Dual Hop MIMO OSTBC for LMS Communication

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***Abstract*—For a multiple antenna land mobile satellite (LMS) communication system with a terrestrial relay node, the system performance when channel state information (CSI) is not available at the source and the relay is analyzed. The system employs orthogonal space time block code (OSTBC) encoding with fixed gain amplify and forward (AF) relaying. The moment generating function (MGF) and first and second moments of the signal-to-noise ratio (SNR) at the destination are derived in exact closed form. The average symbol error rate (SER) and the amount of fading (AoF) of the system are also derived.**

***Index Terms*—Hybrid satellite-terrestrial, cooperative relay, amplify and forward, fading channels, MIMO, moment generating function, outage probability, symbol error rate.**

I. Introduction

INTEGRATING satellite networks with terrestrials networks improve coverage for low-density populations and high data rate services for high-density populations in urban environments [1], [2]. The concept and advantages of cooperative communication in hybrid satellite-terrestrial networks were originally presented in [3]. Several hybrid satellite-terrestrial cooperative systems have thus been analyzed in [2], [4]. In [1], transmit diversity, which adapts to different channel environments, utilizing orthogonal space time block coding (OSTBC) and turbo codes is proposed. In [2], several cooperative techniques for hybrid satellite-terrestrial networks are investigated. In [4], a comprehensive performance analysis of such networks in Shadowed-Rician and Rayleigh fading channels is developed.

The integration of multiple input multiple output (MIMO) relays with a land mobile satellite (LMS) communication system is shown in [5] where authors employed beamforming with variable gain amplify and forward (AF) relaying, hence exploiting channel state information (CSI) at source, relay and destination. However, when no CSI is available, orthogonal space time block coding (OSTBC) is a classical MIMO technique for achieving full spatial diversity. Reference [6] hence proposed and evaluated OSTBC MIMO dual hop relay systems with non-coherent AF relaying over independent and identically distributed (i.i.d.) Rayleigh fading.

The concept of an LMS relay system [5] is based on the key assumption of full CSI availability. In this paper, we remove this assumption and extend this LMS relay system to more realistic one without the availability of CSI. In this case, as in [6], both the source and relay use an OSTBC transmission. The performance is then evaluated for the satellite-relay channel with Shadowed Rician fading and the relay-destination channel with Rayleigh fading. The moment generating function (MGF), first two moments, average symbol error rate (SER) and amount of fading (AoF) are derived.

Manuscript received December 13, 2011. The associate editor coordinating the review of this letter and approving it for publication was B. S. Rajan.

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Digital Object Identifier 10.1109/WCL.2012.020712.110250

II. System and Channel Model

We consider an LMS downlink where the satellite (source (S) node) transmits to a land mobile user terminal (destination (D) node) via a terrestrial relay node (R). The direct link from the satellite to the destination $S \rightarrow D$ is not available due to several reasons such as the excessive atmospheric attenuation, shadowing and the limited reception capability of the mobile receiver. Hence the relay enables the communication link. The $S \rightarrow R$ channel is modeled as a shadowed-Rician fading channel while the $R \rightarrow D$ channel as Rayleigh fading channel. S, R and D nodes are equipped with N_s, N_r and N_d antennas, respectively. The $S \rightarrow R$ channel matrix $\mathbf{H}_1 \in \mathbb{C}^{N_r \times N_s}$ with i.i.d. Shadowed-Rician fading entries can be modeled as $\mathbf{H}_1 = \bar{\mathbf{H}}_1 + \tilde{\mathbf{H}}_1$. Adopting the Shadowed-Rician model proposed in [7], the entries of the line-of-sight component $\bar{\mathbf{H}}_1$ can be modeled as i.i.d. Nakagami-m random variables (RVs) with average power Ω where m describes the severity of shadowing varying over the range $m \geq 0$. The entries of the scattered component $\tilde{\mathbf{H}}_1$ and $R \rightarrow D$ channel matrix $\mathbf{H}_2 \in \mathbb{C}^{N_d \times N_r}$ are i.i.d. complex Gaussian RVs with zero mean and unit variance.

