Soft-Output Extension of an SNR-adaptive Sphere Decoder for Coded MIMO Systems

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***Abstract*—In this paper, a list sphere decoding method based on an extension of the signal noise ratio (SNR)-adaptive sphere decoder (SSD) is proposed to perform joint iterative detection and decoding in coded multiple-input multiple-output (MIMO) systems. The SSD offers almost optimal symbol error rate (SER) performance over the entire range of SNRs, while keeping its complexity roughly constant for uncoded MIMO systems. The proposed list SSD (LSSD) further improves the complexity of detection schemes in coded MIMO systems, which can greatly reduce the number of nodes visited when generating the candidate list. The simulation results show that the LSSD achieves mostly equivalent performance to the original list sphere decoder (LSD), and much lower complexity compared to the LSD.**

***Index Terms*—Sphere decoder, MIMO, ML, soft information**

I. Introduction

With the large demand for high rates wireless communications, multiple-input multiple-output (MIMO) systems are capable of providing high speed transmission. To achieve the capacity on a channel normally requires the help of channel coding that provides redundancy to improve the reliability. The sphere decoder (SD) is used to provide soft information for coded MIMO systems. One jointly iterative detection and decoding method has been proposed [1], which uses a list version of the SD (LSD) to provide a combined detection method for coded MIMO systems. In this scheme, the error correction code (ECC) could be any code that can be decoded by using soft inputs and outputs, such as convolutional codes and turbo codes. There are several papers [2]–[4] focusing on the improvements of the soft detection for coded MIMO systems.

The optimal detector for spatial multiplexing MIMO systems is the maximum likelihood (ML) detector. However, the complexity of the ML detector by exhaustive search grows exponentially with the number of transmit antennas and with the order of the signal constellation. As an alternative, the SD has been developed to attain the ML performance with a low complexity, especially for the high signal noise ratio (SNR) region. The Fincke-Pohst (FP) SD and the more efficient Schnorr-Euchner (SE) SD [5] are alternatives, which achieve ML performance with a reduced complexity, especially for the high SNR region. Nevertheless, the SD faces two challenges over its complexity: (i) which is high in the low SNR region, and (ii) which varies with the SNR. To address these challenges, many SD variants have been developed [6]–[9]. Two proposed SDs [8], [9] obtain fixed complexity, and are also capable of supporting soft outputs in the LSD.

An SNR-adaptive SD (SSD) was proposed previously for the MIMO detection to achieve a low and roughly fixed level of complexity over the whole SNR region, with a near-ML performance [10]. The benefits of the SSD in uncoded MIMO systems have been demonstrated. Moreover, this SSD also can be extended to support the soft detection for coded MIMO systems. In this paper, the soft extension of the SSD for coded MIMO systems is developed with a list SSD (LSSD), which generates a list of candidates and further reduces the complexity of iterative detection at a negligible performance loss. The proposed LSSD generates the candidate list for iterative detection and decoding with a significantly reduced complexity compared to the original LSD, while achieving a similar performance.

The rest of this paper is organized as follows. Section II of the paper describes the iterative detection and decoding MIMO system model. Section III addresses the principle of the soft MIMO detection. The LSSD and the complexity measurement are discussed in Section IV. Simulation results for both performance and complexity are drawn in Section V, followed by the conclusions in Section VI.

II. System Model

We consider a coded spatial multiplexing MIMO system (Fig. 1). Information bits $\mathbf{b}$ as a frame of M_b are encoded by the ECC module, whose output $\mathbf{c}$ goes through an interleaver Π. The ECC can be convolutional code or turbo code in particular with code rate R, thus the length of the coded sequence $\mathbf{c}$ is $M_c = M_b/R$. The interleaver here ensures the statistical independence. The interleaved bits $\mathbf{x}$ are then modulated to the channel symbols $\mathbf{s}$ and transmitted. M_x and M_s are the frame length of $\mathbf{x}$ and $\mathbf{s}$, respectively, where $M_x = M_s \log_2(|\mathcal{Q}|)$. Therefore, for a $N \times N$ MIMO channel, a frame of M_s symbols requires the transmission of $M_{ch} = M_s/N$ blocks of data, corresponding to M_{ch} different channel realizations. For simplicity, the modulator is not depicted in Fig. 1.

