Tone Injection for PAPR Reduction Using Parallel Tabu Search Algorithm in OFDM Systems

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***Abstract*—The main drawback of orthogonal frequency division multiplexing (OFDM) systems is the high peak-to-average power ratio (PAPR), which leads to performance degradation and power inefficiency. Tone injection (TI) is a distortionless technique that can reduce PAPR efficiently without incurring data rate loss or extra side information. However, optimal TI requires an exhaustive search over all combinations of possible constellations, which is an NP-hard problem. Suboptimal algorithms, achieving different tradeoffs between the PAPR reduction and complexity, have thus been developed. In this paper, we introduce a novel parallel tabu search algorithm for TI. Simulation results show that the proposed algorithm achieves significant PAPR reduction while maintaining low complexity.**

***Index Terms*—orthogonal frequency division multiplexing, tone injection (TI), parallel tabu search, peak-to-average power ratio.**

I. Introduction

Despite many advantages of orthogonal frequency division multiplexing (OFDM) over multipath fading [1], [2], it suffers several drawbacks such as large peak-to-average power ratio (PAPR). To avoid operating the transmitter power amplifier with extremely large back-offs, numerous methods are available (see [3], [4], and the references therein) that trade off computational complexity, data rate, signal power, and bit error rate (BER) to reduce the high PAPR of the transmitted OFDM signal.

One class of PAPR reduction techniques uses nonbijective constellations. In a nonbijective constellation, N bits are mapped to more than 2^N different signal points. Thus, a given set of data bits can be mapped to multiple constellation points. Thus, appropriately choosing the suitable constellation points among the allowable set of points, the PAPR can be significantly reduced without a data rate loss or requiring extra side information. One implementation of this idea is tone injection (TI) [5]-[10], which uses a cyclic extension of quadrature amplitude modulation (QAM) constellations to offer alternative encoding with a lower PAPR. However, the TI technique requires solving a hard integer-programming problem, whose complexity grows exponentially with the number of subcarriers. Therefore, suboptimal solutions are typically used.

On the other hand, the tabu search (TS) algorithm [12]-[14] is an iterative heuristic searching based on intelligent problem solving principles, which has been successfully applied to solve complicated combinatorial optimization problems. In this paper, a new parallel TS tone injection scheme is proposed for the reduction of the PAPR. Moreover, by limiting the number of equivalent constellation, the power increase can be further reduced. Simulation results show that this parallel TS-TI scheme can achieve a superior PAPR reduction while maintaining less complexity than the previous schemes.

The outline of the paper is organized as follows. Section II describes the general tone injection technique for OFDM. Section III introduces the proposed parallel TS-TI scheme for PAPR reduction. A new parameter for further reduction of the power increase is also introduced in this section. The performance of the proposed scheme is evaluated in Section IV and Section V concludes the paper.

II. Tone Injection for PAPR Reduction

A. PAPR Definition

OFDM maps a block of input bits to a set of N possible complex symbols X_k chosen from an M-ary signal constellation, e.g., phase shift keying (PSK), or QAM. The set of N symbols is converted to time-domain samples via an inverse discrete Fourier transform (IDFT). For an input OFDM block $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]^T$, the discrete-time baseband equivalent signal oversampled L-times is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} X_k e^{j2\pi kn/LN}, n = 0, 1, \cdots, LN-1, \quad (1)$$

and the PAPR of the OFDM signal in terms of power is defined as

$$PAPR = 10 \cdot \log_{10} \frac{\max\limits_{0 \le n \le LN-1} |x_n|^2}{E\left[|x_n|^2\right]} (dB), \quad (2)$$

where $|x_n|$ returns the magnitude of x_n and $E[\cdot]$ denotes the expectation operation. It has been shown in [11] that the oversampled factor L = 4 is enough to provide a sufficiently accurate estimate of the PAPR.

B. Tone Injection

The main idea of the tone injection technique [5] is to expand the original QAM constellation with several equivalent points so that the same information can be mapped to several

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points. Thus, these extra degrees of freedom can be exploited to generate OFDM symbols with low PAPR. This method is called tone injection because replacing a basic constellation to a new larger constellation is equivalent to injecting a tone with appropriate frequency and phase in the OFDM signal.

