Spatial Multipath Resolution with Space Time Block Codes

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***Abstract*—The use of space time block codes (STBCs) over frequency selective multipath multiple-input multiple-output (MIMO) channels is investigated in combination with *spatial multipath resolution* (SMR) [1], a novel spatial signal processing technique proposed by the authors for mitigating inter-symbol interference. Neither STBC nor SMR requires transmit channel state information. This fact, along with SMR not requiring any modification to the transmitter, makes the MIMO STBC-SMR hybrid effective. Numerical results are presented, illustrating how the scheme fares under the Alamouti STBC scheme, and highlighting the advantage of *adaptive* SMR - i.e., adapting the number of multipaths resolved based on the channel state. A practical multipath MIMO channel based on the IEEE 802.15.3c NLOS (CM4) model is used for simulation.**

***Index Terms*—STBC, SMR, spatial multipath resolution, frequency selective fading.**

I. Introduction

FREQUENCY selective multipath fading in multiple-input multiple-output (MIMO) channels is a significant transmission impairment. A potential solution is *spatial multipath resolution* (SMR) [1] - i.e., using the space dimension for suppressing the inter-symbol interference (ISI). SMR, a non-conventional use of the space dimension, is feasible, when more antennas than the minimal required to achieve the desired quality of service are available. The resulting excess degrees of freedom (DoFs) at the receiver are exploited in SMR to suppress the ISI. SMR employs a rake-receiver structure, but exploits the spatial DoFs the receiver has in excess to those of the transmitter to extract multipath components at its fingers. Mitigation of the ISI is typically achieved with orthogonal frequency division multiplexing (OFDM) [2], time-domain equalization or time-reversal [3]. SMR could be an alternative to those, or hybrid schemes of those and SMR may be developed. For example, hybrid SMR-OFDM requires fewer subcarriers than OFDM. Reference [1] confines to *eigenmode transmission* implemented over the thus 'flattened' effective MIMO channel, and investigates the error performance both when the receiver's DoFs are sufficient to resolve all the paths and not.

Eigenmode transmission is infeasible where accurate channel state information (CSI) is not available at the transmitter. This challenge is overcome by *space time block codes* (STBCs) [4], the Alamouti code [5], the simplest of which is already adopted for wireless standards including the IEEE 802.11n [6]. Since neither SMR nor STBCs requires transmit CSI, this letter thus investigates joint MIMO STBC-SMR configurations. However, with STBCs, since the symbols are transmitted in the form of *space-time blocks*, intra space-time block interference and inter space-time block interference occur. These interferences can be mitigated by SMR. As shown in Section III, the performance gain of STBC-SMR is highly impressive.

The maximum achievable rate of an N_t-transmit antenna orthogonal STBC is given by $1/2 + 1/n$ [7], where $n = 2\lceil N_t/2 \rceil$. This restriction not only makes, the Alamouti code ($N_t = 2$), the only possible full-rate complex orthogonal STBC, but also makes it desirable to use fewer transmit antennas. Nevertheless the diversity order of the STBC improves linearly with the number of receiver antennas. Consequently, if the total number of antennas in a MIMO system is limited, then the most of the antennas can be deployed at the receiver. Such receivers have more spatial DoFs than the transmitter, a requirement for SMR. Therefore, a transition from conventional MIMO STBC to hybrid MIMO STBC-SMR appears feasible.

Contribution of this paper: The use of SMR with MIMO STBC is investigated here for the first time. Unlike with eigenmode transmission examined in [1], the effect of SMR does not manifest in transmitter signal processing. Hence, SMR can be adaptively employed at the receiver, depending on the severity of multipath fading. For instance, where error detection is possible, the receiver can choose the extent of SMR to employ, based on the observed error rates. Therefore, MIMO STBC-SMR appears practicable, and easily deployable in conventional MIMO STBC systems.

Although SMR mitigates the ISI, the price paid is a loss of achievable diversity. The tradeoff between ISI reduction and diversity loss was not investigated in [1]. In the present work, this tradeoff is examined through simulation of the symbol error rate (SER). It is shown here, that utilizing a set of strongest paths could be more effective SER-wise, than trying to resolve as many multiple paths as possible. The paper also proposes and investigates the performance of *adaptive* SMR - i.e., adapting the number of paths resolved based on the relative strength of the multipath components.

