

Genetic Algorithm Based Nearly Optimal Peak Reduction Tone Set Selection for Adaptive Amplitude Clipping PAPR Reduction

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Abstract—In tone reservation (TR) based OFDM systems, the peak to average power ratio (PAPR) reduction performance mainly depends on the selection of the peak reduction tone (PRT) set and the optimal target clipping level. Finding the optimal PRT set requires an exhaustive search of all combinations of possible PRT sets, which is a nondeterministic polynomial-time (NP-hard) problem, and this search is infeasible for the number of tones used in practical systems. The existing selection methods, such as the consecutive PRT set, equally spaced PRT set and random PRT set, perform poorly compared to the optimal PRT set or incur high computational complexity. In this paper, an efficient scheme based on genetic algorithm (GA) with lower computational complexity is proposed for searching a nearly optimal PRT set. While TR-based clipping is simple and attractive for practical implementation, determining the optimal target clipping level is difficult. To overcome this problem, we propose an adaptive clipping control algorithm. Simulation results show that our proposed algorithms efficiently obtain a nearly optimal PRT set and good PAPR reductions.

Index Terms—Genetic Algorithm, OFDM, PAPR, tone reservation.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is widely used in high-speed wireless communication systems because of its inherent robustness against multipath fading and resistance to narrow-band interference [1]. However, OFDM suffers from the high peak to average power ratio (PAPR) of the transmitted signal. This issue can cause serious problems including a severe power penalty at the transmitter. Conventional solutions to reduce the PAPR are to use a linear amplifier or to back-off the operating point of a nonlinear amplifier. But both these solutions result in a significant loss of power efficiency. Many methods have thus

been proposed to reduce the PAPR by modifying the signal itself. The simplest one is clipping the OFDM signal below a PAPR threshold level [2], [3], but it degrades the bit-error-rate (BER) of the system and results in out-of-band noise and in-band distortion. Coding [4] is another technique. Although it can offer the best PAPR reductions, the associated complexity and data rate reduction limit its application. Selected mapping (SLM) technique [5] and the partial transmit sequence (PTS) [6] are based on multiple signal representation method. These methods [7], [9], [11], [12] improve the PAPR statistics of the OFDM signals, but side information may be transmitted from the transmitter to the receiver, which results in a loss of data throughput.

By modifying the modulation constellation, the active set extension (ASE) method [13], the adaptive active set extension [15] and the constellation extension method [14] reduce PAPR, but these algorithms require increased power and computational complexity at the transmitter.

The tone reservation (TR) technique [16]–[18] proposed by Tellado is a distortionless method based on using a small subset of subcarriers, called peak reduction tones (PRTs), to generate a peak-canceling signal for PAPR reduction. The method is simple, efficient and does not require transmission of side information. The tone reservation technique can be divided into two classes: 1) TR-gradient-based technique; 2) TR-clipping-based technique, which is our major focus in this paper. The PAPR reduction performance of the TR-clipping-based technique mainly depends on the selection of peak reduction tone (PRT) set and the optimal target clipping level. The optimal PRT set will result in the best PAPR reduction. However, finding the optimal PRT set is a nondeterministic polynomial-time (NP-hard) problem and cannot be solved for the number of tones envisaged in practical systems. So suboptimal solutions are typically preferable, such as the consecutive PRT set, equally spaced PRT set and random PRT set. Although the performance of random PRT set outperforms those of consecutive PRT set and equally spaced PRT set, it requires enough larger PRT set sampling to obtain better PAPR reduction. The cross entropy (CE)-PRT algorithm in [20], [33] obtains better secondary peak, but it requires larger population or sampling. On the other hand, determining the optimal target clipping level, which directly affects the PAPR reduction of the TR-clipping-based technique, is also difficult, because many factors, such as the number of OFDM subcarriers, the location of PRT set and the modulation scheme significantly influence the selection of the optimal target clipping level.

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In this paper, we first propose a new suboptimal PRT set selection scheme based on the genetic algorithm (GA), which can efficiently solve the NP-hard problem. An adaptive amplitude clipping (AAC-TR) algorithm is also developed to obtain good PAPR reduction performance regardless of the initial target clipping level. Simulation results show that the GA optimization scheme achieves a nearly optimal PRT set and requires far less computational complexity than the random PRT set method. The proposed AAC-TR algorithm also achieves good PAPR reduction performance.

This paper is organized as follows. In Section II, the system model based on the TR method is introduced and the principles of TR techniques are described. The GA algorithm for the nearly optimal PRT set is proposed in Section III. In Section IV, the adaptive amplitude clipping (AAC-TR) algorithm is developed. The performances of GA algorithm, AAC-TR and other algorithms for PRT selection and PAPR reduction are evaluated by computer simulations in Section V. Conclusions are made in Section VI.

In this paper, $\|\cdot\|$ denotes the mean square norm of a vector. $\|\cdot\|_\infty$ denotes the l^∞ norm of a vector. $E[\cdot]$ denotes the expectation of a random variable. \bar{x} denote the complex conjugate of a complex number x . $(\cdot)^T$ denotes the transpose of a matrix. $(\cdot)^H$ denotes the conjugate transpose of a matrix.

II. OFDM SYSTEMS AND TONE RESERVATION TECHNIQUE

This section will describe the orthogonal frequency division multiplexing (OFDM) signal, the peak-to-average power ratio (PAPR), and the tone reservation technique.

A. OFDM Systems and PAPR

An OFDM signal is the sum of N independent, modulated tones (subcarriers) of equal bandwidth with frequency separation $1/T$, where T is the time duration of the OFDM symbol. For a complex-valued phase-shift keying (PSK) or quadrature amplitude modulation (QAM) input OFDM block $\mathbf{X} = [X_0, X_1, \dots, X_{N-1}]^T$ of length N , the inverse discrete Fourier transform (IDFT) generates the ready-to-transmit OFDM signal. The discrete-time OFDM signal is expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{j\frac{2\pi nk}{N}}, \quad n = 0, 1, \dots, N-1, \quad (1)$$

which can also be written in matrix form $\mathbf{x} = [x_0, \dots, x_{N-1}]^T = \mathbf{Q}\mathbf{X}$, where \mathbf{Q} is the IDFT matrix with the (n, k) th entry $q_{n,k} = (1/\sqrt{N})e^{j2\pi nk/N}$.

