Partial Expansion Sphere Decoder with Reduced Branching Factor for MIMO Systems

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***Abstract*—Multiple-input multiple-output (MIMO) detection could be modeled to a tree search problem. This paper proposes one sphere decoder algorithm called partial expansion sphere decoder (PESD) by pruning the search tree using a reduced branching factor. The main idea of the proposed PESD is to reduce the detection complexity by decreasing the branching factor for the search tree. The trade-off between the performance and complexity can be easily controlled by the branching factor. In order to further improve the PESD, a hybrid PESD is proposed by combining the full enumeration for the first several layers of the search tree and the proposed PESD. The simulation results demonstrate that the PESD achieves a flexible trade-off depending on the branching factor and shows that the hybrid PESD obtains performance gains than the pure PESD with the same branching factor.**

I. Introduction

The maximum likelihood (ML) detector is the optimal detection method for spatial multiplexing in multiple-input multiple-output (MIMO) systems. However, the computational complexity of the ML detection (i.e. exhaustive search) grows exponentially with the number of transmit/receive antennas and with the order of the signal constellation [1]. Alternatively, the sphere decoder (SD) has been developed to attain low complexity with the ML performance, especially for the high signal noise ratio (SNR) region [2]. The Fincke-Pohst SD (FP-SD) and the more efficient alternative Schnorr-Euchner SD (SE-SD) are alternatives [3] [4].

However, there are still challenges when the SD based MIMO detection is implemented practically, such as its high computational complexity in the low SNR region. To address this disadvantage, several SD variants have been developed [5]–[11]. [5] selects more reliable symbol candidates according to their conditional probabilities by exploiting the minimum mean square error (MMSE) criterion. However, the complexity is still high for quasi-optimal performance or for a high order constellation. Probabilistic tree pruning approaches [6] [10] sacrifice performance for complexity reduction by pruning the candidates by probabilistic assumption. In order to make the SD more realizable, the K-best SD [7] and the fixed complexity SD (FSD) [11] are proposed, both of which can provide fixed computational complexity. The K-best SD traverses the tree in a breadth-first way by only considering the best K nodes at each level. The FSD ensures a fixed complexity by combining a channel matrix ordering and a search through a small subset of the transmit constellation. However, their complexity is considerably much higher than the complexity of the SD in order to approach the optimal performance, especially in the high SNR region.

In this paper, we propose a new partial expansion sphere decoder (PESD), which efficiently reduces the complexity by choosing a smaller branching factor and also provides a flexible trade-off between performance and complexity.

Main Contributions:

1) The main idea of the proposed PESD is to expand the parent node partially in the search tree instead of expanding all the child nodes, i.e. resulting in the complexity reduction by setting a smaller branching factor [12] for the search tree. The branching factor is a simple and effective parameter to show the size of the search tree and also intuitively shows the complexity of a MIMO detection method. By choosing different branching factors for the detection tree, the PESD achieves a flexible trade-off, and also gives an upper bound of the number of nodes visited by the detection methods.
2) When the branching factor is a small value, the performance of the PESD is degraded. For example, its performance will be the same to Zero-Forcing detection when the branching factor is set to be 1. Full enumeration for the nodes in first several layers could definitely improve the possibility of keeping the ML solution. Therefore, in order to further improve the performance of the proposed PESD, full enumeration could be used in the first several layers, and then the PESD is used for each subtree generated by the previous step. This algorithm is called hybrid PESD, which obtains performance gains than the pure PESD when choosing the same branching factor.

