Resource Allocation for Two-Way AF Relaying with Receive Channel Knowledge

Hossein Bagheri, Masoud Ardakani, and Chintha Tellambura

***Abstract*—The resource allocation problem for two sources communicating via an amplify-forward relay is studied from an outage perspective. Analog network coding is considered for half-duplex nodes with perfect receiver-side channel knowledge. Under a sum power constraint, an optimal power allocation that minimizes an approximate outage probability is derived and shown to improve the performance upto 4.77 dB. A cut-set bound is also optimized to serve as a comparison reference. When such a power allocation is not feasible, two novel resource-optimized schemes, which exploit conventional one-way relaying, are proposed to reduce the outage at low multiplexing gains[1].**

***Index Terms*—Analog network coding, convex optimization, multiplexing gain, resource allocation, two-way relay.**

I. Introduction

Bidirectional relaying, in which two nodes simultaneously exchange independent messages through a relay, is an active research area due to its capability in recovering the spectral efficiency loss resulted from half-duplex operation of network nodes [1]. Bidirectional relaying, however, suffers from reliability degradation in fading [2]. This paper improves the reliability by properly distributing communication resources (i.e., time and power) among network nodes.

Analog network coding (ANC) is a simple, yet important type of bidirectional relaying, in which the relay amplifies and forwards the noisy linear combination of signals simultaneously received from the nodes [3]. Optimum power allocation (OPA) strategies have been proposed in previous studies for such a setup under some restrictive assumptions [2], [4]–[6]. In [4], an OPA is obtained based on separate outage calculation for each traffic flow, assuming equal power for the sources. However, in some scenarios, taking into account the effect of both traffic flows simultaneously is more desirable [7]. In [5], [6], OPA strategies are derived based on the assumption of channel state information available at the transmitters (CSIT). Using CSIT, an opportunistic source selection protocol with OPA has been proposed to support one traffic flow at a time in order to improve the reliability [2]. Other relevant references concerning power allocation for two-way multi-user/multi-relay configurations are [8]–[10].

***Contributions and relation to previous work*.** We study the resource allocation problem in the amplify-forward two-way relay channel (AF-TWRC) with an equal target rate for both users [2], [5]. In regard to previous power allocation studies [2], [4]–[6], we make less limiting assumptions. In particular, we assume channel state information is available at the receivers (CSIR) rather than the more stringent CSIT assumption, and build our analysis based on the probability of either of users being in outage instead of separate outage calculation. The main contributions of this paper are as follows:

1) Under a sum-power constraint on nodes' powers, we obtain a closed form OPA for ANC protocol that minimizes the outage probability for a given target rate[2]. The OPA can improve the outage performance up to 4.77 dB compared to the equal power allocation.
2) When the sum-power constraint is not meaningful, for target rates with low multiplexing gains, we suggest to use one-way AF in addition to/instead of ANC. Specifically, we find an optimal source transmission schedule for a combination of ANC and one-way AF or a combination of two separate one-way AF transmissions. For the latter case, we optimally distribute the relay power between the two separate relay-to-source transmissions. Simulations show 1.6 dB improvement over ANC.

To understand how much the power allocation can make ANC operate close to the fundamental limits of TWRC, we develop a cut-set type bound on the outage performance, and find its OPA. It is worth mentioning that the bound is sometimes tighter than the bound due to ANC with CSIT [5].

Throughout the paper, $i, j \in \{1, 2\}$, and $i \neq j$. A complex Gaussian random variable (r.v.) z with mean m and variance v is represented by $z \sim \mathcal{CN}(m, v)$. Finally, all logarithms are in base 2.

II. System Model and Preliminaries

In this work, a dual-hop communication system, wherein two sources S_1 and S_2 exchange their messages via relay R, is studied (Fig. 1). All terminals are half-duplex. The channel noise on each link is an independent and additive r.v. with $\mathcal{CN}(0, 1)$. The channel gain between S_i and R is denoted by $h_i \sim \mathcal{CN}(0, \Omega_i)$. Without loss of generality, it is assumed that $\Omega_2 \leq \Omega_1$. The channels are independent, frequency-flat, and constant over the signaling duration. Moreover, the uplink and downlink channels are reciprocal. In addition, the channel realization is perfectly known by the receiving end of each transmission[3]. An equal target rate $\frac{R_t}{2}$ is considered for users, and the total transmit power is assumed to be P_T.