For an M-ary square QAM, the real and imaginary parts of X_k can take values from the set $\{\pm d/2, \pm 3d/2, \cdots, \pm(\sqrt{M} - 1)d/2\}$, where $\sqrt{M}$ and d represent the number of levels per dimension and the minimum distance between constellation points, respectively. Mathematically, the objective of tone injection is to send the symbols

$$\tilde{x}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} (X_k + p_k \cdot D + j q_k \cdot D) \exp\left(\frac{j2\pi kn}{LN}\right), \tag{3}$$

where p_k and q_k are integers. In order not to increase BER at the receiver, the value of D should be at least $d\sqrt{M}$ [6]. Note that the amount of PAPR reduction depends on the number of modified symbols in a data block. However, finding the optimal solution for the values of p_k and q_k to obtain the lowest PAPR for $\tilde{x}_n$ becomes an integer-programming problem that is known to be an NP-hard problem. Therefore, it is sufficient to reach very good but suboptimal solutions efficiently for a real-time system.

In order to apply the parallel tabu search algorithm, we need to transform the PAPR reduction problem into a combinatorial optimization problem. Similarly to [7], for the purpose of preventing a greater power increase, only constellations located on the outer ring could be shifted, and the corresponding equivalent constellations are nearly symmetrical about the origin. Therefore, the transmitted signal with the modified TI scheme can be written as

$$\tilde{x}_n(\mathbf{b}) = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} M(X_k, b_k) \exp\left(\frac{j \cdot 2\pi kn}{LN}\right), \tag{4}$$

where $\mathbf{b} = [b_0, \cdots, b_{N-1}]$ is a binary selection sequence whose entries $b_k \in \{0,1\}$ determine whether the corresponding signal symbol X_k is shifted or not, and $M(X_k, b_k)$ is given by

$$M(X_k, b_k) = \begin{cases} S(X_k) & b_k = 1 \\ X_k & b_k = 0 \end{cases}. \tag{5}$$

Note that $M(X_k, b_k)$ shows the new mapping relationship between the original QAM constellation $X_k = (d/2)\, p_k + j\,(d/2)\, q_k$ and the corresponding equivalent point, which can be expressed as [7]

$$S(X_k) = \begin{cases} -\frac{d}{2}p_k - j\frac{d}{2}M'' & (p_k > -M', q_k = M') \\ -\frac{d}{2}p_k + j\frac{d}{2}M'' & (p_k < -M', q_k = -M') \\ -\frac{d}{2}M'' - j\frac{d}{2}q_k & (p_k = M', q_k < M') \\ \frac{d}{2}M'' - j\frac{d}{2}q_k & (p_k = -M', q_k > M') \\ X_k & otherwise \end{cases}, \tag{6}$$

where $M' = \sqrt{M} - 1$ and $M'' = \sqrt{M} + 1$. An example of this extended constellation for 16-QAM is illustrated in Fig. 1, where the original constellation located on the outer ring is duplicated cyclically onto its surrounding region in the complex plane. The resulting constellation includes 12 alternative subconstellations in which the equivalent points are spaced by the extension-size d along the real and/or imaginary axes.

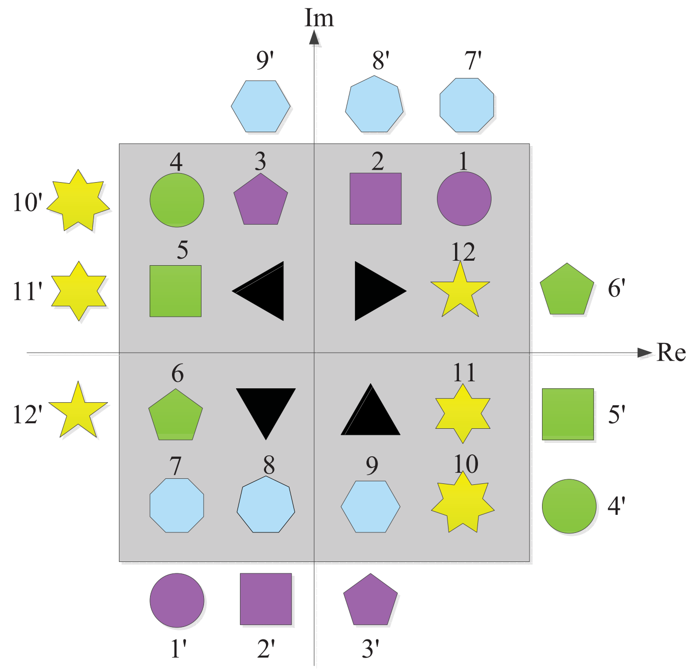

Fig. 1. The cyclically extended 16-QAM constellation diagram.