The MIMO channel is a Rayleigh fading channel matrix $\mathbf{H}$ with independent identically distributed (i.i.d.) elements $\tilde{h}_{ij} \sim \mathcal{CN}(0,1)$, a complex Gaussian variable with zero mean and unit variance 1. As usual, the channel matrix is assumed to be perfectly known by the receiver. The received signal vector after the MIMO channel can be written as $\mathbf{y} = \mathbf{Hs} + \mathbf{n}$,

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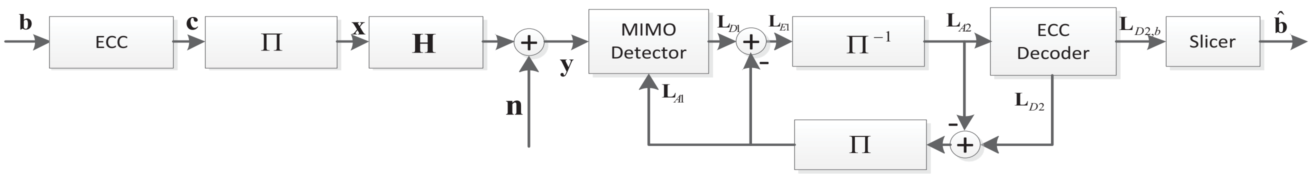

Fig. 1. The system model of iterative detection and decoding

where the transmitted symbol vector $\mathbf{s} = (s_1, s_2, \ldots, s_N)^T$ consists of N symbols from a constellation $\mathcal{Q}$ (a complex constellation such as 16-QAM). $\mathbf{y} = (y_1, y_2, \ldots, y_N)^T$, and y_i is the signal received at the ith antenna ($i = 1, 2, \ldots, N$). $\mathbf{n} = (n_1, n_2, \ldots, n_N)^T$ is the vector of i.i.d. additive white Gaussian noise (AWGN) where $n_i \sim \mathcal{CN}(0, \sigma^2)$ ($i = 1, 2, \ldots, N$).

At the receiver, several iterations of soft information exchange [1] occur between the ECC decoder and MIMO detector. The MIMO detector in this case generates soft *a posteriori* information $\mathbf{L}_{D1}$ by processing the received signal $\mathbf{y}$ and the *a priori* information $\mathbf{L}_{A1}$ from the ECC decoder. This reliability information is expressed by *a posteriori* probability (APP) in the form of log-likelihood ratios (LLR). For example, The LLR of bit $x_k (k = 1, 2, \ldots, M_x)$ is defined as

$$L(x_k) = \ln \frac{\Pr[x_k = +1]}{\Pr[x_k = -1]}. \tag{1}$$

Note that the amplitude levels -1 and $+1$ represent binary 0 and 1, respectively.

For the first iteration, the $\mathbf{L}_{A1}$ is initialized to be $\mathbf{0}$, and the *extrinsic* information $\mathbf{L}_{E1} = \mathbf{L}_{D1} - \mathbf{L}_{A1}$ generated by the MIMO detector is deinterleaved by Π^{-1} to serve as the *a priori* information for the ECC decoder. The ECC decoder then generates the extrinsic information for the next iteration. This process continues until a stopping criterion is met, such as a predefined iteration number or a performance bound. In the final iteration, the ECC decoder obtains the *a posteriori* information $\mathbf{L}_{D2,b}$ on the uncoded bits $\mathbf{b}$, which is sent to the slicer that outputs the final bit estimates $\hat{\mathbf{b}}$.

III. Soft MIMO detector

For simplicity, we assume that we are working on a block of bits $\mathbf{x}$ with $N_B = N \log_2(|\mathcal{Q}|)$, where N_B is the number of bits in one block. The optimal detector obtains the exact APP for each bit x_k ($k = 1, 2, \ldots, N_B$)

$$\begin{aligned} L_{D1}(x_k|\mathbf{y}) &= \ln \frac{\Pr[x_k = +1|\mathbf{y}]}{\Pr[x_k = -1|\mathbf{y}]} \\ &= L_{A1}(x_k) + L_{E1}(x_k|\mathbf{y}). \end{aligned} \tag{2}$$

