Aggregate Interference Analysis for Underlay Cognitive Radio Networks

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***Abstract*—This paper investigates the aggregate interference on a primary user caused by a random number of cognitive radio transmitters distributed in a finite ring. A composite model involving path loss, Rayleigh fading, and shadowing is considered. The exact closed-form moment generating function and an accurate approximation are derived. The aggregate interference is shown to be accurately approximated by a Gamma distribution. The exact outage and an asymptotic approximation are derived.**

***Index Terms*—Cognitive radio, aggregate interference, shadowing, outage probability.**

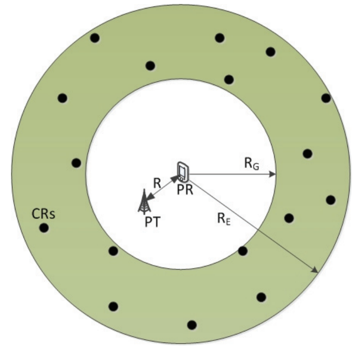

Fig. 1: CRs are distributed in the shaded region around the PR.

I. Introduction

In cognitive radio (CR), desired for the efficient utilization of the available frequency spectrum, secondary users opportunistically access the underutilized frequency spectrum of licensed primary users. In the underlay cognitive set-up [1], the secondary users must ensure that their aggregate interference on the primary user is minimal in order to guarantee a certain quality of service.

Therefore, the modelling of the aggregate interference at the primary receiver (PR) is critical to characterize performance degradation. The aggregate interference is random and depends on several factors such as channel parameters, the spatial distribution of the CRs, activity factors, and power control. The statistics of the aggregate interference have received much attention recently. Reference [2] analyses the average aggregate interference when transmission constraints among CRs are considered, while [3] analyses the capacity-outage of a CR network due to aggregate interference. Reference [4] shows that under certain conditions, the aggregate interference is not lognormal. In reference [5], the authors suggest that the aggregate interference can be modelled as the sum of a normal random variable and a lognormal random variable. Reference [6] considers different activity models for the CRs and obtains the cumulants of the aggregate interference by using Campbell's theorem. Reference [7] derives the moment generating function (MGF) of the combined interference without shadowing for a number of different path loss exponent values.

This paper's objective is to provide a unified analysis of the aggregate interference. Some previous works do not consider one or more of shadowing, path loss or fading. Others either do not have closed-form results or lack results for arbitrary path loss exponent values.

In this paper, we consider the interfering CRs evenly distributed according to a Poisson point process in a finite ring-shaped area around the PR (Fig. 1). These interfering signals are assumed to undergo shadowing, path loss (arbitrary exponents), and Rayleigh fading. The exact closed-form MGF of the aggregate interference and an accurate approximation are derived. Through first- and second-order moment matching, we show that the aggregate interference can be accurately modelled as Gamma distributed. A closed-form expression for the outage and an asymptotic performance analysis are provided. The effect of shadowing is significantly lower at higher path loss exponent values, as shown by our numerical results.

Notations: $\Gamma(x,a) = \int_a^\infty t^{x-1}e^{-t}dt$ and $\Gamma(x) = \Gamma(x,0)$, ${}_2\mathcal{F}_1(,;;)$ and ${}_2\mathcal{F}_2(,;;)$ are the Gauss [8, (eq. 9.10)] and generalized Hypergeometric functions [8, (eq. 9.14)], respectively. $\mathcal{K}_\nu(\cdot)$ is the modified Bessel function of the second kind [8, (eq. 8.407)], $f_X(\cdot)$ is the probability density function (PDF), $F_X(\cdot)$ is the cumulative distribution function (CDF), $M_X(\cdot)$ is the MGF, and $E_X[\cdot]$ denotes expectation with respect to X.