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The authors are with the Department of Electrical and Computer Engineering, University of Alberta (e-mail: {bagheri2, ardakani, tellambura}@ualberta.ca).

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[1]Part of the material in this paper has been presented at the IEEE PIMRC conference, Toronto, 2011 [14].

[2]Our results can be modified to accommodate per node maximum power constraints as well as different target rates.

[3]This task can be accomplished by broadcasting from R the quantized version of both channel coefficients to the users [11].

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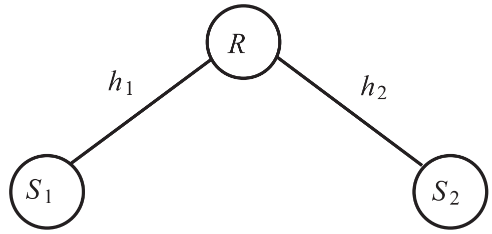

Fig. 1. Two-way relay channel (TWRC) model.

The multiplexing gain r is defined as $r \triangleq \frac{R_t}{\mathcal{C}(P_T)}$, where $\mathcal{C}(P) \triangleq \log(1+P)$. In addition, $g_i \triangleq |h_i|^2$, and $\omega \triangleq \frac{\Omega_2}{\Omega_1}$.

ANC Protocol. In time slot 1, S_i transmits a unit-power signal x_i to R. In time slot 2, R amplifies and forwards its received signal to both users. The received signals by R, and S_i are

$$\begin{aligned} y_r &= h_1\sqrt{P_1}x_1 + h_2\sqrt{P_2}x_2 + n_r \\ y_i &= h_i\sqrt{P_r}x_r + n_i, \end{aligned} \tag{1}$$

where n represents the noise signal at the corresponding receiver, P_r and P_i are the transmit powers for R and S_i, respectively, and $x_r = \frac{1}{\sqrt{g_1P_1+g_2P_2+1}}\, y_r$. Each user receives a copy of its own signal as interference. After removing the known interference, the SNR at user i is

$$\tilde{\gamma}_i = \frac{g_1g_2P_rP_j}{g_i(P_r+P_i)+g_jP_j+1} \overset{(\star)}{\approx} P_r \min\{g_i, \frac{g_jP_j}{P_r+P_i}\}, \tag{2}$$

where $(\star)$ is due to ignoring 1 in the denominator (high SNR approximation, cf. [5], [12], [13]), and using $\frac{xy}{x+y} \approx \min\{x,y\}$ [5][4], which is termed harmonic-to-min approximation (HMA) in this paper. Hereafter, $\overset{(\star)}{=}$ is used to refer to the above approximation. Using Gaussian input signals, the outage probability becomes

$$\begin{aligned} P_{\text{out}}^{\text{ANC}}(R_t) &= \text{Prob}(\min\{\tilde{\gamma}_1, \tilde{\gamma}_2\} < \Gamma \triangleq 2^{R_t}-1) \overset{(\star)}{=} \\ &\text{Prob}(P_r \min\{g_1, \frac{g_2P_2}{P_r+P_1},\ g_2, \frac{g_1P_1}{P_r+P_2}\} < \Gamma). \end{aligned} \tag{3}$$

Lower Bound on the Outage Probability. A lower bound on the outage probability of all two-phase schemes comprising multiple-access (MAC) and broadcast (BC) phases is now provided. The MAC and BC phases respectively take β and $\bar{\beta}$ fractions of time. Cut-set bounds are calculated by considering the relay and transmitter i as a single transmitter or the relay and receiver i as a single receiver. This leads to:

$$R_i^{\text{up}} \le \min\{\bar{\beta}\mathcal{C}(g_i\frac{P_r}{\bar{\beta}}), \beta\mathcal{C}(g_j\frac{P_j}{\beta})\}. \tag{4}$$

[4]The approximation in (2) is different than the actual value by at most 3 dB due to $\frac{1}{2} \le \frac{\frac{xy}{x+y}}{\min\{x,y\}} \le 1$. However, as mentioned in [5] and also shown in Fig. 4, it is quite tight in outage calculations. Please refer to [14] for a detailed accuracy analysis.

