Relay Selection and Performance Analysis in Multiple-User Networks

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***Abstract*—This paper investigates the relay selection (RS) problem in networks with multiple users and multiple common amplify-and-forward (AF) relays. We first give an optimality measure for RS in multiple-user relay networks. An optimal RS (ORS) algorithm is then provided, which is an extension of an RS scheme in the literature that maximizes the minimum end-to-end receive signal-to-noise ratio (SNR) of all users. The complexity of the ORS is quadratic in both the number of users and the number of relays. A suboptimal RS (SRS) scheme is also proposed, which has linear complexity in the number of relays and quadratic complexity in the number of users. Furthermore, diversity orders of both the ORS and the proposed SRS are derived and compared with those of a naive RS scheme and the single-user case. The ORS is shown to achieve full diversity, while the diversity order of the SRS decreases with the number of users. For two-user networks, the closed-form outage probabilities and array gains corresponding to the minimum SNR of the users in the RS schemes are derived. It is proved that the advantage of the SRS over the naive RS scheme increases as the number of relays in the network increases. Simulation results are provided to corroborate the analytical results.**

***Index Terms*—Array gain, diversity order, multiple-user networks, outage probability, relay selection.**

I. Introduction

COOPERATIVE communication, a concept that takes advantage of the possible cooperation among multiple nodes in a network to form virtual multiple-input-multiple-output (MIMO) configuration, has received significant attention in the wireless community [1], [2]. For cooperative networks with multiple relays, relay selection (RS) is an important and effective technique because properly designed RS can achieve full spatial diversity with low complexity and overhead.

RS problems have been extensively studied in the open literature for networks with single source-destination pair, referred to as *single-user networks*, e.g., [3]–[7]. Recently, there is increasing interest in relay networks with multiple source-destination pairs, referred to as *multiple-user networks*. Typical multiple-user networks include ad-hoc, sensor, and mesh networks. However, RS schemes proposed for single-user networks cannot be extended to multiple-user networks straightforwardly due to the challenges in the performance evaluation, the competition among users, and the increased complexity [8]. In the literature, research efforts on RS in multiple-user networks are limited, a brief review of which is provided in the following.

For a multiple-user multiple-relay network, in [9] and [10], under amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying respectively, joint user selection and relay selection is considered. The user with the best direct link quality is first selected, and then a relay is selected for this user to obtain the maximum end-to-end receive signal-to-noise ratio (SNR). Other users are not allowed to transmit. So in [9] and [10], only one user with its best relay is selected at a time, and there is no user competition. In [11], for a multiple-user multiple-relay network, a relay grouping algorithm is proposed to maximize the network sum-rate or the minimum achievable rate among users.

There are some limited works [12]–[16] on multiple-user multiple-relay networks in which multiple users can select relays simultaneously. In this case, due to the user competition for relays, the RS problem becomes more challenging since the RS for one user may impact the choices of others. In [12], grouping and partner selection for cooperative networks with DF relaying are considered. It investigates how to allocate relays to assist users and analyzes the effect of allocation policies on network performance. For each user, the relays are selected based on the strength of the user's channels to the relays. In [13], a single-user network is first considered. Ensuring that relaying can achieve a larger channel capacity than direct transmission, a sufficient condition based on channel quality is derived to find a feasible set of relays for the user. Then the work is extended to the multiple-user case, in which a semi-distributed RS is proposed to maximize the minimum user capacity. However, the proposed scheme does not guarantee optimality because each user chooses a relay in its feasible set randomly. In [14], an RS scheme that maximizes the minimum achievable rate among all users is proposed. The complexity of the scheme is linear in the number of users and quadratic in the number of relays. The work in [14] focuses on the proof of the optimality of the RS scheme, but analytical performance evaluation is not provided. Reference [15] introduces further adjustments to the RS scheme in [14] such that a user is allowed to be helped by multiple relays, assuming that the relays use orthogonal channels. Again analytical performance evaluation is not provided. Reference [16] considers a more general case that a user can be helped by multiple relays and each relay

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can help multiple users, where all users and relays employ orthogonal channels. Game theory is used for relay selection to achieve social optimality. The complexity of the proposed algorithm is linear in the number of relays and quadratic in the number of users.

In this research, we consider a multiple-user multiple-relay network where each user can only be helped by a single relay and one relay can help at most one user. The new contributions of this paper are listed as follows.

- We specify a new optimality measure of RS for multiple-user relay networks. Compared with the optimality used in [14] (maximizing the minimum receive SNR among users), the new measure guarantees the uniqueness of the optimal solution and takes into account the performance of all other users in addition to the worst one.
- An optimal relay selection (ORS) scheme is provided, which is a straightforward extension of the minimum-SNR-maximizing RS scheme proposed in [14]. The complexity of the ORS is quadratic in both the number of users and the number of relays. We also propose a sub-optimal relay selection (SRS) scheme, whose complexity is linear in the number of relays and quadratic in the number of users.
- Diversity orders of the ORS and the SRS are analyzed theoretically using order statistics. For a network with N users, $N_r(\geq N)$ relays, and no direct links, for the ORS, all users have diversity order N_r, which is the full diversity order of a single-user network with N_r relays. Thus, user competition for relays does not affect diversity order if optimally designed. For the SRS, the diversity order of all users is shown to be $N_r - N + 1$. When there are direct links in the network (for which $N_r \geq N$ is not required), the users have diversity order $N_r + 1$ and $\max(N_r - N + 2,\ 1)$ for the ORS and the SRS, respectively.
- For two-user networks, tight upper bounds on the outage probabilities of the ORS and the SRS are derived. It is shown analytically that the SRS achieves better array gain than a naive RS, and its advantage increases as the number of relays increases.
- Simulated outage probabilities are illustrated to justify our analytical results and compare the ORS, the SRS, and a naive RS.

The rest of the paper is organized as follows. The system model and order statistics of receive SNRs are provided in Section II. RS schemes are introduced and discussed in Section III including the ORS scheme (an extended version of the RS scheme in [14]), the propose SRS scheme, and the naive RS scheme for performance benchmark. Diversity orders of the schemes are analyzed in Section IV, and outage probability bounds for two-user networks are derived in Section V. The case with direct links from users to destinations is investigated in Section VI. Numerical/simulation results and conclusion are presented in Section VII and Section VIII, respectively.

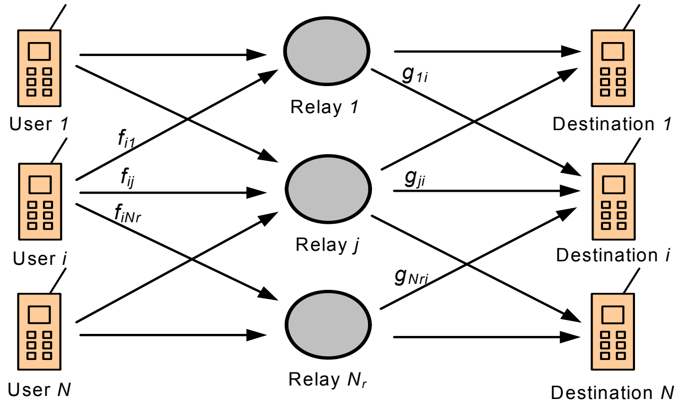

Fig. 1. A multiple-user multiple-relay network model.

II. System Model and Order Statistics

A. System Model

Consider a wireless relay network with N users sending information to their destinations via N_r relay nodes, as shown in Fig. 1. Each node has a single antenna. The power budget is P for each user and Q for each relay. The fading coefficients from the ith user to the jth relay and from the jth relay to the ith destination are denoted as f_{ij} and g_{ji}, respectively. There are no direct links between the users and destinations. Networks with direct links are considered in Section VI. All channels are assumed to be independent and identically distributed (i.i.d.) complex Gaussian with zero-mean and unit-variance, i.e., $f_{ij}, g_{ji} \sim \mathcal{CN}(0,1)$. The channel amplitudes, $|f_{ij}|$ and $|g_{ji}|$, thus follow the Rayleigh distribution.

The users need the relays' help to send information. We assume that each user will be helped by one and only one relay. This assumption minimizes the synchronization requirement on the network. We also assume that each relay can help at most one user to avoid overloading one relay and to potentially prolong the network lifetime [2], [13]. Thus, we need $N_r \geq N$.

A conventional half-duplex two-phase transmission protocol is used [2]. The first phase is the transmission from the users to the relays, and the second phase is the transmission from the relays to the destinations. To avoid interference, the users are assigned orthogonal channels using frequency-division or time-division multiple access. Without loss of generality, the transmission of User i helped by Relay j is elaborated here. Denote the information symbol of User i as x_i, which has unit average energy. Applying AF relaying with coherent power coefficient [17], the receive signal at Destination i is

$$y_{ij} = \sqrt{\frac{PQ}{P|f_{ij}|^2+1}} f_{ij} g_{ji} x_i + \sqrt{\frac{Q}{P|f_{ij}|^2+1}} g_{ji} n_{r_j} + n_{d_i}, \tag{1}$$

where n_{r_j} and n_{d_i} are the additive noises at Relay j and Destination i, respectively, which are assumed to be i.i.d. following $\mathcal{CN}(0,1)$. The end-to-end receive SNR, or SNR in short, of User i thus equals

$$\gamma_{ij} = \frac{PQ|f_{ij}g_{ji}|^2}{P|f_{ij}|^2 + Q|g_{ji}|^2 + 1}. \tag{2}$$

To help the RS procedure, we consider all relay choices for the users and construct a receive SNR matrix as

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N_r} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2N_r} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NN_r} \end{pmatrix}. \quad (3)$$

The main problem of this paper is to find high-performance RS schemes. Nevertheless, low complexity is also desired for practical consideration. For RS in single-user networks, the performance criterion is straightforward, and the competition is only among relays, not users. In contrast, RS in multiple-user networks is a lot more challenging: (i) the multiple communication tasks of the users complicate the performance criterion specification and performance analysis; (ii) in addition to the competition among relays, there is competition among users to select their best relays in order to maximize their individual advantages; and (iii) the complexity of exhaustive search is $\mathcal{O}(N_r^N)$, which is very high for large networks. A good RS scheme should take into account the overall network quality-of-service, the fairness among users, and the complexity.