II. System Model

Suppose the CRs are distributed in an annular area encircling the PR (Fig. 1), with an inner radius (guard distance) of R_G and an outer radius of R_E. The guard distance ensures a minimum performance on the PR. The interference from the CR nodes beyond R_E is assumed to be negligible due to path loss. The primary transmitter (PT) to PR distance is denoted as R . The number of CRs N, which are distributed uniformly according to a Poisson point process [9], has the distribution

$$P(N=n) = \frac{(\beta A_I)^n}{n!}e^{-\beta A_I}, \quad n = 0, \dots \qquad (1)$$

where β is the CR density, and $A_I = \pi(R_E^2 - R_G^2)$ is the total area encompassing the CRs. Not all CRs may be active at a given time, and some may be inhibited from transmitting due to interference between the CRs themselves. This system is modelled as a thinned Poisson point process with a density $p_a p_k \beta$, where p_a and p_k model the activity and inhibition factors, respectively [10]. Therefore, without

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the loss of generality, we will take the CR density to be β. An ad-hoc network of CRs where all the nodes present engage in transmission is assumed. We do not consider any medium access protocol for the CRs. In such a case, our analysis would be a worst-case upper bound for the aggregate interference and outage.

The total interference power received at the PR is

$$I = \sum_{i=1}^{N} I_i, \tag{2}$$

where I_i is the interference caused by the i-th CR, and N is the number of CRs. The interference power I_i is given by

$$I_i = P_s {r_i}^{-\alpha} X_i, \tag{3}$$

where P_s is the power level of a CR, and r_i is the distance between the i-th CR and PR. P_s is a constant, and no power control occurs. X_i characterizes the composite effects of small-scale fading and shadowing. With Rayleigh fading and Gamma shadowing, the PDF of X_i is obtained as [11]

$$f_{X_i}(x) = \frac{2}{\Gamma(\lambda)} \left(\frac{b}{2}\right)^{1+\lambda} x^{\frac{1+\lambda}{2}} \mathcal{K}_{\lambda-1}\left(b\sqrt{x}\right), \tag{4}$$

which is the Generalized-K distribution, where $b = 2\sqrt{\frac{\lambda}{\Omega_s}}$. It has been shown in [11] that $\Omega_s = \sqrt{\frac{\lambda+1}{\lambda}}$, and $\lambda = \frac{1}{e^{\sigma^2}-1}$, where σ^2 is the variance of corresponding log-normal shadowing. When expressed in the decibel scale, $\sigma_{dB} = 8.686\,\sigma$. Gamma shadowing is considered due to the mathematical intractability of conducting further analysis with the log-normal PDF. Apart from the lower tail region, the two distributions show a close match.

III. Interference Statistics

In this section, the exact MGF of the aggregate interference, an approximate MGF, and a Gamma approximation to the aggregate interference are derived.

A. MGF of the aggregate interference

Let $M_I^i(s)$ be the MGF of the i-th interferer ($i = 1 \ldots N$).

$$M_I^i(s) = E_{X_i,r_i}[e^{-sI_i}] = E_{X_i}[E_{r_i}[e^{-sI_i}]] \tag{5}$$

The PDF of r_i can be written as

$$f_R(r_i) = \begin{cases} 2\frac{\pi r_i}{A_I} & , R_G < r_i < R_E \\ 0 & , \text{otherwise} \end{cases}. \tag{6}$$

By averaging e^{-sI_i} using the the PDF of interferer distance (6), and the composite fading model (4), the MGF of the interference from the i-th CR is derived as

$$M_I^i(s) = \frac{\pi\left(P_s s\right)^{\frac{2}{\alpha}-2} b^{-\frac{4}{\alpha}-2}}{(-1)^{\lambda+\frac{2}{\alpha}} 4\alpha^2(1+\alpha)\Gamma(\lambda)A_I}\left(\mathcal{Q}(R_E) - \mathcal{Q}(R_G)\right), \tag{7}$$