The PAPR of \mathbf{x} is defined as the ratio of the maximal instantaneous power to the average power; that is

$$PAPR(\mathbf{x}) = \frac{\max_{0 \leq n < N} |x_n|^2}{E[|x_n|^2]}. \quad (2)$$

The complementary cumulative distribution function (CCDF) is one of the most frequently used performance measures for PAPR reduction, representing the probability that

the PAPR of an OFDM symbol exceeds the given threshold $PAPR_0$, which is denoted as

$$CCDF = Pr(PAPR > PAPR_0). \quad (3)$$

B. Tone Reservation Technique

In the TR-based OFDM scheme, peak reduction tones (PRT) are reserved to generate PAPR reduction signals. These reserved tones do not carry any data information, and they are only used for reducing PAPR. Specifically, the peak-canceling signal $\mathbf{c} = [c_0, c_1, \dots, c_{N-1}]^T$ generated by reserved PRT is added to the original time domain signal $\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T$ to reduce its PAPR. The PAPR reduced signal can be expressed as

$$\mathbf{a} = \mathbf{x} + \mathbf{c} = \mathbf{Q}(\mathbf{X} + \mathbf{C}), \quad (4)$$

where $\mathbf{C} = [C_0, C_1, \dots, C_{N-1}]^T$ is the peak-canceling signal vector in frequency domain. To avoid signal distortion, the data vector \mathbf{X} and the peak reduction vector \mathbf{C} lie in disjoint frequency domains, i.e.

$$X_n + C_n = \begin{cases} X_n, & n \in \mathcal{R}^C \\ C_n, & n \in \mathcal{R}, \end{cases} \quad (5)$$

where $\mathcal{R} = \{i_0, i_1, \dots, i_{M-1}\}$ is the index set of the reserved tones, \mathcal{R}^C is the complementary set of \mathcal{R} in $\mathcal{N} = \{0, 1, \dots, N-1\}$, and $M < N$ is the size of PRT set.

The PAPR of the peak-reduced OFDM signal $\mathbf{a} = [a_0, a_1, \dots, a_{N-1}]^T$ is then redefined [18] as

$$PAPR(\mathbf{a}) = \frac{\max_{0 \leq n < N} |x_n + c_n|^2}{E[|x_n|^2]}. \quad (6)$$

Thus \mathbf{c} must be chosen to minimize the maximum of the peak-reduced OFDM signal \mathbf{a} , i.e.

$$\mathbf{c}^* = \arg \min_{\mathbf{c}} \max_{0 \leq n < N} |x_n + c_n|^2. \quad (7)$$

To obtain the optimum \mathbf{c}^* , (7) can be reformulated as the following optimization problem:

$$\begin{aligned} \min_{\mathbf{c}} \quad & e, \\ \text{subject to} \quad & e \geq 0, \\ & |x_n + c_n|^2 \leq e, \end{aligned} \quad (8)$$

which is a Quadratically Constrained Quadratic Program (QCQP) problem [18] and e is an optimization parameter. Although the optimum of a QCQP exists, the solution requires a high computational cost. To reduce the complexity of the QCQP, a simple gradient algorithm proposed by Tellado in [16]–[18] iteratively updates the vector \mathbf{c} as follows:

$$\mathbf{c}^{(i+1)} = \mathbf{c}^{(i)} - \alpha_i \mathbf{P}[\mathbf{c}^{(i)}], \quad (9)$$

where α_i is a scaling factor, $\mathbf{p} = [p_0, p_1, \dots, p_{N-1}]^T$ is called the time domain kernel, and $\mathbf{p}[(k - k_i)_N]$ denotes a circular shift of \mathbf{p} to the right by a value k_i calculated by

$$k_i = \arg \max_k |x_k + c_k^{(i)}|. \quad (10)$$

The time domain kernel \mathbf{p} is obtained by the following formula:

$$\mathbf{p} = \mathbf{Q}\mathbf{P}, \quad (11)$$

where $\mathbf{P} = [P_0, P_1, \dots, P_{N-1}]^T$ is called the frequency domain kernel whose elements are defined by

$$P_n = \begin{cases} 0, & n \in \mathcal{R}^C, \\ 1, & n \in \mathcal{R}, \end{cases} \quad (12)$$

After J iterations of this algorithm, the peak-reduced OFDM signal is obtained:

$$\mathbf{a} = \mathbf{x} + \mathbf{c}^{(J)} = \mathbf{x} - \sum_{i=1}^J \alpha_i \mathbf{p}[(k - k_i)_N]. \quad (13)$$

From (9)–(13), it can be found that the PAPR reduction performance of the TR-based OFDM system depends on the selection of the time domain kernel \mathbf{p} , which is only a function of PRT set \mathcal{R} . When \mathbf{p} is a single discrete pulse, the best PAPR reduction performance can be obtained because the maximal peak at location k_i can be canceled without distorting other signal sampling. But it is impractical because a single discrete pulse will result in that all tones should be assigned to the PRT set. So we should select the time domain kernel \mathbf{p} such that the \mathbf{p} not only reduces the peak at location k_i but also suppresses the other big values at location $k \neq k_i$.

To find the optimal PRT set, in mathematical form, we require to solve the following combinatorial optimization problem:

$$\mathcal{R}^* = \arg \min_{\mathcal{R}} \left\| [p_1, p_2, \dots, p_{N-1}]^T \right\|_{\infty}, \quad (14)$$

which requires an exhaustive search of all combination of possible PRT set \mathcal{R} , i.e. $|\mathcal{R}| = C_N^M$ possible combination numbers of PRT set are searched, where $C_N^M = N!/M!(N-M)!$ denotes the binomial coefficient. It is an NP-hard problem and cannot be solved for the number of tones envisaged in practical systems. In [16], [18], the consecutive PRT set, the equally spaced PRT set and the random PRT set optimization were proposed as the candidates of PRT set. Although the consecutive PRT set and the equally spaced PRT set are the simplest selections of PRT set, their PAPR reduction performance are inferior to that of the random PRT set optimization. But the random PRT set optimization requires larger PRT set sampling, and the associated complexity limits the application of such a technique. A variance minimization method in [19] is developed to solve the NP-hard problem, and it is just a modified version of the random PRT set optimization, which also has the drawback of high computational cost. In [20], a cross entropy method was proposed to solve the problem. It obtains better results than the existing selection methods, but it requires larger population or sampling sizes. These limitations of the existing methods motivate us to

find an efficient method to obtain a nearly optimal PRT set. As mentioned before, we propose a genetic algorithm (GA) based PRT set selection method for the purpose with very low computational complexity.

III. GENERIC ALGORITHM BASED PRT SET SELECTION

In this section, we will briefly introduce the GA and use it to search for a nearly optimal PRT set. The resulting PRT set will be used along with our proposed adaptive amplitude clipping technique in the next section.

