Impact of Beacon Misdetection on Aggregate Interference for Hybrid Underlay-Interweave Networks

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***Abstract*—The impact of beacon misdetection on the aggregate interference from a hybrid underlay-interweave network is analyzed, for a Poisson field of cognitive radio (CR) nodes distributed over an annular region. This network consists of two types of nodes: underlay, and interweave. The underlay nodes are allowed to transmit anytime, whereas the interweave nodes must first sense an out-of-band beacon. When this sensing is erroneous, interweave node transmissions increase the interference. We analyze the interference statistics by deriving the exact moment generating function, the mean, and the outage probability of the primary receiver, for path loss and Rayleigh fading. Our analysis suggests that hybrid underlay-interweave CR systems are more suitable for areas with low path loss exponents such as rural/suburban environments.**

***Index Terms*—Cognitive radio, aggregate interference, outage probability, beacon transmission.**

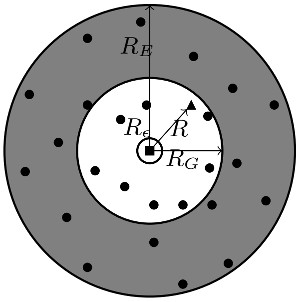

Fig. 1: Hybrid sytem model. The underlay nodes are located in the shaded region, while the interweave nodes occupy the inner region. Respectively, R_G, R_E, and R denote the guard distance, outer distance, and the distance between PR and PT. Legend: dot = interefers, square = PR, triangle = PT.

I. INTRODUCTION

INTERWEAVE and underlay networks are two different cognitive radio (CR) paradigms. The interweave paradigm relates to opportunistic spectrum access, which requires dynamic knowledge on spectrum usage by licensed primary users. Though this paradigm increases spectrum efficiency, interference to the licensed network may occur. Thus, to mitigate interference, the interweave nodes transmit only when a beacon signal from the primary is not detected. Conversely, the underlay nodes are always allowed to transmit granted that they are a guard distance away from the primary receiver. The advantages of both these paradigms can be harnessed via a hybrid underlay-interweave network, where the nodes outside the guard region are always permitted to transmit, whereas the inside nodes only transmit when the beacon is not present (Fig. 1). A hybrid network increases the throughput and the geographical distribution of the secondary network, but may cause additional interference. In such a network, interference on the primary receiver (PR) is from the underlay (shaded region) nodes, and from the interweave (clear region) nodes within the guard region misdetecting the beacon and subsequently transmitting. Practical applications include wireless sensor networks, ad-hoc networks, and broadband users accessing the spectrum allocated to television or WiMAX.

Aggregate interference analysis for underlay and interweave networks has received wide attention. For example, [1] has analyzed the aggregate interference of finite area CR networks for specific path loss exponents using a moment generating function (MGF) based approach, while [2] obtains cumulants of the aggregate interference under different sensing protocols, and develops a statistical model for the interference in spectrum sensing cognitive networks. In [3], a statistical model of interference aggregation is proposed, where a beacon transmission by the primary receiver is exploited for spectrum sensing. Furthermore, [4] analyzes the capacity-outage probability with beacon misdetection for Rayleigh and Nakagami-m fading while the beacon transmitter is located either at the primary transmitter or receiver. The mean, variance, and bounds for the aggregate interference given beacon misdetection is investigated in [5]. Reference [6] has investigated the trade-off between node density and outage probability under a random number of CR nodes. Moreover, [7] has developed interference models for Poisson and Poisson clustered interferer node distributions, while [8] derives the probability density functions (PDFs) of the aggregate interference under power and contention control schemes.

However, no prior work has analyzed the impact of beacon misdetection on the aggregate interference for a Poisson point process (PPP) of CR nodes around the PR for a hybrid underlay-interweave network. Thus, the primary motivation of this letter is to characterize the impact of beacon misdetection on the aggregate interference. Beacon misdetection is random, depending on channel effects and node locations. We will assume path loss and Rayleigh fading. The MGF and the mean of the aggregate interference, and the outage are derived. Furthermore, our numerical results will show that the effects of misdetection are more pronounced in environments with higher path loss exponents.

Notations: $\Gamma(x,a) = \int_a^\infty t^{x-1}e^{-t}dt$ and $\Gamma(x) = \Gamma(x,0)$, ${}_2\mathcal{F}_1(,;;)$ is the Gauss Hypergeometric function [9, (eq. 9.10)]. $\Pr[\cdot]$ denotes the probability, $f_X(\cdot)$ is the PDF, $F_X(\cdot)$ is the cumulative distribution function (CDF), $M_X(\cdot)$ is the MGF, and $E_X[\cdot]$ denotes expectation over X.

