On Approximating the Cognitive Radio Aggregate Interference

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***Abstract*—The aggregate interference caused on a primary receiver by Cognitive Radio (CR) nodes distrbuted uniformly in an annular region around it is investigated, assuming Rayleigh fading and path loss. By approximating the aggregate interference with that caused by the nearest CR interferer, the exact analytical moment generating function for the nearest interferer is derived for cases of no power control and distance based power control. The outage probability based on the approximation is derived, which is a tight lower bound under certain conditions.**

***Index Terms*—Cognitive radio, aggregate interference, outage probability, nearest interferer.**

I. Introduction

COGNITIVE Radio (CR) nodes, which opportunistically access the vacant frequency spectrum allocated to primary licensed users, may generate co-channel interference. Therefore, statistical characterization of the CR interference is of paramount importance. Exact calculations and approximations for the aggregate interference have thus been widely investigated. For example, [1] proposes a model for the aggregate interference, approximating it as the sum of normal and log-normal random variables. In [2], the aggregate interference is analysed when the CRs employ power control, contention control, and hybrid power and contention control schemes. Reference [3] proposes a new statistical model for aggregate interference by using cumulants obtained via the Campbell's theorem, while [4] shows that when the CR exclusion region around the primary receiver (PR) is zero, the aggregate interference can be accurately modelled as a heavy tailed α-stable distribution. Deployment issues for CRs were considered in [5] where the authors compute the exclusion zone radii and the number of allowable CRs when the radio environment is apriori known. In addition, reference [6] investigates the tradeoff between outage probability and node density using the interference from the closest interferer and further studies the effects of interference cancellation.

This paper's main objective is to approximate the total aggregate interference with the interference of the nearest CR. The exact expressions for aggregate interference are usually complex [7] and do not give easy insights into CR network design parameters. The nearest CR approximation may be of help when designing the exclusion zone parameters and allowable node densities. In addition, this approximation may help when designing certain interference cancellation schemes. Although this approximation was previously used in [8], reference [8] does not derive the distribution of the nearest CR distance under a Poisson point process nor consider an exclusion zone. Also, [8] does not investigate how the accuracy of the approximation depends on system parameters, and nor does [8] consider any power control schemes.

This paper assumes a Poisson field of interfering CR nodes distributed in an annular region around the PR. An exclusion zone around the PR is enforced for added protection. We derive the probability density function (PDF) of the nearest CR interferer distance. By using the nearest CR approximation, the moment generating function (MGF) of the interference is obtained in closed-form for two cases: namely, all CRs transmitting at a constant power, and them employing a basic distance based power controlling scheme [9]. By using the MGF, the outage probability is derived in closed-form. Finally, we show that the nearest CR approximation is a lower bound to the aggregate interference, which becomes tight as either the CR density or the width of the annular region decreases, or when the path loss exponent value increases.

This paper is organized as follows. In Section II the system and channel models are introduced. Section III derives the PDF of the distance of the nearest CR, and derives the MGF for two power allocation schemes. Section IV provides the outage analysis, while Section V shows numerical and simulation results. Section VI concludes the paper.

Notations: ${}_2\mathcal{F}_1(,;;)$ is the Gauss Hypergeometric function [10, (eq. 9.10)], $E_n(\cdot)$ is the generalized exponential integral with respect to n, $\lfloor x \rfloor$ is the largest integer less than the x, $f_X(\cdot)$ is the PDF, $F_X(\cdot)$ is the cumulative distribution function (CDF), $M_X(\cdot)$ is the MGF, and $\mathbb{E}_X[\cdot]$ is the expectation with respect to X.

II. System Model

The CRs are assumed to be distributed uniformly in an annular area surrounding the PR, which has inner outer radii R_G and R_E. The inner radius is termed the guard distance, which is designed to limit the interference on the PR. The additional interference from CRs beyond R_E is negligible due to propagation losses and can be ignored. The number of CRs (N) is assumed to be distributed according to a Poisson point process [11], and has the distribution

$$P(N=n)=\frac{(\beta A_I)^n}{n!}e^{-\beta A_I}, \quad n=0,\ldots \tag{1}$$

where $A_I=\pi(R_E^2-R_G^2)$ is the total area in which CR activity is present, and β is the CR density. The aggregate interference at the PR can be written as

$$I=\sum_{i=1}^{N} I_i, \tag{2}$$

where I_i is the interference caused by the i-th CR, and N is the number of CRs present in the annular region. From the general path loss model, I_i is expressed as

$$I_i=P_i {r_i}^{-\alpha} X_i, \quad i=1\ldots N \tag{3}$$

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where P_i is the power of the i-th CR, r_i is the distance from the i-th CR to the PR. For Rayleigh fading, X_i is an exponential random variable with $f_{X_i}(x) = e^{-x}$ $(0 \le x \le \infty)$. The interfering signals undergo independent fading.