A. A Brief Introduction to Genetic Algorithm

The GA introduced by Holland [24] is a stochastic search method inspired from the principles of biological evolution observed in nature. GA uses a population of candidate solutions initially distributed over the entire solution space. Based on the principle of Darwinian survival of the fittest, GA produces a better approximation to the optimal solution by evolving this population of candidate solutions over successive iterations or generations. The GA's evolution uses the following genetic operators:

- 1) *Selection* is a genetic operator that chooses a chromosome from the current generation's population in order to include in the next generation's mating pool. In general, chromosomes with a high fitness (merit) should be selected and at the same time chromosomes with a low fitness should be discarded.
- 2) *Crossover* is a genetic operator that exchanges the elements between two different chromosomes (parents) to produce new chromosomes (offsprings). The new population of the next generation consists of these offsprings.
- 3) *Mutation* is a genetic operator that refers to the alteration of the value of each element in a chromosome with a probability.

GA has been applied to extensive optimization problems, such as pilot location search of OFDM timing synchronization waveforms [27], joint multiuser symbol detection for synchronous CDMA systems [28], the search of low autocorrelated binary sequences [29] and thinned arrays [30]. For a complete understanding of the GA, the reader is referred to [24]–[26].

B. PRT Position Search Based on Genetic Algorithm

A detailed description of the GA used for searching the nearly optimal PRT set positions is described in what follows.

An initial population of S chromosome (parent) sequences is randomly generated. Each parent sequence is a vector of length N , and each element of the vector is a binary zero or one depending on the existence of a PRT at that position (one denotes existence and zero denotes non-existence). The number of the PRT in each binary vector is $M < N$. Denote the S parent sequences as $\{\mathbf{A}_1, \dots, \mathbf{A}_S\}$. Then each \mathbf{A}_ℓ is a binary vector of length N .

For each parent sequence \mathbf{A}_ℓ , the PRT set \mathcal{R}_ℓ is the collection of the locations whose elements are one. Then the frequency domain kernel \mathbf{P}_ℓ corresponding to the PRT set \mathcal{R}_ℓ is obtained by (12), and the time domain kernel $\mathbf{p}_\ell = [p_0^\ell, \dots, p_{N-1}^\ell]$ is

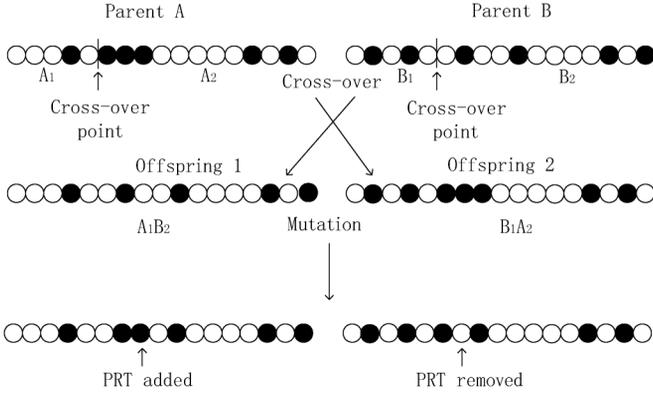


Fig. 1. Illustration of crossover and mutation operations for $M = 6$.

obtained by (11). The merit (secondary peak) of the sequence \mathbf{A}_ℓ is defined as

$$m(\mathbf{A}_\ell) = \left\| [p_1^\ell, \dots, p_{N-1}^\ell]^T \right\|_\infty. \quad (15)$$

The T sequences (called elite sequences) with the lowest merits are maintained for the next population generation. The best merit of the S sequences is defined as

$$m^* = \min_{1 \leq \ell \leq N} m(\mathbf{A}_\ell). \quad (16)$$

Then all S sequences are crossed-over with a probability denoted by p_c . For simplicity, one point crossover is used in this paper. In order to prevent local minima, mutation operator controlled by a probability p_m is applied by changing randomly selected elements in a chromosome sequence. Due to the crossover and mutation operations, the number of PRT in a newly generated sequence (offspring) may be different to M (the size of the PRT set), so that the PRT set is infeasible. Hence one or more zero (zeros) or one (ones) will replace the randomly selected elements in the offspring to guarantee that the PRT set is feasible (the number of the PRT in the offspring is M).

An illustration of the crossover and mutation operations is presented in Fig. 1, where black circles represent PRT positions. Chromosomes from two parents are separated from a randomly selected point and crossed-over to generate new offsprings. Due to crossed-over operation, the size of PRT set of an offspring can be more or less than the required M PRTs. If an offspring has PRTs more than the required M , then several randomly selected PRTs will be removed. If an offspring has PRTs less than the required M , then several PRTs will be added to the randomly selected positions.

The merits of all offspring sequences are evaluated using (15). Each sequence competes for the next generation pool. The T elite sequences obtained from the previous generation replace the T worst sequences with the highest merits in the current generation. This increases the probability of generating better solution and prevents the loss of the optimal solution because of crossover and mutation operations. The best merit of the offsprings is evaluated using (16), which will replace the best merit of the previous generation if it is smaller than it.

The cycle is repeated until a predetermined number of times or a solution with a predefined fitness threshold (the merit is less than some predefined threshold) is achieved. Therefore the proposed GA-based PRT position search algorithm can be summarized in Algorithm 1:

Algorithm 1 GA-PRT Algorithm

- 1: Set the population size S , the PRT set size M , the number T of elite sequences, crossover probability p_c , mutation probability p_m and the maximal iteration number K .
 - 2: Randomly generate an initially feasible population of size S , and find the PRT set \mathcal{R} for each sequence. Calculate the frequency domain kernel \mathbf{P} using (12) and the time domain kernel \mathbf{p} using (11) for each sequence.
 - 3: Calculate the merits (secondary peaks) using (15), select T elite sequences with the lowest merits, and find the best merit m^* using (16) and the corresponding PRT set \mathcal{R}^* .
 - 4: Crossover and mutate all sequences by probabilities p_c and p_m respectively, to generate a new feasible population by randomly adding or removing PRTs.
 - 5: Evaluate the merits (secondary peaks) and the best merit of the new population. If the best merit of the new population is smaller than that of the previous generation, then update the best merit and the corresponding PRT set. Otherwise, remain the previous best merit and the corresponding PRT set unchanged.
 - 6: Replace the T worst sequences with the highest merits in the current generation by the T elite sequences from the previous generation and reselect T elite sequences.
 - 7: If the maximal iteration number K is achieved, output the PRT set and the corresponding secondary peak, and terminate the algorithm; Otherwise, go to Step 4.
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IV. ADAPTIVE AMPLITUDE CLIPPING PAPR REDUCTION ALGORITHM