The paper is organized as follows: Section II details the system model, outlining how receiver processing flattens the effective channel, and how the received space-time blocks corresponding to this effective channel can be computed prior to detection. SER simulation results are presented in Section III for the Alamouti STBC scheme, assuming a practical multipath MIMO channel based on the IEEE 802.15.3c NLOS (CM4) model [8, p.16].

Notation: An $N_r \times N_t$ MIMO system has N_t transmit antennas and N_r receiver antennas. $\mathbf{A} \in \mathbb{C}^{m \times n}$ is an $m \times n$ matrix. $\{\mathbf{A}\}_{\mathcal{C}(m:n)}$ and $\{\mathbf{A}\}_{\mathcal{R}(p:q)}$ are respectively the sub-

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matrices of $\mathbf{A}$ formed with its columns m through n, and rows p through q. The transpose, conjugate transpose and Frobenius norm [9, p.291] of $\mathbf{A}$ are $\mathbf{A}^T$, $\mathbf{A}^H$ and $||\mathbf{A}||_F$. The rank [9, p.12] of $\mathbf{A} \in \mathbb{C}^{n\times n}$ is given by $\mathrm{rank}\,(\mathbf{A})$. $\mathbf{I}_n$ is the identity matrix of rank n. $\mathcal{E}\{X\}$ is the expected value of X. a^* is the complex conjugate of scalar a.

Assumptions: Perfect CSI at the receiver and block fading are assumed.

II. System Model

A. STBC over multipath MIMO:

Consider a multipath MIMO channel given by

$$\mathbf{H}(k) = \sum_{l=0}^{L-1} \mathbf{H}_l \delta(k-\tau_l), \tag{1}$$

where $\mathbf{H}_l \in \mathbb{C}^{N_r\times N_t}$ represents the channel matrix tap of the l-th strongest multipath component, such that $||\mathbf{H}_k||_F \geq ||\mathbf{H}_l||_F$ for $k < l$ and $k, l \in \{0, \ldots, L-1\}$. Let τ_l be corresponding discretized delay in *time units*, each equal to a symbol duration, and define $m = \arg\min_l (\tau_l)$. Conventionally, $\mathbf{H}_0$ is deemed the desired path, and the others, the interfering paths.

Suppose a symbol vector $\mathbf{S}^{(k)} \in \mathbb{C}^{N_s\times 1}$ is transmitted once every T time units - i.e., during time units kT through $((k+1)T-1)$, $k \geq 0$ - in the form of space-time blocks $\mathbf{X}^{(k)} \in \mathbb{C}^{N_t\times T}$. The rate of the code is N_s/T. Denote by $\mathbf{x}^{(j)} \in \mathbb{C}^{N_t\times 1}$, $j \geq 0$, the sub-block of space-time coded symbols transmitted during the j-th time instance. Thus we have $\mathbf{X}^{(k)} = \left[\mathbf{x}^{(kT)}, \mathbf{x}^{(kT+1)}, \ldots, \mathbf{x}^{((k+1)T-1)}\right]$, for $k \geq 0$.

The sub-block of received symbols at the point of first reception of a replica of $\mathbf{x}^{(j)}$ is given by

$$\mathbf{y}^{(j)} = \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{x}^{(j-\tau_l+\tau_m)} + \mathbf{n}^{(j)}, \tag{2}$$

where $\mathbf{n}^{(j)} \in \mathbb{C}^{N_r\times 1}$ is the corresponding additive noise at the receiver. Note that $\mathbf{y}^{(j)}$ lags the transmission of $\mathbf{x}^{(j)}$ by τ_m time units. The conventional STBC receiver processes $\mathbf{y}^{(j)}$s corresponding to each transmitted $\mathbf{X}^{(k)}$, in order to obtain the estimates of $\mathbf{S}^{(k)}$. The ISI manifests as both inter- and intra-space-time block interference, making symbol detection quite challenging.

B. Receiver Design:

The proposed SMR stage preceding the STBC detection involves a rake receiver structure as in [1], and considers the $\tilde{L}$ strongest multipath components, where $\tilde{L} \leq L$. Each of its fingers is extracted through spatial signal processing, represented in terms of a matrix $\mathbf{R}_l \in \mathbb{C}^{\hat{N}_r\times N_r}$, such that

$$\mathbf{R}_l\mathbf{H}_n = \mathbf{0}, \forall l \neq n, \quad \text{and} \quad \mathbf{R}_l\mathbf{H}_l \neq \mathbf{0}, \tag{3}$$

for $l, n \in \{0, \ldots, \tilde{L}-1\}$. The *effective number* $\hat{N}_r$ of receiver antennas after SMR depends on the choice of the combiner weights $\mathbf{C}_l$s introduced below.