In our analysis, the mutual interference between the CRs themselves and the activity factors of the CRs are not considered. However, the activity factors can be readily included by representing the CRs as a thinned Poisson point process [12] with node density $\mathcal{C}\beta$, where $\mathcal{C}$ is a constant depending on the activity factors.

III. Approximating to the Nearest Interferer

Our aim is to approximate the aggregate interference $I = \sum_{i=1}^{N} I_i$, with the interference from the nearest CR. Therefore, the aggregate interference can be written as

$$I_{app} = P_{min} r_{min}^{-\alpha} X, \tag{4}$$

where P_{min}, r_{min} and X_{min} respectively denote the power level, distance to the PR and the fading coefficient, of the nearest CR node.

The MGF of the aggregate interference can be obtained as $M_I(s) = \mathbb{E}[e^{-sI}]$. In order to derive the MGF, we need to obtain the PDF of the nearest CR distance:

$$r_{min} = \min\left(r_1, r_2, \ldots r_N\right).$$

We can obtain the PDF of r_{min} as (see Appendix I for proof)

$$f_{r_{min}}(x) = 2\pi\beta x e^{\pi\beta\left(R_G^2 - x^2\right)}, \quad R_G < x < R_E. \tag{5}$$

Note that the PDF (5) is not proper. This is because when $N = 0$, the PDF does not exist with probability $e^{-A_I\beta}$. While this factor will be considered in calculating the MGF, for the purpose of illustration, we will refer to (5) as the PDF. It is more insightful to obtain the PDF of the normalized distance $\frac{r_{min}}{R_G}$. $f_{\frac{r_{min}}{R_G}}(x)$ for $N \neq 0$ can be obtained through a simple variable change of (5) as

$$f_{\frac{r_{min}}{R_G}}(y) = 2\pi\eta y e^{\pi\eta\left(1-y^2\right)}, \quad 1 < y < \frac{R_E}{R_G}, \tag{6}$$

where the composite distance-density parameter $\eta = \beta R_G^2$. Fig. 1 shows the distribution of the normalized distance under differing η for $\frac{R_E}{R_G} = 4$. Also, note that the shapes of the curves are not dependent on the normalized distance. It is observed that when η is large, the PDF of $f_{r_{min}}(x)$ shows a sharp drop-off. Conversely, for small η values, the PDF drops more gradually with respect to the normalized distance. When η gets significantly low, the PDF does not sharply decrease and has a more or less flat shape. Thus, the nearest CR would dominate the PR's aggregate interference for small η. In practice, when designing a CR system at higher η values, increasing the inner radius instead of reducing the node density is a more effective technique to ensure a guaranteed PR performance while maximizing the amount of CRs employed.

In the following subsections, we will derive the MGF of the aggregate interference when the CRs employ constant transmit power and when the CRs employ a distance based power control scheme.

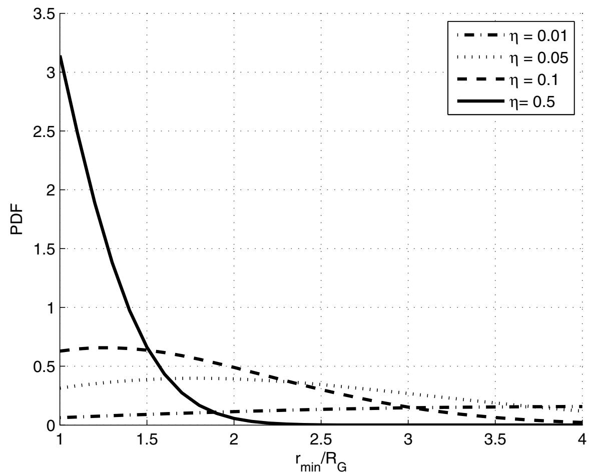

Fig. 1. $f_{\frac{r_{min}}{R_G}}(y)$ for varying η values when $\frac{R_E}{R_G} = 4$.