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II. SYSTEM MODEL

We consider an annular network with the PR at the center (Fig. 1). The positions of the CR nodes are static. However, time dependent random motion is an interesting topic to be considered for future research. The spatial distribution of the CR nodes in the ring of radius R_E is modeled as a homogeneous PPP [10]–[12]. The probability of N nodes within a specified area of A is thus given by

$$\Pr(N=n)=\frac{(\beta A)^n}{n!}e^{-\beta A}, \quad n=0,\dots \tag{1}$$

where β is the average number of CR nodes per unit area (node density). The radius R_G is termed the guard distance. All underlay CR nodes (shaded region) can always transmit, and the worst case scenario when all of them are active will be considered for the analysis. An interweave CR node (inner) transmits only if the out-of-band beacon from the primary receiver indicating its spectrum usage is misdetected. Thus, when this occurs, the interweave node transmissions create additional interference.

For the hybrid system considered, location awareness is necessary for the nodes to know whether they are inside or outside the guard region. This knowledge can be acquired through either global positioning systems or through a CR control node [13].

All the signals including the beacon signals are assumed to undergo path loss and Rayleigh fading. From the simplified path loss model [14], the received power at a distance r from the transmitter is given by $P_r = Pr^{-\alpha}$, where $P = P_0 r_0^{\alpha}$, and α is the path loss exponent. P_0 is the power received at a distance r_0 from the transmitter. Because of Rayleigh fading, the random channel gain $|h_i|^2$ of the i-th channel can be represented without the loss of generality by an exponential random variable where $f_{|h_i|^2}(x) = e^{-x}$, $0 \le x < \infty$.

III. INTERFERENCE ANALYSIS

To assess the impact of beacon misdetection, the exact MGF of the aggregate interference and its mean will be derived. These in turn will help to derive the outage performance.

The total interference at the primary receiver I can be written as

$$I=\sum_{i=1}^{N_1} I_{1,i}+\sum_{j=1}^{N_2} I_{2,j}, \tag{2}$$

where $I_{1,i}$ and $I_{2,j}$ are the interference from the i-th ($i = 1\dots N_1$) interweave and j-th ($j = 1\dots N_2$) underlay CR nodes. N_1 and N_2 denote the number of interweave and underlay nodes.

Let $M_I(s) = E[e^{-sI}]$ be the MGF of the aggregate interference. Suppose $M_{I_1}(s)$ and $M_{I_2}(s)$ denote the MGFs of the interference from interweave and underlay CR nodes respectively. Because of the independence of each group of interferers, $M_I(s)$ can be written as $M_I(s) = M_{I_1}(s)M_{I_2}(s)$.

A. Interference from the interweave CR nodes

The interference $I_{1,i}$ is given by

$$I_{1,i}=Q_i P_{CR}|h_{1,i}|^2 r_{1,i}^{-\alpha}, i=1\dots N_1, \tag{3}$$

where P_{CR} is the power level of the CR nodes, $r_{1,i}$ is the distance to the i-th interweave CR node from the PR, and $|h_{1,i}|^2$ is the channel gain between the i-th interweave CR node and the PR. The misdetection factor Q_i is a Bernoulli random variable with parameter q_i, where q_i is the probability of beacon misdetection for the i-th interweave CR node. A Bernoulli random variable is used because a CR node can only be either transmitting or idle. The probability q_i depends on fading, path loss, and the node's position. The definition of q_i is as follows. Let the power level of the beacon transmission be defined as P_b, the channel gain from the PR to the i-th CR as $|h_{b,i}|^2$, and the threshold level required for correct reception at the CR nodes as P_{T_b}. Then, q_i given $r_{1,i}$ becomes;

$$q_i{=}\Pr\left[P_b|h_{b,i}|^2 r_{1,i}^{-\alpha}<P_{T_b}\right]{=}1-e^{-\frac{P_{T_b}r_{1,i}^{\alpha}}{P_b}}, i{=}1\dots N_1. \tag{4}$$