Each l-th path signal thus extracted is given by $\mathbf{y}_l^{(j)} = \mathbf{R}_l\mathbf{y}^{(j)}$, i.e.,

$$\mathbf{y}_l^{(j)} = \mathbf{R}_l\left(\mathbf{H}_l\mathbf{x}^{(j-\tau_l+\tau_m)} + \mathbf{n}^{(j)} + \underbrace{\sum_{k=\tilde{L}}^{L-1}\mathbf{H}_k\mathbf{x}^{(j-\tau_k+\tau_m)}}_{\text{residual ISI}}\right), \tag{4}$$

where $m = \arg\min_{l\in\{0,\ldots,\tilde{L}-1\}} (\tau_l)$.

The extracted signals are delayed, each $\mathbf{y}_l^{(j)}$ by $(\tau_l - \tau_m)$ time units, and then, combined as follows, to form a set of ISI reduced symbols $\tilde{\mathbf{y}}^{(j)}$ corresponding to a single $\mathbf{x}^{(j)}$.

$$\begin{aligned}\tilde{\mathbf{y}}^{(j)} &= \sum_{l=0}^{\tilde{L}-1}\mathbf{y}_l^{(j+\tau_l-\tau_m)} \\ &= \underbrace{\sum_{l=0}^{\tilde{L}-1}\mathbf{R}_l\mathbf{H}_l}_{\mathbf{H}_{\text{eff}}}\cdot\mathbf{x}^{(j)} + \underbrace{\sum_{l=0}^{\tilde{L}-1}\mathbf{R}_l\mathbf{n}^{(j+\tau_l-\tau_m)}}_{\tilde{\mathbf{n}}^{(j)}} \\ &\qquad + \underbrace{\sum_{l=0}^{\tilde{L}-1}\sum_{k=\tilde{L}}^{L-1}\mathbf{R}_l\mathbf{H}_k\mathbf{x}^{(j-\tau_k+\tau_l)}}_{\text{residual ISI}}\end{aligned} \tag{5}$$

Note that the elimination of ISI requires choosing $\tilde{L} = L$. However, even smaller $\tilde{L}$s cut down a significant portion of the ISI.

$\mathbf{H}_{\text{eff}} \in \mathbb{C}^{\hat{N}_r\times N_t}$ in (5) is the effective flattened MIMO channel. The space-time block $\mathbf{Y}^{(k)} = \left[\tilde{\mathbf{y}}^{(kT)}, \tilde{\mathbf{y}}^{(kT+1)}, \ldots, \tilde{\mathbf{y}}^{((k+1)T-1)}\right]$, represents the received signal over $\mathbf{H}_{\text{eff}}$ corresponding to each $\mathbf{X}^{(k)}$, for $k \geq 0$. Thus we have

$$\mathbf{Y}^{(k)} = \mathbf{H}_{\text{eff}}\mathbf{X}^{(k)} + \mathbf{N}^{(k)} + \text{residual ISI}, \tag{6}$$

where $\mathbf{N}^{(k)} = \left[\tilde{\mathbf{n}}^{(kT)}, \tilde{\mathbf{n}}^{(kT+1)}, \ldots, \tilde{\mathbf{n}}^{((k+1)T-1)}\right]$.

Each $\mathbf{R}_l$ can be computed to satisfy (3) provided the receiver has sufficient number of antennas. Let $\tilde{\mathbf{H}}_l = \left[\mathbf{H}_0 \ \ldots \ \mathbf{H}_{l-1}\ \mathbf{H}_{l+1} \ \ldots \ \mathbf{H}_{\tilde{L}-1}\right]$, $l \in \{0, \ldots, \tilde{L}-1\}$; $\tilde{\mathbf{H}}_l = \mathbf{U}_l\mathbf{\Sigma}_l{\mathbf{V}_l}^H$ its singular value decomposition (SVD); and $m_l = \mathrm{rank}\,(\mathbf{H}_l)$. Define $\tilde{\mathbf{U}}_l = \left[\{\mathbf{U}_l\}_{\mathcal{C}(m_l+1:N_r)}\ \mathbf{0}_l\right]$, where $\mathbf{0}_l \in \mathbb{C}^{N_r\times m_l}$ is a zero matrix. Thus we get $\tilde{\mathbf{U}}_l$ orthogonal to each $\mathbf{H}_k, k \neq l$. Therefore, $\mathbf{R}_l = \mathbf{C}_l\left(\tilde{\mathbf{U}}_l\right)^H$, $l \in \{0, \ldots, \tilde{L}–1\}$ satisfy the orthogonality requirement (3) for arbitrary *combiner weights*, represented by matrices $\mathbf{C}_l \in \mathbb{C}^{\hat{N}_r\times N_r}$.