Based on PRT set, some PAPR reduction methods have been developed. The time domain gradient-based method proposed by Tellado (TR-Gradient-Based Technique) [16]–[18] is of low complexity, but it increases the signal average power and requires very large iterations to obtain the better solution. Because the basic idea of the gradient-based method comes from clipping, TR-Clipping-Filtering-Based Technique is developed in [8]. This scheme iteratively clips the OFDM signal to a predefined threshold A . The clipped signal is then filtered such that the clipping noise appears on the reserved tones only. But the convergence speed of the method is slow. An improved adaptive-scaling TR (AS-TR) algorithm was proposed in [21]–[23]. The basic principle of the AS-TR consists of two processes, i.e. clipping in the time domain and filtering in the frequency domain to suppress the peak regrowth of the OFDM signal. Although the AS-TR provides a better PAPR reduction for predetermined clipping level, the drawback of the AS-TR is that the selection of the optimal clipping level is very difficult. In practice, the optimal clipping level can not be predetermined either. To overcome this drawback of the AS-TR, we propose a

new TR-Clipping-Filtering-Based Technique for PAPR reduction. Our method involves adaptive amplitude clipping control, which allows the determination of the optimal clipping level. Simulation results demonstrate that our scheme can obtain better PAPR reduction regardless of the initial clipping level.

A. Brief Introduction to Adaptive-Scaling TR (AS-TR) Algorithm

The AS-TR method is an iterative clipping and filtered technique. It firstly uses a soft limiter [32] to the input OFDM signal to get the clipping noise $f_n^{(i)}$,

$$f_n^{(i)} = \begin{cases} x_n^{(i)} - Ae^{j\theta_n^{(i)}}, & |x_n^{(i)}| > A \\ 0, & |x_n^{(i)}| \leq A, \end{cases} \quad (17)$$

where A is the target clipping threshold which is relevant to the clipping ratio $\gamma = A^2/E|x_n|^2$, $\theta_n^{(i)}$ is the phase of $x_n^{(i)}$ and i denotes the iteration number. The filtered clipping noise $\hat{f}_n^{(i)}$ is obtained by making $f_n^{(i)}$ through a filter whose passbands are only on reserved tones. Let $\mathbf{f}^{(i)} = [f_0^{(i)}, f_1^{(i)}, \dots, f_{LN-1}^{(i)}]^T$ and $\hat{\mathbf{f}}^{(i)} = [\hat{f}_0^{(i)}, \hat{f}_1^{(i)}, \dots, \hat{f}_{LN-1}^{(i)}]^T$, where L is an oversampling factor. The peak-reduction signal of the AS-TR method is iteratively updated as follows:

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - \beta \hat{\mathbf{f}}^{(i)}, \quad (18)$$

where β is a positive step size that determines the convergence rate. The optimal β is calculated by the following formula:

$$\beta^{(opt)} = \frac{\Re \left[\sum_{n \in \mathbf{S}_p} f_n^{(i)} \overline{\hat{f}_n^{(i)}} \right]}{\sum_{n \in \mathbf{S}_p} |f_n^{(i)}|^2}, \quad (19)$$

where $\Re[x]$ represents the real part of x , $\mathbf{S}_p = \{n : n \in \mathbf{S}_1, |x_n^{(i)}| > |x_{n-1}^{(i)}| \text{ and } |x_n^{(i)}| \geq |x_{n+1}^{(i)}|\}$ is the index set of the peak of $f_n^{(i)}$, and $\mathbf{S}_1 = \{n : |f_n^{(i)}| > 0\}$ is the index set of all clipping pulse.

In general, the larger PAPR reduction obtained by a lower target clipping level is expected. But the PAPR reduction performance of the AS-TR method is very sensitive to the target clipping level. In other words, different clipping ratio γ results in different PAPR reduction performances, as will be demonstrated in Section V. However, the optimal target clipping level or clipping ratio can not be predetermined at the initial stage. In the next section, an adaptive clipping scheme is proposed to get better PAPR reduction regardless of the initial clipping ratio γ .

B. Adaptive Amplitude Clipping Algorithm for TR-Based OFDM Systems

In this section, we propose an adaptive amplitude clipping algorithm for TR-based OFDM systems. The main objective is to control both the target clipping level A and the convergence factor β at each iteration. The objective function is denoted as

$$P = \min_{\beta, A} \sum_{\mathbf{S}_1} \left\| x_n^{(i)} - Ae^{j\theta_n^{(i)}} - \beta \hat{f}_n^{(i)} \right\|^2, \quad (20)$$

where $\mathbf{S}_1 = \{n : |f_n^{(i)}| > 0\}$ is the index of all clipping pulses. The reason that we select (20) as the objective function is based on the following inequality.

$$\left\| x_n^{(i)} - Ae^{j\theta_n^{(i)}} - \beta \hat{f}_n^{(i)} \right\| \leq \left\| x_n^{(i)} - Ae^{j\theta_n^{(i)}} - \beta f_n^{(i)} \right\|. \quad (21)$$

By least square method, (20) shows that the optimal convergence factor is

$$\beta^{(i)} = \frac{\langle |\mathbf{f}^{(i)}|, |\hat{\mathbf{f}}^{(i)}| \rangle}{\|\hat{\mathbf{f}}^{(i)}\|^2}, \quad (22)$$

where $\langle \cdot, \cdot \rangle$ represents the real inner-product. This implies that the calculation of β involves real domain, rather than complex domain, which is another advantage of our proposed algorithm.

Let $\mathbf{S}_2 = \{n : |f_n^{(i+1)}| > 0\}$ and $\Omega = \mathbf{S}_1 \cup \mathbf{S}_2$. Suppose that the size of Ω is N_1 . Then the gradient is updated as follows:

$$\nabla_A = \frac{\sum_{n \in \Omega} |f_n^{(i+1)}|}{N_1}. \quad (23)$$

Then the proposed adaptive amplitude clipping (AAC-TR) algorithm is stated in Algorithm 2.