A. Constant power CRs

When the power levels of each CR is constant, and not dependent on distance, the MGF can be written as $M_{I_{app}}(s) = \mathbb{E}_{X,r_{min}}[e^{-sI_{app}}] = \mathbb{E}_{r_{min}}[\mathbb{E}_X[e^{-sP_s r_{min}^{-\alpha} X}]]$, where P_s is the constant power of a CR. Averaging with respect to X, we find $M_{I_{app}}(s) = \mathbb{E}_{r_{min}}\left[\frac{1}{1+P_s s r_{min}^{-\alpha}}\right]$. When $P_s s r_{min}^{-\alpha} < 1$, we can expand $M_{I_{app}}(s)$ as an infinite series, and get

$$M_{I_{app}}(s) = \mathbb{E}_{r_{min}}\left[\sum_{t=0}^{\infty}\left(-P_s s r_{min}^{-\alpha}\right)^t\right]. \tag{7}$$

For most practical values of system parameters, the summation will converge. Averaging (7) over (5), we obtain the MGF as,

$$M_{I_{app}}(s) = e^{-A_I\beta} + \pi\beta e^{\pi\beta R_G^2} \times \sum_{t=0}^{\infty}\left((-P_s s)^t\left(\frac{E_{\frac{t\alpha}{2}}\left(\pi\beta R_G^2\right)}{R_G^{t\alpha-2}} - \frac{E_{\frac{t\alpha}{2}}\left(\pi\beta R_E^2\right)}{R_E^{t\alpha-2}}\right)\right). \tag{8}$$

The expected value of the interference from the nearest CR can be obtained from (8) as

$$\mathbb{E}[I_{app}] = \pi\beta P_s e^{\pi\beta R_G^2}\left(\frac{E_{\frac{\alpha}{2}}\left(\pi\beta R_E^2\right)}{R_E^{\alpha-2}} - \frac{E_{\frac{\alpha}{2}}\left(\pi\beta R_G^2\right)}{R_G^{\alpha-2}}\right). \tag{9}$$

Using a similar procedure as in [7], the exact MGF can be obtained as

$$M_{I_{ex}}(s) = e^{\beta A_I\left(\frac{\pi}{A_I}\left(R_G^2\left(\mathcal{W}\left(\frac{R_G^\alpha}{P_s s}\right)-1\right) - R_E^2\left(\mathcal{W}\left(\frac{R_E^\alpha}{P_s s}\right)-1\right)\right)-1\right)}. \tag{10}$$

where $\mathcal{W}(x) = {}_2\mathcal{F}_1(1, 2/\alpha; 1 + 2/\alpha; -x)$.

B. CRs employing power control

In this subsection, we consider a distance dependent power allocation scheme [2] [9] where the CRs control their power in order to ensure a constant average received power level to a single CR receiver. A practical example for such a scenario would be wireless sensor nodes transmitting information to their base station.

Without loss of generality, we can take the PR to be located at the origin and the CR receiver be located a distance R_{CR}

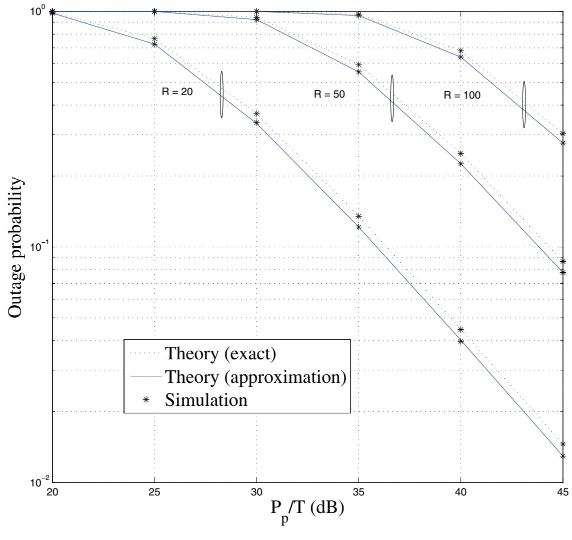

Fig. 2. Outage probability vs $\frac{P_p}{T}$ for different values of R, with $R_E = 500$, $R_G = 50$, $P_s = 20\,\text{dB}$, $\beta = 10^{-4}$,$\alpha = 2$, $\sigma_n^2 = 1$.

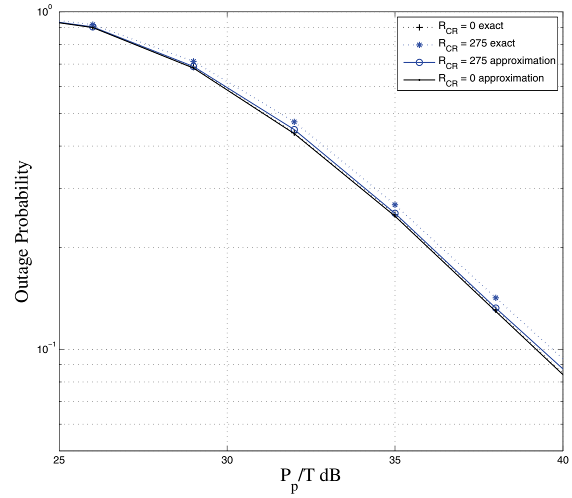

Fig. 3. Outage probability vs $\frac{P_p}{T}$ for power controlled CRs under $R_{CR} = 0$ and $R_{CR} = 275$ for $\beta = 10^{-4}$, $\alpha = 2$, $R_E = 200$, $R_G = 25$, $R = 30$, $P_{rec} = -30\,\text{dB}$, $\sigma_n^2 = 1$