As highlighted in Section III, the choice of $\mathbf{C}_l$s affects not just the error performance, but even the diversity order. Outlined below are three possibilities.

1) Choosing $\mathbf{I}_{N_r}$ for combiner weights appears the simplest. But it causes the rows $(N_r - m_l)$ and onwards of each $\mathbf{R}_l$ to be zero, thus making $\hat{N}_r = N_r - \min_l (m_l)$ and each $\mathbf{C}_l = \{\mathbf{I}_{N_r}\}_{\mathcal{R}(1:\hat{N}_r)}$. Note that this $\hat{N}_r$ is the smallest possible.
2) Another possibility is using $\mathbf{C}_l = \mathbf{P}_l$ where, each $\mathbf{P}_l \in \mathbb{C}^{N_r\times N_r}$ is a randomly chosen *permutation matrix* [9, p.25].
3) Cascading the resolved paths to make $\hat{N}_r = (\tilde{L}\cdot N_r) -$

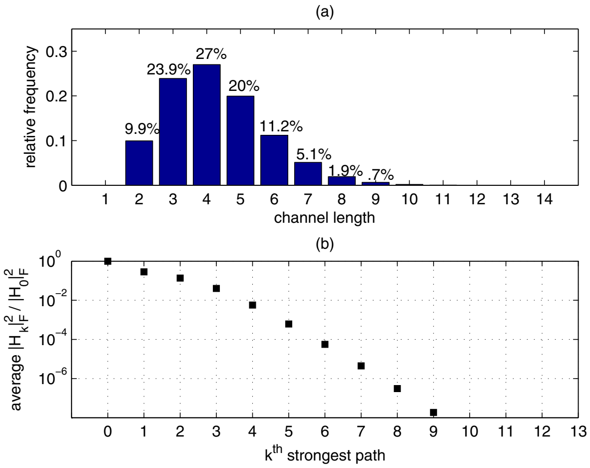

Fig. 1. (a) Histogram of channel length L corresponding to the simulated 10×2 multipath MIMO channel. Mean length $\mathcal{E}\{L\} = 4.25$, and perfect SMR is possible for $L \leq N_r/N_t = 5$ (i.e., $\approx 81\%$ the time). (b) Average relative strength $\mathcal{E}\left\{||\mathbf{H}_k||_F^2 \,/\, ||\mathbf{H}_0||_F^2\right\}$ of the k-th strongest multipath component.

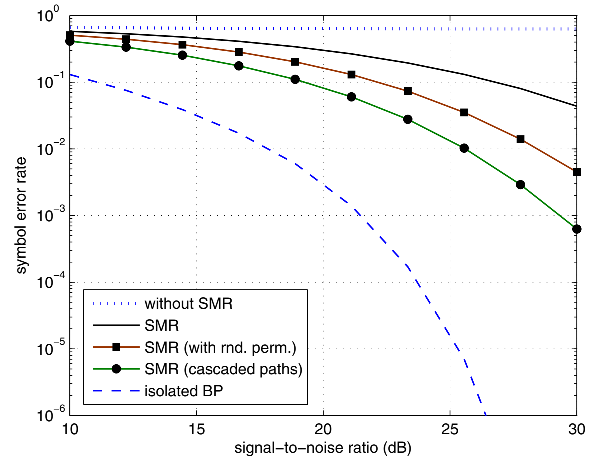

Fig. 2. SER performance of Alamouti code for $N_r = 10$. SER vs. average transmit SNR curves are shown for the cases: (i) best path without SMR (dotted), (ii) SMR (solid), (iii) SMR with randomly permuted combiner weights (solid, with ■ markers), (iv) SMR with cascaded resolved paths (solid, with ● markers), and (v) isolated best path (dashed).