The relay operates in half duplex mode and hence cooperation takes place over two time slots. During the first time slot, the source employing OSTBC encoding transmits a block of n_s symbols in T symbol periods according to the code matrix $\mathbf{X} \in \mathbb{C}^{N_s \times T} = [\mathbf{x}_1, \ldots, \mathbf{x}_T]$ where $\mathbf{x}_k \in \mathbb{C}^{N_s \times 1}$ and hence the code rate is $R_s = n_s/T$. The received signal at the relay in the k-th symbol period, $\mathbf{y}_{R_k} \in \mathbb{C}^{N_r \times 1}$ can be expressed as

$$\mathbf{y}_{R_k} = \sqrt{\tfrac{\rho}{\delta}}\mathbf{H}_1\mathbf{x}_k + \mathbf{n}_{R_k}, k = 1, 2, \ldots, T \qquad (1)$$

where $E\big[\,\|\mathbf{x}_k\|^2\,\big] = 1$, $\mathbf{n}_{R_k} \in \mathbb{C}^{N_r \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_r})$, $\mathbf{I}_{N_r}$ is $N_r \times N_r$ identity matrix. The normalization factor δ ensures that $\mathbb{E}[||\mathbf{H}_1||_F^2]/\delta = N_r N_s$ where $\|.\|_F$ denotes the Frobenius norm of a matrix. The average SNR per channel at the receive end of the relay is ρ.

The relay operating according to fixed gain AF protocol applies a linear transformation $\mathbf{F} = \sqrt{\alpha}\mathbf{I}_{N_r} = \sqrt{\frac{\beta}{(\rho+1)N_r}}\mathbf{I}_{N_r}$ on the received signal and transmits to the destination in the second time slot. The power normalization factor $(\rho + 1)N_r$ ensures that the relay node transmits an average total power β. The received signal at the destination in the k^{th} symbol period, $\mathbf{y}_{D_k} \in \mathbb{C}^{N_d \times 1}$ is thus given by

$$\mathbf{y}_{D_k} = \sqrt{\tfrac{\alpha\rho}{\delta}}\mathbf{H}_2\mathbf{H}_1\mathbf{x}_k + \sqrt{\alpha}\mathbf{H}_2\mathbf{n}_{R_k} + \mathbf{n}_{D_k},\ k = 1, 2, \ldots, T \qquad (2)$$

where $\mathbf{n}_{D_k} \in \mathbb{C}^{N_d \times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_d})$. The colored Gaussian noise $\sqrt{\alpha}\mathbf{H}_2\mathbf{n}_{R_k} + \mathbf{n}_{D_k}$ with conditional covariance matrix $\mathbb{E}[(\sqrt{\alpha}\mathbf{H}_2\mathbf{n}_{R_k} + \mathbf{n}_{D_k})(\sqrt{\alpha}\mathbf{H}_2\mathbf{n}_{R_k} + \mathbf{n}_{D_k})^H|\mathbf{H}_2] = \alpha\mathbf{H}_2\mathbf{H}_2^H + \mathbf{I}_{N_d} = \mathbf{K}$ must be whitened before standard linear OSTBC processing at the destination [6] which is achieved by

$$\mathbf{y}_k = \mathbf{K}^{-\frac{1}{2}}\mathbf{y}_{D_k} = \sqrt{\tfrac{\alpha\rho}{\delta}}\mathbf{H}\mathbf{x}_k + \mathbf{n}_k \qquad (3)$$

where $\mathbf{H} = \mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2\mathbf{H}_1$ and $\mathbf{n}_k = \mathbf{K}^{-\frac{1}{2}}(\sqrt{\alpha}\mathbf{H}_2\mathbf{n}_{R_k} + \mathbf{n}_{D_k})$ is the equivalent white Gaussian noise $\sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_d})$. The end-to-end transmission equation for the overall code matrix $\mathbf{X}$ can be written as

$$\mathbf{Y} = \sqrt{\tfrac{\alpha\rho}{\delta}}\mathbf{H}\mathbf{X} + \mathbf{N} \tag{4}$$

where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_T]$ and $\mathbf{N} = [\mathbf{n}_1, \ldots, \mathbf{n}_T]$. OSTBC transmission splits the MIMO channel into n_s parallel single-input single-output channels with the instantaneous SNR per symbol per channel given by [6]

$$\gamma = \tfrac{\alpha\rho}{\delta} c \left\|\mathbf{H}\right\|_F^2 = \tfrac{\alpha\rho}{\delta} c \mathrm{Tr}\{\mathbf{H}_1^H\mathbf{H}_2^H\mathbf{K}^{-1}\mathbf{H}_2\mathbf{H}_1\} \tag{5}$$

where $c = 1/(R_s N_s)$.