The high SNR approximation of the bound on outage becomes

$$\begin{aligned} P_{\text{out}}^{\text{LB}}(R_t) &= \text{Prob}(\min\{R_1^{\text{up}}, R_2^{\text{up}}\} < \frac{R_t}{2}) = 1- \\ &\text{Prob}(g_1 \ge \max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_1}\}, g_2 \ge \max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_2}\}) \\ &\approx \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_1}\}}{\Omega_1} + \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_2}\}}{\Omega_2}, \end{aligned} \tag{5}$$

where $\Gamma_1 \triangleq (2^{\frac{R_t}{2\bar{\beta}}}-1)\bar{\beta}$, and $\Gamma_2 \triangleq (2^{\frac{R_t}{2\beta}}-1)\beta$. We remark that for $P_1 = P_2 = P_r$, the optimal $\beta^* = \frac{1}{2}$.

III. Optimal Power Allocation for TWRC with Sum-Power Constraint

In this section, the optimal transmit power vector $\vec{P} \triangleq \{P_1, P_2, P_r\}$ that minimizes the outage probabilities (3) and (5), subject to a sum-power constraint is derived as a function of the statistical properties of the channel. In particular, we want $\min\limits_{\vec{P}(\Omega_1,\Omega_2,P_T)} P_{\text{out}}$, subject to $P_1+P_2+P_r \le P_T$.

OPA for ANC. For $\Omega_2 \le \Omega_1$, a suitable power allocation satisfies $P_1 \le P_2$. The reason is to balance (on average) the second and fourth terms in Eq. (3) as much as possible. The formal proof is straightforward, and hence, omitted. Therefore, the arguments of the $\min\{\cdot\}$ operator in (3) can be simplified to either of the following:

$$\begin{cases} \frac{g_2P_2}{P_r+P_1}, \frac{g_1P_1}{P_r+P_2}, & \text{for } P_2 \le \frac{P_T}{2}, \text{ case I;} \\ g_2, \frac{g_1P_1}{P_r+P_2}, & \text{for } \frac{P_T}{2} \le P_2, \text{ case II.} \end{cases} \tag{6}$$

For case I, (3) can be written as

$$P_{\text{out}}^{\text{Case I}}(R_t) \overset{(\star)}{=} 1 - e^{-\frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'}\Gamma} \approx \frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'}\Gamma, \tag{7}$$

where $\Omega_i' \triangleq \frac{P_iP_r}{P_r+P_j}\Omega_i$. It can be shown that the outage probability (7) is convex. Therefore, the optimal $\vec{P}$ is obtained by forming the following Lagrange cost function with parameter λ

$$\mathcal{J}(\vec{P}, \lambda) = \frac{\Omega_1'+\Omega_2'}{\Omega_1'\Omega_2'} + \lambda(P_1+P_2+P_r-P_T), \tag{8}$$

which leads to the following for the range of ω that satisfies $P_2 \le \frac{P_T}{2}$

$$P_2 = \frac{P_T}{1+\sqrt{\omega}+\sqrt[4]{4\omega}}, \quad P_1 = \sqrt{\omega}P_2. \tag{9}$$

For case II, the outage probability is

$$\begin{aligned} P_{\text{out}}^{\text{Case II}}(R_t) &\overset{(\star)}{=} 1 - e^{-(\frac{1}{\Omega_1P_1}+\frac{P_2}{\Omega_1P_1P_r}+\frac{1}{\Omega_2P_r})\Gamma} \\ &\approx (\frac{1}{\Omega_1P_1}+\frac{P_2}{\Omega_1P_1P_r}+\frac{1}{\Omega_2P_r})\Gamma. \end{aligned} \tag{10}$$

Since $\frac{P_T}{2} \le P_2$, then it is clear that choosing $P_2 = \frac{P_T}{2}$ minimizes the outage expression, and gives

$$P_1 = \frac{\sqrt{1+\frac{1}{2}(\frac{1}{\omega}-1)}-1}{\frac{1}{\omega}-1}P_T, \quad P_2 = \frac{P_T}{2}. \tag{11}$$

For both cases, $P_r = P_T - P_1 - P_2$. It is remarked that the power allocation is only a function of the ratio of average fading powers ω. A similar observation has been made for the