Algorithm 2 AAC-TR Algorithm

- 1: Set the initial clipping level A , the maximal iteration number K , the step size ρ and the reserved tone set \mathcal{R} obtained by Algorithm 1.
 - 2: Set $i = 0$, $\mathbf{x}^{(0)} = \mathbf{x}$ and $A^{(0)} = A$, where $\mathbf{x}^{(i)} = [x_0^{(i)}, x_1^{(i)}, \dots, x_{LN-1}^{(i)}]^T$.
 - 3: Calculate the clipping noise $f_n^{(i)}$ using (17). If no clipping noise, transmit signal $\mathbf{x}^{(i)}$ and terminate the program.
 - 4: Filter $f_n^{(i)}$ to satisfy the tone reservation constraints:
 - a) Convert $\mathbf{f}^{(i)}$ to $\mathbf{F}^{(i)} = \text{DFT}\{\mathbf{f}^{(i)}\}_2$,
 - b) Obtain the filtered clipping noise $\hat{\mathbf{F}}^{(i)}$ by projecting $\mathbf{F}^{(i)}$ to the PRT set and remove the out-of band parts of $\mathbf{F}^{(i)}$,
 - c) Convert $\hat{\mathbf{F}}^{(i)}$ to the time domain to obtain $\hat{\mathbf{f}}^{(i)}$ by carrying out the IDFT,
 - 5: Calculate the optimal step size $\beta^{(i)}$ using (22), $\mathbf{x}^{(i+1)}$ using (18), and $\mathbf{f}^{(i+1)}$ using (17).
 - 6: Calculate ∇_A using (23), and update the clipping level A by
$$A^{(i+1)} = A^{(i)} + \rho \nabla_A. \quad (24)$$
 - 7: where ρ is the step size with $0 \leq \rho \leq 1$.
 - 8: Set $i = i + 1$, if $i < K$, go to Step 3; Otherwise, transmit $\mathbf{x}^{(i+1)}$ and terminate the program.
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C. Complexity Analysis for AAC-TR Algorithm

The complexity of the AAC-TR algorithm in oversampling case (oversampling factor $L \geq 4$) for accurate PAPR [10] is measured by using the number of real multiplications. A complex multiplication is counted as four real multiplications. We

only consider the runtime complexity. Step 1 and step 2 are not counted because all clipping-based methods must carry out these two steps.

In step 3, $f_n^{(i)}$ can be calculated as $f_n^{(i)} = x_n^{(i)} - x_n^{(i)}(A/|x_n^{(i)}|)$, where $n \in \mathbf{S}_1 = \{n : |f_n^{(i)}| > 0\}$, and $N_{\mathbf{S}_1}$ is the size of \mathbf{S}_1 . Although $N_{\mathbf{S}_1}$ is a random variable, it is roughly a constant in all iterations and its mean can be calculated as follows [21].

$$\bar{N}_{\mathbf{S}_1} = LN e^{-A^2/2\sigma^2}. \quad (25)$$

So the complexity of computing $f_n^{(i)}$ can be estimated as $2\bar{N}_{\mathbf{S}_1}$ real multiplications, and $\bar{N}_{\mathbf{S}_1}$ real divisions.

In step 4, the number of real multiplications for computing an LN -point DFT with $N_{\mathbf{S}_1}$ nonzero inputs and N in-band outputs (other outputs are not needed) can be computed as follows [23], [31]:

$$\begin{aligned} \mathcal{M}_{LN} &= \mathcal{M}_{LN/2} + 2\mathcal{M}_{LN/4} \\ &\quad + \max(0, \min(6N_{\mathbf{S}_1}, 3LN/2 - 8)) \end{aligned} \quad (28)$$

$$\mathcal{M}_L = 0 \quad (27)$$

$$\mathcal{M}_{2L} = \max(0, \min(3N_{\mathbf{S}_1}, 3L/2 - 4)) \quad (28)$$

where \mathcal{M}_k represents the number of real multiplications for computing a k -point DFT. Based on (26)–(28) and replacing $N_{\mathbf{S}_1}$ by $\bar{N}_{\mathbf{S}_1}$, the average complexity \mathcal{M}_{DFT} of the DFT is calculated. Similarly, replacing $N_{\mathbf{S}_1}$, N and L by M , LN and 1 respectively, the average complexity \mathcal{M}_{IDFT} of the IDFT can be also calculated.

In (22), the calculation of β requires $2\bar{N}_{\mathbf{S}_1}$ real multiplications and 1 real division. Note that the update of A in (24) and the calculation of ∇_A in (23) only require 1 real multiplication and 1 real division, respectively.

The complexity of the AAC-TR algorithm mainly depends on the LN -point DFT/IDFT pair and weighting the clipping noise in (18). The latter requires $2LN$ real multiplications. Based on the above analysis, the total complexity of the AAC-TR algorithm for K iterations is

$$\mathcal{M} = K(4\bar{N}_{\mathbf{S}_1} + \mathcal{M}_{DFT} + \mathcal{M}_{IDFT} + 2LN + 1) \quad (29)$$

real multiplications and $K(\bar{N}_{\mathbf{S}_1} + 2)$ real divisions.

If DFT/IDFT used in the AAC-TR algorithm is replaced by FFT/IFFT to compute the peak-reduced signal, the computational complexity of the AAC-TR algorithm is evaluated as $\mathcal{O}(LN \log(LN))$, which is consistent with the AS-TR algorithm. But it is better than the gradient algorithm [16]. The latter is with complexity of order $\mathcal{O}(LN^2)$ [23]. On the other hand, the AAC-TR algorithm can counteract all large peaks above the clipping level in each iteration, while the gradient algorithm can only mitigate one peak in each iteration.

Compared to the AS-TR algorithm, the complexity of the AAC-TR algorithm slightly increases due to the following factors. The calculation of β for the AS-TR algorithm in (19) is operated over \mathbf{S}_p , which requires $5\bar{N}_{\mathbf{S}_p}$ real multiplications. Nevertheless, the calculation of β for the AAC-TR algorithm in (22) is over \mathbf{S}_1 , which requires $2\bar{N}_{\mathbf{S}_1}$ real multiplications. From [21], we have $\bar{N}_{\mathbf{S}_1} = L\sqrt{6/\pi}(\sigma/A)\bar{N}_{\mathbf{S}_p}$. For example, when $L = 4$, $N = 512$, and $\gamma = 5$ dB, we have $\bar{N}_{\mathbf{S}_1} = 86.6902$

and $\bar{N}_{\mathbf{S}_p} = 39.4389$. So $5\bar{N}_{\mathbf{S}_p} - 2\bar{N}_{\mathbf{S}_1} = 23.8139$ real multiplications are reduced in each iteration. Although the adaptive update of clipping level in (24) and (23) will result in the increment of calculation (mainly real additions), the operation reduction of computing β can compensate some of such increment so that the complexity of the AAC-TR algorithm slightly increases compared to that of the AS-TR algorithm.

V. SIMULATION RESULTS

To evaluate and compare the performance of GA based nearly optimal PRT set positions searching and the AAC-TR algorithm for OFDM PAPR reduction, extensive simulations have been conducted. In our simulations, an OFDM system of 16-QAM (quadrature amplitude modulation) with $N = 512$ sub-carriers is used. The number of reserved PRT set is $M = 32$. In order to get CCDF, 10^5 random OFDM symbols are generated. The transmitted signal is oversampled by a factor of $L = 4$ for accurate PAPR estimation.