III. Statistics of the SNR γ

The statistical properties of the SNR γ are obtained by deriving the exact MGF, first and second moments.

Theorem 1: The MGF of γ is given by

$$M_\gamma(s) = \mathcal{K}^{-1}\det[\mathbf{A}(s)] \tag{6}$$

where $\mathcal{K} = \prod_{i=1}^{p}\Gamma(p-i+1)\Gamma(q-i+1)$, $p = \min(N_r, N_d)$, $q = \max(N_r, N_d)$ and $\mathbf{A}$ is a $p \times p$ matrix with its (k,l)-th entry given by

$$\mathbf{A}_{k,l}(s) = \int_0^\infty e^{-\lambda}\lambda^{\omega_{kl}-1}(1+\alpha\lambda)^{N_s} \frac{\left(1+\alpha(1+\eta cs)\lambda\right)^{(m-1)N_s}}{\left(1+\alpha\left(1+(1+\frac{\Omega}{m})\eta cs\right)\lambda\right)^{mN_s}}d\lambda \tag{7}$$

where $\eta = \dfrac{\rho}{\delta}$ and $\omega_{kl} = k + l + q - p - 1$.

Proof: See Appendix A. ■

While (7) is applicable for any $m \geq 0$, it is not in closed form. To derive one, we assume m to take integer values $0, 1, 2, \ldots$ and address $m = 0$ and $m \geq 1$ cases separately.

Case I: $m \geq 1$: For integer m with $m \geq 1$, $(m-1)N_s$ is an integer ≥ 0 and hence applying binomial theorem for $(1+\alpha(1+\rho cs)\lambda)^{(m-1)N_s}$ and $(1+\alpha\lambda)^{N_s}$ in (7), and solving the resulting integral using [8, eq. (13.2.5)], closed-form expression for $\mathbf{A}_{k,l}(s)$ can finally be obtained as

$$\begin{aligned}\mathbf{A}_{k,l}(s) = &\sum_{u=0}^{N_s}\binom{N_s}{u}\sum_{v=0}^{(m-1)N_s}\binom{(m-1)N_s}{v}(1+\eta cs)^v \\ &\times \frac{\Gamma(u+v+\omega_{kl})}{\alpha^{\omega_{kl}}(1+(1+\frac{\Omega}{m})\eta cs)^{u+v+\omega_{kl}}} \\ &\times U\left[u{+}v{+}\omega_{kl}, u{+}v{+}1{+}\omega_{kl}{-}mN_s, \frac{1}{\alpha(1+(1+\frac{\Omega}{m})\eta cs)}\right]\end{aligned} \tag{8}$$

where U[.,.,.] is the confluent hypergeometric function of the second kind [8], $\eta = \frac{\rho}{1+\Omega}$ since $\delta = 1+\Omega$ when $m \geq 1$.

Case II: $m = 0$: Closed-form expression for $\mathbf{A}_{k,l}(s)$ for $m = 0$ can similarly be obtained as

$$\mathbf{A}_{k,l}(s) = \frac{\Gamma(\omega_{kl})}{\alpha^{\omega_{kl}}(1+\eta cs)^{\omega_{kl}+N_s}}\sum_{u=0}^{N_s}\binom{N_s}{u}(\eta cs)^u \mathrm{U}\left[\omega_{kl}, \omega_{kl}+1-u, \frac{1}{\alpha(1+\eta cs)}\right] \tag{9}$$

where $\eta = \rho$ since $\delta = 1$ when $m = 0$.

Our proposed model is versatile because the Rayleigh-Rayleigh case addressed in [6] is a special case ($m = 0$ in our Shadowed-Rician fading model corresponds to Rayleigh fading). Note that MGF of γ in (6) with entries of $\mathbf{A}$ given by (9) is exactly the same as [6, Theorem 1].

Theorem 2: The first moment of γ is given by

$$\mathbb{E}[\gamma] = -\mathcal{K}^{-1}\sum_{j=1}^{p}\det[\mathbf{B}(j)] \tag{10}$$

where $\mathbf{B}(j)$ is a $p \times p$ matrix with its (k,l)-th entry given by

$$\mathbf{B}(j)_{k,l} = \begin{cases} -\mu_{kl}\, U[\omega_{kl}+1; \omega_{kl}+1, \frac{1}{\alpha}] & l = j \\ \Gamma(\omega_{kl}) & l \neq j \end{cases} \tag{11}$$

where $\mu_{kl} = \frac{\Gamma(\omega_{kl}+1)}{\alpha^{\omega_{kl}}}N_s\rho c$.