B. Optimality Measure

In [14], an RS scheme is proposed, which maximizes the minimum transmission rate of the users, or equivalently, maximizes the minimum SNR of the users. With this RS criterion, however, only the worst user's performance is optimized and the RS solution may not be unique. We use a modified definition for the optimality of an RS scheme. In specific, for N users, an RS scheme is called *optimal* if it has the following N properties:

- Property 1: the minimal SNR among the users is maximized. This equivalently means that the minimum achievable data rate of all users is maximized and the maximum outage or error rate of all users is minimized.
  Denote the set of users as $\mathcal{U}_1 = \{U_1, U_2, \cdots, U_N\}$ and the SNR of User i as γ_{U_i}. This property can be mathematically represented as $\max\min_{u\in\mathcal{U}_1}\gamma_u$ being achieved.
  We also introduce here an important variable $\gamma_{\min}$, denoting the worst SNR among all users for a given RS scheme, i.e., $\gamma_{\min} = \min_{u\in\mathcal{U}_1}\gamma_u$. This is a crucial notation for the analysis in later sections.
- Property k ($k = 2, ..., N$): conditioned on the preceding $k-1$ properties, the kth minimal SNR of all user SNRs is maximized.
  Let $\mathcal{U}_k$ denote the set of the $N-k+1$ users with the largest $N-k+1$ SNRs. This property can be mathematically represented as $\max\min_{u\in\mathcal{U}_k}\gamma_u$ being achieved.

In contrast to maximizing the minimum receive SNR only (which is used in [14]), the new optimality definition guarantees the uniqueness of the optimal solution and considers all other users in addition to the worst one.

C. Performance Measure

Popular performance measures for cooperative networks are outage probability, diversity order, and array gain. For a user, an outage occurs if the user SNR drops below a predetermined SNR threshold γ_{th}. Denote the outage probability corresponding to $\gamma_{\min}$ (the worst SNR among all users) as $P_{\text{out,upp}}$. For all users, since their SNRs are always not lower than $\gamma_{\min}$, their outage probabilities are upper bounded by $P_{\text{out,upp}}$. Diversity order shows how fast the outage probability decreases with the increase in the transmit power P in the high transmit power region. It is conventionally defined as $d \triangleq -\lim_{P\to\infty}\frac{\log P_{\text{out}}}{\log P}$ [18] where P_{out} is the outage probability. For the same reason, the diversity order derived based on $\gamma_{\min}$ is a lower bound on the diversity orders of all users. When two designs have the same diversity order d, array gain can be used for performance comparison, given as $\lim_{P\to\infty}(P^d P_{\text{out}})^{-1}$.

D. SNR Distribution and Order Statistics

In this subsection, we review results on the SNR distribution and order statistics to be used for theoretical analysis later. Since all channels are i.i.d., γ_{ij}'s are also i.i.d.. Denote their cumulative distribution function (CDF) and probability density function (PDF) as $F_\gamma(x)$ and $f_\gamma(x)$, respectively. From the results in [19], we have

$$F_\gamma(x) = 1 - 2\sqrt{\frac{x(x+1)}{PQ}}e^{-(\frac{1}{P}+\frac{1}{Q})x}\mathcal{K}_1\left(2\sqrt{\frac{x(x+1)}{PQ}}\right), \quad (4)$$

where $\mathcal{K}_1(\cdot)$ is the modified first-order Bessel function of the second kind. Since $t\mathcal{K}_1(t) \approx 1$ for small t [20], when $P, Q \gg \max\{x, 1\}$, $F_\gamma(x)$ can be well-approximated as

$$\begin{aligned} F_\gamma(x) &\approx 1 - e^{-(\frac{1}{P}+\frac{1}{Q})x} \\ &= \left(\frac{1}{P}+\frac{1}{Q}\right)x - \sum_{i=2}^{\infty}\frac{1}{i!}\left[-\left(\frac{1}{P}+\frac{1}{Q}\right)\right]^i x^i. \end{aligned} \quad (5)$$

If we sort γ_{ij}'s in descending order[1] as

$$\gamma_1 > \cdots \gamma_k > \cdots > \gamma_{NN_r}, \quad (6)$$

where γ_k is the kth largest element of $\mathbf{\Gamma}$, and use the results of eq. (7)-(14) in [21] of order statistics, the PDF of γ_k can be given as

$$f_{\gamma_k}(x) = \frac{(NN_r)!F_\gamma(x)^{NN_r-k}[1-F_\gamma(x)]^{k-1}f_\gamma(x)}{(NN_r-k)!(k-1)!}. \quad (7)$$

By using binomial expansion and subsequently applying integration by parts, the CDF of γ_k can be derived from $F_{\gamma_k}(x) = \int_0^x f_{\gamma_k}(t)dt$ to yield

$$F_{\gamma_k}(x) = \sum_{i=0}^{k-1}\frac{(NN_r)!\binom{k-1}{i}(-1)^i F_\gamma(x)^{NN_r-k+i+1}}{(NN_r-k+i+1)(NN_r-k)!(k-1)!}. \quad (8)$$

[1] Since the users' SNRs are continuous random variables, the probability that two users have the same SNR is 0.

E. Discussions on Channel State Information (CSI) and Training

The RS schemes in this paper are centralized. Thus a master node, which can be a destination, controls the RS process and is assumed to have perfect and global CSI. Furthermore, to conduct AF relaying, each relay needs to know its channels with the users. In this subsection, for the case when time-division multiple access[2] is adopted to implement orthogonal channels of the users, we discuss how to obtain the CSI information, and the induced overhead.

We assume that Destination 1 is the master node. A possible three-step training scheme for the CSI requirement is provided in the following.

In Step 1, Relay 1 broadcasts a pilot. Each destination can thus estimate its channel with Relay 1. By repeating this for all relays, all destinations know their channels with the relays, i.e., Destination i knows $g_{1i}, \cdots, g_{N_r i}$. This step takes N_r time slots in total.

In Step 2, a two-phase distributed space-time coding is conducted for training [22]. User 1 sends an $N_r \times 1$ pilot vector to the relays, and the relays conduct distributed space-time coding to forward the information to the master node, Destination 1. By using the estimation scheme in [22], based on the estimated $g_{11}, \cdots, g_{N_r 1}$ in Step 1, Destination 1 can estimate the channels from User 1 to the relays $f_{11}, \cdots, f_{1N_r}$. In the mean while, each relay can estimation its channel with User 1. By repeating this for all users, after this step, Destination 1 knows the channels from all users to all relays, and each relay knows its channels with all users. That is, Destination 1 knows $f_{11}, \cdots, f_{1N_r}, \cdots, f_{N1}, \cdots, f_{NN_r}$, and Relay j $(j = 1, 2, ..., N_r)$ knows $f_{1j}, \cdots, f_{Nj}$. This step takes $2NN_r$ time slots.

In Step 3, Destination i $(i \neq 1)$ uses N_r time slots to forward its estimated $g_{1i}, \cdots, g_{N_r i}$ (obtained in Step 1) to Destination 1. Thus, after this step, Destination 1 knows all channels from the relays to the other destinations, i.e., $g_{12}, \cdots, g_{N_r 2}, \cdots, g_{1N}, \cdots, g_{N_r N}$. This step takes $(N-1)N_r$ time slots.

The required CSI is obtained at the master node and the relays after the aforementioned three steps. The total overhead is $N_r + 2NN_r + (N-1)N_r = 3NN_r$. No cross talks among the relays or among the users are required.

III. RS SCHEMES

For the N-user N_r-relay network, in [14], an RS scheme is developed, in which a "linear marking" mechanism is used to maximize the minimal SNR among all users. First an initial feasible relay node assignment is randomly chosen, by which each user–destination pair communicates with the help of a relay node. The relay assignment algorithm is then adjusted in a number of iterations. During each iteration, the user that has the minimal SNR, denoted as $\gamma_{\min}$, searches a better relay such that its SNR can be increased. If the better relay has been assigned to another user (say User a), User a tries to change to another relay (say Relay b) under the condition that the resulted SNR is higher than $\gamma_{\min}$. If Relay b has been assigned to another user, further adjustment to that user's relay assignment is needed. So the relay adjustment of the user with the minimal SNR may have a chain effect on the relay assignment of multiple users. If there exists such an adjustment, the minimal SNR of all the users is increased, and the scheme moves to the next iteration; otherwise, the scheme terminates, which means that the minimal SNR of all users is maximized. The worst-case complexity (measured by the number of comparison operations) of the RS scheme is $\mathcal{O}(NN_r^2)$. Note that the RS scheme optimizes only the worst user's performance.

[2]For the case of frequency-division multiple access, the training scheme is similar.

Here we consider the following extension to the RS scheme in [14] to obtain the RS that is optimal with the definition specified in Section II. We first apply the RS scheme in [14] to find a solution that maximizes the minimal SNR. Suppose that the minimal SNR is with User i and Relay j. Then we delete User i from the user list and delete Relay j from the relay list, and apply the RS scheme in [14] again to the remaining users and relays. This procedure is repeated until all users find their relays. The resulted relay assignment achieves the optimality defined in Section II. We refer to it as the *optimal relay selection (ORS)* in the sequel. The worst-case complexity of the ORS scheme is $\mathcal{O}(N^2N_r^2)$, which is quadratic in the number of users and the number of relays.

For a network with a large number of relays, quadratic complexity in the number of relays may be undesirable. Thus, we propose a *suboptimal relay selection* (SRS) scheme, described in Algorithm 1, whose complexity is linear in the number of relays.