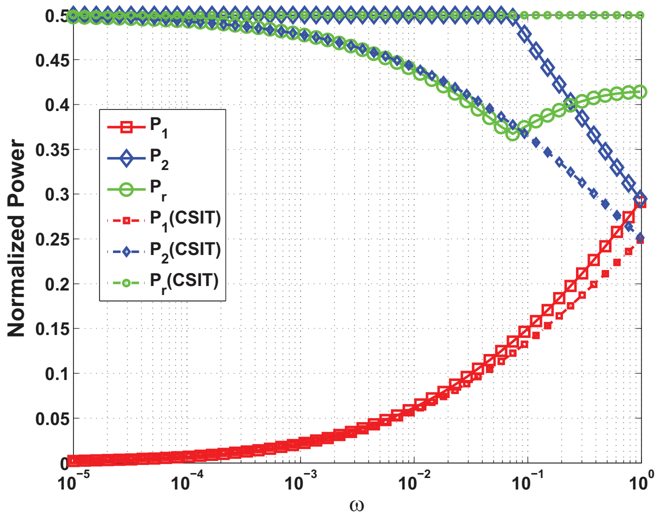

Fig. 2. Optimum share of power for each node. For the case of CSIT, average share of power is plotted.

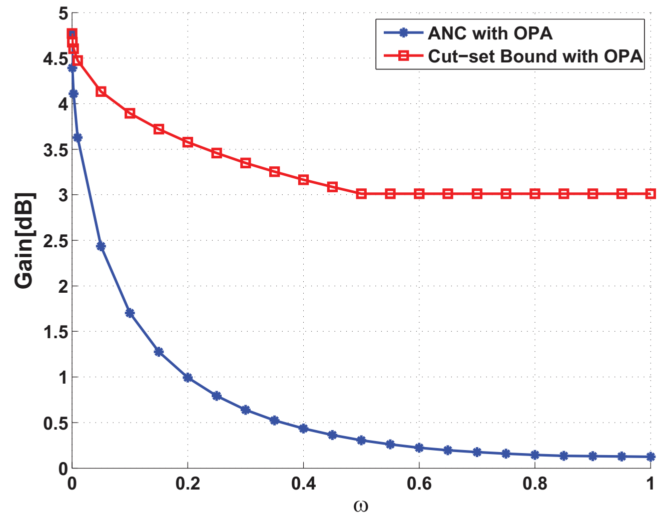

Fig. 3. Power allocation gain with respect to ANC with EPA for a fixed target rate. As ω decreases, optimized ANC approaches the fundamental limits of TWRC.

one-way relay channel [12]. It can be seen that for $\omega \leq \theta$, we have $P_2 = \frac{P_T}{2}$, where $\theta \approx 0.07$ is the switching value between the two cases.

Fig. 2 shows the normalized (to P_T) share of power for each node. The average (over different channel realizations) share of power for each node with CSIT (see [5] for the corresponding PA) is also plotted. An interesting observation is that the behavior of CSIT and CSIR curves is quite similar for each of the source nodes. In the case of CSIT, the relay always gets half of the power, whereas with CSIR, the relay's share varies between 0.36 and 0.5. It is worth mentioning that an exhaustive search to minimize the original outage probability (without HMA) yields similar power shares for outage probabilities less than 0.01.

OPA for the Cut-set Type Lower Bound. The optimization problem associated with the outage probability (5) can be formulated as

$$\begin{aligned} &\min_{\vec{P},\beta} \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_1}\}}{\Omega_1} + \frac{\max\{\frac{\Gamma_1}{P_r}, \frac{\Gamma_2}{P_2}\}}{\Omega_2} \\ &s.t.\ P_1 + P_2 + P_r \leq P_T,\ 0 \leq \beta \leq 1 \end{aligned} \tag{12}$$

We solve the above problem assuming a fixed β. The optimal β can be found by performing an exhaustive search. It is noted that the objective function is convex and the constraints form a convex set. It has been shown in [14] that the optimal allocation based on $\eta \triangleq \frac{\Gamma_2}{\Gamma_1}$ is

$$1)\ \frac{\eta}{\eta+1} < \omega : \qquad P_1 = P_2 = \eta P_r = \frac{\eta}{2\eta+1} P_T. \tag{13}$$

$$2)\ \omega \leq \frac{\eta}{\eta+1} : \quad \begin{cases} P_1 = \dfrac{P_T}{1 + \sqrt{\frac{\eta}{\omega(\eta+1)}} + \sqrt{\frac{1}{\omega\eta(1+\eta)}}}, \\ P_2 = \eta P_r = \dfrac{P_T}{\sqrt{\frac{\omega(1+\eta)}{\eta}} + 1 + \frac{1}{\eta}}. \end{cases} \tag{14}$$

For $\omega = 1$, it can be seen that $P_1 = P_2$.

