Performance Analysis of Antenna Correlation on LMS-Based Dual-Hop AF MIMO Systems

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Abstract—The concept of relay transmission is applied to land mobile satellite (LMS) communication, where a terrestrial relay node facilitates communication from the satellite to the mobile unit. A novel amplify-and-forward (AF) multiple-inputmultiple-output (MIMO) relay system is proposed where the source, the relay, and the destination are all equipped with multiple antennas. The maximal ratio transmission (MRT)-selection combining (SC) diversity technique is used for the first hop and maximal ratio combining (MRC) is used for the second hop. We analyze the outage probability, the average symbol error rate (SER), and the ergodic capacity, for which accurate closed-form approximations are derived by considering the generalized correlated source-to-relay channel and arbitrary-correlated relayto-destination channel. The system is also analyzed for an equally correlated source-to-relay channel, which is the special case of generalized correlation, by deriving exact closed-form expressions for the outage probability and average SER and highly accurate closed-form analytical approximation for the ergodic capacity. From high SNR asymptotic analysis, we show that the system achieves full diversity order. The impact of multiple antennas and their correlation on the performance are assessed via several examples.

Index Terms-Antenna correlation, cooperative communications, land mobile satellite (LMS), multiple-input-multiple-output (MIMO), relay, shadowed Rician.

I. INTRODUCTION

FOURTH-GENERATION (4G) wireless networks offer high data rates and high-quality service at any time and at any place. To realize this aim, researchers have recently adapted relaying and cooperation, which are already used terrestrial communication techniques, to satellite communication via terrestrial relay nodes forming a hybrid satellite/terrestrial network [1], [2]. An example is cooperative satellite communication via terrestrial relay nodes in [2]. Relaying and cooperation via terrestrial relay nodes (either infrastructure-based fixed relays or mobiles) in land mobile satellite (LMS) communication has several advantages. It makes satellite coverage possible in indoor tunnel environments. In the LMS channel, shadowing effects significantly attenuate the received signal [3]. Mitigating

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the signal attenuation in LMS channels is possible with cooperative relaying.

The performance gain through the incorporation of multiple antennas in a relay network has been established [4]-[8]. The capacity improvement of the LMS link with multiple antennas has been empirically [9] and analytically [10] demonstrated. Hence, the adaptation of relay-assisted cooperation coupled with multiple-input-multiple-output (MIMO) technology to a hybrid satellite/terrestrial network becomes imperative for achieving the ambitious throughput and coverage requirements of 4G. When channel state information (CSI) is available, maximal ratio transmission (MRT) [11] and maximal ratio combining (MRC) [12], which are also known as the transmit and receive beamforming, respectively, are frequently used antenna diversity techniques, as they maximize SNR. However, the use of MRC reception increases system hardware complexity, cost, and power consumption compared with, e.g., selection combining (SC), which uses the antenna with the highest SNR for reception.

For terrestrial communication, the performance of the dualhop variable-gain amplify-and-forward (AF) MIMO relay networks has been widely investigated [4]–[7]. Their outage and error performance with beamforming was shown in [4] over independent Rayleigh fading and in [5] over independent Nakagami-m fading. In [6], Louie et al. extended [4] to the case of antenna correlation. The implementation in [7] considered such a network with transmit antenna selection (TAS) at the source and MRC at the destination and analyzed this network over independent Rayleigh fading. However, in all these studies, the relay node was constrained to a single antenna and a symmetric fading scenario was considered. A variable-gain AF relay was considered in [8] with multiple receive antennas implementing SC and a single transmit antenna, and the performance was analyzed over two hops in symmetric Rayleigh fading. Although the effect of the multiple receive antennas at the relay and destination was analyzed by considering the antenna correlation as well, the source was constrained to have a single antenna.

In [13], Dhungana and Rajatheva proposed an AF MIMO relay system that outperforms the configurations in [4]-[6] and [8]. The authors exploited beamforming together with antenna diversity at the relay. In this system, although the relay employs multiple receive antennas, it uses SC reception. The knowledge of the selected relay antenna is informed to the source via the feedback channel. The source then performs transmit beamforming to the selected antenna of the relay. The received signal at the relay is amplified (or decoded) and forwarded through the single transmit antenna of the relay. The destination uses

Manuscript received January 30, 2012; revised May 11, 2012; accepted June 9, 2012. Date of publication July 12, 2012; date of current version October 12, 2012. The review of this paper was coordinated by Prof. H.-F. Lu. This paper was presented in part at the IEEE International Conference on Communications (ICC), Kyoto, Japan, 2011.



Fig. 1. Dual-hop system with hop-by-hop beamforming along with selection diversity.

MRC reception. Since [13] considered an LMS system with a terrestrial relay node, the source-to-relay link was appropriately modeled to experience shadowed Rician fading and the relay-to-destination link to Rayleigh fading.

To the best of our knowledge, the proposed network setup [13] under such an asymmetric fading scenario with such channel models has never been addressed before. In this paper, the same network setup is adopted with the same scenario and channel models but, now, taking the antenna correlation into account. In real radio environments, antennas tend to be correlated due to insufficient spatial separation. The required physical separation is influenced by the surrounding environment. A direct line-of-sight (LOS) wave arriving at the receiving antenna array is found to be highly correlated among the array elements, whereas multipath scattered waves arriving at the receiving array tend to be uncorrelated at a distance of half the wavelength [14]. In [13], the authors derived results for both AF and decode-and-forward (DF). This paper extends the results of [13] for AF relaying only. The DF results easily follow and are omitted here for brevity. We first derive new closedform expressions for the cumulative distribution function (cdf) of the output SNR of the MRT-SC system under generalized and equally correlated shadowed Rician fading. For generalized correlation of the source-to-relay channel, we next derive new integral expressions for the outage probability, the average symbol error rate (SER), and ergodic capacity of the proposed AF system, which are evaluated using a Gauss-Laguerre quadrature (GLQ) rule as in [8]. Under the special case of equal correlation of the LOS components of the source-to-relay channel, the outage probability and average SER are derived in exact closed form without the GLQ approximation. Under this special case, a second-order approximation for the ergodic capacity is derived along with a tight upper bound, and the asymptotic SER and outage probability evaluations are also presented to provide insights into the diversity order and array gain of the system.

The remainder of this paper is organized as follows. Section II presents the system and the channel model. Section III provides a statistical characterization of the sourceto-relay channel. This characterization requires considerable effort and, hence, deserves a separate section. Section IV analyzes the outage probability, average SER, and ergodic capacity. The analytical and simulation results are given in Section V. Section VI concludes the paper.

II. SYSTEM AND CHANNEL MODEL

We consider the downlink of an LMS-based dual-hop AF MIMO relay network where communication between the satellite and a land mobile terminal is assisted by a terrestrial relay node R with the satellite acting as the source node S and the mobile terminal as the destination node D. Since a relay node is placed where mobile users lose their links with the satellite due to heavy shadowing and, hence, relaying is necessary, the $S \rightarrow D$ direct link is assumed to be unavailable. This situation occurs when, for example, mobile users move to places such as tunnels or vegetation areas, where the satellite link may be completely disrupted [15]. This two-hop scenario with no direct link between the satellite and the mobile unit is equally attractive for serving low-cost user terminals with no satellite transmission/reception capabilities.

The source and the destination are equipped with N_s and N_d antennas, respectively, with the source implementing MRT (also known as transmit beamforming), while the destination implementing MRC (also known as receive beamforming) with the help of the CSI available at these nodes (see Fig. 1). Halfduplex transmission is assumed and, hence, cooperation takes place over two time slots. During the first time slot, the source transmits to the relay. The relay then forwards the amplified version of the source signal to the destination in the second time slot. The relay uses N_r antennas with SC for reception and only one antenna for transmission. Although TAS can be used at the transmitting end of the relay, it requires additional CSI feedback, increasing the system overhead and complexity. This particular receive/transmit strategy at the relay is designed to provide a tradeoff between the performance and the complexity/costs/power. The system privileges the satellite link with multiple antennas at both ends for a diversity gain as the link is more vulnerable to fading, while at the same time tries to keep the system cost/complexity as low as possible. If the relay uses two different sets of antennas for satellite and terrestrial links, respectively, optimized for their link-specific purposes, then having a single transmit antenna at the relay makes such relay implementation cost-effective.

A. Source-to-Relay Channel Model

The $S \to R$ channel matrix $\mathbf{H}_{SR} = [\mathbf{h}_{1_{SR}}, \mathbf{h}_{2_{SR}}, \dots, \mathbf{h}_{N_{rSR}}]^T = [\mathbf{g}_{1_{SR}}, \mathbf{g}_{2_{SR}}, \dots, \mathbf{g}_{N_s > R}] \in \mathbb{C}^{N_r \times N_s}$, where $\mathbf{h}_{k_{SR}} = [h_{k,1_{SR}}, \dots, h_{k,N_{sSR}}]^T \in \mathbb{C}^{N_s \times 1}$, $k \in 1, 2, \dots, N_r$ represents

the channel vector from the source to the kth antenna of the relay and $\mathbf{g}_{l_{SR}} = [h_{1,l_{SR}}, \dots, h_{N_r,l_{SR}}]^T \in \mathbb{C}^{N_r \times 1}, \ l \in 1, 2, \dots, N_s$ represents the channel vector from the *l*th antenna of the source to the relay and is assumed to experience correlated shadowed Rician fading. We follow the shadowed Rician model proposed in [16] where a random shadowing LOS component is modeled by Nakagami-m distribution with m describing the severity of shadowing. Unlike the traditional Nakagami-m, where m varies over the range of $0.5 \le m <$ ∞ , in the shadowed Rician model of [16], m is allowed to vary over the range $0 \le m < \infty$. The spatial correlation at the receive antennas of the relay is considered with the uncorrelated transmit antennas at the satellite by assuming the link to be correlated only at the more constrained end. With a further assumption that the decorrelation distance for the scattered component is shorter than that for the LOS component, only the LOS component is considered to be correlated with the spatially white scattered component.