Algorithm 1 The suboptimal RS (SRS) scheme

1: Assign $\mathbf{\Gamma}_0 = \mathbf{\Gamma}$.
2: **for** $k = N : 1$ **do**
3: The number of rows in $\mathbf{\Gamma}_0$ is denoted as k. Find the maximum element of each row of $\mathbf{\Gamma}_0$. Denote the maximum elements of the k rows as $\gamma_{1j_1^*}, \cdots, \gamma_{kj_k^*}$, respectively.
4: Find $\gamma_{i^*j^*} = \min\left(\gamma_{1j_1^*}, \cdots, \gamma_{kj_k^*}\right)$, and assign Relay j^* to User i^*.
5: Delete the j^*th column and the i^*th row of $\mathbf{\Gamma}_0$.

The main idea of the SRS is to find a relay for each user sequentially (not necessarily in the order of the user index) to achieve a complexity that is linear in the number of relays. In Step 3, the best relay for each user that has not yet selected a relay is found. To avoid conflict in RS, in Step 4, the user with the smallest best SNR selects its best relay. This procedure is repeated until all users have made their selections.

The worst-case complexity of the SRS is obtained as follows. If we consider the lth round of RS, the required number of comparison operations in Step 3 to find the maximum elements of $(N-l+1)$ rows is $(N_r - l)(N-l+1)$; the required number of comparison operations in Step 4 is $N-l$. Therefore, the total complexity for the SRS is

$$\begin{aligned} \mathcal{C} &= \sum_{l=1}^{N} \left[(N_r - l)(N-l+1) + (N-l)\right] \\ &= \frac{N(3NN_r + 3N_r - N^2 - 5)}{6}. \end{aligned} \tag{9}$$

By noting that $N \leq N_r$, from (9), the complexity behaves as $\mathcal{O}\left(N^2N_r\right)$, linear in the number of relays and quadratic in the number of users. Therefore, for networks with many more relays than users, the SRS is advantageous in complexity.

The SRS does not always result in the optimal solution. When the best relays of two or more of the users are the same, the SRS may lead to a suboptimal result. To see this, consider the following example of a network with two users and four relays. For one channel realization, we have the SNR matrix: $\mathbf{\Gamma} = \begin{pmatrix} 1.08 & 0.14 & 0.09 & 0.05 \\ 1.07 & 0.15 & 0.50 & 0.04 \end{pmatrix}$. The ORS selects Relay 1 for User 1 and Relay 3 for User 2, with SNR being 1.08 and 0.5 for the two users, respectively. This is the optimal RS solution. The SRS however selects Relay 2 for User 1 and Relay 1 for User 2, with the SNRs being 0.14 and 1.07 for the two users, respectively, which is not optimal.

In this section, we also introduce a naive RS scheme as a benchmark in evaluating the ORS and the SRS schemes. Intuitively, for the multiple-user network, a naive method is to assign the best relays to the users one by one from User 1 to User N. That is, User 1 first selects its best relay (the relay that results in the maximum SNR). Then User 2 selects its best relay among the remaining $N_r - 1$ relays; and so on so forth until User N selects its best relay among the remaining $N_r - N + 1$ relays. As to the complexity, $N_r - k$ comparison operations are needed to find the best relays for User k. Thus, the overall complexity is $\sum_{k=1}^{N}(N_r - k) = \frac{1}{2}(2NN_r - N^2 - N)$, which is linear in both the number of relays and the number of users. Obviously, the naive RS does not always produce the optimal RS result.

IV. Diversity Order Analysis

In this section, we analyze the diversity orders of the RS schemes introduced in Section III.

A. Diversity Order of ORS

To our best knowledge, performance analysis of the ORS is not available in the literature. This paper is the first that derives the diversity order of the ORS.

Recall that the receive SNR matrix for a general network is given in (3), the SNR ordering is given in (6), and $\gamma_{\min}$ is the smallest SNR among users. We have the following lemma and theorem.

Lemma 1: With the ORS, the worst case (i.e., the case when $\gamma_{\min}$ has the lowest position in the ordering of all elements in $\mathbf{\Gamma}$) is $\gamma_{\min} = \gamma_{(N-1)N_r+1}$. And a sufficient and necessary condition for the worst case is: (a) the smallest N_r elements of $\mathbf{\Gamma}$, i.e., $\gamma_{(N-1)N_r+1}, \gamma_{(N-1)N_r+2}, \ldots, \gamma_{NN_r}$, are all in the same row[3], when $N < N_r$; or (b) the smallest N_r elements of $\mathbf{\Gamma}$ are either in the same row or in the same column[4], when $N = N_r$.

Proof: We first prove $\gamma_{\min} \geq \gamma_{(N-1)N_r+1}$ with the ORS by using proof by contradiction. Assume that for a given channel realization, we have $\gamma_{\min} < \gamma_{(N-1)N_r+1}$ in the ORS result. Then in $\mathbf{\Gamma}$, the number of elements smaller than $\gamma_{\min}$ is less than or equal to $N_r - 2$. Without loss of generality, assume that $\gamma_{\min}$ is located in the i^*th row and the j^*th column of $\mathbf{\Gamma}$, i.e., $\gamma_{\min} = \gamma_{i^*j^*}$. Let $\mathcal{R}_<$ denote the set of row indices of the elements (in $\mathbf{\Gamma}$) that are smaller than $\gamma_{\min}$ and are located in the j^* column. Let $\mathcal{C}_\leq$ denote the set of indices of the columns in which $\gamma_{\min}$ and all elements (in $\mathbf{\Gamma}$) smaller than $\gamma_{\min}$ are located. Then we have $|\mathcal{C}_\leq| \leq (N_r - 2) - |\mathcal{R}_<| + 1$, where $|\cdot|$ means the cardinality of a set. Therefore, for the set (of column indices) $\overline{\mathcal{C}_\leq} \triangleq \{1, 2, ..., N_r\} \backslash \mathcal{C}_\leq$, we have $|\overline{\mathcal{C}_\leq}| \geq N_r - ((N_r - 2) - |\mathcal{R}_<| + 1) = |\mathcal{R}_<| + 1$. Note that all elements in any column in $\overline{\mathcal{C}_\leq}$ are larger than $\gamma_{\min}$. Since $|\overline{\mathcal{C}_\leq}| > |\mathcal{R}_<|$, there exists a column index denoted $j^\dagger \in \overline{\mathcal{C}_\leq}$ such that in the ORS result for the given channel realization, Relay $j^\dagger$ is either not assigned to any user or is assigned to a user, denoted User $i^\dagger$, satisfying $i^\dagger \notin \mathcal{R}_<$. If Relay $j^\dagger$ is not assigned to any user, we change User i^* from Relay j^* to Relay $j^\dagger$, which gives User i^* an SNR larger than $\gamma_{\min}$. If Relay $j^\dagger$ is assigned to User $i^\dagger$, we switch the relay assignment for Users i^* and $i^\dagger$ (i.e., assign Relay $j^\dagger$ to User i^* and assign Relay j^* to User $i^\dagger$), and after the switching, both users have SNRs larger than $\gamma_{\min}$. In either case, the new RS result has a minimal user SNR larger than $\gamma_{\min}$. This contradicts that the ORS maximizes the minimal SNR of the users.

The sufficiency of (a) and (b) is straightforward. Thus, it can be concluded that the worst case with the ORS is $\gamma_{\min} = \gamma_{(N-1)N_r+1}$.

Next we prove the necessity of the condition for the case of $N < N_r$, by using proof by contradiction. Assume that for a channel realization, $\gamma_{(N-1)N_r+1}, \gamma_{(N-1)N_r+2}, \ldots, \gamma_{NN_r}$ are not in the same row in $\mathbf{\Gamma}$; but we have $\gamma_{\min} = \gamma_{(N-1)N_r+1}$ in the ORS result. Denote the row index and column index of $\gamma_{\min}$ as i^* and j^*, respectively, i.e., $\gamma_{\min} = \gamma_{i^*j^*}$. Consider two scenarios as follows.

- When none of the elements (in $\mathbf{\Gamma}$) smaller than $\gamma_{\min}$ are located in the j^*th column: In the i^*th row of $\mathbf{\Gamma}$, there exists at least one element larger than $\gamma_{\min}$. Assume such an element is located in the $j^\dagger$th column. If Relay $j^\dagger$ is not assigned to any user, then we change User i^* from Relay j^* to Relay $j^\dagger$, which gives User i^* an SNR larger than $\gamma_{\min}$. If Relay $j^\dagger$ is assigned to a user, denoted as User $i^\dagger$, we switch the relay assignment for Users i^* and $i^\dagger$, and after the switching, both users have SNRs larger than $\gamma_{\min}$. In either case, the new relay selection result has a minimal user SNR larger than $\gamma_{\min}$.
- When at least one of the elements (in $\mathbf{\Gamma}$) smaller than $\gamma_{\min}$ is located in the j^*th column: We remove the j^*th column from $\mathbf{\Gamma}$, and apply the ORS to the remaining $\mathbf{\Gamma}$.[5] The new relay selection result has a minimal user SNR not smaller than $\gamma_{(N-1)N_r}$ (note that $\gamma_{(N-1)N_r} > \gamma_{\min}$).

Since a contradiction is caused in either scenario, the sufficiency of (a) is proved.

Next we prove the necessity of the condition for the case of $N = N_r$, by using proof by contradiction. When $N = N_r$, all relays are assigned. Assume that for a channel realization, $\gamma_{(N-1)N_r+1}, \gamma_{(N-1)N_r+2}, \ldots, \gamma_{NN_r}$ are neither in the same row nor in the same column of $\mathbf{\Gamma}$; but we have $\gamma_{\min} = \gamma_{(N-1)N_r+1}$ in the ORS result. Denote the row index

[3]In other words, the smallest N_r elements of $\mathbf{\Gamma}$ are the SNRs of a particular user helped by the N_r relays, respectively.

[4]In other words, the smallest N elements of $\mathbf{\Gamma}$ are the SNRs of the N users, respectively, if a particular relay is chosen to help them.