The rest of this paper is organized as follows. Section II and Section III describe the MIMO system model and the basic principle of SD algorithm, respectively. Section IV introduces the proposed PESD algorithm and the hybrid PESD, and then analyzes its complexity. Simulation results about performance and complexity are given in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

A spatial multiplexing MIMO system with N_T transmit antennas and N_R receive antennas is considered in this paper. Based on the assumption of a rich scattering memoryless (flat

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fading) channel [1], the received signal vector can be written as [1]

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}, \quad (1)$$

where $\tilde{\mathbf{s}} = (\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_{N_T})^T$ is the transmitted symbol vector and $(.)^T$ denotes the transpose operation; $\tilde{s}_i \in \mathcal{Q}$ ($i = 1, 2, \ldots, N_T$) and $\mathcal{Q}$ is a complex constellation such as a 64-quadrature amplitude modulation (QAM); $\tilde{\mathbf{y}} = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_{N_R})^T$ is the received signal at the receiver and $\tilde{y}_i$ ($i = 1, 2, \ldots, N_R$) is the signal received at the ith antenna. $\tilde{\mathbf{H}}$ denotes the $N_R \times N_T$ Rayleigh fading channel matrix with independent identically distributed (i.i.d.) elements $\tilde{h}_{ij} \sim \mathcal{CN}(0, 1)$, where $\mathcal{CN}(0, 1)$ denotes a complex Gaussian distribution with zero mean and unit variance. $\tilde{\mathbf{n}} = (\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_{N_R})^T$ is the vector of i.i.d. additive white Gaussian noise (AWGN) where $\tilde{n}_i \sim \mathcal{CN}(0, \sigma^2)$ denotes a complex Gaussian distribution with zero mean and variance σ^2. As usual, the channel matrix is assumed to be perfectly known by the receiver, which can be estimated by standard pilot-based channel estimation methods [13]. For brevity, we also assume $N_T = N_R = N$ and uncoded MIMO systems.

By factorizing the channel matrix and preprocessing the received signal appropriately, the complex channel matrix can be transformed to real matrix representation $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$, where $\mathbf{y}, \mathbf{n} \in \mathcal{R}^n$, $\mathbf{H} \in \mathcal{R}^{n \times m}$ and $\mathbf{s} \in \mathcal{R}^m$ with $m = n = 2N$. It will be obtained by

$$\begin{bmatrix} \Re(\tilde{\mathbf{y}}) \\ \Im(\tilde{\mathbf{y}}) \end{bmatrix} = \begin{bmatrix} \Re(\tilde{\mathbf{H}}) & -\Im(\tilde{\mathbf{H}}) \\ \Im(\tilde{\mathbf{H}}) & \Re(\tilde{\mathbf{H}}) \end{bmatrix} \begin{bmatrix} \Re(\tilde{\mathbf{s}}) \\ \Im(\tilde{\mathbf{s}}) \end{bmatrix} + \begin{bmatrix} \Re(\tilde{\mathbf{n}}) \\ \Im(\tilde{\mathbf{n}}) \end{bmatrix}, \quad (2)$$

where $\Re(x)$ and $\Im(x)$ are the real and the imaginary part of x, respectively.

The ML detection rule for the equivalent real system could be given as [14]

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s} \in \Omega^m} \|\mathbf{z} - \mathbf{R}\mathbf{s}\|^2, \quad (3)$$

where $\mathbf{R}$ is an $m \times m$ upper-triangular matrix, which is obtained by the QR decomposition of the real channel matrix $\mathbf{H}$. The symbol $\|\mathbf{x}\|$ denotes the Frobenius norm of $\mathbf{x}$. The m dimensional vector $\mathbf{s}$ of symbols from the real constellation Ω is denoted by Ω^m . For example, a 16-QAM constellation can be decomposed to be two real 4-PAM constellations with $\Omega = \{-3, -1, 1, 3\}$. The details of this well-known model are omitted for brevity, and the interested reader is referred to [14] for further details. Note that exhaustive search of ML detection has complexity $\mathcal{O}(|\mathcal{Q}|^N)$, which increases exponentially with the number of transmit antennas N_T and the size of constellation, the SD algorithm [2] has been proposed to reduce the complexity of the ML detector.

III. Sphere Decoder Algorithm

The real-system formulation (3) is used to briefly explain the basic SD, and the reader is referred to [3], [4] for more details. The main idea of the SD is restricting the search space for detection from all the constellation points to a hypersphere with a certain radius d centered around the received signal. The FP-SD and SE-SD [3] [4] are two efficient methods to achieve the SD. The ML estimate $\hat{\mathbf{s}}$ is the candidate symbol with the minimum squared Euclidean distance in this hypersphere, such that $\|\mathbf{z} - \mathbf{R}\mathbf{s}\|^2 \leq d^2$.