The shadowed Rician faded $S \rightarrow R$ channel with the receiveside correlated LOS component and the spatially white scattered component can be modeled as $\mathbf{H}_{SR} = \bar{\mathbf{H}}_{SR} +$ $\mathbf{H}_{w_{SR}}$, where the entries of the LOS component $\bar{\mathbf{H}}_{SR} =$ $[\bar{\mathbf{h}}_{1_{SR}}^{I}, \bar{\mathbf{h}}_{2_{SR}}^{I}, \dots, \bar{\mathbf{h}}_{N_{rSR}}^{I}]^{T} = [\bar{\mathbf{g}}_{1_{SR}}^{I}, \bar{\mathbf{g}}_{2_{SR}}^{I}, \dots, \bar{\mathbf{g}}_{N_{sSR}}^{I}] =$ $[z_{k,l}]_{k=1,\ldots,N_r;l=1,\ldots,N_s}$ are Nakagami-*m* random variables (RVs) such that the elements of the vector $\mathbf{h}_{k_{SB}}$, where $k \in$ $1, 2, \ldots, N_r$, are uncorrelated, whereas those of $\bar{\mathbf{g}}_{l_{SR}}$, where $l \in 1, 2, \ldots, N_s$, are correlated with a common correlation matrix; the entries of the scattered component $\mathbf{H}_{w_{SR}} = [\mathbf{h}_{w_{1_{SR}}},$ $\mathbf{h}_{w_{2_{SR}}}, \dots, \mathbf{h}_{w_{N_{rSR}}}]^{T}$ are independent complex Gaussian RVs with zero mean and unit variance. Analysis of the proposed relay network with an arbitrary correlation model for the LOS component becomes intractable. Hence, the correlation model proposed in [17] and [18] is used in this paper. Using an approach similar to that in [17] and [18], we can express the entries of $\mathbf{H}_{SR} = [z_{k,l}]_{k=1,\dots,N_r; l=1,\dots,N_s}$ (for a positive integer m) with mN_sN_r number of zero-mean complex Gaussian RVs

$$(G_{kl})_{n} = \sqrt{\frac{\Omega}{m}} \left(\sqrt{1 - \rho_{k}} (X_{kl})_{n} + \sqrt{\rho_{k}} (X_{0l})_{n} \right) + j \sqrt{\frac{\Omega}{m}} \left(\sqrt{1 - \rho_{k}} (Y_{kl})_{n} + \sqrt{\rho_{k}} (Y_{0l})_{n} \right) k = 1, 2, \dots, N_{r}, \ l = 1, 2, \dots, N_{s}, \ n = 1, 2, \dots, m$$
(1)

where $j = \sqrt{-1}, 0 \le \rho_k < 1$, and $(X_{kl})_n, (Y_{kl})_n (k=0,1,\ldots,N_r; l=1,2,\ldots,N_s; n=1,2,\ldots,m)$ are independent Gaussian RVs with zero mean and variance 1/2. For any $k, u \in \{0,1,\ldots,N_r\}; l, v \in \{1,2,\ldots,N_s\}; n, q \in \{1,2,\ldots,m\}, \mathbb{E}[(X_{kl})_n (Y_{uv})_q] = 0, \mathbb{E}[(X_{kl})_n (X_{uv})_q] = \mathbb{E}[(Y_{kl})_n (Y_{uv})_q] = (1/2)\delta(k-u)\delta(l-v)\delta(n-q)$, where $\delta(.)$ is the Kronecker delta function. The cross-correlation coefficient between any $(G_{kl})_n$ and $(G_{uv})_q$ can be calculated as

$$\rho_{(kl)_n,(uv)_q} = \frac{\mathbb{E}\left[(G_{kl})_n (G_{uv})_q^* \right] - \mathbb{E}\left[(G_{kl})_n \right] \mathbb{E}\left[(G_{uv})_q^* \right]}{\sqrt{\mathbb{E}\left[\left| (G_{kl})_n \right|^2 \right] \mathbb{E}\left[\left| (G_{uv})_q \right|^2 \right]}} = \begin{cases} \sqrt{\rho_k \rho_u} & (k \neq u, l = v, \text{ and } n = q) \\ 0 & (l \neq v \text{ or } n \neq q). \end{cases}$$
(2)

Let

$$R_{k,l}^2 = \sum_{n=1}^{m} |(G_{kl})_n|^2.$$
 (3)

 $R_{k,l}^2(k = 1, 2, ..., N_r; l = 1, 2, ..., N_s)$ is the sum of 2mindependent Gaussian RVs with chi-square distribution $\chi_{2m}(0, 1/2)$, where the notation follows from [17] and [18].¹ $R_{k,l}$, therefore, represents the Nakagami-*m* RV with mean square value $\mathbb{E}[R_{k,l}^2] = \Omega$. By using (2), one can show that the cross-correlation between $R_{k,l}^2$ and $R_{u,l}^2$ ($k \neq u$) is $\rho_{R_{k,l}^2,R_{u,l}^2} = \rho_k \rho_u$, whereas the cross-correlation between $R_{k,l}^2$ and $R_{k,v}^2$ ($l \neq v$) is $\rho_{R_{k,l}^2,R_{k,v}^2} = 0$. The cross-correlation between $R_{k,l}$ and $R_{u,v}$ can be calculated as [19]

$$\rho_{R_{k,l},R_{u,v}} = \varphi(m,1) \left\{ {}_{2}F_{1}\left(-\frac{1}{2},-\frac{1}{2};m;\rho_{R_{k,l}^{2},R_{u,v}^{2}}\right) - 1 \right\}$$
(4)

where $\varphi(m, 1) = \Gamma^2(m + 1/2)/(\Gamma(m)\Gamma(m + 1) - \Gamma^2(m + 1/2))$ and $_2F_1(., .; .; .)$ is the Gaussian hypergeometric function [20, pp. 556]. By assigning $z_{k,l} = R_{k,l}$ $(k = 1, 2, ..., N_r; l = 1, 2, ..., N_s)$, the entries of the receive-side-correlated LOS component matrix $\overline{\mathbf{H}}_{SR}$ of the $S \to R$ channel can be successfully modeled.

B. Relay-to-Destination Channel Model

The $R \to D$ SIMO channel is assumed to be an arbitrary correlated Rayleigh fading channel. Hence, the channel vector $\mathbf{h}_{RD} \in \mathbb{C}^{N_d \times 1}$ can be represented as

$$\mathbf{h}_{RD} = [h_{1_{RD}}, \dots, h_{N_{dRD}}]^T = \mathbf{D}^{\frac{1}{2}} \mathbf{h}_{w_{RD}}$$
(5)

where $\mathbf{D} = \mathbb{E}[\mathbf{h}_{RD}\mathbf{h}_{RD}^*]$ is the $N_d \times N_d$ destination correlation matrix. The entries of $\mathbf{h}_{w_{RD}}$ are independent complex Gaussian RVs with zero mean and unit variance.

C. End-to-End SNR

If the $R \to S$ feedback channel is perfect and the CSI is ideally available, the source beamforms to the selected antenna of the relay. If the kth antenna of the relay is selected at time t, the received signal y_{R_k} is given by

$$y_{R_k} = \sqrt{\frac{P_1}{\epsilon}} \mathbf{h}_{k_{SR}}^H \mathbf{w}_{t_k} s + n_{R_k} \tag{6}$$

where s is the useful transmit signal with $\mathbb{E}[|s|^2] = 1$ and n_{R_k} is the zero-mean complex Gaussian noise with a variance σ_1^2 . $(.)^H$ denotes the conjugate transpose and P_1 is the transmitted power at the source. The normalization factor ϵ ensures that $E[||\mathbf{h}_{k_{SR}}||^2]/\epsilon = N_s$, where ||.|| denotes the Euclidean norm of a vector. The transmit weight vector $\mathbf{w}_{t_k} \in \mathbb{C}^{N_s \times 1}$ is chosen according to the principle of MRT [11] as $\mathbf{w}_{t_k} =$

 $^{{}^{1}\}chi_{n}(s,\sigma^{2})$ denotes a noncentral chi-square distribution with *n* degrees of freedom, noncentrality parameter s^{2} , and the common variance of the corresponding Gaussian components σ^{2} . $\chi_{n}(0,\sigma^{2})$ denotes a central chi-square distribution.