[5]Note that the remaining $\mathbf{\Gamma}$ has N rows and $(N_r - 1)$ $(\geq N)$ columns, and thus the ORS can be applied.

and column index of $\gamma_{\min}$ as i^* and j^*, respectively, i.e., $\gamma_{\min} = \gamma_{i^*j^*}$. Note that the total number of elements (in $\mathbf{\Gamma}$) smaller than $\gamma_{\min}$ is $N_r - 1$. Consider two scenarios as follows.

- Among the $2N_r - 1$ elements in the i^*th row and the j^*th column of $\mathbf{\Gamma}$, when the total number of elements smaller than $\gamma_{\min}$ is $N_r - 1$ (in other words, the i^*th row and the j^*th column of $\mathbf{\Gamma}$ contain all elements smaller than $\gamma_{\min}$): Since the i^*th row does not contain all the smallest N_r elements of $\mathbf{\Gamma}$, we can find an element in the i^*th row, say $\gamma_{i^*j^\dagger}$ (i.e., the element in the $j^\dagger$th column), that is larger than $\gamma_{\min}$. Since the j^*th column does not contain all the smallest N_r elements of $\mathbf{\Gamma}$, we can find an element in the j^*th column, say $\gamma_{i^\ddagger j^*}$ (i.e., the element in the $i^\ddagger$th row), that is larger than $\gamma_{\min}$. Then we can have a new relay selection: first assign Relay $j^\dagger$ to User i^*, and assign Relay j^* to User $i^\ddagger$ (note that after the assignments, both Users i^* and $i^\ddagger$ have SNRs larger than $\gamma_{\min}$); then remove the i^*th and the $i^\ddagger$th rows and the j^*th and the $j^\dagger$th columns in $\mathbf{\Gamma}$, and apply the ORS to the remaining $\mathbf{\Gamma}$ (in which all elements are larger than $\gamma_{\min}$) for relay assignments for other users. The new relay selection result has a minimal user SNR larger than $\gamma_{\min}$.
- Among the $2N_r - 1$ elements in the i^*th row and the j^*th column of $\mathbf{\Gamma}$, when the total number of elements smaller than $\gamma_{\min}$ is less than $N_r - 1$: For the i^*th row, let $\mathcal{C}_<$ and $\mathcal{C}_>$ denote the set of column indices of the elements larger than and smaller than $\gamma_{\min}$, respectively. So $|\mathcal{C}_<| + |\mathcal{C}_>| = N_r - 1$. For the j^*th column, let $\mathcal{R}_<$ denote the set of row indices of the elements smaller than $\gamma_{\min}$. Since in the i^*th row and the j^*th column of $\mathbf{\Gamma}$, the total number of elements smaller than $\gamma_{\min}$ is less than $N_r - 1$, we have $|\mathcal{C}_<| + |\mathcal{R}_<| < N_r - 1$. Together with the fact $|\mathcal{C}_<| + |\mathcal{C}_>| = N_r - 1$, we have $|\mathcal{C}_>| > |\mathcal{R}_<|$, which means that, in the ORS result for the given channel realization, there exists a relay assignment, say assignment of Relay j' to User i', such that $j' \in \mathcal{C}_>$ and $i' \notin \mathcal{R}_<$. Then for the ORS result, if we switch the relay assignments for User i^* and User i' (note that after the switching, both User i^* and User i' have SNRs larger than $\gamma_{\min}$), the new relay selection result has a minimal user SNR larger than $\gamma_{\min}$.

Since a contradiction is caused in either scenario, the sufficiency of (b) is proved. ∎

Theorem 1: With the ORS, each user has diversity order N_r.

Proof: With the ORS, the best case (i.e., the case when $\gamma_{\min}$ has the highest position in the ordering of all elements in $\mathbf{\Gamma}$) is when the largest N elements of $\mathbf{\Gamma}$, i.e., $\gamma_1, \gamma_2, \ldots,$ and γ_N, are in different rows and different columns, and therefore, $\gamma_{\min} = \gamma_N$. From Lemma 1, the worst case with the ORS is $\gamma_{\min} = \gamma_{(N-1)N_r+1}$. So $\gamma_{\min}$ can take γ_N, γ_{N+1}, …, or $\gamma_{(N-1)N_r+1}$. Thus, an outage probability upper bound, $P_{\text{out,upp,ORS}}$, which is the outage probability corresponding to $\gamma_{\min}$, can be calculated as $P_{\text{out,upp,ORS}} = \sum_{k=N}^{(N-1)N_r+1} \text{Prob}(\gamma_{\min} = \gamma_k) F_{\gamma_k}(\gamma_{th})$. For presentation simplicity, we assume $Q = P$ in the sequel. The results can be generalized to unequal power case straightforwardly as long as the powers of all nodes have the same scaling. When $P \gg \max\{\gamma_{th}, 1\}$, using (5) and (8), we have the following approximation

$$\begin{aligned} &P_{\text{out,upp,ORS}} \\ &\approx \sum_{k=N}^{(N-1)N_r+1} \sum_{i=0}^{k-1} \left[\frac{\text{Prob}(\gamma_{\min} = \gamma_k)(NN_r)!\binom{k-1}{i}(-1)^i}{(NN_r - k + i + 1)(NN_r - k)!(k-1)!} \right. \\ &\quad \left. \times \left(\frac{2\gamma_{th}}{P} - \sum_{j=2}^{\infty} \frac{(-1)^j (2\gamma_{th})^j}{i! P^j} \right)^{NN_r - k + i + 1} \right]. \quad (10) \end{aligned}$$

Notice that the SNR elements of $\mathbf{\Gamma}$ are i.i.d.. Thus there are in total $(NN_r)!$ possible orderings of the SNR elements, each with probability $1/(NN_r)!$. For any given k, we can count the number of orderings that result in $\gamma_{\min} = \gamma_k$ and denote it as c_k. Thus $\text{Prob}(\gamma_{\min} = \gamma_k) = c_k/(NN_r)!$. Since for the ORS scheme (or the SRS scheme), only the relay ordering matters, c_k depends on N and N_r (the dimensions of $\mathbf{\Gamma}$) and k, and is independent of $F_\gamma(x)$, the PDF of each element of $\mathbf{\Gamma}$. That is, $\text{Prob}(\gamma_{\min} = \gamma_k)$ is independent of $F_\gamma(x)$, and thus, independent of P.

So, with respect to P, the highest order term in the summation in (10) is the term with $k = (N-1)N_r + 1$ and $i = 0$. Then we have

$$\begin{aligned} &P_{\text{out,upp,ORS}} \\ &\approx \frac{\text{Prob}\left(\gamma_{\min} = \gamma_{(N-1)N_r+1}\right) (2\gamma_{th})^{N_r} (NN_r)!}{N_r(N_r-1)!(NN_r - N_r)!} P^{-N_r} \\ &\quad + \mathcal{O}\left(P^{-(N_r+1)}\right) \sim \mathcal{O}\left(P^{-N_r}\right), \quad (11) \end{aligned}$$

which has diversity order N_r. Since the outage probability of each user is not higher than $P_{\text{out,upp,ORS}}$, we conclude that each user has diversity order not less than N_r.

Now we consider the outage probability of a user (say User K), $\text{P}_{\text{out,User-}K\text{,ORS}}$, which can be calculated as in (12) on the top of the next page, where (a) is because every user has the same chance of having the worst SNR and (b) comes from (11). This means that User K has diversity order not more than N_r. Together with the fact that each user has diversity order not less than N_r, it can be concluded that each user has diversity order N_r. ∎

For a single-user network with N_r relays, the best RS has diversity order N_r [7]. So Theorem 1 shows that for multiple-user networks, even with user competition for relays, the ORS can achieve full single-user diversity order.

B. Diversity Order of SRS

For the SRS scheme proposed in Section III, the following diversity order is proved.

Theorem 2: With the SRS, the diversity order of each user is $N_r - N + 1$.

Proof: Consider the SRS described in Algorithm 1 and the SNR ordering in (6). The same as the proof of Theorem 1, the best case is $\gamma_{\min} = \gamma_N$. The worst case happens when the remaining $N_r - (N-1)$ elements of $\mathbf{\Gamma}$ for the last user in the RS are the smallest $N_r - (N-1)$ elements. In this case, $\gamma_{\min} = \gamma_{(N-1)(N_r+1)+1}$. So $\gamma_{\min}$ can take $\gamma_N, \gamma_{N+1}, \ldots,$ or $\gamma_{(N-1)(N_r+1)+1}$. The outage probability

$$\begin{aligned} \mathrm{P}_{\text{out,User-}K\text{,ORS}} &= \textstyle\sum_{k=1}^{N} \big[\text{Prob}\left(\text{User } K \text{ is in outage}|\text{User } K \text{ has the } k\text{th best SNR among all users}\right) \\ &\qquad\qquad \cdot \text{Prob}\left(\text{User } K \text{ has the } k\text{th best SNR among all users}\right)\big] \\ &\geq \text{Prob}\left(\text{User } K \text{ is in outage}|\text{User } K \text{ has the worst SNR}\right)\text{Prob}\left(\text{User } K \text{ has the worst SNR}\right) \\ &\overset{(a)}{=} \tfrac{1}{N}\text{Prob(The user with the worst SNR is in outage)} = \tfrac{1}{N}P_{\text{out,upp,ORS}} \overset{(b)}{\sim} \mathcal{O}\left(P^{-N_r}\right) \end{aligned} \tag{12}$$

corresponding to $\gamma_{\min}$ can be calculated as $P_{\text{out,upp,SRS}} = \sum_{k=N}^{(N-1)(N_r+1)+1} \text{Prob}(\gamma_{\min} = \gamma_k)F_{\gamma_k}(\gamma_{th})$. Similar to the proof of Theorem 1, for any k, $\text{Prob}\left(\gamma_{\min} = \gamma_k\right)$ does not depend on P. When $Q = P \gg \max\{\gamma_{th}, 1\}$, we have

$$\begin{aligned} &P_{\text{out,upp,SRS}} \\ &\approx \frac{\text{Prob}\left(\gamma_{\min} = \gamma_{(N-1)(N_r+1)+1}\right)\left(2\gamma_{th}\right)^{N_r-N+1}(NN_r)!}{(N_r-N+1)(N_r-N)!\left((N-1)(N_r+1)\right)!} \\ &\qquad \times P^{-(N_r-N+1)} + \mathcal{O}\left(P^{-(N_r-N+2)}\right). \end{aligned} \tag{13}$$

This shows that an achievable diversity order of every user is not less than $N_r - N + 1$.