Fig. 3, demonstrates the gain of OPA (Eqs. (9) and (11)) with respect to equal power allocation (EPA), in which $P_1 = P_2 = P_r = \frac{P_T}{3}$. The maximum gain is 4.77 dB, which is similar to the gain of the EPA scenario with total power $3P_T$. We note that for $\omega = 1$, the optimum power vector is $\vec{P}^* \approx \{0.29, 0.29, 0.42\}$. In the figure, the gain (w.r.t. ANC with EPA) of power optimized cut-set bound with equal timing between MAC and BC phases is also plotted. It can be seen that as ω decreases, optimized ANC approaches the best of two-way relaying protocols with $\beta = \frac{1}{2}$.

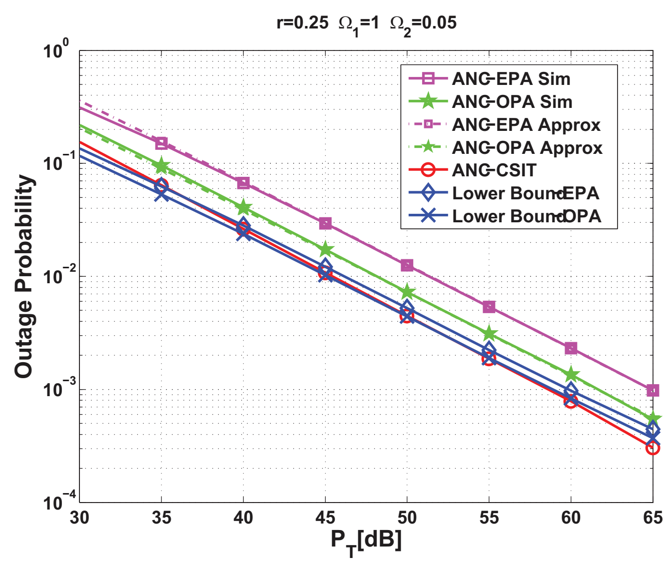

Fig. 4. The role of power allocation on the performance of ANC and the relative lower bound. The transmit power of ANC curves is doubled to compensate for the power scaling in Eq. (4), and hence, to provide a fair comparison with the corresponding lower bound plots.

Fig. 4 evaluates the exact outage expressions of ANC and the cut-set bound under EPA as well as OPA for $\omega = 0.05$, and $r = 0.25$. The approximate outage expressions are also plotted. The plots infer that the approximations are fairly accurate. Furthermore, interestingly, the gap between ANC curves is larger than that of the lower bound in this figure. We see that

ANC and cut-set bound with OPA are closer to each other while ANC and cut-set bound with EPA are much farther apart. The outage probability, when OPA is performed based on CSIT [5], is also provided to appreciate the gain of having CSIT.

IV. Resource Allocation for TWRC without Sum-Power Constraint

We saw that proper distribution of the total transmit power among nodes significantly improves the outage performance for asymmetric configurations. However, when such a power allocation is not possible, we enhance the outage performance by proposing two novel schemes that employ one-way AF in addition to/instead of ANC. The first scheme is based on a combination of one-way and two-way AF schemes, and the other one is composed of two separate one-way transmissions. We find an optimal source transmission schedule for both schemes, and for the latter one, we optimally distribute the relay power between the two separate relay-to-source transmissions. The presence of one-way transmission provides additional advantage of freeing up some resources for other potential neighbors. However, one-way transmission decreases the spectral efficiency which limits the advantages of the proposed schemes to low multiplexing gain regimes. In particular, the latter scheme is well suited for extremely low multiplexing gains.

In the following, we assume all nodes have the same power P. Our results can also be extended to the setup with non-identical powers. Defining $\gamma_i \triangleq g_i P$ and $\xi_i \triangleq \Omega_i P$, the achievable rate destined for user i in ANC protocol is given by $R_i = \frac{1}{2}\mathcal{C}(\frac{\gamma_1\gamma_2}{2\gamma_i+\gamma_j+1})$. It can be seen that for $\omega \leq 1$, on average $R_1 \leq R_2$. To boost R_1, one simple solution is to make transmitter 1 back off it's power to let its receiver get a signal from user 2. In addition to the simple back-off (BO) approach, two new schemes are proposed to enhance R_1, and in turn reduce the outage probability.