Let $M_{I_1}^{i}(s)$ denote the MGF of the interference from one CR node. The channel gains and path loss effects are independent from each other. Therefore, $M_{I_{1,i}}(s)$ can be written as

$$M_{I_{1,i}}(s)=E_{|h_{1,i}|^2,r_{1,i}}[e^{-sI_{1,i}}]=E_{r_{1,i}}[E_{|h_{1,i}|^2}[e^{-sI_{1,i}}]]. \tag{5}$$

The inner expectation is with respect to an exponential random variable. Due to the beacon misdetection probability, $e^{-sI_{1,i}}$ will have two values given $|h_{1,i}|^2$ and $r_{1,i}$. Therefore, averaging with respect to the inner expectation yields

$$M_{I_{1,i}/r_{1,i}}(s)=1-q_i+\frac{q_i}{1+sP_{CR}r_{1,i}^{-\alpha}}. \tag{6}$$

All the interweave CR nodes in Fig. 1 are distributed over the annular area between the rings of radii R_ϵ and R_G. The radius R_ϵ is an artifact so that the simplified path loss model holds [4], [5]. The value of R_ϵ is usually chosen as 1. The CR nodes inside the radius of R_ϵ are assumed to perfectly detect the beacon.

Because a homogeneous PPP is considered, under any given number (N_1) of nodes, these will be distributed uniformly in the annular area (Fig. 1). Therefore, we can write the CDF of the distance from the i-th interweave CR node to the PR $r_{1,i}$ as, $F_{r_{1,i}}(r) = \frac{r^2-R_\epsilon^2}{R_G^2-R_\epsilon^2}$. The PDF of this distance can thus be computed by differentiating the CDF as

$$f_{r_{1,i}}(r)=\begin{cases}\frac{2\pi r}{A_1}, & R_\epsilon<r<R_G;\\ 0, & \text{otherwise,}\end{cases} \tag{7}$$

where $A_1 = \pi(R_G^2 - R_\epsilon^2)$. Substituting the misdetecion probability (4) in (6), and using the expansion $e^{-x} = \sum_{k=0}^{\infty}\frac{(-x)^k}{k!}$, we get

$$M_{I_{1,i}/r_{1,i}}(s)=\sum_{k=0}^{\infty}\frac{\left(-\frac{P_{T_b}r_{1,i}^{\alpha}}{P_b}\right)^k}{k!}-\sum_{l=1}^{\infty}\frac{\left(-\frac{P_{T_b}r_{1,i}^{\alpha}}{P_b}\right)^l}{l!(1+sP_{CR}r_{1,i}^{-\alpha})} \tag{8}$$

Averaging the conditional MGF (8) over the distance from the i-th interweave CR node to the PR $r_{1,i}$ (7) gives

$$\begin{aligned}M_{I_{1,i}}(s)={}&\frac{2\pi}{A_1}\sum_{k=0}^{\infty}\frac{\left(-\frac{P_{T_b}}{P_b}\right)^k}{k!}\frac{1}{\alpha k+2}\left(R_G^{\alpha k+2}-R_\epsilon^{\alpha k+2}\right)\\ &-\frac{2\pi}{A_1}\sum_{l=1}^{\infty}\frac{\left(-\frac{P_{T_b}}{P_b}\right)^l}{l!}\frac{1}{sP_{CR}(2+\alpha+\alpha l)}\\ &\times\left(R_G^{2+\alpha+\alpha l}\mathcal{V}\left(\frac{R_G^{\alpha}}{sP_{CR}}\right)-R_\epsilon^{2+\alpha+\alpha l}\mathcal{V}\left(\frac{R_\epsilon^{\alpha}}{sP_{CR}}\right)\right),\end{aligned} \tag{9}$$

where $\mathcal{V}(x) = {}_2\mathcal{F}_1(1, 1 + l + 2/\alpha; 2 + l + 2/\alpha; -x)$.

Because each interferer is independent, $M_{I_1}(s)$ given N_1 can be written as $M_{I_1/N_1}(s) = \left(M_{I_{1,i}}(s)\right)^{N_1}$. By replacing A in (1) with A_1, and averaging $M_{I_1/N_1}(s)$ with (1), we get $M_{I_1}(s) = e^{\beta A_1 (M_{I_{1,i}}(s) - 1)}$.