where $\mathcal{Q}(R)$ is defined in (8). Due to the complexity of (7), it is desirable to find an accurate approximation. To this end, the Gamma approximation to the generalized-K distribution suggested in [12] may be used, where the scale and shape parameters θ and k are $\left(\frac{2(\lambda+1)}{\lambda} - 1\right)\Omega_s$ and $\frac{1}{\frac{2(\lambda+1)}{\lambda}-1}$, respectively. Using this approximation, we get $M_{I,approx}^i(s)$ (9), where $\mathbb{I}(x) = {}_2\mathcal{F}_1(k, k+2/\alpha; 1+k+2/\alpha; -x)$. Because each interferer is independent, the MGF of I given N can be written as $M_{I/N}(s) = \left(M_I^i(s)\right)^N$. By averaging $M_{I/N}(s)$ over the probability distribution (1), we find

$$M_I(s) = e^{\beta A_I (M_I^i(s) - 1)}. \tag{11}$$

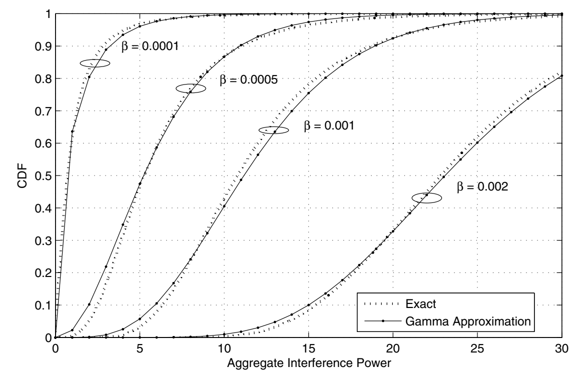

Fig. 2: The CDF of the aggregate interference and the Gamma approximation for different values of β under $P_s = 30\,\text{dB}$, $R_G = 15$, $R_E = 100$, $\alpha = 2$ and no shadowing.

B. Approximating the aggregate interference

The first- and second-order statistics of the aggregate interference are important performance means. $E[I]$, the expected value of the interference is $\beta A_I E[I_i]$. It can be shown that

$$E[I] = 2\pi P_s \beta \sqrt{e^{\sigma^2}} \left(\frac{R_E^{2-\alpha} - R_G^{2-\alpha}}{2-\alpha}\right). \tag{12}$$

The variance can be found out to be

$$Var[I] = \pi\beta P_s^2 k\theta^2(1+k)\left(\frac{R_E^{2-2\alpha} - R_G^{2-2\alpha}}{1-\alpha}\right). \tag{13}$$

These 1^{st} and 2^{nd} moments can be matched with the respective moments of a Gamma distribution to obtain a simple approximation for the aggregate interference. The shape parameter and the scale parameter of the Gamma approximation can be found out to be $\frac{(E[I])^2}{Var[I]}$, and $\frac{Var[I]}{E[I]}$ respectively. Fig. 2 shows the CDF of the simulated aggregate interference, and the CDF of the Gamma equivalent aggregate interference. This figure reveals the aggregate interference roughly follows a skewed alpha-stable distribution [13], where the skewness parameter reduces as β is increased. The two curves show a tight fit for both large and small β values, and the Gamma approximation of the aggregate interference at the PR is accurate.

IV. Performance Analysis

This section derives the outage probability, the asymptotic outage and the diversity/coding gains.

A. Receiver SINR characteristics

Here, we derive the CDF and the PDF . The SINR γ at the PR can be written as

$$\gamma = \frac{P_p R^{-\alpha} Y}{I + \sigma_n^2}, \tag{14}$$

$$\mathcal{Q}(R) = \frac{b^{4+\frac{4}{\alpha}}(-1)^{\frac{2}{\alpha}+\lambda}\alpha^2\Gamma(\lambda-1)R^{2\alpha+2}}{(P_s s)^{\frac{2}{\alpha}}}\,{}_2F_2\left(2, 2+\frac{2}{\alpha}; 3+\frac{2}{\alpha}, 2-\lambda; \frac{R^\alpha b^2}{4P_s s}\right) + \frac{2^{5+\frac{4}{\alpha}}\left(P_s s\right)^2(1+\alpha)\pi}{sin(\pi\lambda)} \times \left((2+\alpha)\Gamma(1+\lambda+\frac{2}{\alpha}) + \alpha\left(\lambda\Gamma(1+\lambda+\frac{2}{\alpha}, -\frac{R^\alpha b^2}{4P_s s}) - \Gamma(2+\lambda+\frac{2}{\alpha}, -\frac{R^\alpha b^2}{4P_s s})\right)\right) \quad (8)$$