from the PR along the x-axis. Let θ be the angle from the positive x-axis to the nearest CR transmitter. We assume θ to be uniformly distributed between 0 and 2π. Let P_{rec} be the average power level ensured at the CR receiver. The average transmit power of the nearest CR can be written as

$$P_{nr} = P_{rec}\left(r_{min}^2 + R_{CR}^2 - 2r_{min}R_{CR}\cos\theta\right)^{\frac{\alpha}{2}}. \qquad (11)$$

We write the MGF as

$$M_{I_{app}}^{PC}(s) = \mathbb{E}_\theta[\mathbb{E}_{r_{min}}[\mathbb{E}_X[e^{-sP_{nr}r_{min}^{-\alpha}X}]]].$$
$$= \sum_{t=0}^{\infty}(-sP_{rec})^t$$
$$\times\mathbb{E}_\theta\left[\mathbb{E}_{r_{min}}\left[r_{min}^{-\alpha t}(r_{min}^2+R_{CR}^2- 2r_{min}R_{CR}\cos\theta)^{\frac{\alpha t}{2}}\right]\right] \qquad (12)$$

When α is an even number, we can simplify (12) using the binomial expansion and obtain (13) for $M_{I_{app}}^{PC}(s)$ after averaging with respect to θ and r_{min}. The convergence of (12) is relatively lower compared to (8).

IV. Outage Analysis

In the previous section, we obtained the MGF of the aggregate interference for two different scenarios. In this section, the outage probability at the PR due to the interference and noise will be derived using the previously obtained MGFs.

The signal to interference and noise ratio γ at the PR is denoted as

$$\gamma = \frac{P_pR^{-\alpha}Z}{I + \sigma_n^2}, \qquad (14)$$

where P_p denotes the power level of the primary transmitter, R is the distance between PR and its primary transmitter, and σ_n^2 is the noise variance. Z is the random channel gain dependent on fading. We consider Rayleigh fading only in our analysis for the primary network. Although the analysis may be extended to other fading and shadowing models, it is omitted due to space constraints. Thus, Z is an exponential random variable. The CDF of γ given I can be written as $F_{\gamma/I}(x) = \Pr\left(\frac{P_pR^{-\alpha}Z}{I+\sigma_n^2} \le x\right) = 1 - e^{\left(-\frac{x(I+\sigma_n^2)}{P_pR^{-\alpha}}\right)}$. Integrating with respect to I gives the $F_\gamma(x)$ as

$$F_\gamma(x) = 1 - e^{\left(-\frac{x\sigma_n^2}{P_pR^{-\alpha}}\right)}M_I\left(\frac{x}{P_pR^{-\alpha}}\right). \qquad (15)$$

Substituting the threshold level T for x gives us the outage probability.

V. Numerical Results

In this section, we compare the exact outage probability, and the nearest CR approximation for different system parameters, and obtain insights as to when the nearest CR approximation is appropriate.

Fig. 2 compares the outage probability of the constant powered CR's scenario under different primary tranceiver distance R values. When the primary tranceiver distance increases, the outage probability increases significantly. Interestingly, R does not affect the approximation error. This is because R is not a parameter affecting the aggregate interference. In addition, Fig. 2 compares the theoretical and simulated values, and the match is excellent.

The outage with respect to the normalized primary transmit power $\frac{P_p}{T}$ for the CRs with basic power control is plotted in Fig. 3. The approximation is tight for low average ensured power levels P_{rec} at the CR receiver. When the distance between the PR and the CR receiver R_{CR} increases, the outage probability increases correspondingly, and the approximation diverges from the exact value.