In the GA-PRT algorithm, the population size $S = 30$, the maximum iteration number $K = 170$, the crossover probability $p_c = 0.9$, the mutation probability $p_m = 0.05$, and the elite sequences $T = 2$. For comparison, the random set optimization (RSO) and CE algorithm are also tested. The optimal PRT set of the RSO is obtained by generating 10^5 random sets and selecting the best PRT set. The parameters used in the CE algorithm basically follow [20], i.e. the population size or the number of samples is $U = 120$, the step size $\rho = 0.1$, the smoothing factor $\lambda = 0.8$, and the maximum iteration number $K = 170$.

The corresponding PRT sets obtained by the proposed GA-PRT algorithm and the existing methods for $M = 32$ are as follows.

$$\begin{aligned} \text{GA-PRT} &= \{10, 11, 28, 42, 43, 61, 95, 107, 115, 120, 131, 155, \\ &\quad 156, 160, 176, 193, 202, 215, 232, 254, 273, 316, \\ &\quad 321, 337, 370, 384, 403, 412, 416, 447, 484, 485\}, \end{aligned} \quad (30)$$

$$\begin{aligned} \text{CE-PRT} &= \{8, 49, 75, 76, 111, 117, 127, 134, 145, 156, 159, \\ &\quad 163, 164, 202, 214, 223, 258, 268, 273, 322, 342, \\ &\quad 350, 366, 412, 427, 438, 455, 457, 458, \\ &\quad 467, 488, 504\}, \end{aligned} \quad (31)$$

$$\begin{aligned} \text{CS-PRT} &= \{225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, \\ &\quad 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, \\ &\quad 246, 247, 248, 249, 250, 251, 252, 253, \\ &\quad 254, 255, 256\}, \end{aligned} \quad (32)$$

$$\begin{aligned} \text{ES-PRT} &= \{15, 31, 47, 63, 79, 95, 111, 127, 143, 159, 175, 191, \\ &\quad 207, 223, 239, 255, 271, 287, 303, 319, 335, 351, \\ &\quad 367, 383, 399, 415, 431, 447, 463, 479, 495, 511\}, \end{aligned} \quad (33)$$

$$\begin{aligned} \text{RS-PRT} &= \{16, 45, 57, 61, 63, 80, 81, 104, 105, 118, 134, 155, \\ &\quad 159, 167, 184, 187, 198, 200, 201, 203, 241, 250, \\ &\quad 276, 284, 329, 394, 408, 459, 466, 481, 495, 498\}, \end{aligned} \quad (34)$$

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY (CC) AND SECONDARY PEAK (SP) FOR DIFFERENT METHODS, AND DIFFERENCE OF THE SECONDARY PEAKS OBTAINED BY THE CE METHOD AND OTHER METHODS

methods	CC	SP	differences
CS-PRT	-	0.9936	0.7131
ES-PRT	-	1	0.7195
GA-PRT	$SK = 30 * 170 = 5100$	0.2996	0.0191
CE-PRT	$UK = 120 * 170 = 20400$	0.2805	0
RS-PRT	10^5	0.3207	0.0402

where GA-PRT, CE-PRT, CS-PRT, ES-PRT and RS-PRT respectively represent GA optimization based PRT set, CE optimization based PRT set, consecutive PRT set, equally spaced PRT set and random set optimization based PRT set.

A. Computational Complexity Versus Normalized Secondary Peak

Table I compares the computational complexity and the normalized secondary peak among different methods. For comparison, the differences between the normalized secondary peak of the CE algorithm and those of other methods are also calculated. It can be seen that the secondary peaks achieved by consecutive PRT set and equally spaced PRT set are the worst among all methods. The secondary peak obtained by the CE algorithm excels those of other methods. The secondary peak by random set optimization (RSO) with 10^5 randomly chosen PRT set is 0.0402 larger than that of CE algorithm with 20400 searches. So the CE algorithm with lower computational complexity gets better performance than RSO. On the other hand, the computational complexity of CE algorithm is five times greater than that of GA-PRT algorithm. But the difference between the secondary peaks of the two methods is only 0.0191. In other words, the complexity reduction by GA-PRT algorithm is $(20400 - 5100) = 15300$ operations with payment of only 0.0191 in the secondary peak. So the proposed GA-based PRT set selection algorithm is more efficient than the CE algorithm for solving the secondary peak. The fact that the PAPR reduction performances of CE and GA are almost the same will be demonstrated in Fig. 2.

B. PAPR Reduction Versus Different PRT Sets

In Fig. 2, the comparison of PAPR reduction performance based on Tellado's gradient algorithm (GD-TR) [16], [18] with the above different PRT sets is shown. Here the iteration number of the gradient algorithm is set to 10. When $P_r(PAPR > PAPR_0) = 10^{-4}$, the PAPR of the original OFDM is 12 dB. The PAPRs of CS-PRT set and ES-PRT set are 10.8 dB and 10.5 dB, respectively. Using the random set optimization in [18], when the number of randomly selected PRT sets is 10^5 , the PAPR is reduced to 9.2 dB. The PAPR obtained by the CE-PRT with the search complexity $UK = 120 * 170 = 20400$ in [20] is 9.1 dB. The PAPR obtained by the GA-PRT with the search complexity $SK = 30 * 170 = 5100$ is 9.1 dB. There is a negligible gap between the PAPRs obtained by CE-PRT and by GA-PRT. But from Table I, we see that the search complexity of the GA-PRT is only $SK/UK = 30/120 = 25\%$ of that by the CE-PRT.

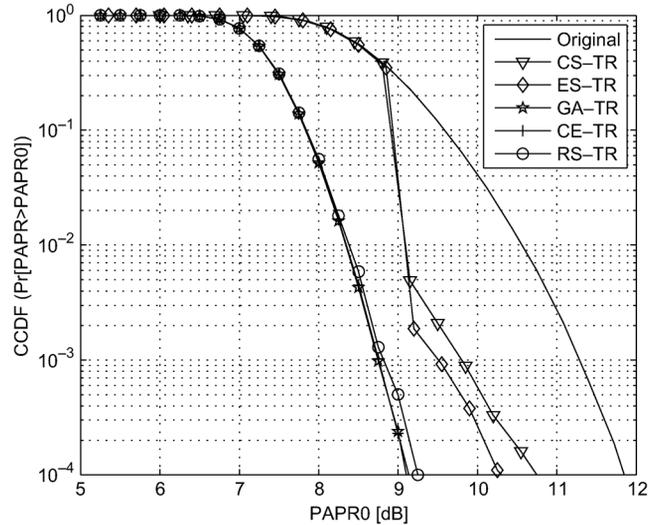


Fig. 2. Comparison of PAPR reduction based on the Tellado's gradient algorithm with different PRT sets.