 $\mathbf{h}_{k_{SR}}/\|\mathbf{h}_{k_{SR}}\|$. The output SNR at the *k*th antenna of the *A* relay is then given by

$$\gamma_{k_{SR}} = \frac{\bar{\gamma_1}}{\epsilon} \cdot \|\mathbf{h}_{k_{SR}}\|^2 \tag{7}$$

where $\bar{\gamma_1} = P_1/\sigma_1^2$ is the average SNR per receive antenna of the relay. In SC, the branch with the largest instantaneous SNR is selected as the output of the combiner. If K denotes the index of the selected branch, then $K = \arg_k \max \gamma_{k_{SR}}$. The received signal at the selected branch is then multiplied by the gain G and forwarded to the destination. The received signal vector at the destination $\mathbf{y}_D \in \mathbb{C}^{N_d \times 1}$ thus can be written as

$$\mathbf{y}_D = \mathbf{h}_{RD} G\left(\sqrt{\frac{P_1}{\epsilon}} \|\mathbf{h}_{K_{SR}}\|s + n_{R_K}\right) + \mathbf{n}_D \qquad (8)$$

where $\mathbf{n}_D \in \mathbb{C}^{N_d \times 1}$ is a zero-mean complex Gaussian noise vector with covariance matrix $\sigma_2^2 \mathbf{I}_{N_d}$, where \mathbf{I}_{N_d} is the $N_d \times N_d$ identity matrix. Let us denote the instantaneous SNR of the first hop, i.e., $\gamma_{K_{SR}}$, by γ_1 (i.e., $\gamma_1 = \gamma_{K_{SR}}$). The variable gain G can be chosen as [4], [21]

$$G^2 = \frac{P_2}{(\gamma_1 + \tau)\sigma_1^2} \tag{9}$$

where $\tau = 1$ for the channel-noise-assisted AF (CNA-AF) relays and $\tau = 0$ for the channel-assisted AF (CA-AF) relays [4]. P_2 is the power transmitted at the relay.

The received signal at the destination is multiplied by the receive weight vector $\mathbf{w}_r^H \in \mathbb{C}^{1 \times N_d}$ chosen according to the principle of MRC [12] as $\mathbf{w}_r^H = \mathbf{h}_{RD}^H / ||\mathbf{h}_{RD}||$, thus resulting in

$$\tilde{y}_D = \mathbf{w}_r^H y_D = \sqrt{\frac{P_1}{\epsilon}} G \|\mathbf{h}_{RD}\| \|\mathbf{h}_{K_{SR}}\| s$$
$$+ G \|\mathbf{h}_{RD}\| n_{R_K} + \frac{\mathbf{h}_{RD}^H}{\|\mathbf{h}_{RD}\|} \mathbf{n}_D. \quad (10)$$

If we denote $\gamma_2 = \bar{\gamma}_2 ||\mathbf{h}_{RD}||^2$, which is the instantaneous SNR of the second hop, where $\bar{\gamma}_2 = P_2/\sigma_2^2$, then the end-toend SNR γ_{eq} , after some mathematical manipulations, can be expressed as

$$\gamma_{\rm eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + \tau}.$$
 (11)

III. STATISTICAL CHARACTERIZATION OF THE OUTPUT SNR OF THE $S \to R$ Channel

The statistical characterization of the $S \rightarrow R$ channel output SNR is shown here by deriving the exact closed-form expression for the cdf, which will subsequently be used in the perfomance analysis of the proposed dual-hop AF MIMO relay system in the following. In the shadowed Rician channel model, m = 0 corresponds to the complete obstruction of the LOS, reducing the channel to a Rayleigh channel. The $m \ge 1$ and m = 0 cases are separately addressed.

A. $m \ge l$

When $m \geq 1$, we derive the exact closed-form expression for the cdf of the output SNR of the $S \to R$ channel, i.e., $F_{\gamma_1}(y)$, by considering the generalized case of unequal values of ρ_k . In the following, we refer to this case as generalized correlation. When $\rho_k(k = 1, 2, ..., N_r)$ are all equal to ρ , the LOS components of the $S \to R$ channel become equally correlated at the receive side. We refer to this special case as equal correlation. For this special case, the result of generalized correlation is further simplified and a new closed-form expression for $F_{\gamma_1}(y)$ is derived.

Theorem 1: The cdf of γ_1 for a generalized correlation is given by

$$F_{\gamma_1}(y) = \int_0^\infty \frac{1}{\Gamma(mN_s)} t^{mN_s - 1} e^{-t} \prod_{k=1}^{N_r} \\ \times \left[1 - \left(\frac{m}{m + (1 - \rho_k)\Omega} \right)^{mN_s} e^{-\frac{y}{\eta}} e^{-\frac{\rho_k\Omega}{m + (1 - \rho_k)\Omega} t} \\ \times \sum_{i=0}^M \sum_{q=0}^i \frac{1}{q!(i-q)!} \left(\frac{y}{\eta} \right)^q (c_k \theta_k t)^{i-q} \mu_{iq_k} \right] dt$$

$$(12)$$

where

$$c_{k} = \frac{\rho_{k}m}{(1-\rho_{k})(m+(1-\rho_{k})\Omega)}, \quad \theta_{k} = \frac{(1-\rho_{k})\Omega}{m+(1-\rho_{k})\Omega}.$$
(13)

$$\mu_{iq_k} = \frac{1}{2\pi j} \oint_{\phi} \frac{z^{q+N_s(m-1)}}{(1-z)(z-\theta_k)^{mN_s+i-q}} dz$$
(14)

is a contour integral that can be evaluated by using a residue theorem as

$$\mu_{iq_{k}} = \lim_{z \to \theta_{k}} \frac{1}{(mN_{s}+i-q-1)!} \frac{d^{mN_{s}+i-q-1}}{dz^{mN_{s}+i-q-1}} \left(\frac{z^{q+N_{s}}(m-1)}{(1-z)}\right)$$
$$= \sum_{g=0}^{mN_{s}+i-q-1} \binom{q+N_{s}(m-1)}{g} \frac{\theta_{k}^{q+N_{s}}(m-1)-g}{(1-\theta_{k})^{mN_{s}+i-q-g}}.$$
(15)

Proof: See Appendix A.

Corollary 1: For equal correlation, the cdf of γ_1 can be expressed as

$$F_{\gamma_{1}}(y) = 1 - \sum_{p=1}^{N_{r}} {N_{r} \choose p} (-1)^{p+1} \sum_{i=0}^{pM} \sum_{q=0}^{i} \beta_{iqp} \frac{(c\theta)^{i-q}}{\eta^{q}} y^{q} e^{-p\frac{y}{\eta}}$$
$$\times \frac{\Gamma(mN_{s}+i-q)}{\Gamma(mN_{s})} \frac{\left(1 + (1-\rho)\frac{\Omega}{m}\right)^{i-q-mN_{s}(p-1)}}{\left(1 + (1-\rho)\frac{\Omega}{m} + p\rho\frac{\Omega}{m}\right)^{mN_{s}+i-q}} \quad (16)$$

where

$$c = \frac{\rho m}{(1-\rho)(m+(1-\rho)\Omega)}, \quad \theta = \frac{(1-\rho)\Omega}{m+(1-\rho)\Omega}.$$
 (17)

The coefficients β_{iqp} are recursively computed by

$$\beta_{iqp} = \sum_{u=i-M}^{i} \sum_{l=q-i+u}^{q} \beta_{ul(p-1)} \frac{\mu_{(i-u)(q-l)}}{(q-l)!(i-u-q+l)!} \times \mathcal{I}_{[0,(p-1)M]}(u) \mathcal{I}_{[0,u]}(l) \quad (18)$$

where $\beta_{iq1} = \mu_{iq}/(q!(i-q)!)$, $\mathcal{I}_{[f,h]}(o)$ is the indicator function defined by

$$\begin{split} \mathcal{I}_{[f,h]}(o) &= \begin{cases} 1 & f \le o \le h \\ 0 & \text{otherwise} \end{cases} \text{and} \\ \mu_{iq} &= \frac{1}{2\pi j} \oint_{\phi} \frac{z^{q+N_s(m-1)}}{(1-z)(z-\theta)^{mN_s+i-q}} \, dz \\ &= \sum_{g=0}^{mN_s+i-q-1} \binom{q+N_s(m-1)}{g} \frac{\theta^{q+N_s(m-1)-g}}{(1-\theta)^{mN_s+i-q-g}}. \end{split}$$

Proof: When $\rho_k(k = 1, 2, ..., N_r) = \rho$, (12) reduces to

$$F_{\gamma_1}(y) = \int_0^\infty \frac{1}{\Gamma(mN_s)} t^{mN_s - 1} e^{-t}$$

$$\times \left[1 - \left(\frac{m}{m + (1 - \rho)\Omega}\right)^{mN_s} e^{-\frac{y}{\eta}} e^{-\frac{\rho\Omega}{m + (1 - \rho)\Omega}t} \right]_{i=0}^{N_r} \frac{1}{q!(i - q)!} \left(\frac{y}{\eta}\right)^q (c\theta t)^{i-q} \mu_{iq} \left[\int_{0}^{N_r} dt \right]_{i=0}^{N_r} dt$$
(20)

where c and θ are defined in (17) and μ_{iq} in (19). The desired expression (16) is obtained by applying the binomial theorem and then using the expansion

$$\left[\sum_{i=0}^{M}\sum_{q=0}^{i}\frac{\mu_{iq}}{q!(i-q)!}\left(\frac{y}{\eta}\right)^{q}(c\theta t)^{i-q}\right]^{p}$$
$$=\sum_{i=0}^{pM}\sum_{q=0}^{i}\beta_{iqp}\left(\frac{y}{\eta}\right)^{q}(c\theta t)^{i-q} \quad (21)$$

with β_{iqp} computed according to (18) and, finally, using [22, eq. (3.351.3)].