B. Interference from the underlay CR nodes

The interference $I_{2,j}$ can be written as

$$I_{2,j} = P_{CR} |h_{2,j}|^2 r_{2,j}^{-\alpha}, i = 1 \ldots N_2. \tag{10}$$

In a similar manner to the above subsection, the MGF of the interference from a single underlay CR node ($M_{I_{2,j}}(s)$) can be written as

$$\begin{aligned} M_{I_{2,j}}(s) &= E_{r_{2,j}}[E_{|h_{2,j}|^2}[e^{-sI_{2,j}}]] \\ &= E_{r_{2,j}}[\frac{1}{1 + sP_{CR} r_{2,j}^{-\alpha}}]. \end{aligned} \tag{11}$$

The PDF of $r_{2,j}$ ($R_G < r_{2,j} < R_E$) can be obtained as $f_{r_{2,j}}(r) = \frac{2\pi r}{A_2}$, where $A_2 = \pi(R_E^2 - R_G^2)$. Performing the expectation with respect to $r_{2,j}$ [15] yields

$$\begin{aligned} M_{I_{2,j}}(s) = \frac{\pi}{A_2} \Bigg(& R_E^2 \left(1 - \mathcal{W}\left(\frac{R_E^\alpha}{sP_{CR}} \right) \right) \\ & - R_G^2 \left(1 - \mathcal{W}\left(\frac{R_G^\alpha}{sP_{CR}} \right) \right) \Bigg), \end{aligned} \tag{12}$$

where $\mathcal{W}(x) = {}_2\mathcal{F}_1(1, 2/\alpha; 1 + 2/\alpha; -x)$.

The MGF of the interference from CR nodes beyond R_G can thus be written as $M_{I_2}(s) = e^{\beta A_2 (M_{I_{2,j}}(s) - 1)}$.

Moreover, if several additional layers of homogeneous nodes are present, the MGFs of those can be obtained separately to derive the aggregate MGF.

When no misdetection occurs, the MGF of the aggregate interference $M_I(s) = M_{I_2}(s)$.

C. Mean interference power

The mean power of the aggregate interference is an important performance measure. The mean interference $E[I]$ can be written as

$$\begin{aligned} E\left[I\right] &= \beta A_1 E[I_{1,i}] + \beta A_2 E[I_{2,j}] \\ &= \beta \left(A_1 E[Q_i P_{CR} |h_{1,i}|^2 r_{1,i}^{-\alpha}] + A_2 E[P_{CR} |h_{2,j}|^2 r_{2,j}^{-\alpha}] \right). \end{aligned} \tag{13}$$

After performing the expectations, we can obtain $E[I]$ as (14).

IV. OUTAGE PROBABILITY ANALYSIS

This section will derive the CDF of the signal to interference and noise ratio (SINR).

The power level of the primary transmitter is P_p, and the distance from the receiver is R. With Rayleigh fading and path loss taken into account, the received primary signal power (P_R) is $P_R = P_p R^{-\alpha} |h|^2$, where $|h|^2$ is the exponentially distributed channel power gain. Let γ denote the SINR, which can be written as $\gamma = \frac{P_R}{I + \sigma_n^2}$, where σ_n^2 is the noise variance. Thus, the CDF of the SINR is obtained as [1]

$$F_\gamma(x) = 1 - e^{\left(-\frac{x\sigma_n^2}{P_p R^{-\alpha}}\right)} M_{I_1}\left(\frac{x}{P_p R^{-\alpha}}\right) M_{I_2}\left(\frac{x}{P_p R^{-\alpha}}\right) \tag{15}$$

The outage is obtained simply by substituting γ_{Th} in place of x, where γ_{Th} is the SINR threshold value needed for correct reception.

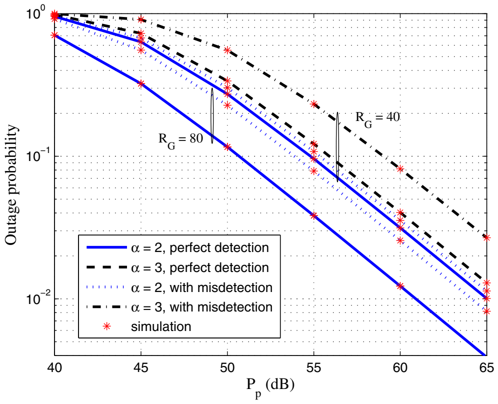

Fig. 2: Outage probability vs the primary system power level P_p for different path loss exponents and guard distances. CR node power level $P_{CR} = 30$ dB, and the beacon transmit power level $P_b = 37$ dB.