The Euclidean distance for one candidate symbol $\mathbf{s} \in \Omega^m$ is

$$\|\mathbf{z} - \mathbf{R}\mathbf{s}\|^2 = \sum_{i=1}^{m} \left(z_i - \sum_{j=i}^{m} r_{i,j} s_j \right)^2, \quad (4)$$

where z_i, $r_{i,j}$ and s_j are the ith element of $\mathbf{z}$, the (i, j)th element of $\mathbf{R}$ and the jth element of $\mathbf{s}$, respectively. The Euclidean distance can be obtained by summing up all the partial Euclidean distance (PED) from $i = 1$ to $i = m$. In the search tree, the root node is at the mth layer. Thus, the Euclidean distance can be calculated by summing up all the PEDs from $i = m$ to $i = 1$. For each layer,

$$PED_i = PED_{i+1} + D_i, \quad (5)$$

where PED_i is the PED at the ith layer, and the incremental Euclidean distance is given as

$$D_i = \left(z_i - \sum_{j=i+1}^{m} r_{i,j} s_j - r_{i,i} s_i \right)^2. \quad (6)$$

We will sometimes refer to the PED as the cost.

Therefore, the SD performs a weighted tree search. The tree has $m + 1$ levels with the initialization of the root $PED_{m+1} = 0$. The task of the SD algorithm is to find the leaf node ($i = 1$) satisfying the radius constraint. Whenever the PED_i exceeds d^2, the node and its child nodes are pruned. As a result, significant parts of the tree can be eliminated without affecting the optimality from the search.

The FP-SD and the more efficient SE-SD [3] [4] achieve the ML-performance with a reduced complexity, especially in the high SNR region [2]. For the original FP-SD, the initial radius can be chosen based on the noise level [15]. However, the initial radius for the SE-SD is typically set as $d = \infty$ [4] to make sure that at least one point is included in the hypersphere.

As discussed in [4], although the FP-SD and SE-SD achieve the optimal performance with much lower complexity compared to the ML detector, the computational complexity is still very high in the low SNR region. We propose the PESD based on the branching factor, which efficiently reduces the complexity and obtains a flexible trade-off between performance and complexity.

IV. Partial Expansion Sphere Decoder

Before demonstrating the proposed PESD, we firstly introduce the branching factor for the search tree.

A. The branching factor

The complexity of various searching algorithms can be adequately measured in terms of the branching factor b [12]. It is defined to be the number of child nodes at each node of the search tree. In a tree where every node has the same branching factor b, it is also the branching factor of the tree. If b is not uniform, an average branching factor (effective branching factor β) can be calculated, which is the average number of child nodes for each parent node. β measures the increase of the average complexity because of extending the search depth by one extra level.

For the MIMO detection, the branching factor of the searching tree is uniform $b = |\Omega|$ with exhaustive search (ML detection). The number of nodes expanded in this search tree is

$$b + b^2 + \ldots + b^{m-1} + b^m. \tag{7}$$

For example, b is 4 for a 4×4 16-QAM MIMO system, this means 4 nodes are expanded for each node in the search tree. However, the branching factor is not uniform with SD algorithm because some nodes are pruned in the search process. Thus, the effective branching factor β could be an interesting measurement of the complexity for different SD algorithms. For the detection of a $N \times N$ MIMO system, there are $2N$ levels in the tree with a real space transformation. Assuming the number of visited nodes C is known, the effective branching factor may be acquired by

$$C = \beta + \beta^2 + \beta^3 + \ldots + \beta^{2N} = \frac{\beta(1-\beta^{2N})}{1-\beta}, \tag{8}$$

when $\beta \neq 1$. According to the above equality, the effective branching factor β could be computed, which would be bounded by