Note that the expression (16) encounters 0^0 for the uncorrelated scenario. If we use $0^0 = 1$, the desired result for the uncorrelated case can be obtained. Expression (16) can be efficiently evaluated for $\rho = 0$ by using MATLAB since it has implemented $0^0 = 1$.

B. m = 0

Theorem 2: For m = 0, the cdf of γ_1 is given by

$$F_{\gamma_1}(y) = 1 - \sum_{p=1}^{N_r} {N_r \choose p} (-1)^{p+1} \sum_{i=0}^{p(N_s-1)} \frac{\beta_{ip}}{\eta^i} y^i e^{-p\frac{y}{\eta}} \quad (22)$$

with β_{ip} computed as

$$\beta_{ip} = \sum_{u=i-N_s+1}^{i} \frac{\beta_{i(p-1)}}{(i-u)!} \mathcal{I}_{[0,(p-1)(N_s-1)]}(u)$$
(23)

where $\beta_{i1} = 1/i!$.

Proof: Since the LOS component is absent, by using (52) the cdf of γ_1 can be written as

$$F_{\gamma_1}(y) = \left[1 - Q_{N_s}\left(0, \sqrt{\frac{2y}{\eta}}\right)\right]^{N_r}.$$
 (24)

By using contour integral representation for the Marcum-Q function and applying the binomial theorem, we get

$$F_{\gamma_1}(y) = 1 - \sum_{p=1}^{N_r} {N_r \choose p} (-1)^{p+1} e^{-p\frac{y}{\eta}} \mu^p$$
(25)

where $\mu = 1/(2\pi j) \int_{\phi} e^{(y/\eta)z}/(z^{N_s}(1-z))dz$ is the contour integral, which is again evaluated by using the residue theorem as

$$\mu = \lim_{z \to 0} \frac{1}{(N_s - 1)!} \frac{d^{N_s - 1}}{dz^{N_s - 1}} \left(\frac{e^{\frac{y}{\eta}z}}{1 - z}\right) = \sum_{i=0}^{N_s - 1} \frac{1}{i!} \left(\frac{y}{\eta}\right)^i.$$
(26)

The theorem is finally proved after applying the expansion

$$\left[\sum_{i=0}^{N_s-1} \frac{1}{i!} \left(\frac{y}{\eta}\right)^i\right]^p = \sum_{i=0}^{p(N_s-1)} \beta_{ip} \left(\frac{y}{\eta}\right)^i \tag{27}$$

where β_{ip} is defined in (23).

The statistics derived here for the $S \rightarrow R$ channel, which is a correlated shadowed Rician fading MIMO LMS channel, can be also useful for the performance analysis of single-hop MIMO LMS communication. The MRT-SC is a very interesting diversity technique for LMS communication due to its lowhardware-complexity and cost-effective qualities, although it has suboptimal performance compared with the MIMO-MRC diversity analyzed in [10]. To our knowledge, this diversity technique for LMS communication has not been analyzed earlier. With the help of the cdf of the SNR thus derived, important performance measures such as outage probability, SER, and ergodic capacity for MRT-SC LMS MIMO channel can be easily derived.

IV. PERFORMANCE ANALYSIS

Using the statistics of γ_1 derived in Section III, we present a comprehensive performance analysis of the proposed AF MIMO relay network under both generalized correlation and the special case of equal correlation models of the first hop when $m \ge 1$. The performance measures include the outage probability, SER, and ergodic capacity. Due to similarity of (22) and (16), the results for the m = 0 case follow similarly to those of the equal correlation case and, hence, can easily be derived. For the sake of brevity, results for the m = 0 case are omitted in this paper.

A. Generalized Correlation of the LOS Components of the $S \rightarrow R$ Channel

1) Outage Probability: Outage probability is a crucial performance measure defined as the probability of failing to achieve a predefined threshold SNR, $\gamma_{\rm th}$. It is obtained by evaluating the cdf of $\gamma_{\rm eq}$ at $\gamma_{\rm th}$ as

$$P_{\rm out} = Pr(\gamma_{\rm eq} < \gamma_{\rm th}) = F_{\gamma_{\rm eq}}(\gamma_{\rm th}).$$
(28)

Theorem 3: An accurate closed-form approximation for the cdf of γ_{eq} in (11) for the generalized correlation of the LOS components of the first hop can be obtained as

$$F_{\gamma_{eq}}(y) \approx 1 - \frac{1}{\Delta(\mathbf{D})} \sum_{r=1}^{N_d} \phi_r^{N_d - 1} C(N_d, r) e^{-\frac{y}{\bar{\gamma}_2 \phi_r}} \\ \times \left[1 - \frac{1}{\Gamma(mN_s)} \sum_{p=1}^{N} \sum_{l=1}^{N} \omega_p \vartheta_l \prod_{k=1}^{N_r} \right] \\ \times \left\{ 1 - \left(\frac{m}{m + (1 - \rho_k)\Omega} \right)^{mN_s} \right\} \\ \times e^{-\frac{\rho_k \Omega}{m + (1 - \rho_k)\Omega} t_p} e^{-\frac{y(\zeta_l \bar{\gamma}_2 \phi_r + y + \tau)}{\zeta_l \bar{\gamma}_2 \phi_r \eta}} \\ \times \sum_{i=0}^{M} \sum_{q=0}^{i} \frac{1}{q!(i-q)!} \left(\frac{y(\zeta_l \bar{\gamma}_2 \phi_r + y + \tau)}{\zeta_l \bar{\gamma}_2 \phi_r \eta} \right)^q \\ \times (c_k \theta_k t_p)^{i-q} \mu_{iq_k} \right\} \right]$$
(29)

where $\phi_1 > \phi_2 > \cdots > \phi_{N_d}$ are the eigenvalues of **D**. $C(N_d, r)$ is the (N_d, r) cofactor of the Vandermonde matrix, $V_{uv} = \phi_v^{u-1}$. $\Delta(\mathbf{D})$ is the determinant of V_{uv} . t_p , $\zeta_l(p = 1, \ldots, N)$ $N; l = 1, \ldots, N)$ and $\omega_p, \vartheta_l(p = 1, \ldots, N; l = 1, \ldots, N)$ are the abscissas and weights, respectively, of GLQ. $t_p(p = 1, \ldots, N)$ are the roots of the generalized Laguerre polynomial, $L_N^{mN_s-1}(t)$, with the corresponding weights $\omega_p(p = 1, \ldots, N)$ given by [23, eq. (1)]

$$\omega_p = \frac{\Gamma(N+mN_s)t_p}{N!(N+1)^2 \left[L_{N+1}^{mN_s-1}(t_p)\right]^2}$$
(30)

which can be easily computed by using Mathematica. Similarly, $\zeta_l (l = 1, ..., N)$ are the roots of the Laguerre polynomial $L_N(\zeta)$ with the corresponding weights $\vartheta_l (l = 1, ..., N)$ given by [20, p. 890]

$$\vartheta_l = \frac{\zeta_l}{(N+1)^2 \left[L_{N+1}(\zeta_l) \right]^2}.$$
(31)

Proof: The proof is given in Appendix B.

The outage probability of CNA-AF for the given correlation structure is obtained by substituting $y = \gamma_{\text{th}}$ and $\tau = 1$ into (29) and that for CA-AF by substituting $y = \gamma_{\text{th}}$ and $\tau = 0$ into (29).

2) Average SER: We consider the modulation formats with conditional SER expression of the form

$$P_s = aE_\gamma \left[Q(\sqrt{2b\gamma})\right] \tag{32}$$

where Q(.) is the Gaussian-Q function, and a and b are modulation-specific constants. For example, a = 1 and b = 1for binary phase-shift keying (BPSK); a=1 and b=0.5 for coherently detected orthogonal binary frequency-shift keying (BFSK); and a=2(M-1)/M and $b=3/(M^2-1)$ for M-ary pulse amplitude modulation (PAM). Approximate SER for M-ary PSK can also be found with a = 2 and $b = \sin^2(\pi/M)$.