Since there is only one equivalent constellation for the symbol to choose to be shifted or not, the resulting combinatorial optimization problem can be written as

$$\begin{aligned} &\min\ f(\mathbf{b}) = |\tilde{x}_n(\mathbf{b})|^2 \\ &\text{subject to} : \mathbf{b} \in \{0,1\}^N. \end{aligned} \tag{7}$$

The optimization problem (7) is an integer optimization problem, which has been proven to be a nondetermined-polynomial-time complete problem [15]. Therefore, a computationally efficient parallel TS-based scheduling algorithm will be proposed to solve the optimization problem (7) in the next section.

III. The Proposed Parallel TS-TI Scheme and Its Complexity

A. *Basics of the Tabu Search Algorithm*

Tabu search [12] is a high-level procedure which can be used for solving many combinatorial optimization problems; it uses information from the search history to drive the search into regions that might have better solutions. Intensification and diversification strategies are thus two highly important elements of the tabu search. The former encourages more extensive search near good visited solutions, whereas the latter redirect the search to unvisited regions of the solution space. Short-term memory (usually implemented by means of tabu list) is designed to prevent repetition of some moves considered to be forbidden. However, the use of the tabu list decreases the possibility of cycling (because it prevents returning), in a certain number of iterations, to a solution visited recently. In principle, the tabu search algorithm iterates between two phases:

- *generating acceptance set beyond the tabu list;*
- *updating the tabu list so as to select the admissible solution.*

Note that the search starts with an empty tabu list and the initial solution is randomly generated. After each iteration, the different bit position between the previous and current solution is recorded in the tabu list and it will not be flipped in the next iterations. Since all adjacent moves are allowed, the role of this tabu list is to avoid possible cycles of visited solutions.

B. Proposed Parallel TS-TI Scheme

In [16], the tabu search algorithm has been introduced for partial transmit sequences (PTS). However, in this section, we apply a parallel tabu search algorithm to tone injection scheme that only utilizes short term history to avoid the ineffective cyclic move sequences.

For the PAPR problem in a TI-OFDM system, $\mathbf{b}$ is defined as shown in (7). For each solution $\mathbf{b}_i$, set $\mathcal{N}(\mathbf{b}_i)$ contains all feasible solutions that can be achieved from $\mathbf{b}_i$ with one bit flip. That is, this neighborhood set $\mathcal{N}(\mathbf{b}_i)$ is the set of vectors that have Hamming distance one from $\mathbf{b}_i$. In principle, TS begins from an initial feasible solution $\mathbf{b}_i$, and then moves to a solution $\mathbf{b}_{i'} \in \mathcal{N}(\mathbf{b}_i)$. This movement must meet the condition that $\mathbf{b}_{i'}$ ($\mathbf{b}_i \neq \mathbf{b}_{i'}$) has the minimum cost among all the allowable solutions $\mathcal{N}(\mathbf{b}_i)$. Through repeated iteratively optimization, the solutions with lower PAPR values than those previously encountered are recorded. The final PAPR recorded, when the search is completed, is the output solution. Therefore, TS is a variable neighborhood method, where each step redefines the neighborhood until obtain the optimal result or the terminal condition.

The idea of parallel TS algorithm [13] is derived from the genetic algorithm (GA). The GA uses the crossover operation to create two new solutions from two existing solutions. By using the binary representation of the TI-based PAPR problem (7), the crossover operation of the parallel TS can be applied as follows: two current basic TS solutions are selected as parents and cut at a randomly selected point. The fragments are swapped, and two new solutions are produced. This crossover operation can yield better solutions by combining the advantages of parent solutions. The pseudocode of the TS algorithm is described in detail below.