For a particular user, say User K, similar to the proof of Theorem 1, the outage probability can be lower bounded as

$$\mathrm{P}_{\text{out,User-}K\text{,SRS}} \geq \frac{1}{N}P_{\text{out,upp,SRS}} \sim \mathcal{O}\left(P^{-(N_r-N+1)}\right). \tag{14}$$

This means that User K has diversity order not more than $N_r - N + 1$. Together with the fact that each user has diversity order not less than $N_r - N + 1$, it can be concluded that each user has diversity order $N_r - N + 1$. ■

C. Diversity Order of Naive RS

Now, we analyze the diversity order of the naive RS. We consider the RS for User k ($k \in \{1, \cdots, N\}$). User k selects the best relay that results in the maximum SNR from the remaining $N_r - k + 1$ available relays. Denote the maximum SNR of User k as $\gamma_{\max,k}$. Since all SNRs, γ_{ij}'s, are i.i.d., the CDF of $\gamma_{\max,k}$ is $F_{\gamma_{\max,k}}(x) = [F_\gamma(x)]^{N_r-(k-1)}$. The minimum SNR of the first k users is $\min_{l=1,\ldots,k}\{\gamma_{\max,l}\}$, the CDF of which is $1 - \prod_{l=1}^{k}\left[1 - F_{\gamma_{\max,l}}(x)\right]$. Therefore, an upper bound on the outage probabilities of the first k users for the naive RS scheme is

$$P_{\text{out,upp,naive,first } k \text{ users}} = 1 - \prod_{l=1}^{k}\left[1 - F_\gamma(\gamma_{\text{th}})^{N_r-(l-1)}\right]. \tag{15}$$

When $Q = P \gg \max\{\gamma_{th}, 1\}$, from (5), we have

$$\begin{aligned} P_{\text{out,upp,naive,first } k \text{ users}} &\approx \left(2\gamma_{th}\right)^{N_r-k+1} P^{-(N_r-k+1)} \\ &\quad + \mathcal{O}\left(P^{-(N_r-k+2)}\right). \end{aligned} \tag{16}$$

Since the outage probability of each of the first k users is not higher than $P_{\text{out,upp,naive,first } k \text{ users}}$, the diversity order of all the first k users is not less than $N_r - k + 1$.

On the other hand, since User k has $N_r - k + 1$ relays to select, its diversity order is not higher than $N_r - k + 1$. Thus, User k has diversity order $N_r - k + 1$. In other words, User 1 has diversity order N_r, while User N has diversity order $N_r - N + 1$, which is the same as that of the SRS.

V. Outage Probability Analysis for Two-User Networks

In this section, we provide outage probability analysis for two-user relay networks with $N_r \geq 2$ relays. The receive SNR matrix of the network can then be written as

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1j} & \cdots & \gamma_{1N_r} \\ \gamma_{21} & \cdots & \gamma_{2j} & \cdots & \gamma_{2N_r} \end{pmatrix}. \tag{17}$$

A. Outage Probability Bound of ORS

As mentioned in Section II, we calculate the outage probability based on the minimum SNR, $\gamma_{\min}$, which provides an upper bound on both users' outage probabilities. The following theorem is proved.

Theorem 3: For a two-user network, with the ORS, the outage probabilities of both users are upper bounded by

$$\begin{aligned} P_{\text{out,upp,ORS}} &= \frac{N_r - 1}{2N_r - 1}F_{\gamma_2}(\gamma_{th}) + \frac{N_r + 2}{2(2N_r - 1)}F_{\gamma_3}(\gamma_{th}) \\ &\quad + \sum_{i=4}^{N_r+1} \frac{2N_r\binom{N_r}{i-1}}{(2N_r - (i-1))\binom{2N_r}{i-1}}F_{\gamma_i}(\gamma_{th}), \end{aligned} \tag{18}$$

where $F_{\gamma_k}(x)$ is the CDF of γ_k given in (8).

Proof: With the ORS, $\gamma_{\min}$ can take $\gamma_2, \gamma_3, \cdots$, or γ_{N_r+1}. The outage probability upper bound, $P_{\text{out,upp,ORS}}$, can be calculated as

$$\begin{aligned} P_{\text{out,upp,ORS}} &= \text{Prob}(\gamma_{\min} \leq \gamma_{th}) \\ &= \textstyle\sum_{k=2}^{K} \text{Prob}(\gamma_{\min} = \gamma_k)\text{Prob}(\gamma_k \leq \gamma_{th}) \\ &= \textstyle\sum_{k=2}^{K} \text{Prob}(\gamma_{\min} = \gamma_k)F_{\gamma_k}(\gamma_{th}), \end{aligned} \tag{19}$$

where $K = N_r + 1$. We now calculate the probability of $\gamma_{\min} = \gamma_k$ ($k = 2, \cdots, N_r + 1$) by considering the following three cases.

- $\gamma_{\min} = \gamma_2$ happens when γ_1 and γ_2 are in two distinct rows and two distinct columns of $\mathbf{\Gamma}$. Thus $\text{Prob}(\gamma_{\min} = \gamma_2) = \frac{N_r-1}{2N_r-1}$.
- $\gamma_{\min} = \gamma_3$ happens when γ_1 and γ_2 are in the same column, or γ_1 and γ_2 are in the same row and γ_3 is in the other row. Thus $\text{Prob}(\gamma_{\min} = \gamma_3) = \frac{1}{2N_r-1} + \frac{N_r}{2(2N_r-1)} = \frac{N_r+2}{2(2N_r-1)}$.
- $\gamma_{\min} = \gamma_k$ ($k = 4, \cdots, N_r + 1$) happens when all $\gamma_1, \gamma_2, \cdots, \gamma_{k-1}$ are in the same row and γ_k is in the other row. Then $\text{Prob}(\gamma_{\min} = \gamma_k) = \frac{2N_r\binom{N_r}{k-1}}{(2N_r-(k-1))\binom{2N_r}{k-1}}$.

Using these probabilities in (19), we can obtain (18). ■

By using (8) and with some straightforward algebraic manipulations, (18) can be rewritten as (20) on the top of the next page.

Now we consider the high-power approximation of the outage probability for the special case that $Q = P$. This is useful in the array gain discussion.

$$P_{\text{out,upp,ORS}} = F_\gamma(\gamma_{th})^{N_r}\left[\frac{(N_r-1)(2N_r)!}{(2N_r-1)(2N_r-2)!}\sum_{i=0}^{1}\frac{\binom{1}{i}(-1)^i F_\gamma(\gamma_{th})^{N_r+i-1}}{2N_r+i-1} + \frac{(N_r+2)(2N_r)!}{4(2N_r-1)(2N_r-3)!}\sum_{i=0}^{2}\frac{\binom{2}{i}(-1)^i F_\gamma(\gamma_{th})^{N_r+i-2}}{2N_r+i-2} + \sum_{j=4}^{N_r+1}\sum_{i=0}^{j-1}\frac{2N_r(2N_r)!\binom{N_r}{j-1}\binom{j-1}{i}(-1)^i F_\gamma(\gamma_{th})^{N_r-j+i+1}}{(2N_r-j+1)(2N_r-j+i+1)(2N_r-j)!(j-1)!\binom{2N_r}{j-1}}\right]. \tag{20}$$

When $N_r > 2$, the highest order term of P in (20) is the term with $j = N_r + 1$ and $i = 0$ in the double summation, which equals 2. Therefore, by using (5), for $P \gg \max\{\gamma_{th}, 1\}$, $P_{\text{out,upp,ORS}}$ can be written as

$$P_{\text{out,upp,ORS}} \approx 2^{N_r+1}\gamma_{th}^{N_r}P^{-N_r} + \mathcal{O}\left(P^{-(N_r+1)}\right), \tag{21}$$

from which it can be seen that the array gain of $P_{\text{out,upp,ORS}}$ is $2^{-(N_r+1)}\gamma_{th}{}^{-N_r}$.

When $N_r = 2$, the double summation in (20) does not appear because $\gamma_{\min} = \gamma_4$ does not happen, and the highest order term in (20) is the term with $i = 0$ in the second summation, which equals 4. Thus,

$$P_{\text{out,upp,ORS}} \approx 2^{N_r+2}\gamma_{th}^{N_r}P^{-N_r} + \mathcal{O}\left(P^{-(N_r+1)}\right). \tag{22}$$

So its array gain is $2^{-(N_r+2)}\gamma_{th}{}^{-N_r}$.

B. Outage Probability Bound of SRS

For the SRS, the outage probability based on $\gamma_{\min}$ is similarly derived and given by the following theorem.

Theorem 4: For a two-user network, with the SRS, the outage probabilities of both users in the network are upper bounded by

$$P_{\text{out,upp,SRS}} = \frac{N_r-1}{2N_r-1}F_{\gamma_2}(\gamma_{th}) + \frac{N_r+1}{2(2N_r-1)}F_{\gamma_3}(\gamma_{th}) + \sum_{i=4}^{N_r+1}\frac{2N_r\binom{N_r}{k-1}}{(2N_r-(k-1))\binom{2N_r}{k-1}}F_{\gamma_i}(\gamma_{th}) + \sum_{i=4}^{N_r+2}\frac{2(N_r-1)\binom{N_r}{k-2}F_{\gamma_i}(\gamma_{th})}{(2N_r-(k-2))(2N_r-(k-1))\binom{2N_r}{k-2}}. \tag{23}$$

Proof: With the SRS, $\gamma_{\min}$ can take $\gamma_2, \gamma_3, \cdots$, or γ_{N_r+2}. Therefore, the outage probability upper bound can be written as (19) where $K = N_r + 2$. In the following, we calculate the probability of $\gamma_{\min} = \gamma_k$ where $k = 2, \cdots, N_r + 2$.