V. NUMERICAL RESULTS

This section shows the effects of beacon misdetection on the outage probability for several different parameter values, and compares the theoretical and simulation results. We will use the parameters $R = 30$, $R_G = 40$, $R_E = 120$, $R_\epsilon = 1$, $\gamma_{Th} = 1$, $P_{T_b} = 1$, and $\beta = 5 \times 10^{-3}$ in all the plots for consistency. For Fig. 2, an additional R_G value will be used for comparisons. Prior research [1] has considered similar ratios for $\frac{R_E}{R_G}$, and $\frac{R}{R_G}$. Higher $\frac{R_E}{R_G}$ ratios will not significantly change the results due to path loss, and a different $\frac{R}{R_G}$ value would primarily shift the curves.

We will first compare the theoretical results with the simulations, and observe the characteristics of the outage probability over P_p in Fig. 2. Cases where perfect detection occurs (interference exclusively from underlay nodes), and where misdetection occurs, are plotted for different path loss exponent (α) and guard distance (R_G) values. The tight match between simulation and the theoretical results confirms our analysis. The impact of misdetection is stronger under higher path loss exponent values because higher path loss exponents increase the probability beacon misdetection. For example, the outage probability under $\alpha = 3$ and $R_G = 40$ shows around a 4 dB shift, while for $\alpha = 2$ and $R_G = 40$, this is around 0.5 dB. When $R_G = 80$ and $\alpha = 2$, the outage reduces because a portion of the underlay zone is replaced by an interweave zone. However, the performance gap between the cases of misdetection and perfect detection increases significantly.

In Fig. 3, the outage probability is plotted against the common power level of interfering nodes P_{CR}. The curves for $\alpha = 3$ show a large shift in outage for a given value of P_{CR} than for $\alpha = 2$. The path loss exponent's (α) effect on the outage for a hybrid underlay/interweave network is complex. With higher α, the received primary power will decrease while the probability of beacon misdetection will increase. However, the interference from the misdetected interweave nodes and the underlay nodes will decrease with α. From the results, it is obvious that proper detection plays a crucial role regarding the outage performance (lower α values enable this). Moreover, for low P_{CR}, the interference is negligible, and the outage is larger for higher α due to lower received primary signal

$$E[I] = \beta \left(2\pi P_{CR} \frac{R_E^{2-\alpha} - R_\epsilon^{2-\alpha}}{2-\alpha} - \frac{2\pi P_{CR}}{\alpha - 2} \left(R_G^2 \left(\frac{P_{T_b} \left(\frac{R_G^\alpha P_{T_b}}{P_b} \right)^{-\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{R_G^\alpha P_{T_b}}{P_b} \right)}{P_b} - R_G^{-\alpha} e^{-\frac{R_G^\alpha P_{T_b}}{P_b}} \right) \right.\right.$$
$$\left.\left. - R_\epsilon^2 \left(\frac{P_{T_b} \left(\frac{R_\epsilon^\alpha P_{T_b}}{P_b} \right)^{-\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}, \frac{R_\epsilon^\alpha P_{T_b}}{P_b} \right)}{P_b} - R_\epsilon^{-\alpha} e^{-\frac{R_\epsilon^\alpha P_{T_b}}{P_b}} \right) \right) \right) \tag{14}$$

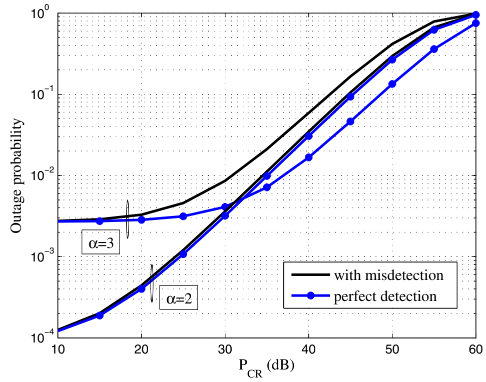

Fig. 3: Outage probability vs CR node power level P_{CR} for different path loss exponents. The primary power level $P_p = 70\,\text{dB}$, and the beacon transmit power $P_b = 37\,\text{dB}$.