Algorithm 1 : *Parallel Tabu Search for PAPR Reduction*

```
Initial tabu list L(B) = ∅
Select initial vectors b^now to get crossover vectors b^now'
b^best = arg min{f(b^now), f(b^now')}
for t = 1 to iter − 1 do
    Create acceptance set: A = N(b^now) ∉ L(B)
    Obtain the parallel vectors b^now
    Perform crossover on b^now and put them into b^now
    Evaluate the new solution: b^best = arg min{f(b^now)}
    if f(b^now) < f(b^best) then
        b^best = b^now
    end if
    Update tabu list L(B)
end for
Return b^best
```

C. Parameter for Further Reduce the Power Increase

Inspired by the tone reservation ratio (TRR) in tone reservation[1], we suggest a technique to limit the power increase of TI. Existing TI techniques allow symbols in each subcarrier mapped to the equivalent constellation points. While in the proposed technique, only the symbols in the specific subcarriers can be shifted to the equivalent points.

Mathematically, we can limit the number of '1' (denote as N_r) in the binary selection sequence $\mathbf{b}_i$ of Eqs. (5) and (7) to achieve this goal,

$$\tilde{x}_n(\mathbf{b}') = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} M'(X_k, b'_k) \exp\left(\frac{j \cdot 2\pi kn}{LN}\right), \quad (8)$$

where

$$\mathbf{b}' = [b'_0, b'_1, \cdots, b'_{RN-1}, \underbrace{0, 0, \cdots, 0}_{(1-R)N\ zeros}].$$

Note that the position of $(1-R)N\ zeros$ can be selected randomly. The proportion of '1' in one OFDM block can be expressed as $R = N_r/N$, and $M'(X_k, b_k)$ is defined as

$$M'(X_k, b'_k) = \begin{cases} S(X_k) & b'_k = 1 \\ X_k & b'_k = 0 \end{cases}. \quad (9)$$

Therefore, the resulting combinatorial optimization problem can be rewritten as

$$\min\ f(\mathbf{b}') = |\tilde{x}_n(\mathbf{b}')|^2$$
$$\text{subject to}: \mathbf{b}' \in \{0,1\}^N, \quad (10)$$

where $\{0,1\}^N$ is the set of N-dimensional binary vectors.

It should be noted that this new parameter R can be applied to any TI-based OFDM system. The effect of this parameter on the tone injection system will next be analyzed by simulation.

D. Analysis of the Computational Complexity

In this section, we will analyze the computational complexity of the proposed scheme. For the conventional TI, finding the optimal values of p_k and q_k in (3) requires solving an integer programming problem, which has exponential complexity. Assume there are L candidates per constellation, if K

[1]The tone reservation technique [17] reserves N_r tones for PAPR reduction and utilizes the remaining $(N - N_r)$ tones for data transmission. The tone reservation ratio (TRR), defined as $R = N_r/N$, is closely related with the data transmission efficiency (or throughput).

dimensions are to be shifted, we must search over all [6]

$$C_N^K \cdot L^K \approx \frac{N^K}{K!} \approx (NL)^K \tag{11}$$

combinations for the vectors $\{p_k\}$ and $\{q_k\}$. Each combination is calculated using IFFT.

On the other hand, for a general population-based search method, the complexity can be expressed in terms of the number of samples. If the iteration number and the population size in one generation are $iter$ and pop, respectively, then the number of samples is $S = iter \times pop$. Note that the complexity of each sample to find a suboptimal solution is $\mathcal{O}(N \log N)$ multiplications due to the N-point IFFT operations.

Parallel TS is composed of the parallel search structures and the crossover operation. The parallel number of the TS algorithm is pop = 4 and each TS algorithm searches the constellations until the termination condition. Parallel TS uses $move$ = 11. Hence, the search number of parallel structures of the TS algorithms is $move \times pop$. Crossover is applied for all candidates of the parallel search structures with each other. Accordingly, the search number of the crossover in one iteration is $c = C_4^2 = 6$, where 4 and 2 are equal to the parallel number pop and the number of alternative constellations, respectively. The total search number of the parallel TS for one iteration is equal to $move \times pop + c = 50$. The iterations are repeated until they reach the predetermined search samples S. Therefore, the complexity for the parallel TS-TI scheme with samples S for finding a suboptimal solution is $\mathcal{O}(SN \log N)$, which is of the same order as the CE-TI [10] scheme. However, simulations show that the parallel TS-TI requires less samples (i.e., lower complexity) to obtain the same PAPR as the CE-TI. In Section IV, we will compare the PAPR reduction performance of these schemes by simulations.