$$1 \leq \beta \leq |\Omega|. \tag{9}$$

For example, for a 2×2 MIMO system using 16-QAM, the search tree has 4 levels, and each node could be fully expanded to have 4 child nodes if the branching factor is equal to $|\Omega| = 4$, which is shown in Fig. 1. The SE-SD needs to examine all the branches in the tree, where there are $4^1 + 4^2 + 4^3 + 4^4 = 340$ nodes. However, if the branching factor $b = 2$, the search tree is given as Fig. 2, where the gray branches and nodes are pruned. There are only $2^1 + 2^2 + 2^3 + 2^4 = 30$ nodes in the pruned search tree, which significantly reduce the detection complexity.

B. Partial Expansion Sphere Decoder

According to the discussion in section IV-A, it is intuitive to apply the idea of different branching factors for the search tree in MIMO detection. Our main idea is to reduce the searching complexity by decreasing the branching factor for the search tree. However, for the PESD, the nodes kept in the search tree are only those nodes with $\|\mathbf{z} - \mathbf{R}\mathbf{s}\|^2 \leq d^2$. The expanded number of nodes C will be upper bounded by

$$C \leq b + b^2 + \ldots + b^{m-1} + b^m. \tag{10}$$

As shown in Algorithm 1, by choosing a different branching factor, the proposed PESD achieves different performance and complexity. When b is greater, the performance approaches to the optimal ML performance, while the complexity increases. With a smaller b, the complexity is significantly reduced. The SE-SD becomes one special case of the proposed PESD when the branching factor $b = |\Omega|$.

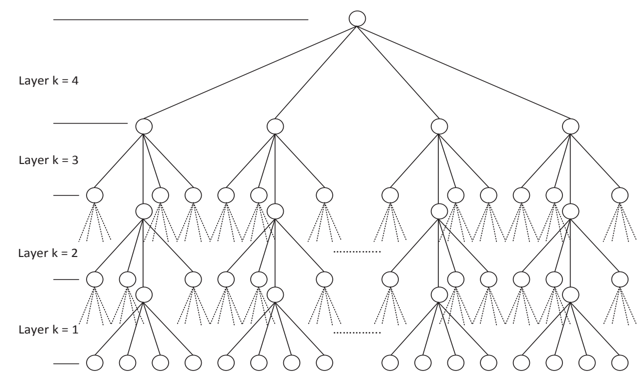


Fig. 1. The exhaustive search tree (branching factor $b = 4$) for a 2×2 16-QAM system.

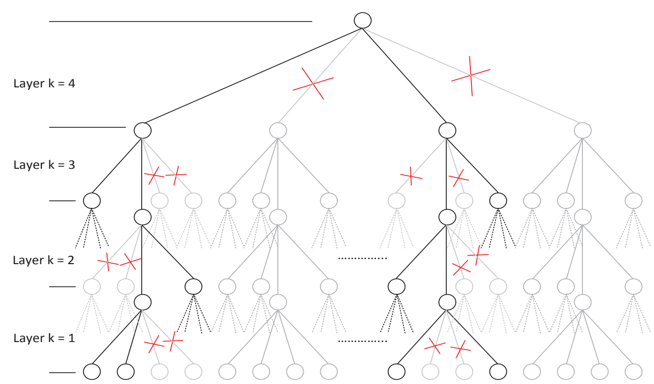


Fig. 2. The search tree with branching factor $b = 2$ for a 2×2 16-QAM system.

C. Hybrid PESD

Full numeration for the search tree is more likely to increase the possibility of obtaining the optimal ML solution. Thus, in order to further improve the proposed PESD, a hybrid PESD is proposed here which has two main steps in the search process.

Firstly, the hybrid PESD expands all the branches of the early K_F levels in the search tree, i.e. full enumeration, where K_F is the number of levels being fully expanded. The choice of K_F depends on the number of antennas in MIMO systems (the levels of the search tree). Intuitively, K_F may be greater when the number of antennas increases. The number of all the child nodes after the first step is $N_F = |\Omega|^{K_F}$. All these child nodes are ordered by an increasing accumulative partial cost, and become the new roots of all the generated subtrees with $m - K_F$ levels. By this process, the probability of keeping the ML solution is increased. Moreover, the hybrid PESD becomes the pure PESD when no levels are fully expanded, i.e. $K_F = 0$.