$\sum_{l=0}^{\tilde{L}-1} m_l$ is yet another possibility. It seems to yield the best error rates. Corresponding $\mathbf{C}_l$s are of form $[\mathbf{0}_{l,1}, \mathbf{I}_{N_r - m_l}, \mathbf{0}_{l,2}]^T$, where each $\mathbf{0}_{l,1}$ has $\sum_{k=0}^{l-1}(N_r - m_k)$ zeros columns, and each $\mathbf{0}_{l,2}$, $\sum_{k=l+1}^{\tilde{L}-1}(N_r - m_k)$ zero columns. $\hat{N}_r$ could exceed N_r. However, $\mathbf{R}_l$s already being correlated, that would not necessarily increase MIMO diversity.

Note that all three above are forms of *equal gain* combining. Other forms of combination are also possible. Given (6), the estimation of $\mathbf{X}^{(k)}$, and then, $\mathbf{S}^{(k)}$, requires only conventional STBC receiver processing.

III. Numerical Results

This section investigates the SER performance of SMR with MIMO STBC, by using Monte-Carlo simulation. The Alamouti scheme [5], which has $N_t = N_s = T = 2$ and

$$\mathbf{X}^{(k)} = \begin{bmatrix} \mathbf{s}_1^{(k)} & \mathbf{s}_2^{(k)*} \\ \mathbf{s}_2^{(k)} & -\mathbf{s}_1^{(k)*}, \end{bmatrix}, \quad \text{where} \quad \mathbf{S}^{(k)} = \begin{bmatrix} s_1^{(k)} \\ s_2^{(k)} \end{bmatrix}, \tag{7}$$

is used for the purpose. Perfect CSI at the receiver and block fading are assumed. 10^6 realizations of an $N_r \times 2$, IEEE 802.15.3c NLOS (CM4) model based (see [1] for model parameters) multipath MIMO channel, whose histogram of channel length for $N_r = 10$ is given in Fig. 1(a), are simulated. 100 quadrature phase shift keying (QPSK) modulated symbol pairs $(s_1^{(k)}, s_2^{(k)})$ are transmitted per each channel realization.

- Fig. 2 assumes $N_r = 10$. Dotted curve corresponds to conventional STBC decoding, by ignoring multipath interference (i.e., without SMR). Detection then fails utterly owing to ISI, with SERs exceeding 60% irrespective of the average transmit signal-to-noise ratio (SNR). Dashed line represent conventional STBC reception of the best path, assuming the interfering paths are non-existent. It is given as an unachievable lower bound on the SER, for performance comparison. It fares the best, because all 10 receiver antennas contribute to MIMO diversity. Moreover, ordering of the paths by strength adds selection diversity.

  Solid lines correspond to SMR schemes, attempting to resolve as many best paths as possible, up to a maximum of $N_r/N_t = 5$ paths. The poor error performance as well as the loss of diversity corresponds to spending spatial DoFs for resolving the paths. The choice of combiner weights $\mathbf{C}_l$s seems to affect the performance significantly. Cascading the resolved paths (solid curve with ● markers) performs the best. Randomly permuted $\mathbf{C}_l$s (solid curve with ■ markers) too outperforms the use of $\mathbf{C}_l = \mathbf{I}_{N_r}$ (solid curve without markers). Their relative merits can be explained in terms of different $\hat{N}_r$s they yield. Error floors, as in [1, Fig. 3], are not observed with SMR here, because all paths are resolved $\approx 81\%$ of the times, and the strongest disregarded path is on average about 30 dB weaker than the best path - see Fig. 1(b).
- Fig. 3 represents the same simulation as with Fig. 2, repeated with the channel length restricted to be at most 4, so that perfect SMR happens. SER performance of the three aforementioned SMR schemes are compared[1] here for the cases $N_r = 10$ (solid lines) and $N_r = 12$ (dashed lines). For case $N_r = 10$, the relative merits observed in Fig. 2 prevail, with cascaded resolved paths producing the best performance. However, lower error rates are observed, because the best path is stronger and the number of interfering paths is lower than before ($\approx 19\%$ of the times). SER results for the case $N_r = 12$ are observed to be lower than those for $N_r = 10$, agreeing with the intuition that having higher receiver DoFs is better.
- Since each multipath resolved reduces the effective number of receiver antennas by $N_t = 2$, and thus limits the

[1] Since the MIMO taps produced by the channel model do not scale linearly with N_r, the cases $N_r = 10$ and $N_r = 12$ are not strictly comparable. Nevertheless, the effect of N_r may be qualitatively compared.