IV. Simulation Results

To evaluate the PAPR performance of the proposed scheme, the complementary cumulative distribution function (CCDF) of the PAPR is used as given by

$$CCDF_x(PAPR_0) = Prob(PAPR > PAPR_0). \tag{12}$$

This is the probability that the PAPR of a symbol exceeds the threshold level $PAPR_0$. The simulations below are performed for random QAM modulated OFDM symbols, with the subcarriers N under the condition of an oversampling factor $L = 4$. The legends "Original" and "CE-TI" denote ordinary OFDM and OFDM with TI and cross-entropy optimization, while the two "Parallel TS-TI" denote OFDM with the parallel tabu search TI technique with R = 1 and 1/3, respectively.

Figs. 2 and 3 depict the CCDF curves of two PAPR reduction schemes with different subcarriers N = 128 and N = 256. As seen, the PAPR performance of the proposed parallel TS-TI scheme with R = 1 and 1/3 are both better than the CE-TI with the same computational complexity. Specially, when the maximum samples is set to fifty, we see that the two parallel TS-TI give more PAPR reduction, about 0.6 dB and 0.8 dB, as compared to the CE-TI. Similar situations also occurred in the condition of Fig. 3. Furthermore, to achieve a PAPR of 8 dB at 10^{-3}, the computational load of the parallel TS-TI with R = 1 and CE-TI schemes are 50 and 180 (3.6 $times$ less searching is required), respectively.

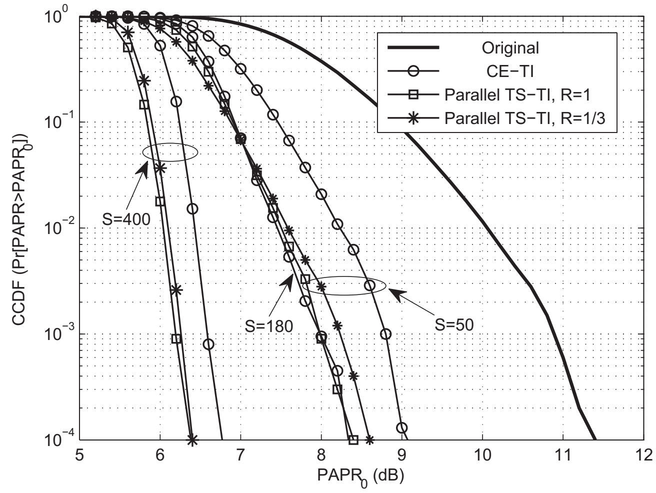

Fig. 2. CCDFs of the PAPR for various TI schemes with 128 subcarriers.

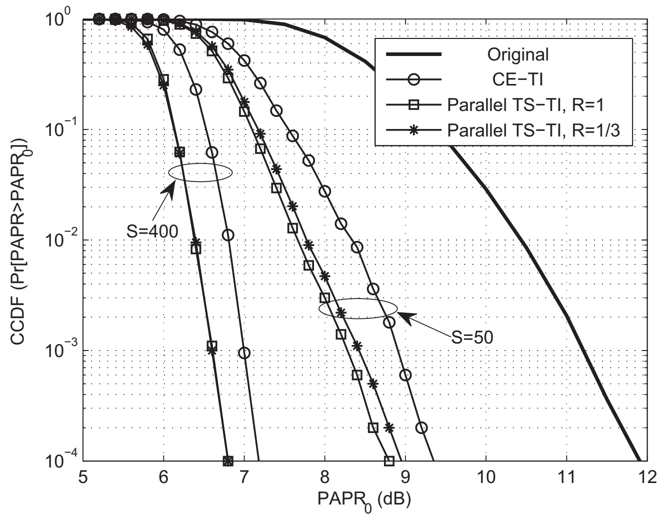

Fig. 3. CCDFs of the PAPR for various TI schemes with 256 subcarriers.

Next, we compare CE-TI and parallel TS-TI for the same complexity. In this case, the number of subcarriers is set to 128. According to [10], we also choose that λ = 0.8 and ρ = 0.1 for the CE-TI with S = 50, 100, 200, 400 and 1000, respectively. Fig. 4 illustrates this comparison. It can be observed that, with the same complexity, the parallel TS-TI scheme with R = 1 and 1/3 leads to much smaller average PAPR than CE-TI.