Theorem 4: An accurate closed-form approximation for the average SER of the AF (CNA-AF and CA-AF) MIMO relay under the generalized correlation structure of the first hop can be expressed as

$$P_{s} \approx \frac{a}{2} - \frac{a\sqrt{b}}{2\Delta(\mathbf{D})} \sum_{r=1}^{N_{d}} \phi_{r}^{N_{d}-1} C(N_{d}, r) \sqrt{\Upsilon_{r}}$$

$$\times \left[1 - \frac{1}{\sqrt{\pi}\Gamma(mN_{s})} \sum_{p=1}^{N} \sum_{l=1}^{N} \sum_{s=1}^{N} \omega_{p} \vartheta_{l} \varpi_{s} \prod_{k=1}^{N_{r}} \right]$$

$$\times \left\{ 1 - \left(\frac{m}{m + (1 - \rho_{k})\Omega} \right)^{mN_{s}} e^{-\frac{\rho_{k}\Omega}{m + (1 - \rho_{k})\Omega} t_{p}} \right]$$

$$\times e^{-\frac{x_{s}\Upsilon_{r}(\zeta_{l}\bar{\gamma}_{2}\phi_{r}\eta)}{\zeta_{l}\bar{\gamma}_{2}\phi_{r}\eta}} \sum_{i=0}^{M} \sum_{q=0}^{i} \frac{1}{q!(i - q)!}$$

$$\times \left(\frac{x_{s}\Upsilon_{r}(\zeta_{l}\bar{\gamma}_{2}\phi_{r} + x_{s}\Upsilon_{r} + \tau)}{\zeta_{l}\bar{\gamma}_{2}\phi_{r}\eta} \right)^{q}$$

$$\times (c_{k}\theta_{k}t_{p})^{i-q}\mu_{iq_{k}} \right\} \right]$$

$$(33)$$

where $\Upsilon_r = \bar{\gamma}_2 \phi_r / (1 + b \bar{\gamma}_2 \phi_r)$. ω_p , ϑ_l , t_p , and ζ_l are defined in Theorem 3. x_s (s = 1, ..., N) are the roots of the generalized Laguerre polynomial $L_N^{-1/2}(x)$, with the corresponding weights $\varpi_s(s = 1, ..., N)$ given by

$$\varpi_s = \frac{\Gamma(N+\frac{1}{2})x_s}{N!(N+1)^2 \left[L_{N+1}^{-1/2}(x_s)\right]^2}.$$
 (34)

Proof: With γ replaced by $\gamma_{\rm eq}$, (32) can be expressed as [24]

$$P_s = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{e^{-by}}{\sqrt{y}} F_{\gamma_{\rm eq}}(y) \, dy. \tag{35}$$

Expression (33) can then be derived by substituting (65) into (35) and again solving the triple integral of the form $\int_0^\infty x^{\beta_2} e^{-x} \int_0^\infty e^{-\zeta} \int_0^\infty t^{\beta_1} e^{-t} f(t,\zeta,x) dt d\zeta dx$ by using the GLQ rule.

3) Ergodic Capacity: The capacity of a communication channel characterizes the maximum amount of error-free information that can be transmitted over that channel in a given time over a given bandwidth. The ergodic capacity is obtained by averaging the mutual information between the source and the destination. The ergodic capacity of a dual-hop AF MIMO relay can be written as

$$\begin{aligned} \mathcal{C} &= \frac{1}{2} \mathbb{E} \left[\log_2(1 + \gamma_{\rm eq}) \right] \\ &= \frac{1}{2(\ln 2)} \int_0^\infty \ln(1 + y) f_{\gamma_{\rm eq}}(y) \, dy \end{aligned} (36) \\ &= \frac{1}{2(\ln 2)} \int_0^\infty \frac{1}{1 + y} \left(1 - F_{\gamma_{\rm eq}}(y) \right) dy. \end{aligned} (37)$$

Theorem 5: By considering the generalized correlation model for the first hop, an accurate closed-form approximation for the ergodic capacity is given by

$$\mathcal{C} \approx \frac{1}{2(\ln 2)\Delta(\mathbf{D})} \sum_{r=1}^{N_d} \phi_r^{N_d - 1} C(N_d, r)$$

$$\times \left[-e^{\frac{1}{\bar{\gamma}_2 \phi_r}} \operatorname{Ei} \left(-\frac{1}{\bar{\gamma}_2 \phi_r} \right) - \frac{1}{\Gamma(mN_s)} \right]$$

$$\times \sum_{p=1}^N \sum_{l=1}^N \sum_{s=1}^N \omega_p \vartheta_l \upsilon_s \frac{1}{(\varrho_s + \frac{1}{\bar{\gamma}_2 \phi_r})} \prod_{k=1}^{N_r} \right]$$

$$\times \left\{ 1 - \left(\frac{m}{m + (1 - \rho_k)\Omega} \right)^{mN_s} e^{-\frac{\rho_k \Omega}{m + (1 - \rho_k)\Omega} t_p} \right\}$$

$$\times e^{-\frac{\varrho_s (\zeta_l \bar{\gamma}_2 \phi_r + \varrho_s \bar{\gamma}_2 \phi_r + \tau)}{\zeta_l \eta}} \sum_{i=0}^M \sum_{q=0}^i \frac{1}{q!(i-q)!} \left\{ \left(\frac{\varrho_s (\zeta_l \bar{\gamma}_2 \phi_r + \varrho_s \bar{\gamma}_2 \phi_r + \tau)}{\zeta_l \eta} \right)^q \right\} \right\}$$

$$\times (c_k \theta_k t_p)^{i-q} \mu_{iq_k} \right\} \right]$$
(38)

where Ei(.) is the exponential integral function [22, Sec. (8.2)]. ω_p , ϑ_l , t_p , and ζ_l are defined in Theorem 3. $\varrho_s(s = 1, ..., N)$ are the roots of the Laguerre polynomial $L_N(\varrho)$ with the corresponding weights $\upsilon_s(s = 1, ..., N)$ given by

$$v_s = \frac{\varrho_s}{(N+1)^2 [L_{N+1}(\varrho_s)]^2}.$$
(39)

Proof: The theorem is proved by substituting (65) into (37) and then following the same approach as we did in Theorem 4 and further using [22, eq. (3.352.4)].

B. Equal Correlation of the LOS Component of the $S \rightarrow R$ Channel

When the LOS components of the $S \rightarrow R$ channel are equally correlated, the performance metrics that we derived for generalized correlation by using GLQ approximation are now obtained in exact closed form without any approximation and, hence, can be accurately and efficiently computed.

1) Outage Probability:

Theorem 6: The exact closed-form expression for the cdf of γ_{eq} for the equal correlation of the LOS components of the first hop is given by

$$F_{\gamma_{eq}}(y) = 1 - \frac{2}{\Delta(\mathbf{D})} \sum_{p=1}^{N_r} {N_r \choose p} (-1)^{p+1} \sum_{i=0}^{pM} \sum_{q=0}^{i} \beta_{iqp} (c\theta)^{i-q} \\ \times \frac{\Gamma(mN_s + i-q)}{\Gamma(mN_s)} \frac{\left(1 + (1-\rho)\frac{\Omega}{m}\right)^{i-q-mN_s(p-1)}}{\left(1 + (1-\rho)\frac{\Omega}{m} + p\rho\frac{\Omega}{m}\right)^{mN_s + i-q}} \\ \times \sum_{r=1}^{N_d} C(N_d, r) \sum_{s=0}^{q} {q \choose s} \frac{\phi_r^{\frac{2N_d - s - 3}{2}}}{p^{\frac{s-1}{2}} \eta^{\frac{2q-s+1}{2}}} y^{\frac{2q-s+1}{2}} \\ \times (y+\tau)^{\frac{s+1}{2}} e^{-y\left(\frac{p}{\eta} + \frac{1}{\bar{\gamma}_2 \phi_r}\right)} K_{s-1} \left(2\sqrt{\frac{p(\tau y + y^2)}{\eta \bar{\gamma}_2 \phi_r}}\right)$$
(40)

where $K_v(x)$ is the modified Bessel function of the second kind of order v [22, Sec. 8.407].

Proof: By substituting (16) and (64) into (63) and then solving the resultant integral by applying the binomial theorem and using [22, eq. (3.471.9)], (40) is obtained.

The outage probability is obtained by evaluating (40) at $y = \gamma_{\rm th}$.

2) Average SER: By using the equal correlation model for the LOS components of the first hop, we evaluate the average SER of the system, taking $\tau = 0$ (CA-AF) for the mathematical tractability, which is also a tight upper bound for that of CNA-AF [4], [21].

Theorem 7: The average SER of the CA-AF system when the LOS components of the $S \rightarrow R$ channel are equally correlated at the receive side is given by

$$P_{s} = \frac{a}{2} - \frac{a\sqrt{b}}{\Delta(\mathbf{D})} \sum_{p=1}^{N_{r}} {N_{r} \choose p} (-1)^{p+1} \\ \times \sum_{i=0}^{pM} \sum_{q=0}^{i} \beta_{iqp} (c\theta)^{i-q} \frac{\Gamma(mN_{s}+i-q)}{\Gamma(mN_{s})} \\ \times \frac{\left(1 + (1-\rho)\frac{\Omega}{m}\right)^{i-q-mN_{s}(p-1)}}{\left(1 + (1-\rho)\frac{\Omega}{m} + p\rho\frac{\Omega}{m}\right)^{mN_{s}+i-q}} \\ \times \sum_{r=1}^{N_{d}} C(N_{d},r) \sum_{s=0}^{q} {q \choose s} \frac{\phi_{r}^{\frac{2N_{d}-s-3}{2}}}{p^{\frac{s-1}{2}}\eta^{\frac{2q-s+1}{2}}\bar{\gamma}_{2}^{\frac{s+1}{2}}} \\ \times \frac{\Gamma\left(q+s+\frac{1}{2}\right)\Gamma\left(q-s+\frac{5}{2}\right)}{\Gamma(q+2)} \frac{(2B)^{s-1}}{(b+A+B)^{q+s+\frac{1}{2}}} \\ \times {}_{2}F_{1}\left(q+s+\frac{1}{2},s-\frac{1}{2};q+2;\frac{b+A-B}{b+A+B}\right)$$
(41)

where $A = p/\eta + 1/(\bar{\gamma}_2 \phi_r)$ and $B = 2\sqrt{p/(\eta \bar{\gamma}_2 \phi_r)}$.