Here, the Bayes' theorem and independence of the bits x_k due to the interleaver are used to obtain the *a priori* LLRs $L_{A1}(x_k)$ and the *extrinsic* LLRs $L_{E1}(x_k|\mathbf{y})$. From [1], the *extrinsic* information can be denoted by

$$L_{E1}(x_k|\mathbf{y}) = \ln \frac{\sum\limits_{\mathbf{x} \in \mathbb{X}_{k,+1}} p(\mathbf{y}|\mathbf{x}) \cdot \exp(\frac{1}{2}\mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]})}{\sum\limits_{\mathbf{x} \in \mathbb{X}_{k,-1}} p(\mathbf{y}|\mathbf{x}) \cdot \exp(\frac{1}{2}\mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]})}, \tag{3}$$

where $\mathbb{X}_{k,+1}$ and $\mathbb{X}_{k,-1}$ denote the sets of bit vectors $\mathbf{x} = (x_1, x_2, \ldots, x_{N_B})^T$ having $x_k = +1$ and $x_k = -1$, respectively. $\mathbf{x}_{[k]}$ represents the subvector of $\mathbf{x}$ by omitting the kth bits x_k; $\mathbf{L}_{A1[k]}$ denotes the subvector of $\mathbf{L}_{A1} = (L_{A1}(x_1), L_{A1}(x_2), \ldots, L_{A1}(x_{N_B}))^T$ by omitting the $L_{A1}(x_k)$.

By applying (3) and the Max-log approximation, the *extrinsic* information becomes

$$\begin{aligned} L_{E1}(x_k|\mathbf{y}) \approx\ & \frac{1}{2} \max_{\mathbf{x} \in \mathbb{X}_{k,+1}} \left\{ -\frac{1}{\sigma^2/2} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 + \mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]} \right\} \\ & - \frac{1}{2} \max_{\mathbf{x} \in \mathbb{X}_{k,-1}} \left\{ -\frac{1}{\sigma^2/2} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 + \mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]} \right\}. \end{aligned} \tag{4}$$

In spite of these simplifications, the computing of $L_{E1}(x_k|\mathbf{y})$ has an exponential complexity $O(|\mathcal{Q}|^N)$, and is prohibitively complex for the systems with a large number of antennas and with high-order modulations. The main task here is to find the candidate list in (4). The LSD [1] is proposed to quickly find the candidate list by using the SD. Therefore, in this paper, the new LSSD further reduces this complexity.

IV. List SNR-adaptive Sphere Decoder

The LSSD is a soft extension of the SSD [10] that efficiently reduces the complexity. The SSD focuses on finding the near-ML estimate, while the LSSD is used to obtain the set of candidates around the ML estimate that can be exploited to calculate the soft extrinsic information of (4) for the iterative detection and decoding. Next, the basic idea of the SSD is briefly discussed and then extended to be the LSSD.

A. SNR-adaptive Sphere Decoder

Compared with the conventional SDs (FP and SE), the SSD [10] further reduces the complexity and obtains a roughly fixed complexity at a negligible performance loss.

The traditional SDs prune only the nodes but do not lie in the ML path. This type of pruning is called admissible pruning. However, admissible pruning achieves only a small reduction in the complexity, especially in the low SNR region. Therefore, to achieve substantial complexity savings, more than admissible pruning is required. The SSD is to scale the search radius of the hypersphere base on the SNR, which is defined as

$$d_{SSD}^2 = \frac{\rho}{\rho + C_0} \times d^2, \tag{5}$$

where d_{SSD} is the radius in the SSD, ρ is the SNR of the MIMO system, d is the radius of the hypersphere, and C_0 is a predefined constant to guarantee that more nodes are pruned in the low SNR region and fewer points are pruned in the high SNR region. As a result of

$$\lim_{\rho \to \infty} \frac{\rho}{\rho + C_0} = 1, \tag{6}$$

the performance of the SSD reverts to that of the original SD when the SNR is sufficiently high.

B. List Extension of the SSD

In this section, the soft extension of the SSD in coded MIMO systems is obtained. The LSSD generates a list $\mathcal{L}$ of $N_{\mathcal{L}}$ candidates by searching the tree by a rule. This list includes the ML estimate, but the size of the list satisfies $1 \leq N_{\mathcal{L}} < 2^{N_c \cdot N}$, where $N_c = \log_2(|\mathcal{Q}|)$ is the number of bits per modulated symbol.