In Section III-C, we introduce the parameter R to further reduce the power increase. However, the important issue is how to choose a proper value of R, which determines the number of subcarriers that constellations shifted to the equiv-

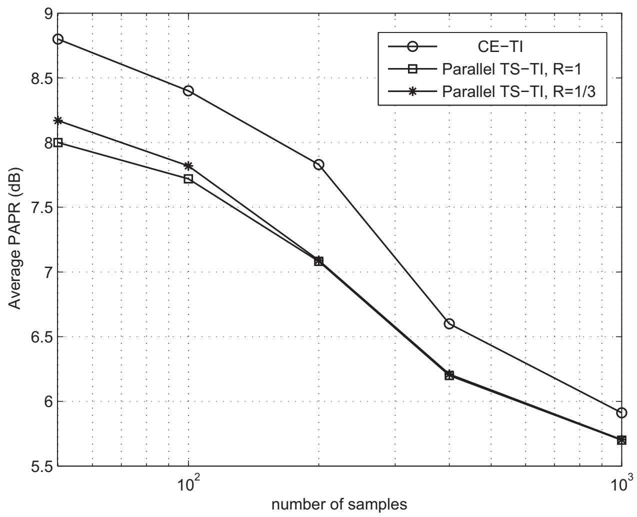

Fig. 4. Average PAPR reduction comparison of the CE-TI and Parallel TS-TI schemes for the same complexity.

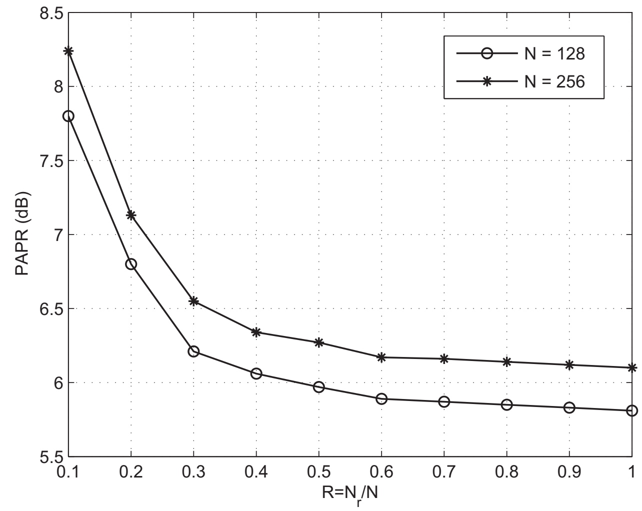

Fig. 6. PAPR reduction vs. proposed parameter R in the Parallel TS-TI scheme.

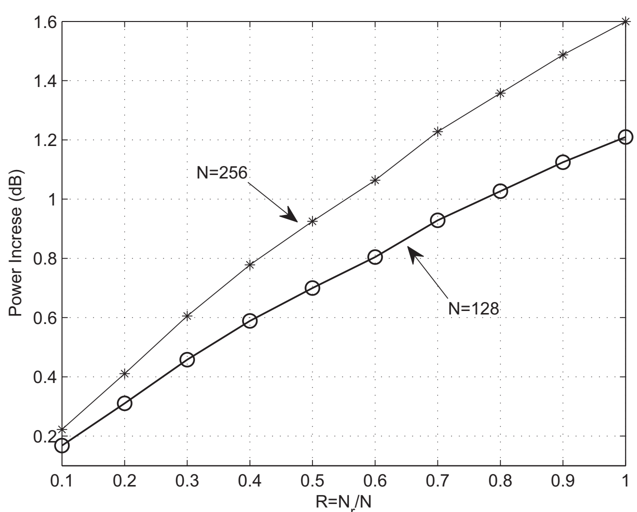

Fig. 5. Power increase vs. proposed parameter R in the Parallel TS-TI scheme with the 6 dB PAPR threshold.

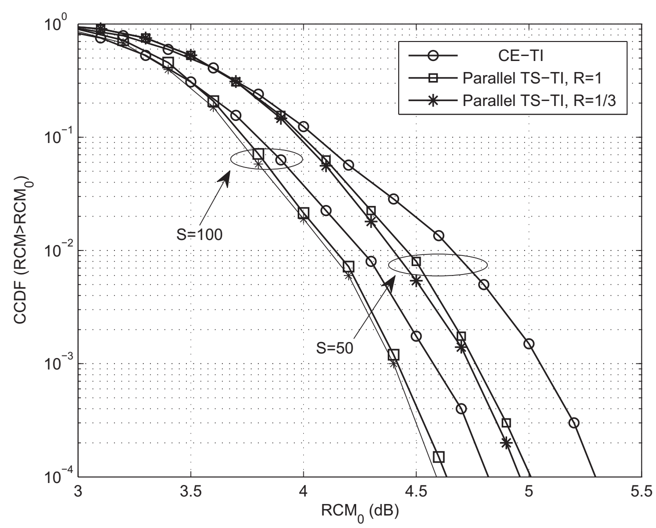

Fig. 7. CCDFs of the raw cubic metric comparison of the CE-TI and Parallel TS-TI schemes.