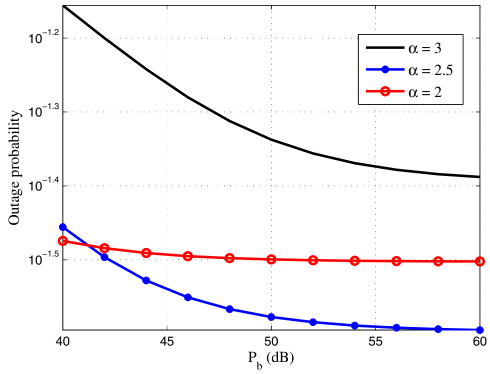

Fig. 4: Outage probability vs the beacon power level P_b under CR node power level $P_{CR} = 30\,\text{dB}$, and the primary power level $P_p = 60\,\text{dB}$.

powers. But, when P_{CR} is increased, a lower α increases the received interference, and will results a higher outage.

Another important trend to analyze is the effect of beacon transmit power level. When this power level increases, the outage probability drops, and levels off (Fig. 4). This level is the outage probability under perfect detection, where the aggregate interference is purely due to the interference from underlay CR nodes. Moreover, for the system parameters used, the outage probability of $\alpha = 2.5$ drops below the outage of $\alpha = 2$ when P_b is increased. As mentioned earlier, because both primary and interferer signals undergo path loss, lower α increases the received primary power as well as the interference (detection probability is not affected significantly due to high P_b values), and vice-versa for higher α. Therefore, a minima will occur on the outage as α changes. The exact value of α at the minima will change depending on other parameters.

VI. CONCLUSION

For a hybrid underlay-interweave network with a Poisson field of CR interferers, this paper analyzed the impact of beacon misdetection on the PR's aggregate interference. Path loss and Rayleigh fading were considered for the beacons, interferers, and the primary user transmissions. The exact MGF and mean of the aggregate interference, and the outage probability were derived. Beacon misdetection causes significant performance losses, which are more pronounced at higher path loss exponent values. Due to the effects of beacon misdetection, we suggest that hybrid underlay-interweave networks are less suitable for areas with high path loss exponents such as dense urban areas.

REFERENCES

[1] L. Vijayandran, P. Dharmawansa, T. Ekman, and C. Tellambura, "Analysis of aggregate interference and primary system performance in finite area cognitive radio networks," *IEEE Trans. Commun.*, vol. PP, no. 99, pp. 1–12, 2012.
[2] A. Rabbachin, T. Q. S. Quek, H. Shin, and M. Z. Win, "Cognitive network interference," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 480–493, Feb. 2011.
[3] A. Ghasemi and E. Sousa, "Interference aggregation in spectrum-sensing cognitive wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 2, no. 1, pp. 41–56, Feb. 2008.
[4] M. Derakhshani and T. Le-Ngoc, "Aggregate interference and capacity-outage analysis in a cognitive radio network," *IEEE Trans. Veh. Technol.*, vol. 61, no. 1, pp. 196–207, Jan. 2012.
[5] M. Vu, S. Ghassemzadeh, and V. Tarokh, "Interference in a cognitive network with beacon," in *Proc. 2008 IEEE WCNC*, pp. 876–881.
[6] V. Mordachev and S. Loyka, "On node density—outage probability tradeoff in wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1120–1131, 2009.
[7] K. Gulati, B. Evans, J. Andrews, and K. Tinsley, "Statistics of co-channel interference in a field of Poisson and Poisson-Poisson clustered interferers," *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6207–6222, Dec. 2010.
[8] Z. Chen, C.-X. Wang, X. Hong, J. Thompson, S. Vorobyov, X. Ge, H. Xiao, and F. Zhao, "Aggregate interference modeling in cognitive radio networks with power and contention control," *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 456–468, Feb. 2012.
[9] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press, 2007.
[10] E. Salbaroli and A. Zanella, "Interference analysis in a Poisson field of nodes of finite area," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1776–1783, May 2009.
[11] E. Sousa and J. Silvester, "Optimum transmission ranges in a direct-sequence spread-spectrum multihop packet radio network," *IEEE J. Sel. Areas Commun.*, vol. 8, no. 5, pp. 762–771, Jun. 1990.
[12] J. Ilow and D. Hatzinakos, "Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers," *IEEE Trans. Signal Process.*, vol. 46, no. 6, pp. 1601–1611, Jun. 1998.
[13] K. Ali, S. Neogy, and P. Das, "Optimal energy-based clustering with gps-enabled sensor nodes," in *Proc. 2010 IEEE SENSORCOMM*, pp. 13–18.
[14] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.
[15] S. Kusaladharma and C. Tellambura, "Aggregate interference analysis for underlay cognitive radio networks," *IEEE Wireless Commun. Lett.*, vol. 1, no. 6, pp. 641–644, 2012.