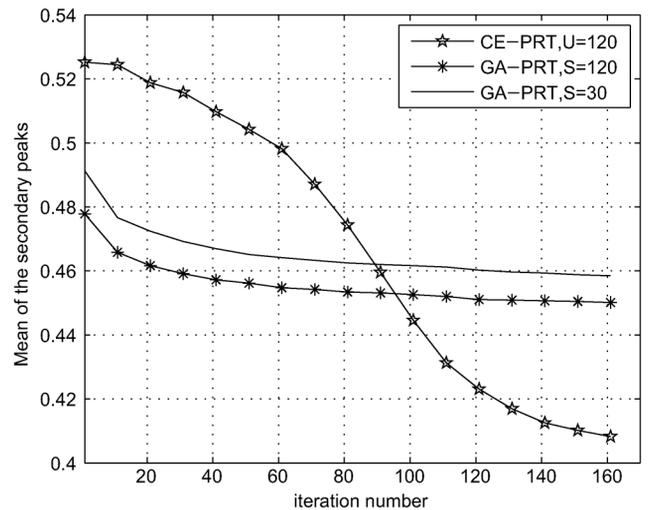


Fig. 3. Comparison of the mean of the best secondary peak between the GA-PRT algorithm and the CE-PRT one.

C. Comparison of the Secondary Peak Between GA-PRT Algorithm and CE-PRT Algorithm

In Fig. 3, 100 experiments are performed to compare the means of the best secondary peak obtained by GA-PRT algorithm and CE-PRT algorithm. According to the original CE algorithm proposed by Rubinstein, the sample size U is very large to get better performance for the CE-PRT algorithm, so we only adopt $U = 120$ used in the [20] for comparison. It can be found that the performance of the GA-PRT algorithm is better than that of the CE-PRT one in approximately 1-90 iterations. As the increase of iterations, the secondary peaks of the CE-PRT algorithm are improved. This displays that the convergence of the CE-PRT algorithm is slower than that of the GA-PRT one, so that the proposed GA-PRT algorithm can get a better sub-optimal PRT set. On the other hand, the maximal difference of the secondary peak gained by the GA-PRT algorithm between $S = 30$ and $S = 120$ is only 0.0134, so $S = 30$ is a better choice for the proposed GA-PRT algorithm.

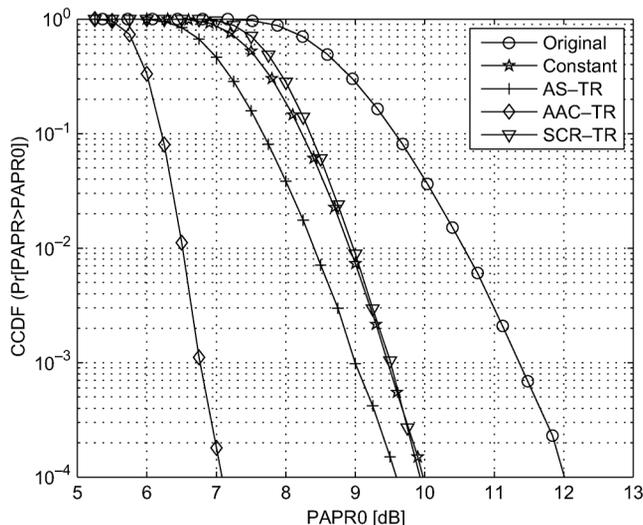


Fig. 4. Comparison of PAPR reduction for different methods with the same GA-PRT set.

D. PAPR Reduction Versus Different Methods

Fig. 4 compares the PAPR reduction performance of the proposed AAC-TR algorithm with the constant scaling (Constant) algorithm, AS-TR method in [21], [22], signal to clipping noise ratio (SCR-TR) algorithm and gradient descent (GD-TR) algorithm [17] for the same GA-PRT set. Here the iteration number of the constant scaling algorithm is 40. The same maximum iteration number is set to 10 for AS-TR, AAC-TR, GD-TR and SCR-TR. When $P_r(PAPR > PAPR_0) = 10^{-4}$, the PAPR of the original OFDM is 11.9 dB. Using the constant scaling algorithm to SCR-TR algorithm GD-TR algorithm and AS-TR algorithm, the PAPRs are approximately reduced to 9.33 dB, 9.67 dB, 9.22 dB, 8.56 dB, respectively. The PAPR is approximately reduced to 7.05 dB by using the proposed AAC-TR algorithm. Compared to the PAPR of the original OFDM, an approximate 4.85 dB reduction gain is obtained, which is 1.51 dB larger than AS-TR algorithm, 2.62 dB larger than SCR-TR algorithm, 2.17 dB larger than GD-TR algorithm and 2.28 dB larger than constant scaling algorithm for the same 10 iterations.

E. Average PAPR Reduction Versus Iteration

Fig. 5 compares the average PAPR reduction performance of the AS-TR, AAC-TR and GD-TR with clipping ratio $\gamma = 4$ dB for the same iteration numbers. Fig. 4 shows that the average PAPR reduction performance of the AAC-TR algorithm is better than those of AS-TR and GD-TR. When the iteration number equals 11, the AAC-TR algorithm converges to 6 dB PAPR. Although the GD-TR algorithm is simple, its convergence speed is the slowest among the three methods. When the iteration number is 20, its average PAPR is approximately 7.4 dB, which is 1.4 dB larger than AAC-TR algorithm in 11 iterations. The AS-TR algorithm converges to 7.1 dB PAPR in 7 iterations, however, which is approximately 0.9 dB larger than AAC-TR algorithm in the same iterations.

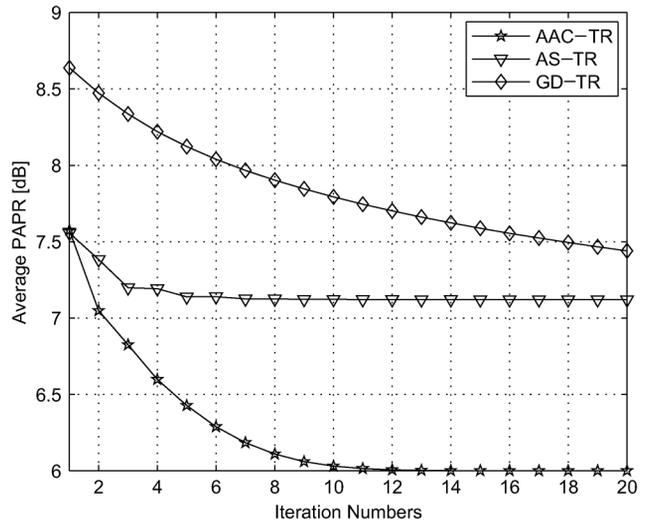


Fig. 5. Relationship of PAPR reduction with iteration numbers for different methods.