Proof: The theorem can be proved by substituting $F_{\gamma_{eq}}(y)$ in (40) with $\tau = 0$ into (35) and solving the resultant integral by using [22, eq. (6.621.3)].

3) Asymptotic Evaluations of the Average SER and Outage Probability: To provide further insights into the parameters governing the system performance, we present high SNR approximations for the average SER and the outage probability of CA-AF since CA-AF relay provides a tight upper bound for the CNA-AF relay at high SNR [21]. By using the results from [25], at high SNR, the average SER of CA-AF can be approximated as

$$P_s \approx (G_a \eta)^{-G_d} \tag{42}$$

where G_d is the diversity gain. The array gain G_a is given by

$$G_a = 2b \left(\frac{a2^{G_d - 1}\Lambda\Gamma(G_d + 1/2)}{\sqrt{\pi}G_d}\right)^{-\frac{1}{G_d}}$$
(43)

where

$$\Lambda = \frac{1}{(G_d - 1)!} \frac{\partial^{G_d - 1}}{\partial x^{G_d - 1}} f_X(0)$$

= $\frac{1}{(G_d - 1)!} \frac{\partial^{G_d}}{\partial x^{G_d}} F_X(0), \qquad X = \frac{\gamma_{\text{eq}}}{\eta}.$

In [4] and [6], where a single antenna relay is employed, the diversity order is given by the minimum number of antennas at S and D. However, we have multiple receive antennas at the relay with SC. By using the fact that MRT-SC can achieve a full diversity order [26], the diversity order of the proposed AF relay network, i.e., G_d , is given by

$$G_d = \min(N_s N_r, N_d) \tag{44}$$

which is the maximum possible diversity order. To obtain Λ , we first substitute $y = x\eta$ into (40). After expressing the Bessel function by its series expansion about x = 0, we then find its G_d th order derivative with respect to x and, finally, evaluate it at x = 0. Λ is then given by

$$\begin{split} \Lambda &= \frac{1}{\Delta(\mathbf{D})} \sum_{p=1}^{N_r} \binom{N_r}{p} \sum_{i=0}^{pM} \sum_{q=0}^{i} \beta_{iqp} (c\theta)^{i-q} \\ &\times \frac{\Gamma(mN_s + i - q)}{\Gamma(mN_s)(G_d - 1)!} \frac{\left(1 + (1 - \rho)\frac{\Omega}{m}\right)^{i-q - mN_s(p-1)}}{\left(1 + (1 - \rho)\frac{\Omega}{m} + p\rho\frac{\Omega}{m}\right)^{mN_s + i-q}} \\ &\times \sum_{r=1}^{N_d} C(N_d, r) \sum_{s=0}^{q} \binom{q}{s} \sum_{l=0}^{|s-1|-1|} \frac{\left(|s-1|-l-1\right)!}{l!} \\ &\times \frac{\phi_r^{\frac{2N_d - s + |s-1| - 2l - 3}{2}}}{p^{\frac{s + |s-1| - 2l - 1}{2}} \kappa^{\frac{s - |s-1| + 2l + 1}{2}}} \sum_{h=0}^{G_d} \binom{G_d}{h} (-1)^{p+l+h} \\ &\times \left(p + \frac{1}{\kappa\phi_r}\right)^h \prod_{o=1}^{G_d - h} (q + 2l + 2 - |s-1| - o) \\ &\times \delta(q + 2l + 1 - |s-1| - G_d + h) \end{split}$$
(45)

where $\kappa = \bar{\gamma}_2/\eta$ and $\delta(.)$ is the Kronecker delta function.

As we successfully derived the first term of the Taylor series expansion of the probability density function (pdf) $f_X(x)$ of $X = \gamma_{eq}/\eta$ at x = 0 for CA-AF system $f_X(x) \approx \Lambda x^{G_d-1} + O(x^{G_d-1})$ as $x \to 0$, by using the definition of outage probability (28), the asymptotic outage probability can be expressed as

$$P_{\rm out} = F_X(\gamma_{\rm th}/\eta) \approx \frac{\Lambda}{G_d} \left(\frac{\gamma_{\rm th}}{\eta}\right)^{G_d}.$$
 (46)

4) Ergodic Capacity: To find the exact ergodic capacity of the dual-hop AF MIMO relay under the equal correlation structure of the first hop by substituting (40) into (37), one needs to solve the integral of the form $\int_0^\infty y^u (y+\tau)^v (y+1)^{-1} e^{-\alpha y} K_\beta (\zeta \sqrt{\tau y+y^2}) dy$. The integral seems very difficult to be solved. In (36), if we use the second-order Taylor series approximation of $\ln(1+y)$ about the mean value of γ_{eq} , $\mathbb{E}(\gamma_{eq})$, we can obtain the approximated closed-form solution for C as [27]

$$\mathcal{C} \approx \frac{1}{2(\ln 2)} \left[\ln \left(1 + \mathbb{E}[\gamma_{\rm eq}] \right) - \frac{\mathbb{E}\left[\gamma_{\rm eq}^2 \right] - \left(\mathbb{E}[\gamma_{\rm eq}] \right)^2}{2 \left(1 + \mathbb{E}[\gamma_{\rm eq}] \right)^2} \right]. \quad (47)$$

Theorem 8: The *n*th moment of γ_{eq} with $\tau = 0$ is given by

$$\mathbb{E}[\gamma_{eq}^{n}] = \frac{2\sqrt{\pi}n}{\Delta(\mathbf{D})} \sum_{p=1}^{N_{r}} {N_{r} \choose p} (-1)^{p+1} \sum_{i=0}^{pM} \sum_{q=0}^{i} \beta_{iqp} (c\theta)^{i-q} \\ \times \frac{\Gamma(mN_{s}+i-q)}{\Gamma(mN_{s})} \frac{\left(1+(1-\rho)\frac{\Omega}{m}\right)^{i-q-mN_{s}(p-1)}}{\left(1+(1-\rho)\frac{\Omega}{m}+p\rho\frac{\Omega}{m}\right)^{mN_{s}+i-q}} \\ \times \sum_{r=1}^{N_{d}} C(N_{d},r) \sum_{s=0}^{q} {q \choose s} \frac{\phi_{r}^{\frac{2N_{d}-s-3}{2}}}{p^{\frac{s-1}{2}}\eta^{\frac{2q-s+1}{2}}\bar{\gamma}_{2}^{\frac{s+1}{2}}} \\ \times \frac{\Gamma(q+n+s)\Gamma(q+n-s+2)}{\Gamma(q+n+\frac{3}{2})} \frac{(2B)^{s-1}}{(A+B)^{q+n+s}} \\ \times {}_{2}F_{1}\left(q+n+s,s-\frac{1}{2};q+n+\frac{3}{2};\frac{A-B}{A+B}\right)$$
(48)

where A and B are defined in Theorem 7.

Proof: The *n*th moment of γ_{eq} can be written as

$$\mathbb{E}\left[\gamma_{\rm eq}^n\right] = \int_0^\infty n y^{n-1} \left(1 - F_{\gamma_{\rm eq}}(y)\right) dy.$$
(49)

Expression (48) results after substituting (40) into (49) with $\tau = 0$ and following the same approach that we used in Theorem 7.

The second-order approximated ergodic capacity of the dualhop CA-AF MIMO system can now be obtained by substituting the first- and second-order moments computed by using (48) into (47).

V. NUMERICAL RESULTS

Here, we validate our analytical results through Monte Carlo simulations and assess the impact of antenna correlation on the performance of the proposed AF MIMO relay system by providing several illustrative examples. Our analytical expressions involve infinite series representation effectively truncated by M number of terms. We set M to be 10 in the following numerical



Fig. 2. Outage probability of CNA-AF with m = 2, $\eta = 10$ dB, $\bar{\gamma}_2 = 5$ dB, and $\Omega = 0.2$ for antenna configurations (3,3,2) with a generalized correlation model and (4,4,4) with an equal correlation model.

examples. In computing the GLQ approximation, we choose N to be 30. For generalized correlated $S \to R$ channel, we denote the ρ_k $(k = 1, 2, ..., N_r)$ values used in the following numerical computations by the vector $\bar{\rho} = [\rho_1, \rho_2, ..., \rho_{N_r}]$. The correlation of the equally correlated $S \to R$ channel is parameterized by ρ . Although any arbitrary correlation model is valid for the second hop, for our numerical results, we construct a correlation matrix **D** by using the practical channel model of [28] as in [6], [8], and [24], with the $(u, v)^{\text{th}}$ entry of **D** given by

$$\mathbf{D}_{u,v} = e^{-j2\pi(v-u)d\cos(\Theta)} e^{-\frac{1}{2}(2\pi(v-u)d\sin(\Theta)\Sigma)^2}$$
(50)

where d is the relative antenna spacing between the adjacent antennas (measured in number of wavelengths) of the uniform linear antenna array at the destination, and Θ and Σ are the mean angle of arrival (AoA) and the destination angle spread, respectively. The actual AoA is given by $\underline{\Theta} = \Theta + \hat{\Theta}$, where $\hat{\Theta}$ is a Gaussian RV with zero mean and variance Σ^2 .