Back-Off Scheme. One can realize from (3) that reducing P_1 from P to a suitable fraction of P enhances the outage probability. To obtain the optimal back-off value, it is noticed that

$$P_{\text{out}}^{\text{BO}}(R_t) = \text{Prob}(\min\{\frac{g_2P^2}{P+P_1}, \frac{g_1P_1}{2}\} < \Gamma) = 1 - e^{-(\frac{P+P_1}{P^2\Omega_2}+\frac{2}{P_1\Omega_1})\Gamma}. \quad (15)$$

The outage probability is then minimized by selecting $P_1 = P\min\{\sqrt{2\omega}, 1\}$.

Hybrid One-Way Two-Way AF Scheme. This scheme uses the fact that one-way transmission achieves a better individual rate compared to ANC which gives a higher sum-rate. Hence, to improve R_1 (i.e., data flow from user 2 to user 1), ANC is modified to accommodate a one-way relaying from S_2 to S_1 with $\bar{\lambda}$ fraction of the transmission time (λ percent of the time is devoted to ANC). In fact, the new scheme tries to balance R_1 and R_2 by proper scheduling between one-way and ANC strategies. Using HMA, the achievable rates are approximated as

$$\begin{aligned} R_1 &= \frac{\lambda}{2}\mathcal{C}(\frac{\gamma_1\gamma_2}{2\gamma_1+\gamma_2+1}) + \frac{\bar{\lambda}}{2}\mathcal{C}(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}) \\ &\overset{(\star)}{=} \frac{\lambda}{2}\mathcal{C}(\min\{\gamma_1,\gamma_2/2\}) + \frac{\bar{\lambda}}{2}\mathcal{C}(\min\{\gamma_1,\gamma_2\}), \\ R_2 &= \frac{\lambda}{2}\mathcal{C}(\frac{\gamma_1\gamma_2}{\gamma_1+2\gamma_2+1}) \overset{(\star)}{=} \frac{\lambda}{2}\mathcal{C}(\min\{\gamma_1/2,\gamma_2\}). \end{aligned} \quad (16)$$

The aim is to find λ that minimizes the outage probability. To calculate the probability of outage, three SNR regions are considered with their corresponding outage events:

$$\begin{aligned} \gamma_2 \leq \frac{\gamma_1}{2}: &\quad \min\{\lambda\mathcal{C}(\frac{\gamma_2}{2}) + \bar{\lambda}\mathcal{C}(\gamma_2), \lambda\mathcal{C}(\gamma_2)\} \leq R_t, \\ \frac{\gamma_1}{2} \leq \gamma_2 \leq \gamma_1: &\quad \min\{\lambda\mathcal{C}(\frac{\gamma_2}{2}) + \bar{\lambda}\mathcal{C}(\gamma_2), \lambda\mathcal{C}(\frac{\gamma_1}{2})\} \leq R_t, \\ \gamma_1 \leq \gamma_2: &\quad \lambda\mathcal{C}(\frac{\gamma_1}{2}) \leq R_t. \end{aligned}$$

Using $(1+\frac{\gamma_2}{2})^{\lambda}(1+\gamma_2)^{\bar{\lambda}} \approx 1+\frac{\gamma_2}{2^{\lambda}}$, which gives a lower bound on SNR of S_1, the outage probability becomes

$$P_{\text{Out}}^{\text{Hyb}} = 1 - \Big(e^{-a(\frac{1}{\xi_1}+\frac{1}{2\xi_2})} + \frac{\omega}{\omega+1}\big(e^{-d(\frac{1}{\xi_1}+\frac{1}{\xi_2})} + e^{-a(\frac{1}{\xi_1}+\frac{1}{\xi_2})}\big)\Big), \quad (17)$$

where $d = 2((1+\Gamma)^{\frac{1}{\lambda}} - 1), a = \max\{2b, d\}, b = 2^{\lambda}\Gamma$. The optimum λ can be simply found by minimizing (17). For highly asymmetric case (i.e., $\omega \ll 1$), $P_{\text{Out}}^{\text{Hyb}} \approx a(\frac{1}{\xi_1} + \frac{1}{2\xi_2})$, and therefore,

$$\lambda_{\text{Hyb}}^* = \underset{\lambda}{\operatorname{argmin}} \max\{2^{\lambda}(2^{R_t} - 1), 2^{\frac{R_t}{\lambda}} - 1\}, \quad (18)$$

which is found by equating the $\max(\cdot)$ arguments[5].