Secondly, the proposed PESD is applied for each subtree, and it updates the radius by the new cost of the estimate for the symbol when finding one leaf node in this subtree. By

Algorithm 1: The PESD Algorithm

Input : b, $\mathbf{z}$, $\mathbf{H}$, d
Output: $\hat{\mathbf{s}}$

1 Initial the sphere radius $d = \infty$, and take the root s_0 (level $k = m$, where k denotes the kth layer being examined) as the start Current Node;
2 **for** *depth* $k \leftarrow m$ **to** 1 **do**
3   Expand the Current Node, generate its successors $\forall s \in \Omega$ satisfying $\left(z_k - \sum_{j=k}^{m} r_{k,j} s_j\right)^2 \leq d^2 - \sum_{i=k+1}^{m} \left(z_i - \sum_{j=i}^{m} r_{i,j} s_j\right)^2$; keep the successors in the candidate list $\mathcal{L}_k$;
4   Sort these successors in an increasing order of their PEDs;
5   **if** $\text{length}(\mathcal{L}_k) \leq b$ **then**
6   Keep all the candidates in $\mathcal{L}_k$ as the child nodes of this node;
7   **else**
8   Keep the first b candidates in $\mathcal{L}_k$ for the next searching and prune other successors;
9   **end**
10   **for** *every element* $s_j \in \mathcal{L}_k$ **do**
11   **if** s_j *is not a leaf node,* **then** set s_j as the Current Node; let $k = k - 1$ and then go back to line 2;
12   **else** if s_j is a leaf node ($k = 1$), and if its cost is lower than d^2, keep it as the best solution and update d^2 to be $\|\mathbf{z} - \mathbf{R}\mathbf{s}\|^2$;
13   **end**
14 **end**

TABLE I
THE NUMBER OF THE EXPANDED NODES FOR PESD WITH DIFFERENT BRANCHING FACTORS b IN DIFFERENT MIMO SYSTEMS

PESD	4×4 16-QAM	4×4 64-QAM	8×8 64-QAM
$b = 1$	8	8	16
$b = 2$	510	510	131070
$b = 4$	87380	87380	5.7266×10^9
$b = 8$		1.917396×10^7	3.2169×10^{14}

using this new radius as the cost bound of the next search, it is more likely to prune more nodes or to discard a whole subtree. Moreover, because the cost of the roots of all subtrees are already in an increasing order, once the ith subtree is pruned, all the following $N_F - i$ subtrees will be pruned. Therefore, the hybrid PESD needs a smaller branching factor b than the PESD for the similar performance.

D. Complexity Analysis

1) The Number of Nodes Expanded: Firstly, we discuss the impact of the branching factor for the search tree of different MIMO detection, as shown in Table I. For 16-QAM MIMO systems, the SE-SD is a special case of the proposed PESD with $b = 4$; while the branching factor would be $b = 8$ for any 64-QAM MIMO system. The number of the expanded nodes for the MIMO detection straightforwardly demonstrates the complexity reduction of the proposed PESD with a reduced branching factor. For example, in a 4×4 16-QAM system, the number of the expanded nodes by choosing $b = 2$ is 510, which is only 0.58% of the SE-SD ($b = 4$ for the PESD). Furthermore, for a larger MIMO system such as a 8×8 64-QAM system, the number of expanded nodes are 1.3×10^5 and 5.7×10^9 for $b = 2$ and $b = 4$, respectively. They achieves nine and five order of magnitude complexity savings compared to the SE-SD ($b = 8$). This table demonstrates that the proposed PESD could obtain more significant complexity reduction for larger MIMO systems.

The size of the search tree is clearly indicated by the number of expanded nodes, which is an upper bound of the complexity because the proposed PESD prunes more nodes according to the hypersphere rule.