Fig. 2 shows the outage probability versus threshold SNR $\gamma_{\rm th}$ plots of the CNA-AF system in different correlation scenarios for both the generalized correlation and the equal correlation models. The analytical plots are from (29) for generalized correlation and from (40) for equal correlation with $\tau = 1$ and $y = \gamma_{\rm th}$. These plots clearly agree with the Monte Carlo simulations. The detrimental effect of correlation at low SNR thresholds and the beneficial effect at high SNR thresholds are clearly evident. Such behavior was also observed in [6].

Fig. 3 exhibits the outage probability performance of the CA-AF system against average SNR for a given threshold SNR $\gamma_{\rm th} = 1$ in different correlation scenarios by using the equal correlation model. The analytical curves are plotted by using (40) with $\tau = 0$ and $y = \gamma_{\rm th}$, and they clearly agree with the Monte Carlo simulations. Two different antenna configurations have been shown to illustrate that the correlation does not allow



Fig. 3. Outage probability of CA-AF with m = 2, $\Omega = 0.2$, $\gamma_{\rm th} = 1$, and $\bar{\gamma}_2 = \eta$ for antenna configuration (N_s, N_r, N_d) using the equal correlation model.



Fig. 4. Average SER of CNA-AF with m = 3, $\Omega = 0.2$, and $\bar{\gamma}_2 = 1/4\eta$ using BPSK modulation and a generalized correlation model.

the system to fully exploit the benefits of multiple antennas. The high SNR analytical curves are from (46). These asymptotic curves, as they converge with the exact outage probability curves in the high SNR regime, verify that the system achieves maximum diversity order possible.

Fig. 4 assesses the impact of spatial correlation on the average SER of the CNA-AF system for the generalized correlated $S \rightarrow R$ link by using BPSK modulation, for which the analytical plots are from (33) with a = 1 and b = 1. Of the two antenna configurations shown, both of which were plotted by taking L = 30, the analytical and Monte Carlo simulation plots match better for those with fewer antennas. The result can be otherwise improved by increasing the value of L. The SER performance improves by employing more antennas at the terminals. However, the resulting performance gain significantly degrades if antennas are highly correlated.

Fig. 5 shows the average SER of the CA-AF system for an equally correlated $S \rightarrow R$ link with BPSK and 4PAM in different correlation scenarios reflecting the detrimental effect of



Fig. 5. Average SER of CA-AF with m = 3, $\Omega = 0.2$, and $\bar{\gamma}_2 = \eta$ for antenna configuration (N_s, N_r, N_d) using BPSK and 4PAM, and equal correlation model.



Fig. 6. Ergodic capacity of CNA-AF with m = 3, $\Omega = 0.2$, and $\bar{\gamma}_2 = 1/4\eta$ using a generalized correlation model.

correlation. The average SERs of the system in a low correlation scenario are plotted for two different antenna configurations for both BPSK and 4PAM to depict the benefits of employing multiple antennas when they are less correlated. The analytical curves are obtained from (41) with a = 1 and b = 1 for BPSK, and a = 1.5 and b = 0.5 for 4PAM, which perfectly match the Monte Carlo simulations. The high SNR analytical curves are plotted by using (42). The clear convergence of these curves with the exact SER in the high SNR regime again verifies that the system achieves the maximum possible diversity order.

Fig. 6 shows the ergodic capacity of the CNA-AF system for generalized correlation. The analytical plots are from (38) with $\tau = 1$, clearly matching the Monte Carlo simulations.



Fig. 7. Ergodic capacity of CA-AF with m = 3, $\Omega = 0.2$, and $\bar{\gamma}_2 = 1/4\eta$ for antenna configuration (N_s, N_r, N_d) using an equal correlation model.

Higher correlation resulting in loss of capacity, and capacity improvement by increasing the number of selection diversity antennas at the relay, are visible in the figure.

Fig. 7 shows the ergodic capacity of the CA-AF system for the equal correlation case. The second-order approximated ergodic capacity expression of (47) is seen to be highly accurate in the SNR regime of interest with a negligible difference between the analytical and simulation plots. The simple upper bound of the capacity obtained from Jensen's inequality as $C \leq 1/2 \log_2(1 + \mathbb{E}[\gamma_{eq}])$ is also plotted in the figure and is fairly tight in the SNR regime of interest. The capacity gain from multiple antennas and the loss from their correlation are also depicted in the figure.

VI. CONCLUSION

We have investigated the performance of a new AF MIMO relay system for LMS communication in the scenario of correlated antennas in terms of the outage probability, average SER, and ergodic capacity, for which easily computable closedform expressions were derived. The validity and accuracy of the expressions were verified through Monte Carlo simulations. The system was found to perform better with a larger number of antennas at all three terminals. The outage performance may improve or degrade with correlation depending upon the SNR threshold. However, correlation was always found to have a detrimental effect on the SER and capacity performance. It was shown that the system achieves maximum possible diversity order.

APPENDIX A Proof of Theorem 1

Since $\mathbf{h}_{k_{SR}} = \bar{\mathbf{h}}_{k_{SR}} + \mathbf{h}_{w_{k_{SR}}}$, given $\bar{\mathbf{h}}_{k_{SR}}$, $\|\mathbf{h}_{k_{SR}}\|^2 (k = 1, 2, ..., N_r)$ values are independent with noncentral chisquare distribution $\chi_{2N_s}(R_k, 1/2)$, where $R_k^2 = \|\bar{\mathbf{h}}_{k_{SR}}\|^2 =$ $\sum_{l=1}^{N_s} |z_{k,l_{SR}}|^2 = \sum_{l=1}^{N_s} \sum_{n=1}^m |(G_{kl})_n|^2$. The conditional cdf of $||\mathbf{h}_{k_{SR}}||^2$ is hence given by [29, eq. (2-1-124)]

$$F_{\|\mathbf{h}_{k_{SR}}\|^{2}|R_{k}}(y|r_{k}) = Pr\left(\|\mathbf{h}_{k_{SR}}\|^{2} \le y|r_{k}\right)$$
$$= 1 - Q_{N_{s}}\left(\sqrt{2}r_{k},\sqrt{2y}\right) \quad (51)$$

where $Q_N(.,.)$ denotes the generalized Marcum-Q function of order N [29, eq. (2-1-122)]. The conditional cdf of $\gamma_1 = \gamma_{K_{SR}} = \eta \|\mathbf{h}_{K_{SR}}\|^2$, where $K = \arg_k \max \gamma_{k_{SR}}$ and $\eta = \bar{\gamma}_1/\epsilon$, can then be obtained as

$$F_{\gamma_{1}|(R_{1},...,R_{N_{r}})}(y|r_{1},...,r_{N_{r}}) = Pr\left(\|\mathbf{h}_{1_{SR}}\|^{2} \leq \frac{y}{\eta},...\|\mathbf{h}_{N_{r_{SR}}}\|^{2} \leq \frac{y}{\eta}|r_{1},...,r_{N_{r}}\right) = \prod_{k=1}^{N_{r}} \left[1 - Q_{N_{s}}\left(\sqrt{2}r_{k},\sqrt{\frac{2y}{\eta}}\right)\right].$$
(52)

The unconditional cdf of γ_1 can be obtained by averaging (52) over the joint distribution of R_1, \ldots, R_{N_r} , i.e.,

$$F_{\gamma_1}(y) = \int_0^\infty \dots \int_0^\infty F_{\gamma_1|(R_1,\dots,R_{N_r})}(y|r_1,\dots,r_{N_r}) \times f_{(R_1,\dots,R_{N_r})}(r_1,\dots,r_{N_r}) dr_1\dots dr_{N_r}.$$
 (53)

To obtain $f_{(R_1,\ldots,R_{N_r})}(r_1,\ldots,r_{N_r})$, let $X_k = R_k^2$. Given $(X_{0l})_n$ and $(Y_{0l})_n$ $(l = 1, 2, \ldots, N_s, n = 1, 2, \ldots, m)$, X_k $(k = 1, 2, \ldots, N_r)$ values are independent with distribution $\chi_{2mN_s}(S_k, \lambda_k^2)$, where $\lambda_k^2 = (1 - \rho_k)\Omega/(2m)$, $S_k^2 = \rho_k(\Omega/m)T$ and $T = \sum_{l=1}^{N_s} \sum_{n=1}^m \{(X_{0l})_n^2 + (Y_{0l})_n^2\}$. The conditional pdf of X_k can be written as [29, eq. (2-1-118)]

$$f_{X_k|T}(x_k|t) = \frac{1}{2\lambda_k^2} \left(\frac{x_k}{\rho_k \frac{\Omega}{m} t}\right)^{\frac{(mN_s-1)}{2}} \exp\left\{-\frac{(\rho_k \frac{\Omega}{m} t + x_k)}{2\lambda_k^2}\right\} \times I_{mN_s-1} \left(\frac{\sqrt{x_k \rho_k \frac{\Omega}{m} t}}{\lambda_k^2}\right)$$
(54)

where $I_{\alpha}(.)$ is the α th-order modified Bessel function of the first kind [22, section (8.406)]. By using variable transformation, the conditional pdf of R_k can be obtained as

$$f_{R_k|T}(r_k|t) = \frac{r_k^{mN_s}}{\lambda_k^2 \left(\sqrt{\rho_k \frac{\Omega}{m} t}\right)^{mN_s - 1}} \exp\left\{-\frac{\left(\rho_k \frac{\Omega}{m} t + r_k^2\right)}{2\lambda_k^2}\right\} \times I_{mN_s - 1} \left(\frac{r_k \sqrt{\rho_k \frac{\Omega}{m} t}}{\lambda_k^2}\right).$$
(55)