$$M^i_{I,approx}(s) = \frac{1}{2+k\alpha}\left(\frac{2\pi}{A_I\theta^k}\right)\left(\frac{R_E^{\alpha k+2}}{(P_s s)^k}\mathbb{I}\left(\frac{R_E^\alpha}{P_s s\theta}\right) - \frac{R_G^{\alpha k+2}}{(P_s s)^k}\mathbb{I}\left(\frac{R_G^\alpha}{P_s s\theta}\right)\right) \quad (9)$$

$$f_\gamma(x) = \left(A - \frac{BV}{x}\left(\frac{H\left(k+\frac{2}{\alpha}\right)}{(Ux+J)^k} - \mathbb{I}\left(\frac{J}{Ux}\right) + \frac{Hk}{(Ux)^k}\mathbb{I}\left(\frac{J}{Ux}\right) - \frac{F\left(k+\frac{2}{\alpha}\right)}{(Ux+G)^k} + \mathbb{I}\left(\frac{G}{Ux}\right) - \frac{Gk}{(Ux)^k}\mathbb{I}\left(\frac{G}{Ux}\right)\right)\right) \times \mathrm{e}^{\left(-B-Ax+BV\left(\frac{F}{(Ux)^k}\mathbb{I}\left(\frac{G}{Ux}\right) - \frac{H}{(Ux)^k}\mathbb{I}\left(\frac{J}{Ux}\right)\right)\right)} \quad (10)$$

where P_p is the power level of the PT, σ_n^2 is the noise variance, and Y is the channel gain between the primary transmitter and receiver. We only consider the case where the primary signals undergo path loss and Rayleigh fading due to the mathematical complexity of analyzing for other cases. Then, Y is a unit exponential PDF with $f_Y(y) = e^{-y}$ for $y > 0$. The variables Y and I are independent, and the CDF of γ is

$$F_{\gamma/I}(x) = P\left(\frac{P_p R^{-\alpha}Y}{I+\sigma_n^2} \le x\right) = P\left(Y \le \frac{x(I+\sigma_n^2)}{P_p R^{-\alpha}}\right) = 1 - e^{\left(-\frac{x(I+\sigma_n^2)}{P_p R^{-\alpha}}\right)}. \quad (15)$$

Averaging with respect to I, we get

$$F_\gamma(x) = 1 - e^{\left(-\frac{x\sigma_n^2}{P_p R^{-\alpha}}\right)} M_I\left(\frac{x}{P_p R^{-\alpha}}\right). \quad (16)$$

The PDF $f_\gamma(x)$ can be obtained as equation (10), where $A = \frac{\sigma_n^2}{P_p R^{-\alpha}}$, $B = \beta A_I$, $U = \frac{1}{P_p R^{-\alpha}}$, $V = \frac{1}{2+k\alpha}\left(\frac{2}{A_I\theta^k}\right)$, $F = \frac{R_E^{\alpha k+2}}{P_s^k}$, $G = \frac{-R_E^\alpha}{P_s\theta}$, $H = \frac{R_G^{\alpha k+2}}{P_s^k}$ and $J = \frac{-R_G^\alpha}{P_s\theta}$.