One-Way AF Scheme. The scheme is composed of two separate one-way relayings from S_2 to S_1, and from S_1 to S_2, with λ_1 and $\lambda_2 = \bar{\lambda}_1$ given transmission time fractions, respectively. Hence, the rate destined for S_i is $\frac{\lambda_i}{2}\mathcal{C}(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}) \overset{(\star)}{=} \frac{\lambda_i}{2}\mathcal{C}(\min\{\gamma_1,\gamma_2\})$[6]. It is clear that $\lambda_1 = \frac{1}{2}$ minimizes the outage probability. One can easily show the resulting approximate outage probability is always inferior to that of ANC. To modify the scheme, we let the relay spend $\alpha_i P_r$ for forwarding data to S_i, with $\lambda_1\alpha_1 + \lambda_2\alpha_2 = 1$. Noting that the SNR at S_i is $\tilde{\gamma}_i = \frac{g_1g_2P_rP_j}{g_iP_r+g_jP_j+1} \overset{(\star)}{=} \min\{g_iP_r, g_jP_j\}$, the outage probability becomes

$$\begin{aligned} P_{\text{out}}^{\text{OW}} &= \text{Prob} \\ &\Big(\min\{\lambda_1\mathcal{C}(\min\{\gamma_1\alpha_1,\gamma_2\}), \lambda_2\mathcal{C}(\min\{\gamma_2\alpha_2,\gamma_1\})\} \leq R_t\Big) \\ &= 1 - \text{Prob}(\gamma_1 > \hat{\Gamma}_1, \gamma_2 > \hat{\Gamma}_2) = 1 - e^{-(\frac{\hat{\Gamma}_1}{\xi_1}+\frac{\hat{\Gamma}_2}{\xi_2})} \\ &\approx \frac{\hat{\Gamma}_1}{\xi_1} + \frac{\hat{\Gamma}_2}{\xi_2}, \end{aligned} \quad (19)$$

where $\hat{\Gamma}_i = \max\{\frac{\tilde{\Gamma}_i}{\alpha_i}, \tilde{\Gamma}_j\}$, and $\tilde{\Gamma}_i = 2^{\frac{R_t}{\lambda_i}} - 1$. The optimal relay power split is obtained from

$$\begin{aligned} \min_{\alpha_1,\alpha_2,\lambda} &\ \frac{\hat{\Gamma}_1}{\xi_1} + \frac{\hat{\Gamma}_2}{\xi_2} \\ s.t. &\quad \lambda_1\alpha_1 + \lambda_2\alpha_2 = 1 \end{aligned} \quad (20)$$

[5]A more complex approach is to optimize the powers of S_2 and R between one and two-way transmissions, in addition to λ.

[6]The power of S_i could be scaled by $\frac{1}{\lambda_j}$. Our results can be readily extended to this case.

We fix λ_1, and find the optimal α_i^*. The optimum λ_1^* will be attained by an exhaustive search. It can be shown that the above problem is a convex optimization problem, and can be solved similar to the optimization problem (12) to determine α_i based on system parameters:

$$\begin{aligned} &1)\ 1 \le x, y: && \alpha_1 = \frac{\eta}{x}, && \alpha_2 = \frac{1}{\eta y} \\ &2)\ y \le 1 \le x: && \alpha_1 = \frac{1-\frac{\lambda_2}{\eta}}{\lambda_1}, && \alpha_2 = \frac{1}{\eta} \\ &3)\ x \le 1 \le y: && \alpha_1 = \eta, && \alpha_2 = \frac{1-\lambda_1\eta}{\lambda_2} \end{aligned} \qquad (21)$$

where $\eta = \frac{\tilde{\Gamma}_1}{\tilde{\Gamma}_2}$, $x \triangleq \lambda_1\eta + \sqrt{\lambda_1\lambda_2\frac{\eta}{\omega}}$, $y \triangleq \frac{\lambda_2}{\eta} + \sqrt{\lambda_1\lambda_2\frac{\omega}{\eta}}$. We note that $1 \le \lambda_1\eta + \frac{\lambda_2}{\eta}$ always holds and makes $x, y \le 1$ impossible.

Simulation Results and Discussion. Fig. 5 shows the outage performance for $r = 0.01, 0.1$. It can be seen that the one-way approach is better than the hybrid scheme in very low multiplexing gains, whereas the reverse is true for higher multiplexing gains. It is also noticed that the gain of the proposed schemes w.r.t. ANC decreases as r or ω increases. The reason is due to the presence of one-way relaying in the proposed schemes or due to the increased symmetry of the configuration. Furthermore, it is observed that the performance of the back-off scheme is quite well compared to that of other schemes. Nevertheless, it should be remarked that the proposed schemes have the advantage of leaving some resources for other potential users in the network. Particularly, each user is idle (i.e., does not transmit nor receive) for $\frac{\bar{\lambda}}{2}$ and $\frac{1}{2}$ fractions of the time in the hybrid and one-way schemes, respectively. This unoccupied fraction can be used by nearby users. In the back-off approach however, both users are active all the time.