alent constellation. If R is too large, then the transmit power increase is high, degrading the transmission efficiency. In contrast, if R is too small, the number of shifted constellations will be too small to achieve a satisfactory PAPR performance. Figs. 5 and 6 investigate the change of PAPR and the power increase with the proposed parameter R. From the two figures, taking both power increase and PAPR reduction into account, it can be easily found that $R = 1/3$ is a good choice.

The cubic metric (CM) [18] is a effective predictor of the actual reduction in power capability, or power de-rating, of a typical power amplifier in a mobile handset. It has been adopted by the 3GPP members as a method to determine PA power de-rating because of its accuracy over a wide range of devices and signals. This method has proven to be superior, for W-CDMA signals, compared to methods that use the statistical PAPR to predict de-rating [18]. With the introduction of various OFDM-type modulation formats it is crucial that the cubic metric is verified as a valid predictor of power de-rating. This was found to be true for several different power amplifiers using different technologies and various signals with a range of PAPRs. The CM of a signal is defined as

$$CM = \frac{RCM - RCM_{ref}}{K}, \tag{13}$$

where RCM is the raw CM. For a signal $x(t)$, the RCM is given by

$$RCM = 20\log\left[\text{rms}\left[\left(\frac{|x(t)|}{\text{rms}[x(t)]}\right)^3\right]\right]. \tag{14}$$

TABLE I

POWER INCREASE, AVERAGE SAMPLE AND ITS RUNTIME COMPARISON OF THE TI SCHEMES BASED ON CROSS-ENTROPY (CE) AND PARALLEL TABU SEARCH (PARALLEL TS) ALGORITHMS, WITH 6 dB PAPR THRESHOLD AND 128 SUBCARRIERS

Scheme	CE	Parallel TS, R=1	Parallel TS, R=1/3
Power Increase	1.27 dB	1.21 dB	0.47 dB
Average Sample	675.36	540.54	535.68
Sample Runtime	90.407 ms	69.856 ms	61.323 ms
BER Degradation	no	no	no

Note that the RCM_{ref} and K are used to complete estimate of the power de-rating required to meet a given adjacent channel leakage ratio [19]. For example, in downlink of LTE $RCM_{ref} = 1.52$ dB and $K = 1.56$ are used. Fig. 7 shows this comparison. As can be seen, the proposed scheme also achieves a good performance when the CCDF of CM is chosen.

Finally, comparisons of the CE-TI and parallel TS-TI schemes under a threshold of 6 dB (relative to the average power) for a 128-subcarrier OFDM system are listed. 10^5 OFDM blocks are simulated. With these settings, the CE-TI and parallel TS-TI obtain the same PAPR reduction for the target threshold used. In addition, since the TI utilizes the extended constellation to reduce the PAPR, therefore, a transmit power increase instead of the BER degradation occurs. We thus compare their complexity in terms of the average number of IFFTs. Table I shows the perfermance comparison of the two TI schemes. As shown, the power increase is reduce to a little more than 60% when the proposed parameter R is applied (the position of N_r is selected randomly). Taking the PAPR reduction, power increase, and the computational complexity into account, it can be concluded that the parallel TS-TI scheme with R = 1/3 is a good choice.

V. CONCLUSION

This paper presented a parallel tabu search-based TI scheme for reducing computational complexity and improving PAPR performance of OFDM signals. We first formulated the TI technique as a combinatorial optimization problem, and then to solve the problem applied the parallel tabu search algorithm. In addition, in order to prevent the transmit power increase, only the constellations located on the specified subcarriers can be shifted, and the corresponding equivalent constellations are nearly radially symmetrical about the origin. Simulation results showed that the proposed scheme achieved a significant PAPR reduction performance with low complexity.

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