2) The Number of Nodes Visited: The SD has received enormous attraction because of the reduced complexity over the exhaustive search; hence, it is very important to evaluate its complexity for the implementation. The update of radius and the ordering at each level make it unfeasible to analyze the complexity theoretically. Therefore, in this paper, we resort to simulation for the evaluation of the complexity of the PESD and the hybrid PESD.

The complexity of the SD is proportional to the average number of nodes visited by each symbol detection in the searching tree. From [15], the complexity is related to the number of antennas and the SNR ρ. In this paper, we consider the number of nodes visited by all the levels, so the expected complexity of the PESD is given by

$$C(m, \rho) = \sum_{k=1}^{m} \varphi_k, \tag{11}$$

where φ_k is the number of nodes visited at kth level of the search tree.

Remarks:

1) The proposed PESD efficiently reduces the complexity and achieves different trade-offs between performance and complexity by choosing a different branching factor b. Further, the SE-SD is obtained when the branching factor is set to be the size of the constellation $|\Omega|$ for the proposed PESD; while the Zero-Forcing detection is achieved when $b = 1$.
2) In order to improve the performance of the proposed PESD, the hybrid PESD is proposed by combining the full enumeration and the proposed PESD. It achieves performance gains than the pure PESD because the full enumeration of the first several layers increases the possibility of obtaining the optimal ML solution. Moreover, the proposed PESD is one special case of the hybrid PESD when $K_F = 0$.

V. SIMULATION RESULTS

In this section, the PESD and the hybrid PESD are simulated for uncoded MIMO systems over a flat Rayleigh fading

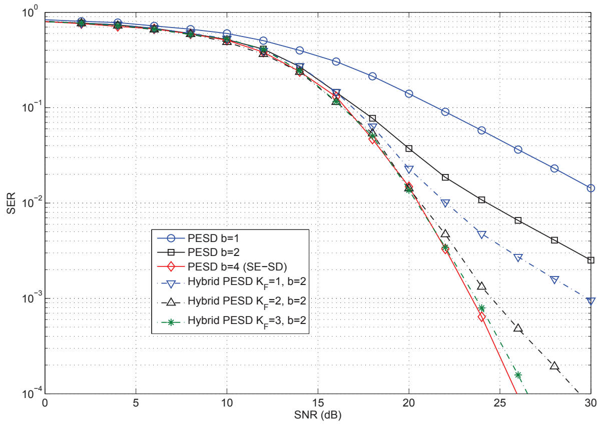


Fig. 3. Performance comparison of the proposed PESD and the hybrid PESD with different branching factors for a 4×4 16-QAM system.

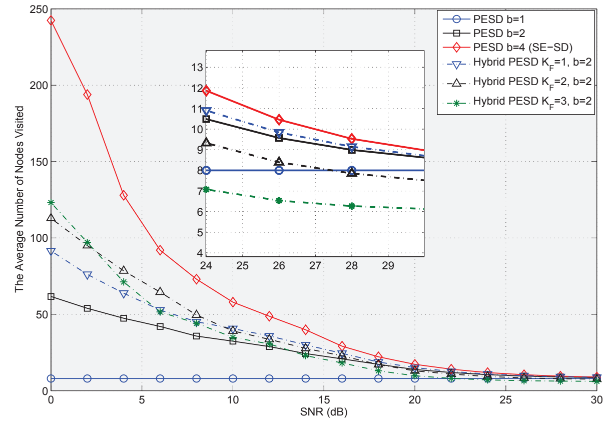


Fig. 4. Complexity comparison of the proposed PESD and the hybrid PESD with different branching factors for a 4×4 16-QAM system.

channel. The performance is evaluated by symbol error rate (SER); while the complexity is measured by the average total number of nodes visited/effective branching factor by each case. After that, the performance and complexity are simulated and compared for different branching factors in 4×4 MIMO systems, where a 16-QAM modulation and a 64-QAM modulation are exploited.