B. Asymptotic Outage

Since the outage probability (16) is complicated, a mathematically tractable asymptotic expression is useful. Our aim here is to obtain the outage probability when $\frac{P_p}{T}$ is significantly larger than P_s and σ_n^2. By expanding $\mathbb{I}(\mathcal{C}x)$, we can obtain

$$\mathbb{I}(\mathcal{C}x) = \frac{2+\alpha k}{2x^k\mathcal{C}^k} + \frac{k(2+\alpha k)}{x^{k+1}\mathcal{C}^{k+1}(\alpha-2)} + O\left(\frac{1}{x^{k+2}}\right), \quad (17)$$

where $\mathcal{C}$ is a constant. Thus, by using equation (17), for the expression of the MGF (9) in equation (16), and with some algebraic manipulations, the CDF for high $\frac{P_p}{T}$ with respect to the noise and interference can be obtained as

$$F_{\gamma_{Asy}}(x) = 1 - e^{\left(-\frac{x\sigma_n^2}{P_p R^{-\alpha}}\right)} e^{\frac{2\beta\pi\theta k R^\alpha}{\alpha-2}\frac{P_s}{P_p}\left(R_E^{2-\alpha} - R_G^{2-\alpha}\right)x}. \quad (18)$$

For small x, e^x can be written as $1+x$. Therefore, (18) can be approximated by

$$F_{\gamma_{Asy}}(x) = 1 - \left(1 - \frac{x\sigma_n^2}{P_p R^{-\alpha}}\right) \times \left(1 + \frac{2\beta\pi\theta k R^\alpha}{\alpha-2}\frac{P_s}{P_p}\left(R_E^{2-\alpha} - R_G^{2-\alpha}\right)x\right). \quad (19)$$

Defining $\mathcal{A} = \frac{\sigma_n^2}{R^{-\alpha}}$ and $\mathcal{B} = \frac{2\beta\pi\theta k R^\alpha}{\alpha-2}P_s\left(R_E^{2-\alpha} - R_G^{2-\alpha}\right)$, we get

$$F_{\gamma_{Asy}}(x) = 1 - \left(1 - \mathcal{A}\frac{x}{P_p}\right)\left(1 + \mathcal{B}\frac{x}{P_p}\right) \approx (\mathcal{A} - \mathcal{B})\frac{x}{P_p}, \quad (20)$$

which is the asymptotic CDF.

C. Diversity Gain and Coding Gain

The diversity gain and coding gain fully characterize the asymptotic performance of a system. The outage probability at high SINR can be written as $P_{out}(\gamma) \approx (G_c\gamma)^{-G_d}$, where G_c is the coding gain and G_d is the diversity gain. The outage probability is the same as the CDF with x replaced by the threshold level T. Increasing the SINR is analogous to increasing P_p while having a constant σ and P_s. Therefore, P_{out} becomes

$$P_{out} \approx \left(\frac{1}{(\mathcal{A}-\mathcal{B})}P_{norm}\right)^{-1}, \quad (21)$$

where $P_{norm} = \frac{P_p}{T}$ is the normalized PT transmit power with respect to the threshold.
From (21), we observe that $G_d = 1$ and $G_c = \frac{1}{(\mathcal{A}-\mathcal{B})T}$.

V. Numerical Results

In this section, we show the exact and asymptotic characteristics of the outage probability with the variation of P_{norm} ($\frac{P_p}{T}$), P_s and β under differing conditions. A Gamma shadowing environment has been considered for the simulation, and $M^i_{I,approx}$ has been used for our theoretical calculations. When $\sigma \to 0$, we have the situation where shadowing is negligible. In Fig. 3, we plot the outage probability with respect to both P_P and P_s for fixed values of R, R_G, R_E and a noise variance $\sigma_n^2 = 1$. We use free space propagation ($\alpha = 2$), shadowing index $\sigma = 2$, and the interferer density $\beta = 0.0001$. The outage probability decreases slowly at high P_s and, approaches the noise limited scenario for low P_s values. Even if P_p is increased, if P_s increases correspondingly, the outage probability remains unchanged.