In Fig. 4, the outage probability is plotted as a function of the CR transmit power. It is seen that when the path loss exponent is high, the approximation is extremely accurate. This is because, at higher path loss factors, the interference is dominated by the nearest CR. In many practical wireless channels, such as in dense urban environments or hilly terrain, the path loss exponent is particularly high and the nearest CR approximation is strongly valid. For a path loss exponent value of 2 (free space propagation), when the CR transmit power is

$$M_{I_{app}}^{PC}(s) = e^{-A_I\beta} + 2\pi^2\beta e^{\pi\beta R_G^2}\sum_{t=0}^{\infty}(-P_{rec}s)^t \sum_{k=0}^{\frac{\alpha t}{2}}\binom{\frac{\alpha t}{2}}{k}\sum_{l=0}^{\lfloor\frac{k}{2}\rfloor}\binom{k}{2l}R_{CR}^{2k-2l}\frac{(4l-1)!!}{(2l)!}\left(\frac{E_{k-l}(\pi\beta R_G^2)}{R_G^{2k-2l-2}} - \frac{E_{k-l}(\pi\beta R_E^2)}{R_E^{2k-2l-2}}\right) \quad (13)$$

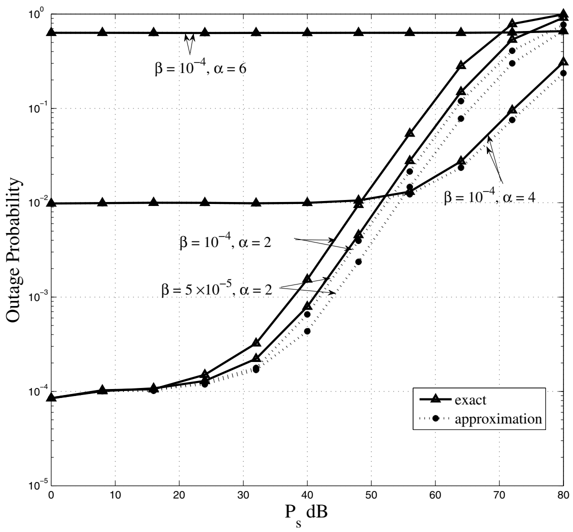

Fig. 4. The exact and approximate outage probability vs P_s for different values of β and α, with $R_E = 200$, $R_G = 20$, $R = 10$, $\frac{P_p}{T} = 60\,$dB, $\sigma_n^2 = 1$. The two curves for $\alpha = 6$ coincide.

low, the aggregate interference can be validly approximated by the interference from the nearest CR. Even when the CR transmit power is increased for free space propagation, the error is minimal, and when we reduce the CR density, the error gets further reduced.

VI. Conclusion

The aggregate interference for a Poisson point process of CR nodes distributed in an annular area encircling the PR was investigated, assuming Rayleigh fading and path loss. The aggregate interference was approximated as the interference from the nearest CR node. The PDF of the distance from the nearest CR to the PR was derived. In addition, the exact MGF of the nearest CR node approximation was derived for two scenarios. Namely, when the CRs employ no power control or a distance based power control scheme. The outage probability was derived by using the MGFs. Simulations confirmed our analysis and we showed that the approximation is extremely tight under certain situations. These include higher path loss exponents, lower node densities, and larger exclusion regions. It was also observed that the transmitter receiver distance of the primary network had minimal bearing on the accuracy of the approximation.

Appendix I: Proof of $f_{r_{min}}(x)$

The conditional CDF of r_{min} given N can be obtained as

$$F_{r_{min}/N}(x) = \Pr[r_{min} < x] = 1 - [\Pr(r_i > x)]^N. \quad (16)$$

The PDF of r_i can be expressed as

$$f_R(r_i) = \begin{cases} 2\frac{\pi r_i}{A_I} & , R_G < r_i < R_E \\ 0 & , \text{otherwise} \end{cases}. \quad (17)$$

From (17), $\Pr(r_i{>}x)$ can be found out to be $\frac{\pi}{A_I}\left(R_E^2 - x^2\right)$. Substituting this in (16) and differentiating, we get

$$f_{r_{min}/N=n}(x) = 2n\left(\frac{\pi}{A_I}\right)^n x\left(R_E^2 - x^2\right)^{n-1}. \quad (18)$$

Averaging (18) with respect to (1) for $N \neq 0$, we get

$$\begin{aligned} f_{r_{min}}(x) &= 2x\sum_{n=1}^{\infty} n\left(\frac{\pi}{A_I}\right)^n\left(R_E^2 - x^2\right)^{n-1}\frac{(\beta A_I)^n e^{-\beta A_I}}{n!} \\ &= 2\pi\beta e^{-\beta A_I} x\sum_{n=1}^{\infty}\frac{\left(\pi\beta\left(R_E^2 - x^2\right)\right)^{n-1}}{(n-1)!}. \end{aligned} \quad (19)$$

The series (19) can be summed as an exponential, and we obtain $f_{r_{min}}(x)$ as (5). When $N = 0$, $f_{r_{min}}(x)$ is non-exixtant with probability $e^{-A_I\beta}$.

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