- $\gamma_{\min} = \gamma_2$ happens when γ_1 and γ_2 are in two distinct rows and two distinct columns of $\mathbf{\Gamma}$. Thus Prob$(\gamma_{\min} = \gamma_2) = \frac{N_r-1}{2N_r-1}$.
- $\gamma_{\min} = \gamma_3$ happens when γ_1 and γ_2 are in the same column and γ_3 is in γ_1's row, or γ_1 and γ_2 are in the same row and γ_3 is in the other row. Thus Prob$(\gamma_{\min} = \gamma_3) = \frac{1}{2(2N_r-1)} + \frac{N_r}{2(2N_r-1)} = \frac{N_r+1}{2(2N_r-1)}$.
- $\gamma_{\min} = \gamma_k$ ($k = 4, \cdots, N_r + 2$) happens when (1) $\gamma_2, \cdots, \gamma_{k-1}$ are in the same row, and γ_1 and γ_k are in the other row, and γ_1 and γ_2 are in the same column. This event happens with probability $\frac{2(N_r-1)\binom{N_r}{k-2}}{(2N_r-(k-2))(2N_r-(k-1))\binom{2N_r}{k-2}}$; or (2) $\gamma_1, \cdots, \gamma_{k-1}$ are in the same row and γ_k is in the other row. This event happens with probability $\frac{2N_r\binom{N_r}{k-1}}{(2N_r-(k-1))\binom{2N_r}{k-1}}$.

Using these probabilities in (19), we can obtain (23). ■

By following the same steps in Section V-A, $P_{\text{out,upp,SRS}}$ can be rewritten as (24) on the top of the next page.

Next, we consider the high-power approximation of the outage probability for the special case that $Q = P$. By noticing that $F_\gamma(\gamma_{th}) = \mathcal{O}(1/P)$, the highest order term of P in (24) is the term with $i = N_r + 2$ and $j = 0$ in the second double summation, which equals $2/(N_r+1)$. Therefore, by using (5), for $P \gg \max\{\gamma_{th}, 1\}$, we have

$$P_{\text{out,upp,SRS}} \approx \frac{2^{N_r}\gamma_{th}^{N_r-1}}{N_r+1}P^{-(N_r-1)} + \mathcal{O}\left(P^{-N_r}\right). \tag{25}$$

So its array gain is $2^{-N_r}\gamma_{th}^{-(N_r-1)}(N_r+1)$.

C. Outage Probability Bound of Naive RS

Now, we consider the naive RS. From Section IV-C, for two-user relay networks, the CDF of $\gamma_{\min}$ of the naive RS scheme is $F_{\gamma_{\min}}(x) = [F_\gamma(x)]^{N_r} + [F_\gamma(x)]^{N_r-1} - [F_\gamma(x)]^{2N_r-1}$. An upper bound on the outage probabilities for the naive RS is thus

$$P_{\text{out,upp,naive}} = F_\gamma(\gamma_{th})^{N_r-1}\left[1 + F_\gamma(\gamma_{th}) - F_\gamma(\gamma_{th})^{N_r}\right]. \tag{26}$$

When $Q = P \gg \max\{\gamma_{th}, 1\}$, we have

$$P_{\text{out,upp,naive}} \approx (2\gamma_{th})^{N_r-1}P^{-(N_r-1)} + \mathcal{O}\left(P^{-N_r}\right). \tag{27}$$

So its array gain is $2^{-(N_r-1)}\gamma_{th}^{-(N_r-1)}$.

D. Discussions

In this subsection, for the two-user network, we discuss the properties of the ORS and SRS schemes and compare with the naive RS scheme (the benchmark).

First, we discuss the performance of the ORS scheme, which is shown to produce the optimal RS result and full diversity with a complexity that is quadratic in the number of relays. We can compare its performance with a single-user N_r-relay network to see the performance degradation due to the competition between the two users. Note that in this work, user diversity is not explored, and only user competition is taken into account. Thus, the two-user N_r-relay network is expected to perform worse than the single-user N_r-relay network.

$$P_{\text{out,upp,SRS}} = F_\gamma(\gamma_{th})^{N_r-1}\Bigg[\frac{(N_r+1)(2N_r)!}{4(2N_r-1)(2N_r-3)!}\sum_{i=0}^{1}\frac{\binom{1}{i}(-1)^i F_\gamma(\gamma_{th})^{N_r+i}}{2N_r+i-1} + \frac{(N_r+1)(2N_r)!}{4(2N_r-1)(2N_r-3)!}$$
$$\times\sum_{i=0}^{2}\frac{\binom{2}{i}(-1)^i F_\gamma(\gamma_{th})^{N_r+i-1}}{2N_r+i-2} + \sum_{i=4}^{N_r+1}\sum_{j=0}^{i-1}\frac{2N_r(2N_r)!\binom{N_r}{i-1}\binom{i-1}{j}(-1)^j F_\gamma(\gamma_{th})^{N_r-i+j+2}}{(2N_r-i+1)(2N_r-i+j+1)(2N_r-i)!(i-1)!\binom{2N_r}{i-1}}$$
$$+\sum_{i=4}^{N_r+2}\sum_{j=0}^{i-1}\frac{2(N_r-1)(2N_r)!\binom{N_r}{i-2}\binom{i-1}{j}(-1)^j F_\gamma(\gamma_{th})^{N_r-i+j+2}}{(2N_r-i+1)(2N_r-i+2)(2N_r-i+j+1)(2N_r-i)!(i-1)!\binom{2N_r}{i-2}}\Bigg]. \quad (24)$$

For the single-user network, with the best RS [23], the outage probability is $P_{\text{out,single}} = F_\gamma(\gamma_{th})^{N_r}$. When $P \gg \max\{\gamma_{th}, 1\}$, we have

$$P_{\text{out,single}} \approx (2\gamma_{th})^{N_r} P^{-N_r} + \mathcal{O}\left(P^{-(N_r+1)}\right).$$

Its diversity order is N_r, and its array gain is $2^{-N_r}\gamma_{th}^{-N_r}$.

Since the single-user network and two-user network have the same diversity order, we compare their array gains. The ratio of the array gain of the single-user network to that of the two-user network is given as

$$c_{\text{single, ORS}} = \lim_{P\to\infty}\frac{P_{\text{out,upp,ORS}}}{P_{\text{out,single}}} = \begin{cases} 2 \approx 3\,\text{dB} & \text{if } N_r > 2 \\ 4 \approx 6\,\text{dB} & \text{if } N_r = 2 \end{cases}. \quad (28)$$

The array gain degradation in the two-user network is due to the competition between the two users.

Compared with the naive RS, the ORS has a higher diversity order with a higher complexity.

Second, we discuss the properties of the SRS. The SRS is suboptimal and loses $N-1$ diversity order in multiple-user networks. But it has a lower complexity, which is linear in the number of relays. The SRS has the same diversity order as that of the naive RS. Now, we discuss the array gain difference of the SRS and the naive RS in a two-user network. By using (25) and (27), the ratio of the array gain of the SRS to that of the naive RS scheme is given as

$$c_{\text{SRS, naive}} = \lim_{P\to\infty}\frac{P_{\text{out,upp,naive}}}{P_{\text{out,upp,SRS}}} = \frac{N_r+1}{2}. \quad (29)$$

Eq. (29) shows that the SRS has a larger array gain due to a clever order of users in selecting relays. If the number of relays increases, the array gain advantage of the SRS increases linearly with the relay number.

VI. Relay Selection in Networks with Direct Links

In this section, the multiple-user relay network with direct links between the users and their destinations is discussed. With a direct link, a user can communicate with its destination via either the direct link or a relay link. Similarly, we assume that each relay can help at most one user, and a user can be helped by at most one relay. The condition $N_r \geq N$ is not required in this case due to the direct links.

A. System Model and RS Schemes

A network model and a two-phase transmission protocol similar to those in Section II are considered. The only difference is that a destination can receive signals from its user during the first phase via a direct link. The receive signal at Destination i via its direct link is

$$y_{id} = \sqrt{P}h_{id}x_i + n_{d_i}, \quad (30)$$

where n_{d_i} is the additive noise at Destination i which follows $\mathcal{CN}(0,1)$ and h_{id} denotes the channel of the direct link for User i which follows $\mathcal{CN}(0,1)$. The receive SNR of User i via the direct link equals $\gamma_{id} = P|h_{id}|^2$. The CDF of γ_{id}, $F_{\gamma_{id}}(x)$, is

$$F_{\gamma_{id}}(x) = 1 - e^{-\frac{x}{P}} = \frac{x}{P} - \sum_{j=2}^{\infty}\frac{(-1)^j x^j}{j! P^j}. \quad (31)$$

We consider the simple case that for each user, either the direct link or a relay link is selected (e.g., [14]), instead of the combination of the two links (e.g.,[16]). To help the RS procedure, considering both the direct links and all relay links, we construct a receive SNR matrix as

$$\mathbf{\Gamma}' = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1N_r} & \gamma_{1d} & 0 & \dots & 0 \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2N_r} & 0 & \gamma_{2d} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \dots & \gamma_{NN_r} & 0 & 0 & \dots & \gamma_{Nd} \end{pmatrix}, \quad (32)$$

which is an $N \times (N_r + N)$ matrix. Elements in $\mathbf{\Gamma}'$ are independent but non-identically distributed.

With this construction of the receive SNR matrix, the ORS, SRS, and naive RS schemes proposed in Section III for networks with no direct links can be straightforwardly applied to networks with direct links. The orders of the worst-case complexity of these schemes will keep the same. In what follows, the diversity orders of the schemes for networks with direct links are analyzed.