Fig. 4 shows the exact and the asymptotic outage probability with respect to P_{norm} for differing values of σ and α. At high

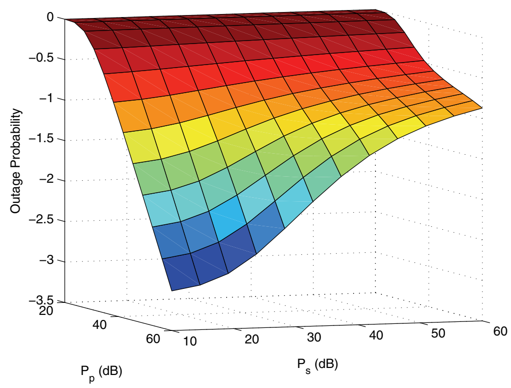

Fig. 3: The outage probability vs P_p and P_s, under $T = 1$, $\alpha = 2$, $\sigma = 2$, $\sigma_n^2 = 1$, $\beta = 0.0001$, $R = 30$, $R_G = 15$, $R_E = 100$

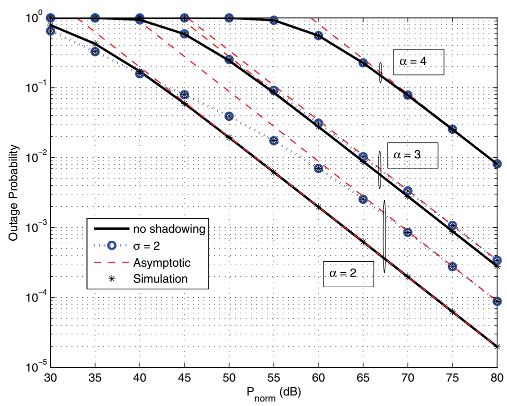

Fig. 4: The exact and asymptotic outage probability vs the normalized transmit power P_{norm}, for different values of σ and α under $\sigma_n^2 = 1$, $\beta = 0.0001$, $P_s = 30\text{dB}$, $R = 30$, $R_G = 15$, $R_E = 100$.

P_{norm}, the asymptotic curves are a perfect match to the exact outage plots. An important observation is that at this value of β, when α increases, shadowing has little effect on the outage. When the path loss exponent is 4, the plots for both σ values show little difference; however, under free space propagation, the plots vary significantly for the two different σ values. This result is consistent with the derivation obtained earlier in (21). For the values for R_G, R_E and R that we selected, the outage probability increases with α. If $R << R_G$, a higher α will ensure a lower outage probability.

Fig. 5 compares the outage probability when the interferer density β varies, for different values of α and σ. This figure shows that the effect of σ on environments with different α depends on the value of β. When the number of CRs increase to very high values, P_{out} approaches 1, while at low β values, P_{out} is governed primarily by noise. It is interesting to note that at high β values, the effect of α on the outage probability is minimal.

VI. Conclusion

This paper investigated the aggregate interference at the primary receiver from CRs distributed in a finite ring encircling the exclusion zone. A Poisson point process of CRs and a composite fading model with the Generalized-K distribution were considered. The exact MGF and approximate MGF of

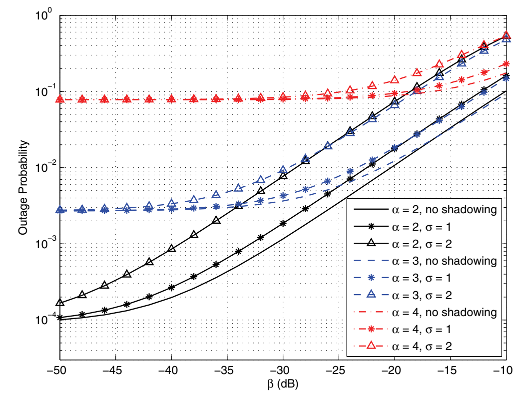

Fig. 5: The outage probability vs interferer density (β) for different values of σ and α under $P_s = 30\,\text{dB}$, $P_p = 70\,\text{dB}$, $R = 30$, $R_G = 15$, $R_E = 100$.

the aggregate interference at the PR were derived. These are valid for arbitrary path loss exponent values. We also showed that the aggregate interference is approximately Gamma distributed. The exact and asymptotic outage probabilities of the PR were derived. Our numerical results confirmed the analysis and showed that the effect of shadowing is significantly lower with higher path loss exponent values.

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