Pruning the nodes by using a smaller branching factor in the proposed PESD results in a suboptimal detection performance. The impact of this suboptimality is quantified in Fig. 3. The proposed PESD and the hybrid PESD are compared with the SE-SD for a 4×4 16-QAM system. The PESD with a greater b achieves better performance. For example, the PESD with $b = 4$ outperforms the PESD with $b = 2$ and $b = 1$ around 3 dB and 10 dB at an SER of 10^{-2}, respectively. When $b = 4$, the PESD becomes the conventional SE-SD. Moreover, the hybrid PESD achieves performance gains than the pure PESD with the same branching factor. For example, the hybrid PESD with $K_F = 2, b = 2$ achieves 3.5 dB performance gain over the PESD with $b = 2$ at an SER of 10^{-2}. The number of layers with full enumeration is greater, the performance gains more, e.g., at an SER of 10^{-3}, the hybrid PESD with $K_F = 3, b = 2$ gains 1 dB and 7 dB than the hybrid PESD with $K_F = 2, b = 2$ and $K_F = 1, b = 2$, respectively.

The complexity comparison for the same set-up shown in Fig. 3 is depicted in Fig. 4. The PESD with a smaller branching factor achieves lower complexity, for example, the number of nodes visited by the PESD with $b = 2$ is 61, which is only 25% of the PESD with $b = 4$ (242 nodes). The PESD with $b = 1$ searches 8 nodes for all SNRs, which confirms that it becomes Zero-Forcing detection when $b = 1$. For the hybrid PESD, there is an interesting result in this figure: the case with $K_F = 3$ has higher complexity than the cases with $K_F = 2$ and $K_F = 1$ in the low SNR region, while it attains lower complexity in the high SNR region; for example, at an SNR of 28 dB, the hybrid PESD with $K_F = 3, b = 2$ search 6.27 nodes on average, while the SE-SD visits approximately 9.53 nodes. However, this case with $K_F = 3$ achieves near optimal performance as shown in Fig. 3.

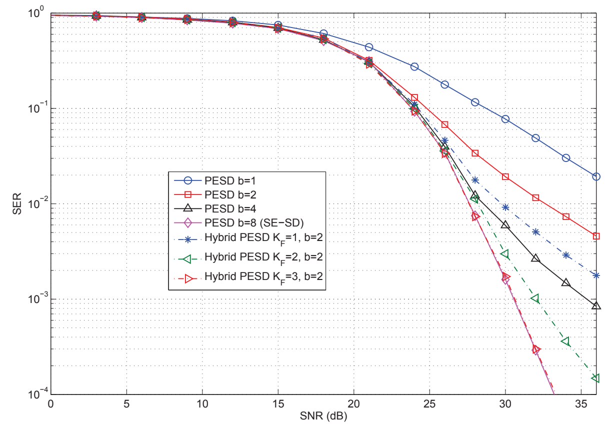
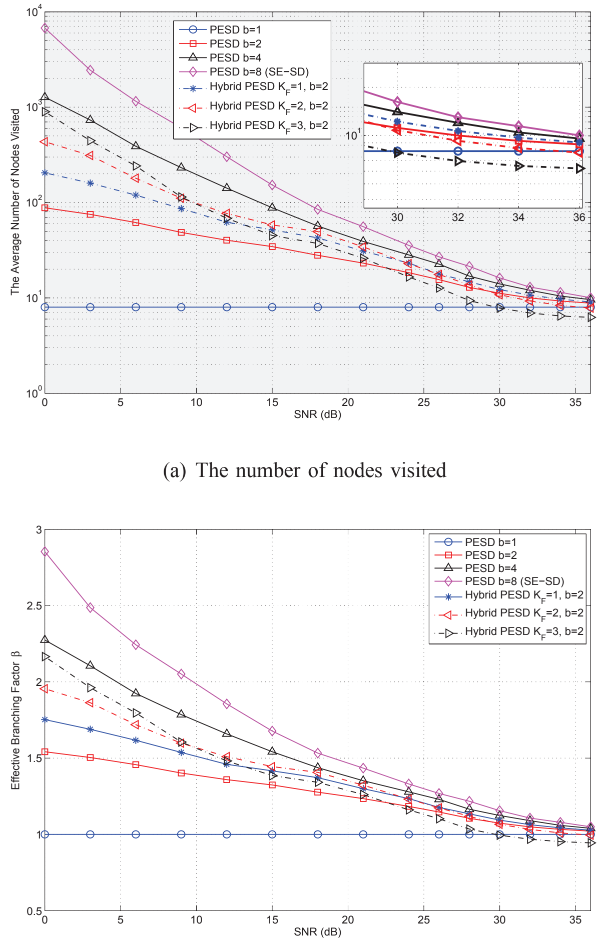