TABLE II
COMPARISON OF AVERAGE POWER INCREASE (API), AVERAGE SIMULATION TIME (AST), AND PAPR FOR DIFFERENT METHODS WITH $\gamma = 5$ dB

methods	API (dB)	AST (ms)	PAPR (dB)
GD-TR	0.3867	7.7485	9.22
AS-TR	0.3854	11.3914	8.56
AAC-TR	0.2596	13.2898	7.05

F. Average Power Increase, Average Simulation Time and PAPR Reduction Versus Different Methods

Table II compares the average power increase (API) (in dB), average simulation time (AST) (in millisecond) and PAPR for AS-TR, AAC-TR and GD-TR with clipping ratio $\gamma = 5$ dB for 10 iterations. We observe that the AST of the GD-TR method is least (Note that the GD-TR method must prestore an $LN \times LN$ IFFT matrix, the calculation does not include in simulation time), but its PAPR is 0.66 dB larger than that of the AS-TR algorithm, and 2.17 dB larger than that of the AAC-TR algorithm. The APIs of AS-TR and AAC-TR are almost the same. Compared to AS-TR algorithm, the AST of the AAC-TR algorithm increases slightly, but its API is less than that of the AS-TR, and PAPR is 1.51 dB smaller than that of AS-TR.

G. PAPR Reduction Versus Different Clipping Ratios

Fig. 6 compares the PAPR reduction performance of AS-TR algorithm and AAC-TR method with 10 iterations for three different target clipping ratios, $\gamma = 0$ dB, 2 dB and 4 dB. For comparison, the original OFDM signal's PAPR is also given. When $P_r(PAPR > PAPR_0) = 10^{-4}$, the AS-TR algorithm for the three different clipping ratios, $\gamma = 0$ dB, 2 dB and 4 dB can obtain 0.9 dB, 1.4 dB and 2.4 dB PAPR reduction from the original OFDM PAPR of 12 dB. It demonstrates that the AS-TR algorithm is sensitive to the target clipping ratio. Different target clipping ratios can result in different PAPR reduction performance. Contrary to the AS-TR algorithm, the proposed AAC-TR algorithm obtains an approximately 5 dB PAPR

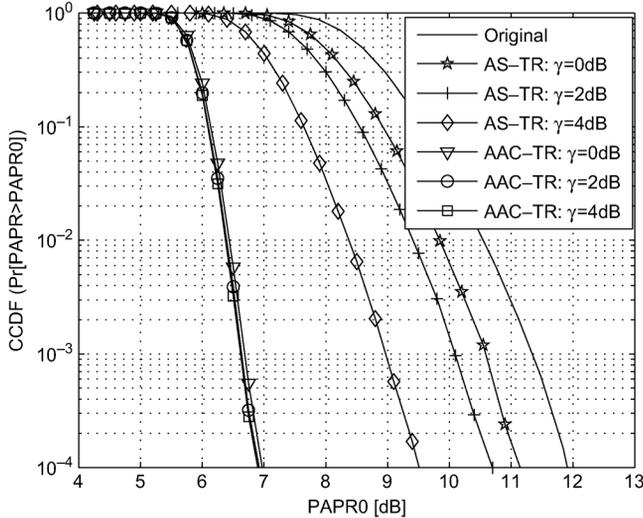


Fig. 6. Comparison of PAPR reduction between AAC-TR and AS-TR with different clipping ratios.

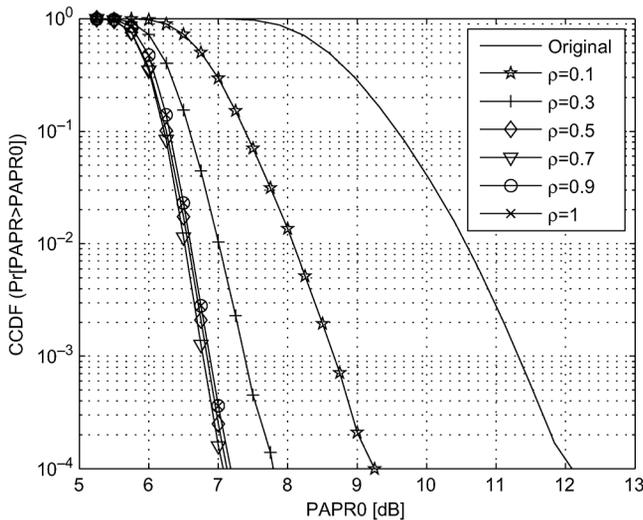


Fig. 7. Comparison of PAPR reduction with different step size ρ .

reduction from the original OFDM PAPR of 12 dB for all three of the target clipping ratios.

H. Different ρ Versus PAPR Reduction

In Fig. 7, we compare the PAPR reduction performance of AAC-TR algorithm for different step size ρ . When $\rho = 0.1$ and $\rho = 0.3$, the PAPRs are 9.3 dB and 7.8 dB. For other choices on ρ , the differences of the PAPRs are very small. The reasons can be that the smaller ρ can not effectively adjust the clipping level A . This corresponds to that A is not updated. So we should select a bigger step size ρ to gain better PAPR performance for the AAC-TR algorithm.

VI. CONCLUSION

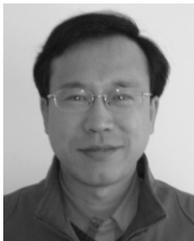
This paper studies PAPR reduction for tone reservation-based OFDM systems. The PAPR reduction mainly depends on the selection of peak reduction tone (PRT) set and the optimal target clipping level. Finding the optimal PRT set is equivalent

to solving the secondary peak minimization problem, which must optimize over all combination of possible PRT sets. It is an NP-hard problem and cannot be readily solved for the number of tones appeared in practical OFDM systems. The existing selection methods, such as the consecutive PRT set, the equally spaced PRT set and the random PRT set, perform poorly compared to the optimal PRT set or require high computational complexity. In this paper, an efficient scheme based on genetic algorithm (GA) was proposed to give a nearly optimal PRT set. Compared to the CE-PRT algorithm, the proposed GA-PRT algorithm has lower computational complexity and achieves a good approximation to the secondary peak of the CE-PRT algorithm. Although the TR-clipping-based technique is simple and attractive for practical implementation, finding the optimal target clipping level is difficult and the optimal target clipping level can not be predetermined in the initial stage. An adaptive clipping control algorithm is proposed to solve this problem. Simulation results show that the proposed adaptive clipping control algorithm achieves good PAPR reductions regardless of the target clipping ratios.

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