The extrinsic information in (4) can be rewritten as Eq. (7), where $\mathcal{L} \cap \mathbb{X}_{k,+1}$ and $\mathcal{L} \cap \mathbb{X}_{k,-1}$ represent the subset of vectors $\mathcal{L}$ having $x_k = +1$ and $x_k = -1$, respectively.

$$L_{E1}(x_k|\mathbf{y}) \approx \frac{1}{2} \max_{\mathbf{x} \in \mathcal{L} \cap \mathbb{X}_{k,+1}} \left\{ -\frac{1}{\sigma^2/2} \|\mathbf{y} - \mathbf{Hs}\|^2 + \mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]} \right\} - \frac{1}{2} \max_{\mathbf{x} \in \mathcal{L} \cap \mathbb{X}_{k,-1}} \left\{ -\frac{1}{\sigma^2/2} \|\mathbf{y} - \mathbf{Hs}\|^2 + \mathbf{x}_{[k]}^T \mathbf{L}_{A1[k]} \right\}. \tag{7}$$

In order to attain the candidate list, the LSSD is needed to constraint the hypersphere. By factorizing channel matrix ($\mathbf{H} = \mathbf{QR}$) and preprocessing the received signal appropriately, the ML detection rule for the equivalent real system may be given as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s} \in \Phi} ||\mathbf{z} - \mathbf{Rs}||^2, \tag{8}$$

where Φ is the set of all points which satisfy $||\mathbf{z} - \mathbf{Rs}||^2 \leq d_{LSSD}^2$, d_{LSSD} is the radius of the hypersphere.

Steps to find the candidates list using the new LSSD can be shown as following:

1) Initialize the radius $d_{LSSD} = \infty$ to guarantee at least one point in the candidate list is found, and set the number of candidates $N_{\mathcal{L}}$. Moreover, let $k = m$ ($m = 2N$ for real matrix) and $p = 1$;
2) Generate all the children denoted by the set $\mathcal{T}$ in the k-th level of the search tree which satisfy

$$\left(z_k - \sum_{j=k}^{m} r_{k,j} s_j \right)^2 \leq d_k^2, \tag{9}$$

where $d_k^2 = d_{LSSD}^2 - \sum_{i=k+1}^{m} \left(z_i - \sum_{j=i}^{m} r_{i,j} s_j \right)^2$ and $k \in \{m, m-1, \ldots, 1\}$;
3) Sort the components in $\mathcal{T}$ according to the ascending order of the branch cost c_i in this level, where

$$c_i = \left(z_i - r_{i,i} s_i - \sum_{j=i+1}^{m} r_{i,j} s_j \right)^2 \tag{10}$$

and $s_i \in \mathcal{T}$, $i \in \{1, 2, \ldots, N'\}$. N' is the number of elements in $\mathcal{T}$;
4) From $i = 1$ to N', let $\hat{s}_k = s_i$.
   a) If $k = 1$, $c_i < d_k^2$ and $p \leq N_{\mathcal{L}}$ (the candidate list is not full), we add this new point to the list as the pth point $Cand_p = \hat{\mathbf{s}}$, and let the radius be

$$d_{Cand_p}^2 = \|\mathbf{z} - \mathbf{R}\hat{\mathbf{s}}\|^2 \tag{11}$$

   and $p = p + 1$;
   b) Otherwise, when the list grows full ($p > N_{\mathcal{L}}$), we compare the radius of this new point $\hat{\mathbf{s}}$ with maximum value in d_{Cand}, replace the point with the biggest radius if the new point has smaller radius, and also set the maximum radius as the radius of $\hat{\mathbf{s}}$. Meantime, the d_{LSSD} is updated to be

$$d_{LSSD}^2 = \frac{\rho}{\rho + C_0} \max(d_{cand}^2); \tag{12}$$

5) If $k > 1$, let $k = k - 1$ and go back to 2).

The LSSD significantly reduces the complexity of generating the candidate list $\mathcal{L}$. First, the radius is updated whenever a better point than the worst point in the list is found. Second, the candidate list of the LSSD does not need to be generated for every iteration. Once computed, it is stored in the memory and used by every iteration. Therefore, for every iteration, the only information needed to be updated is the *a priori* information from the channel decoder.

Similar to the MIMO detector, the *a posteriori* information of the channel decoder can also be decomposed into the *a priori* information and *extrinsic* information for the iterative detection and decoding. Therefore, the details of channel decoder are not shown in this paper.

C. Complexity Measurement

An exact complexity analysis of the LSSD algorithm appears intractable. This paper evaluates the computational complexity of generating the candidate list by resorting to the simulation. Therefore, we use the expected average number of nodes visited at all levels of the search tree as the complexity, which is given by

$$C(m, \sigma^2, d^2) = \sum_{k=1}^{m} \varphi_k, \tag{13}$$

where φ_k is the number of nodes visited at kth level within the hypersphere of radius d.