V. Conclusion

We presented optimal resource (i.e., time and power) allocation policies for the bidirectional relaying problem with perfect CSIR. We derived an optimal power allocation for ANC protocol, which is only a function of the ratio of average fading powers. For asymmetric TWRC, OPA can bring impressive gains (up to 4.77 dB) w.r.t. EPA. We also obtained a power optimized cut-set lower bound on the outage probability as a solution to a convex optimization problem, and compared ANC with the bound. In contrast to ANC, the bound's OPA is a function of the target rate in addition to the ratio of average fading powers. When power distribution between nodes is not allowed, we improved the outage performance for low target multiplexing gains by proposing two schemes that efficiently incorporate one-way relaying. For the hybrid scheme, we optimized the timing between two-way and one-way relayings, while for the one-way approach, we optimized the relay power as well as the transmission time between one-way relayings. The one-way scheme gives a better gain w.r.t. the hybrid scheme for very low multiplexing gains, while the reverse is true for relatively larger multiplexing gains.

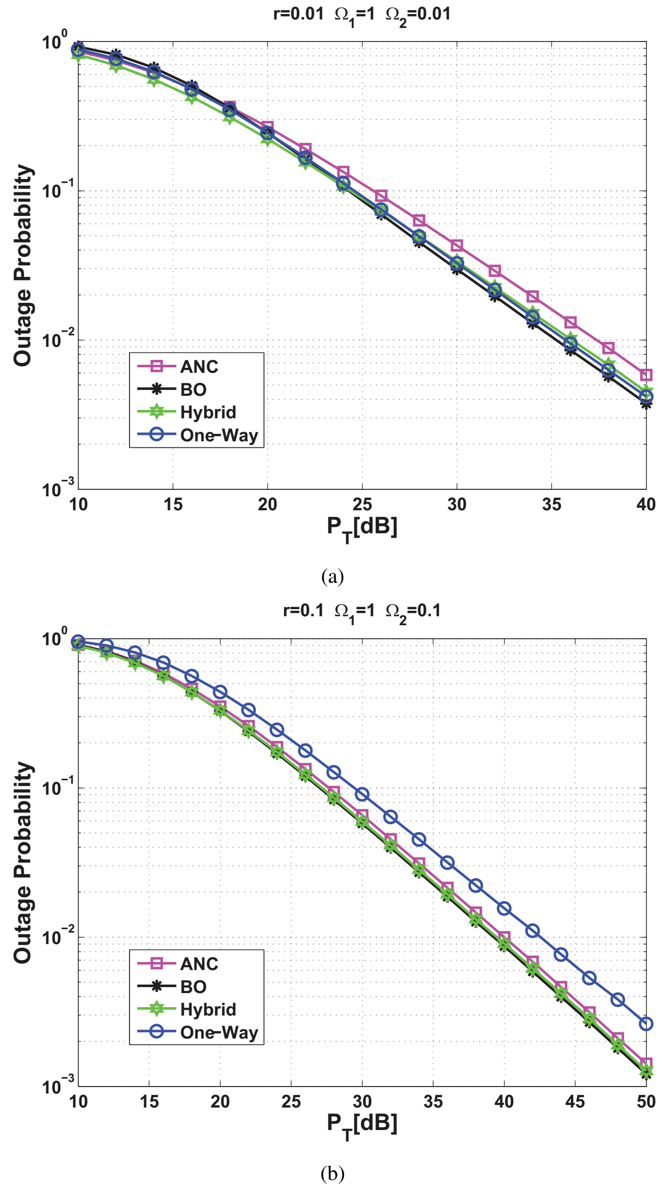

Fig. 5. Outage performance comparison for low multiplexing gains, using exact outage expressions with optimized resources according to (18) and (21). For $r = 0.01$, the hybrid, one-way, and back-off schemes are 1.25, 1.6, and 2.1 dB better than ANC, respectively. For $r = 0.1$, the hybrid and back-off schemes are 0.5, and 0.8 dB better than ANC, respectively, while the one-way scheme is much worse than ANC.

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