B. Diversity Order Analysis

Theorem 5: For a multiple-user network with direct links, with the ORS, each user has diversity order $N_r + 1$; with the SRS, each user has diversity order $\max(N_r - N + 2,\ 1)$.

Proof: We sort the nonzero entries of $\mathbf{\Gamma}'$, γ_{ij}'s and γ_{id}'s, in descending order as $\gamma_1 > \cdots\gamma_k > \cdots > \gamma_{N(N_r+1)}$ where γ_k is the kth largest element of $\mathbf{\Gamma}'$. Since γ_{ij}'s and γ_{id}'s are independent but not necessarily identically distributed (i.n.n.i.d.) random variables, using the Bapat-Beg Theorem

[24] that gives the joint CDF of the order statistics of i.n.n.i.d. random variables, the CDF of γ_k, $F_{\gamma_k}(x)$, is

$$F_{\gamma_k}(x) = \sum_{m=N(N_r+1)-(k-1)}^{N(N_r+1)} \frac{1}{m!(N(N_r+1)-m)!} Perm(\mathbf{A}(m)), \quad k = 1, 2, ..., N(N_r+1), \quad (33)$$

where $\mathbf{A}(m)$ is an $N(N_r+1) \times N(N_r+1)$ matrix given as

$$\mathbf{A}(m) = [\ \underbrace{\mathbf{F(x)} \ \cdots \ \mathbf{F(x)}}_{m \text{ columns}} \ \ \underbrace{\mathbf{1-F(x)} \ \cdots \ \mathbf{1-F(x)}}_{N(N_r+1)-m \text{ columns}}\ ],$$

$\mathbf{F}(\mathbf{x})$ and $\mathbf{1} - \mathbf{F}(\mathbf{x})$ are two $N(N_r+1)$-dimensional column vectors defined as (34) on the top of the next page (where superscript T means transpose operation and $F_{\gamma_{ij}}(x)$ is the CDF of γ_{ij}), and $Perm(\mathbf{B})$ is the permanent of the $n \times n$ matrix $\mathbf{B} = (b_{i,j})$ defined as

$$Perm(\mathbf{B}) = \sum_{\sigma \in \mathcal{S}_n} \prod_{i=1}^{n} b_{i,\sigma(i)},$$

where $\mathcal{S}_n$ is the set of all permutations of the numbers $(1, 2, \cdots, n)$, σ is one such permutation, and $\sigma(i)$ is the ith item in permutation σ.

First we derive the diversity order of the ORS. Since the analysis is similar to that in Section IV, we only show the differences due to the non-identical distributions of the entries in $\mathbf{\Gamma}'$. We consider the outage probability corresponding to the minimum SNR, denoted again as γ_{min}, which is an upper-bound for the outage probability of any user. For ORS, it can be given as $P_{\text{out,upp,ORS}} = \sum_{k=N}^{N(N_r+1)-N_r} \text{Prob}(\gamma_{\text{min}} = \gamma_k) F_{\gamma_k}(\gamma_{th})$ because the best case ($\gamma_{\text{min}} = \gamma_N$) happens when the largest N elements of $\mathbf{\Gamma}'$ are in different rows and different columns, and a worst case ($\gamma_{\text{min}} = \gamma_{N(N_r+1)-N_r}$) happens when the smallest N_r+1 elements (except zeros) of $\mathbf{\Gamma}'$ are all in the same row. Since $\text{Prob}(\gamma_{\text{min}} = \gamma_k) \leq 1$, we have

$$P_{\text{out,upp,ORS}} \leq \sum_{k=N}^{N(N_r+1)-N_r} F_{\gamma_k}(\gamma_{th}). \quad (35)$$

Without loss of generality, we assume $Q = P$. For $P \gg \max\{x, 1\}$, the CDF of any random variable in $\mathbf{\Gamma}'$ is either (5) or (31), and can be written in the form

$$F_\gamma(x) \approx \alpha x P^{-1} + \mathcal{O}(x^2, P^{-2}),$$

where $\alpha = 2$ for relay link or $\alpha = 1$ for direct link. Thus, with respect to P, in (33), the highest order term in the summation is the term with the lowest m, i.e., $m = N(N_r+1) - k + 1$; and in (35), the highest order term in the summation is the term with the largest k, i.e., $k = N(N_r+1) - N_r$. Therefore, the highest order of P in the summation of (35) is $-[N(N_r+1) - (N(N_r+1) - N_r) + 1] = -(N_r+1)$. This means that each user has diversity order not less than N_r+1. Using a similar method to that in proof of Theorem 1, we can show that the diversity order of each user is not more than N_r+1. Thus, each user has diversity order N_r+1.

Now we derive the diversity order of the SRS. We consider the cases of $N_r \geq N$ and $N_r < N$ separately.

If $N_r \geq N$, γ_{min} can take $\gamma_N, \gamma_{N+1}, \ldots,$ or $\gamma_{(N-1)(N_r+2)+1}$. Thus, an upper bound on the outage probability can be calculated as $P_{\text{out,upp,SRS}} \leq \sum_{k=N}^{(N-1)(N_r+2)+1} F_{\gamma_k}(\gamma_{th})$. Similar to the ORS case, with respect to P, the highest order term in $F_{\gamma_k}(x)$ given in (33) is the one with $m = N(N_r+1)-k+1$; and the highest order term in the summation $\sum_{k=N}^{(N-1)(N_r+2)+1} F_{\gamma_k}(\gamma_{th})$ is the one with $k = (N-1)(N_r+2)+1$. Therefore, the highest order of P in the summation is $-[N(N_r+1)-((N-1)(N_r+2)+1)+1] = -(N_r - N + 2)$, which means that the diversity order of each user is not less than $N_r - N + 2$. Using a similar proof to that for Theorem 2, we can show that all users also have diversity order not more than $N_r - N + 2$. Thus, each user has diversity order $N_r - N + 2$.

If $N_r < N$, at least $N - N_r$ users should use their direct links. If all N_r relays are assigned to N_r users within the first N_r steps of the SRS algorithm, the remaining $N - N_r$ users have to use their direct links without any choice. Moreover, if end-to-end SNRs of these $N - N_r$ direct links are the $N - N_r$ worst SNRs of the network, i.e., $\gamma_{N(N_r+1)-(N-N_r)+1}, \gamma_{N(N_r+1)-(N-N_r)+2}, \cdots, \gamma_{N(N_r+1)}$, then $\gamma_{\text{min}} = \gamma_{N(N_r+1)}$, the worst SNR in $\mathbf{\Gamma}'$. Therefore, γ_{min} can take $\gamma_N, \gamma_{N+1}, \ldots,$ or $\gamma_{N(N_r+1)}$. And an upper bound for the outage probabilities can be calculated as $P_{\text{out,upp,SRS}} \leq \sum_{k=N}^{N(N_r+1)} F_{\gamma_k}(\gamma_{th})$. Similar to the $N_r \geq N$ case, with respect to P, the highest order term in the summation is with $k = N(N_r+1)$ and $m = N(N_r+1) - k + 1$. Therefore, the highest order of P is $-[N(N_r+1) - N(N_r+1) + 1] = -1$, which means that the diversity order of each user is not less than 1. Similar to the proof of Theorem 2, a diversity order upper bound is also 1. So the diversity order of the SRS is 1. ■

For the naive RS, similar to Section IV-C, the diversity order of User k can be proved to be $\max\{N_r - k + 2, 1\}$. When the worst SNR among all users is considered, the worst user diversity order is $\max\{N_r - N + 2, 1\}$, corresponding to User N, who has the fewest relay choices.

VII. Numerical and Simulation Results

In this section, we show simulation results to justify our analysis, and to evaluate the performance of the ORS, SRS, and naive RS schemes. All nodes are assumed to have the same power, i.e., $Q = P$. The SNR threshold γ_{th} is set to be 0 dB.

First we verify the derived diversity orders of different RS schemes. Fig. 2 shows the simulated outage probabilities of the ORS, the SRS, the naive RS schemes, and a random RS scheme in a three-user network with four relays. In the random RS, each user randomly chooses a relay without conflict. For the ORS, the SRS, and the random RS scheme, due to the homogeneity of the network, User 2 and User 3 have the same outage probability as User 1, and thus, only the outage probability of User 1 is shown. For the naive RS scheme, outage probabilities of the three users are different. User 1 achieves the performance of the single-user case since it has all N_r relays to choose from. User 2 has worse performance than User 1, but has better performance than User 3. In Fig. 2, reference lines (dashed lines) with slopes 1, 2, 3, and 4 are also

$$\begin{aligned} \mathbf{F}(\mathbf{x}) &= [F_{\gamma_{11}}(x)\ \cdots\ F_{\gamma_{N1}}(x)\ F_{\gamma_{12}}(x)\ \cdots\ F_{\gamma_{N2}}(x)\ \cdots\ F_{\gamma_{1N_r}}(x)\ \cdots\ F_{\gamma_{NN_r}}(x),\ F_{\gamma_{1d}}(x)\ \cdots\ F_{\gamma_{Nd}}(x)]^T, \\ \mathbf{1}-\mathbf{F}(\mathbf{x}) &= [1-F_{\gamma_{11}}(x)\ \cdots\ 1-F_{\gamma_{N1}}(x)\quad 1-F_{\gamma_{12}}(x)\ \cdots\ 1-F_{\gamma_{N2}}(x) \\ &\qquad \cdots\ 1-F_{\gamma_{1N_r}}(x)\ \cdots\ 1-F_{\gamma_{NN_r}}(x),\quad 1-F_{\gamma_{1d}}(x)\ \cdots\ 1-F_{\gamma_{Nd}}(x)]^T \end{aligned} \tag{34}$$

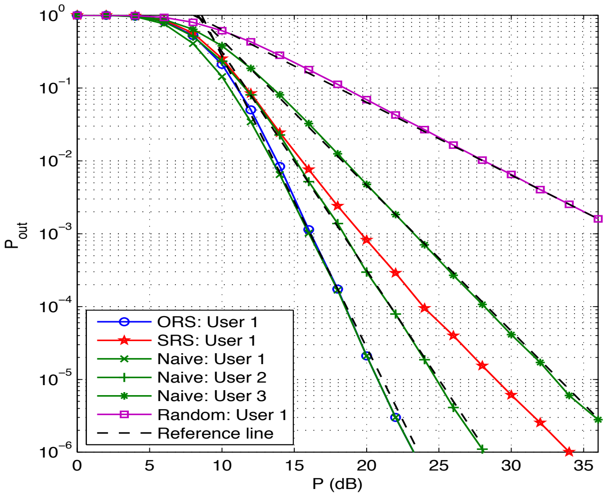

Fig. 2. Outage probabilities for a network with three users and four relays for the ORS, SRS, naive, and random RS.