In (55), $T \sim \chi_{2mN_s}(0, 1/2)$. Therefore, $f_{(R_1, \dots, R_{N_r})}(r_1, \dots, r_{N_r})$ can finally be obtained as

$$f_{(R_1,...,R_{N_r})}(r_1,...,r_{N_r})$$

= $\int_0^\infty f_{(R_1,...,R_{N_r})|T}(r_1,...,r_{N_r}|t)f_T(t) dt$

$$= \int_{0}^{\infty} \frac{t^{mN_s-1}e^{-t}}{\Gamma(mN_s)} \prod_{k=1}^{N_r} \frac{r_k^{mN_s}}{\lambda_k^2 \left(\sqrt{\rho_k \frac{\Omega}{m}t}\right)^{mN_s-1}} \\ \times \exp\left\{-\frac{\left(\rho_k \frac{\Omega}{m}t + r_k^2\right)}{2\lambda_k^2}\right\} I_{mN_s-1} \left(\frac{r_k \sqrt{\rho_k \frac{\Omega}{m}t}}{\lambda_k^2}\right) dt.$$
(56)

By substituting (52) and (56) into (53), we obtain

$$F_{\gamma_1}(y) = \int_0^\infty \frac{t^{mN_s - 1}e^{-t}}{\Gamma(mN_s)} \prod_{k=1}^{N_r} \mathbb{I}_k(t) dt$$
(57)

where

$$\mathbb{I}_{k}(t) = \int_{0}^{\infty} \left\{ 1 - Q_{N_{s}} \left(\sqrt{2}r_{k}, \sqrt{\frac{2y}{\eta}} \right) \right\}$$

$$\times \frac{r_{k}^{mN_{s}} \exp\left\{ -\frac{\left(\rho_{k}\frac{\Omega}{m}t + r_{k}^{2}\right)}{2\lambda_{k}^{2}}\right\}}{\lambda_{k}^{2} \left(\sqrt{\rho_{k}\frac{\Omega}{m}t}\right)^{mN_{s}-1}}$$

$$\times I_{mN_{s}-1} \left(\frac{r_{k}\sqrt{\rho_{k}\frac{\Omega}{m}t}}{\lambda_{k}^{2}} \right) dr_{k}$$
(58)

which can be expressed as in (59) by using $Q_N(\alpha,\beta) = \exp(-\beta^2/2)/(2\pi j) \oint_{\phi} \exp((\alpha^2/2)(1/z-1) + (\beta^2/2)z)/(z^N(1-z))dz$, which is the contour integral representation for the generalized Marcum-Q function given in [30], where ϕ is a circular contour of radius r that encloses the origin (0 < r < 1) as follows:

$$\mathbb{I}_{k}(t) = 1 - e^{-\frac{y}{\eta}} e^{-\frac{\rho_{k}\Omega/m}{2\lambda_{k}^{2}}t} \frac{1}{2\pi j} \oint_{\phi} \frac{e^{\frac{y}{\eta}z}}{z^{N_{s}}(1-z)} \times \underbrace{\int_{0}^{\infty} \frac{r_{k}^{mN_{s}} e^{-r_{k}^{2}\left(\frac{1}{2\lambda_{k}^{2}}-\frac{1}{z}+1\right)}}{\lambda_{k}^{2}\left(\sqrt{\rho_{k}\frac{\Omega}{m}t}\right)^{mN_{s}-1}} I_{mN_{s}-1}\left(\frac{r_{k}\sqrt{\rho_{k}\frac{\Omega}{m}t}}{\lambda_{k}^{2}}\right) dr_{k} dz}_{\mathbf{I}}$$
(59)

where we changed the order of integration. By substituing $1/(2\lambda_k^2) - 1/z + 1 = 1/(2\sigma^2)$ and $\sqrt{\rho_k(\Omega/m)}/\lambda_k^2 = s/\sigma^2$, the last integral I in (59) can be solved as

$$\mathbf{I} = \frac{\sigma^2 s^{mN_s - 1} e^{\frac{s^2}{2\sigma^2}}}{\lambda_k^2 \left(\sqrt{\rho_k \frac{\Omega}{m} t}\right)^{mN_s - 1}} \int_0^\infty \frac{r_k^{mN_s}}{\sigma^2 s^{mN_s - 1}} e^{-\frac{s^2 + r_k^2}{2\sigma^2}}$$
$$\times I_{mN_s - 1} \left(\frac{r_k s}{\sigma^2}\right) dr_k$$
$$= \frac{z^{mN_s}}{\left(1 + 2\lambda_k^2\right)^{mN_s} (z - \theta_k)^{mN_s}} \exp\left(\frac{c_k tz}{z - \theta_k}\right) \quad (60)$$

where c_k and θ_k are as defined in (13). Substituting (60) into (59) and further simplifying, we obtain

$$\mathbb{I}_{k}(t) = 1 - \frac{e^{-\frac{y}{\eta}} e^{-\frac{\rho_{k}\Omega}{(1+2\lambda_{k}^{2})^{m}t}}}{(1+2\lambda_{k}^{2})^{mN_{s}}} \frac{1}{2\pi j} \oint_{\phi} \frac{e^{y\eta z} e^{\frac{c_{k}\theta_{k}t}{(z-\theta_{k})}} z^{N_{s}(m-1)}}{(1-z)(z-\theta_{k})^{mN_{s}}} dz.$$
(61)

If $\lambda_k^2 = (1 - \rho_k)\Omega/(2m)$ is substituted back into (61) and $\mathbb{I}_k(t)$ is further simplified by applying the Taylor series expansion of $\exp((y/\eta)z + c_k\theta_kt/(z - \theta_k))$ followed by the binomial expansion, we obtain

$$\mathbb{I}_{k}(t) = 1 - \left(\frac{m}{m + (1 - \rho_{k})\Omega}\right)^{mN_{s}} e^{-\frac{y}{\eta}} e^{-\frac{\rho_{k}\Omega}{m + (1 - \rho_{k})\Omega}t} \times \sum_{i=0}^{M} \sum_{q=0}^{i} \frac{1}{q!(i-q)!} \left(\frac{y}{\eta}\right)^{q} (c_{k}\theta_{k}t)^{i-q} \mu_{iq_{k}} \quad (62)$$

where the infinite series is accurately truncated by M number of terms and μ_{iq_k} is as defined in (14). The desired result in (12) is finally obtained by substituting (62) into (57).

APPENDIX B Proof of Theorem 3

The cdf of γ_{eq} , i.e., $F_{\gamma_{eq}}(y)$, can be expressed as [4]

$$F_{\gamma_{eq}}(y) = 1 - \int_{0}^{\infty} \left(1 - F_{\gamma_1}\left(\frac{y(w+y+\tau)}{w}\right) \right) f_{\gamma_2}(w+y) \, dw.$$
(63)

By manipulating [31], the pdf of γ_2 , i.e., $f_{\gamma_2}(y)$, can be obtained as

$$f_{\gamma_2}(y) = \frac{1}{\bar{\gamma}_2 \Delta(\mathbf{D})} \sum_{r=1}^{N_d} \phi_r^{N_d - 2} e^{-\frac{y}{\bar{\gamma}_2 \phi_r}} C(N_d, r).$$
(64)

Now, by substituting (12) and (64) in (63), we obtain

$$F_{\gamma_{eq}}(y) = 1 - \frac{1}{\Delta(\mathbf{D})} \sum_{r=1}^{N_d} \phi_r^{N_d - 1} C(N_d, r) e^{-\frac{y}{\gamma_2 \phi_r}} \\ \times \left[1 - \frac{1}{\Gamma(mN_s)} \int_0^\infty e^{-\zeta} \int_0^\infty t^{mN_s - 1} e^{-t} \right] \\ \times \prod_{k=1}^{N_r} \left\{ 1 - \left(\frac{m}{m + (1 - \rho_k)\Omega} \right)^{mN_s} \\ \times e^{-\frac{y(\zeta \gamma_2 \phi_r + y + \tau)}{\zeta \gamma_2 \phi_r \eta}} e^{-\frac{\rho_k \Omega}{m + (1 - \rho_k)\Omega} t} \right\} \\ \times \sum_{i=0}^M \sum_{q=0}^i \frac{1}{q!(i-q)!} \\ \times \left(\frac{y(\zeta \overline{\gamma}_2 \phi_r + y + \tau)}{\zeta \overline{\gamma}_2 \phi_r \eta} \right)^q \\ \times (c_k \theta_k t)^{i-q} \mu_{iq_k} dt d\zeta \right].$$
(65)

For unequal correlation coefficients ρ_k , the closed-form expression for the above double integral is unavailable. However, since the double integral is in the form of $\int_0^{\infty} e^{-\zeta} \int_0^{\infty} t^{\beta} e^{-t} f(t, \zeta) dt d\zeta$, this expression can be accurately approximated as in (29) by using the GLQ rule.

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