-j)\,q_j = \begin{cases} p_{L-2+i} & i = 1,2 \\ 0 & i = 3,\ldots K. \end{cases} \quad (47)$$

It follows from eq. (46) that $c(0)q_0 = p_0$. Thus, if we set $p_0 = 1$, we get $q_0 = 1/c(0)$. After substituting for q_0, p_{L-1} and p_L, eq. (47) can be given in matrix form eq. (15). The solution of eq. (15) completely determines the denominator of $r(x)$. Again, by the comparing the coefficients of x^i on both sides of eq. (46) for $i = 1,2,\ldots,L-2$, we get the values of p_i for $i = 1,2,\ldots L-2$ as expressed in eq. (17).

D. Proof of Proposition 4

For $r(x)$ to match with the single low-SNR term, which is 1, we must have

$$p_0 + \sum_{l=1}^{L} p_l x^l = q_0 + \sum_{k=1}^{K} q_k x^k, \quad (48)$$

which implies that $p_i = q_i$, $i = 0,1,2,\ldots,L$. We can set $p_0 = q_0 = 1$ such that $P_{out}(\gamma_T, 0) = 1$.

With the fading PDF $f(\beta) = \sum_{k=0}^{K-1} a(k)\beta^{(t+k)} + O\left(\beta^{t+K}\right)$ as $\beta \to 0^+$, the outage probability can be expressed as $P_{out}(\gamma_T,\rho) = \sum_{l=0}^{K-1} b(l)x^{-(t+1+l)} + O(x^{t+1+K})$ as $x \to \infty$, where $x = \rho/\gamma_T$, and $b(l) = a(l)/(t+1+l)$. To satisfy this high-SNR expansion, we must have

$$\left(\sum_{l=0}^{K-1} b(l)y^{K-L+l}\right)\left(y^K + \sum_{l=1}^{K} q_l y^{K-l}\right) = y^K + \sum_{l=1}^{L} q_l y^{K-l}, \quad (49)$$

where $y = 1/x$, $K - L = t+1$. By comparing the coefficients of y^i on both sides of eq. (49) for $i = K-L, K-L+1, \ldots, K, K+1, \ldots, 2K-L-1$, we get

$$\sum_{j=1}^{K} b(j-i)\,q_j = \begin{cases} 0 & i = 1,2,\ldots,K-L-1 \\ 1 & i = K-L \\ q_{L-n} & i = K-n, n = 0,1,\ldots,L-1. \end{cases} \quad (50)$$

This set of linear equations is expressed in the form of a matrix in Proposition 3 and completely determines the coefficient vector $\mathbf{q}$.

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