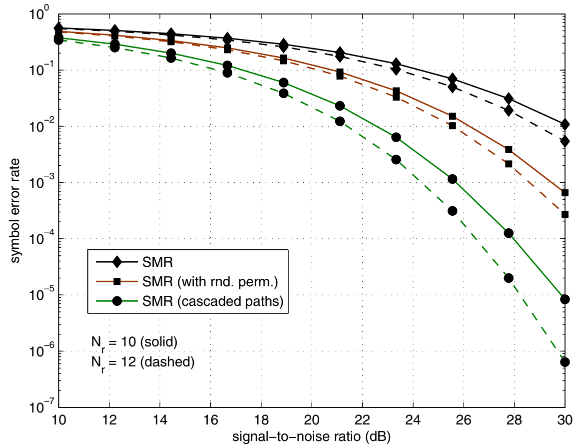

Fig. 3. SER performance of Alamouti code for $N_r = 10$ (solid) and $N_r = 12$ (dashed). Channel length is forced to be no greater than 4. SER vs. average transmit SNR curves are shown for the cases: (i) SMR (with ♦ markers), (i) SMR with randomly permuted combiner weights (with ■ markers), and (iii) SMR with cascaded resolved paths (with • markers).

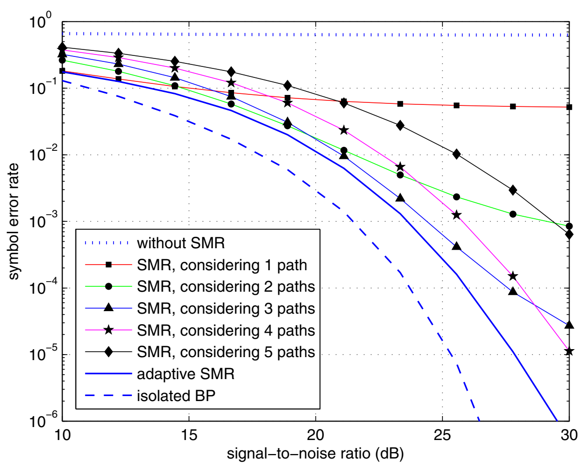

Fig. 4. SER performance of Alamouti code for $N_r = 10$. SER vs. average transmit SNR curves are shown for the cases: (i) best path without SMR (dotted), (ii) SMR, considering $\tilde{L} \in \{1, \ldots, 5\}$ paths (solid, with markers), (iii) adaptive SMR (solid, without markers), and (iii) isolated best path (dashed).

MIMO diversity, it is not always desirable to resolve as many paths as possible. This aspect is investigated in Fig. 4, assuming the same channel as with Fig. 2. Solid curves with markers correspond to resolving at most $\tilde{L} \in \{1, \ldots, N_r/N_t\}$ strongest paths. Note that, for a given curve, $\tilde{L}$ is held fixed for all channel realizations. Resolving fewer paths (e.g. $\tilde{L} = 1, 2$) seems prudent at low SNRs, where additive noise dominates the interference. But the performance yields to residual interference as the SNR improves; and an optimal fixed $\tilde{L}$ seems to exist ($\tilde{L} = 4$, in this case) at high SNRs, highlighting the conflicting effects of multipath and MIMO diversity in SMR.

Adapting $\tilde{L}$ based on the *instantaneous* CSI, i.e., determining the optimal $\tilde{L}$ every time the channel varies (or $\mathbf{R}_l$s are recomputed), results in even better error performance. Corresponding error performance (depicted by the solid curve without markers) is seen better than SMR based on any fixed $\tilde{L}$ value. A practicable algorithm for adapting $\tilde{L}$ is however, yet to be determined.

IV. Conclusion

Spatial multipath resolution (SMR) in multiple-antenna STBC systems was investigated. It suppresses the ISI due to frequency selective fading, exploits that both STBC and SMR require only receiver CSI, and performs better whenever the receiver has significantly higher number of antennas than the transmitter. Numerical results were obtained for the Alamouti scheme, investigating how the choice of combiner weights and adapting the number of multipaths resolved affect the error performance.

As emphasized in [1], SMR can be applied in arbitrary multipath channels and MIMO configurations, provided the receiver has sufficient excess DoFs. As shown here with STBC, and in [1] with eigenmode transmission, SMR can be combined with almost any signal processing scheme designed for flat fading MIMO channels. Nevertheless, its use with schemes not requiring transmit CSI, such as STBC and spatial multiplexing, improves the feasibility of practical implementation. The hybrid MIMO OFDM-SMR systems too merit consideration.

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