Fig. 5. Performance comparison of the proposed PESD and the hybrid PESD with different branching factors for a 4×4 64-QAM system.

In order to measure the effects of the PESD for different MIMO systems, a 64-QAM and 4×4 MIMO system is next assessed in Fig. 5 and Fig. 6, where the parameter setting is the same as the 4×4 16-QAM MIMO system except including the case of $b = 8$ achieving the optimal performance.

Fig. 5 shows the SER performance of the proposed PESD and the hybrid PESD for different branching factors. Again, by varying b, the PESD achieves different SER performance; the hybrid PESD also obtains different SER performance by varying K_F with the same b. For a 64-QAM MIMO system, the SE-SD is obtained when the branching factor b is equal to 8. It is also clear that the performance is getting better with a greater b, e.g. the PESD with $b = 4$ gains about 4.5 dB over the case with $b = 2$ at an SER of 10^{-2}. With the same branching factor $b = 2$, the hybrid PESD with $K_F = 3$ achieves a quasi-optimal performance and outperforms 5 dB than the pure PESD at an SER of 10^{-2}. With more layers fully expanded, the SER performance is improved more at the same branching



(a) The number of nodes visited

(b) Effective branching factor

Fig. 6. Complexity comparison of the proposed PESD and the hybrid PESD with different branching factors for a 4×4 64-QAM system.

factor. For example, the hybrid PESD with 3 full enumeration layers achieves around $1.5\,\mathrm{dB}$ performance gain than the case with $K_F = 2$.

The complexity comparison of the PESD and the hybrid PESD is compared in Fig. 6, where both the number of visited nodes and the effective branching factor are given. From Fig. 6(a), it is shown that the complexity reduction by a smaller branching factor is much more clear in the large MIMO system. For example, at $0\,\mathrm{dB}$, the proposed PESD with $b = 1$, $b = 2$ and $b = 4$ are only 0.12%, 1.3% and 18% of the PESD with $b = 8$ (SE-SD), respectively. Moreover, similar to the results in Fig. 4 for the 16-QAM MIMO system, the complexity of the hybrid PESD is slightly higher than the pure PESD; while the former case brings in the significant performance gain than the latter case as shown in Fig. 5. The hybrid PESD also obtains lower complexity than the SE-SD in the high SNR region. Fig. 6(b) not only shows that the complexity reduction of the proposed SD algorithms, but also gives the comparison of the branching factor b and effective branching factor β. Because of the extra pruning based on the hypersphere rule, β is smaller than b and also shows the complexity reduction of the proposed PESD.

VI. Conclusions

In this paper, we proposed a partial expansion sphere decoder (PESD), which significantly reduces the complexity and also provides flexible trade-offs between SER performance and complexity by choosing different branching factors. It could easily reduce the number of branches in the search tree in order to reduce the complexity of searching the optimal solution. Furthermore, the SE-SD becomes one typical PESD when the branching factor is the size of constellation. Because of the flexible trade-offs between performance and complexity, the proposed PESD could be easily applied for hardware implementation in practice according to the required performance and hardware cost. In order to further improve the performance of the proposed PESD, a hybrid PESD was also proposed by combining the full enumeration and the proposed PESD. The simulation results confirmed that both the PESD and the hybrid PESD achieve a flexible trade-off for the MIMO detection.

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