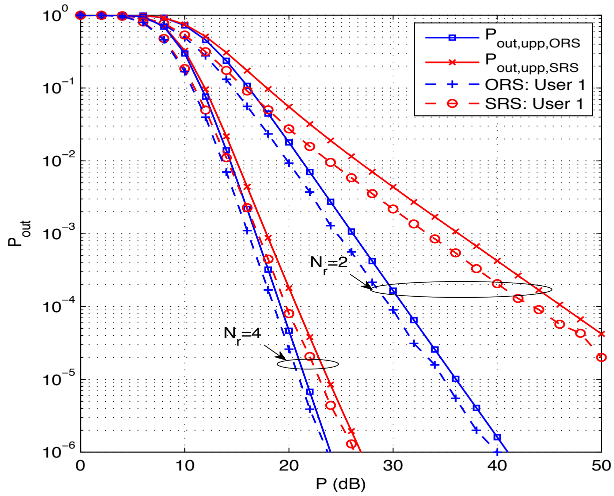

Fig. 4. Outage probabilities corresponding to $\gamma_{\min}$ and of users in networks with two users and two or four relays for the ORS and SRS.

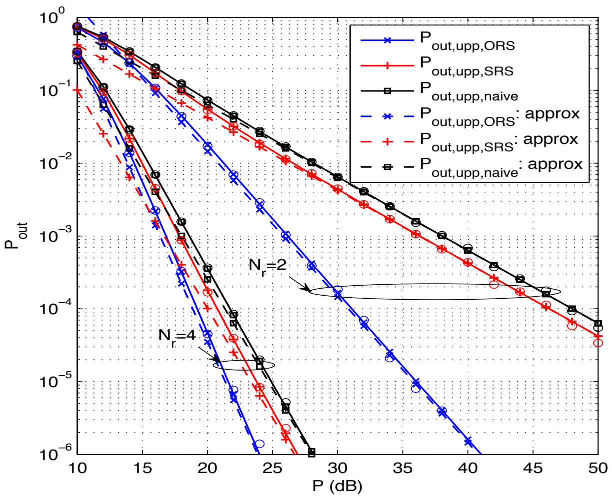

Fig. 3. Outage probabilities corresponding to $\gamma_{\min}$ for networks with two users and two or four relays for the ORS, SRS, and naive RS.

drawn to see the diversity orders clearly. We can see that the ORS has diversity order 4, which is full diversity; the SRS has diversity order 2, which equals $(N_r - N + 1)$. For the naive RS scheme, outage probabilities of User 1, User 2, and User 3 are different, and they have diversity orders 4, 3, and 2, respectively. The random RS scheme has diversity order 1 only. These observations confirm the validity of the diversity order analysis in Section IV. Fig. 2 also shows that the ORS and SRS have better fairness among users than the naive RS.

Next we evaluate the derived outage probability bounds for different RS schemes. Fig. 3 is on two-user networks with two and four relays. It shows simulated outage probability corresponding to $\gamma_{\min}$ (shown in circles), exact analytical outage probability corresponding to $\gamma_{\min}$ in eqs. (20), (24) and (26) (continuous lines), and approximated analytical outage probability corresponding to $\gamma_{\min}$ in eqs. (21), (22), (25) and (27) (dashed lines) for the ORS, SRS, and naive RS schemes. For the entire simulated power range, we can see that our analytical results have good match with the simulation results for all schemes and both network settings. This confirms the accuracy of our analysis. The outage probability approximations are accurate for large P.

Now we compare the outage probability bounds and the outage probabilities of individual users for the ORS and SRS schemes. In Fig. 4, for two-user networks with two and four relays, we show the simulated outage probabilities of User 1 with the ORS and SRS schemes and compare with the outage probability upper bounds derived using the minimum SNR, i.e., in eqs. (20) and (24). It can be seen from the figure that the outage probability upper bounds are tight especially when the number of relays is large. It can be further observed that the ORS and the SRS have almost the same performance at low transmit power region, but the ORS has better performance in the high transmit power region because of its diversity advantage.

We further investigate the array gain differences (1) between the ORS and the single-user best-relay case (which is equivalent to the performance of User 1 in the naive RS scheme), and (2) between the SRS and the naive RS. Fig. 5 shows simulation results of the outage probability bounds of the ORS, SRS, and naive RS ($P_{\text{out,upp,ORS}}$, $P_{\text{out,upp,ORS}}$, $P_{\text{out,upp,naive}}$) and the outage probability of User 1 in the naive RS scheme (equivalent to

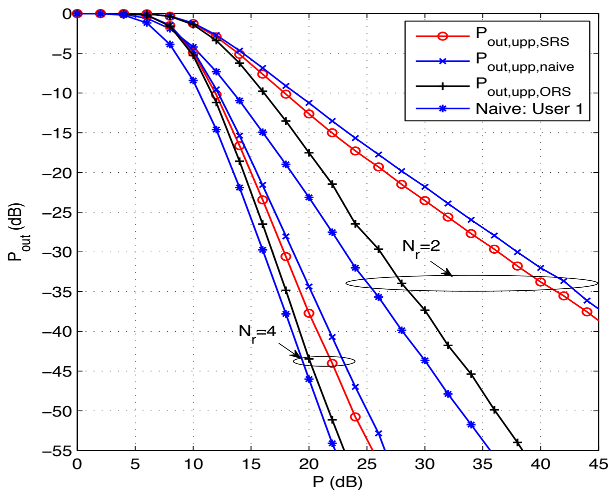

Fig. 5. Array gain difference (observed from outage probabilities) between the ORS and single-user best-relay case, and between the SRS and naive RS.

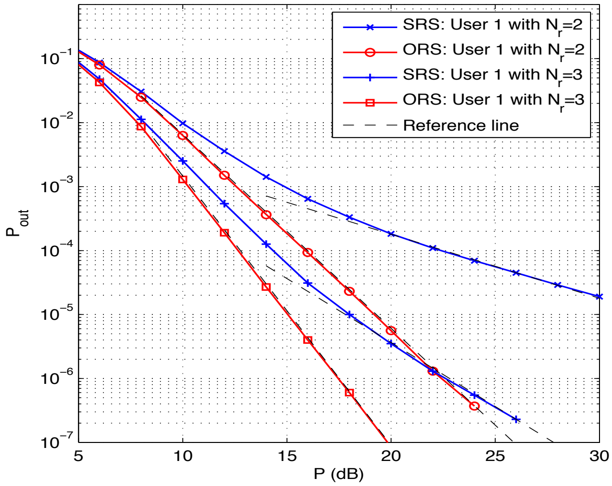

Fig. 6. Outage probabilities for a network with direct links, three users and two or three relays for the ORS and SRS.

$P_{\text{out,single}}$). We have thethe following observations from the figure. Compared with the single-user best-relay case, in the high transmit power region, at the same power, the ORS has 6 dB loss in array gain when $N_r = 2$, and 3 dB loss when $N_r = 4$. Compared with the SRS, in high transmit power region, at the same power, the naive RS scheme has 1.7 dB loss in array gain when $N_r = 2$, and 4 dB loss when $N_r = 4$. These are consistent with our analysis in (28) and (29).

In Fig. 6, we show the simulated user outage probabilities of the ORS and SRS for a three-user network with direct links. We consider two scenarios: $N_r = 2$ (for the case $N > N_r$) and $N_r = 3$ (for the case $N_r \geq N$). Reference lines (dashed lines) with slopes 1, 2, 3, and 4 are also drawn. This figure shows that the ORS has diversity order 3 and 4 for the two scenarios, respectively, which is equal to $(N_r + 1)$; while SRS has diversity order 1 and 2 for the two scenarios, respectively, which is equal to $\max\{N_r - N + 2, 1\}$. This confirms the validity of our analysis on diversity order in Section VI.

VIII. Conclusion

The relay selection problem in a network with multiple users and multiple AF relays is investigated in this paper. A scheme achieving optimal relay selection whose complexity is quadratic in the number of users and in the number of relays is introduced. A suboptimal relay selection scheme is also proposed, whose complexity is quadratic in the number of users and linear in the number of relays. A naive relay selection scheme is also introduced for performance comparison. The diversity orders of the schemes are derived. For two-user networks, outage probabilities corresponding to the minimal SNR are derived for different relay selection schemes. The suboptimal relay selection is shown to achieve a higher array gain than the naive relay selection. Diversity orders for the relay selection schemes in networks with direct links are also derived.

In this research, each user or relay is equipped with a single antenna. However, the RS schemes in this research can be extended to multiple-antenna cases using antenna selection or beamforming/combining. We use antenna selection as an example. The structure of the SNR matrix $\mathbf{\Gamma}$ keeps the same but with higher dimensions. In specific, each entry in $\mathbf{\Gamma}$ is $\gamma_{i,l,k;j,m}$, which represents the SNR of the path from the lth antenna of User i to the kth antenna of Destination i by the help of the mth antenna of Relay j. Then the RS schemes in this research can be employed. The performance analysis, however, is more mathematically complex and is part of our future work.

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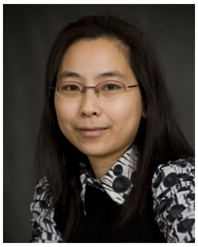

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