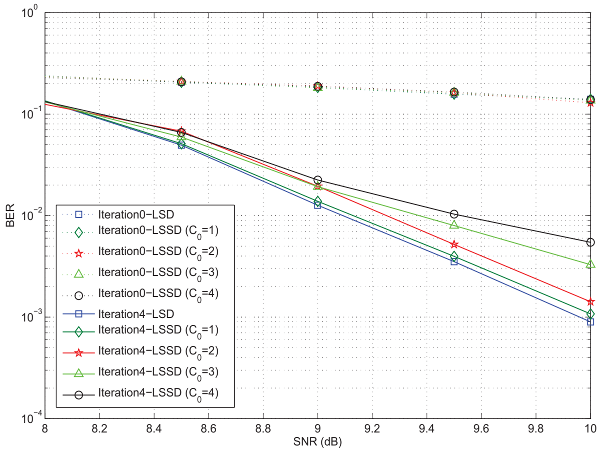

Fig. 2. Performance comparison for different C_0 in a 4×4 16-QAM coded MIMO system with a maximum of 4 iterations.

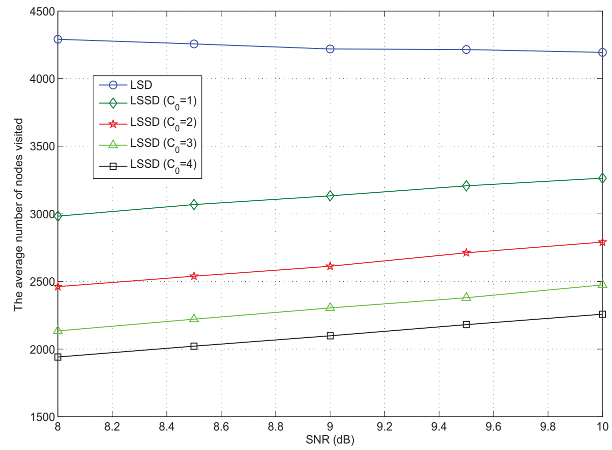

Fig. 3. Complexity comparison for different C_0 in a 4×4 16-QAM coded MIMO system a maximum of 4 iterations.

V. Simulation Results

In this section, we shall assess the advantages of the LSSD for a coded MIMO system. The performance measured by the bit error rate (BER) and the complexity measured by the number of nodes visited for a 4×4 coded MIMO system are shown here. The LSSD is compared to the original LSD with different values of parameter C_0. The systematic recursive convolutional code with rate $R = 1/2$ is exploited to encode the transmitted bits sequence $\mathbf{b}$ with the frame length 8192, where the feed-forward and feedback-generating polynomials are $G_1(D) = 1 + D^2$ and $G_2(D) = 1 + D + D^2$ with memory length 2 [1]. A random interleaver is exploited here.

In order to choose the best C_0, the performance and complexity comparison of the LSSD for different values of C_0 are shown. It is easy to find a proper value for C_0 to attain a nice trade-off between the performance and the complexity. From Fig. 2 the performance is similar for different C_0 without iterations. But for 4 iterations, the performance gets closer to that of the conventional LSD when C_0 decreases, such as $C_0 = 1, 2$. To maintain the performance, a smaller value should be chosen for C_0.

The average number of nodes visited is shown in Fig. 3. The new LSSD obtains a significantly reduced complexity compared to the original LSD. For example, the complexity of the LSSD with $C_0 = 2$ is around 50% of the complexity of the LSD when $\mathrm{SNR} = 8\,\mathrm{dB}$. The complexity for the LSSD with different values of C_0 is also investigated in Fig. 3. As C_0 increases, the complexity decreases more. For example, the average visited nodes are about 2×10^3 by the LSSD with $C_0 = 4$, around 2.5×10^3 with $C_0 = 2$ and approximate 3×10^3 with $C_0 = 1$, respectively. Therefore, considering the performance and the complexity, $C_0 = 2$ should be chosen for a 4×4 16-QAM coded MIMO system. Similarly, an appropriate value for other MIMO systems can also be found after several trials.

VI. Conclusions

For coded MIMO systems, this paper introduced a list SNR-adaptive sphere decoder (LSSD), as a soft extension of the previously proposed SSD. The LSSD uses the SNR-dependent idea in generating the candidate list, and achieves a very close performance to the conventional LSD with a significantly reduced complexity. By iterative detection and decoding, the LSSD further improves the complexity of detection schemes in coded MIMO systems at a negligible performance loss. The simulation results indicate that our proposed LSSD achieves nearly equivalent performance to the conventional LSD with much lower complexity than the latter.

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