Proof: See Appendix B. ■

Theorem 3: The second moment of γ is given by

$$\mathbb{E}[\gamma^2] = \mathcal{K}^{-1}\sum_{j=1}^{p}\sum_{i=1}^{p}\det[\mathbf{D}(j,i)] \tag{12}$$

where $\mathbf{D}(j,i)$ is a $p \times p$ matrix with its (k,l)-th entry given by

$$\mathbf{D}(j,i)_{k,l} = \begin{cases} \nu_{kl}U[\omega_{kl}+2; \omega_{kl}+1, \frac{1}{\alpha}] & l = j = i \\ -\mu_{kl}\, U[\omega_{kl}+1; \omega_{kl}+1, \frac{1}{\alpha}] & l = j \,\text{or}\, l = i; \\ & j \neq i \\ \Gamma(\omega_{kl}) & l \neq j; l \neq i \end{cases} \tag{13}$$

where

$$\nu_{kl} = \frac{\Gamma(\omega_{kl}+2)}{\alpha^{\omega_{kl}}}N_s \times \begin{cases} \left(N_s(1+\Omega)^2 + 1 + 2\Omega + \frac{\Omega^2}{m}\right)\left(\frac{\rho c}{1+\Omega}\right)^2 & m \geq 1 \\ (N_s+1)(\rho c)^2 & m = 0 \end{cases}$$

Proof: The proof is similar to that of Theorem 2. By using the second derivative of (6), Theorem 3 can be proven (details are omitted for brevity). ■

IV. Performance Analysis

With the help of the statistics derived in Section III, we now develop a performance analysis of the system.

A. Average SER

For modulation formats with conditional SER expression of the form $aQ(\sqrt{2b\gamma})$ where $Q(.)$ is the Gaussian-Q function that can be expressed as $Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$ by using Craig's formula, the average SER is given by

$$P_s = aE_\gamma[Q(\sqrt{2b\gamma})] = \frac{a}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left(\frac{b}{\sin^2\theta}\right)d\theta. \tag{14}$$

These modulation formats include binary phase shift keying (BPSK) ($a = 1, b = 1$); coherently detected orthogonal binary frequency shift keying (BFSK)($a = 1, b = 0.5$) ; M-ary pulse amplitude modulation (PAM) ($a = 2(M-1)/M, b = 3/(M^2-1)$). Approximate SER for M-ary Phase shift Keying (PSK) can also be found with $a = 2, b = \sin^2(\pi/M)$.

By using (6) with (8) or (9) into (14) gives the exact SER. However, numerical evaluation is required.

We now show that for systems with $\min(N_r, N_d) = 1$, a closed-form average SER can be obtained. If we use integral expression (7) for the matrix entries in (6), the average SER can be writen using (14) as

$$\begin{aligned}P_s =&\frac{a}{\pi\Gamma(q)}\int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-\lambda}\lambda^{q-1}\left(\frac{1+\alpha\lambda}{1+\alpha(1+\frac{\eta cb}{\sin^2\theta})\lambda}\right)^{N_s}\\ &\times\left(\frac{1+\alpha(1+\frac{\eta cb}{\sin^2\theta})\lambda}{1+\alpha(1+(1+\frac{\Omega}{m})\frac{\eta cb}{\sin^2\theta})\lambda}\right)^{mN_s} d\lambda d\theta\\ =&\frac{a}{\Gamma(q)}\int_0^{\infty} e^{-\lambda}\lambda^{q-1}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\zeta+\sin^2\theta}\right)^{N_s}\\ &\times\left(1+\frac{\Omega}{m}\frac{\zeta}{\zeta+\sin^2\theta}\right)^{-mN_s} d\lambda d\theta \qquad (15)\end{aligned}$$

where $\zeta = \frac{\eta cb\alpha\lambda}{1+\alpha\lambda}$. Further manipulation yields,

$$\begin{aligned}P_s =&\frac{a}{\Gamma(q)}\int_0^{\infty} e^{-\lambda}\lambda^{q-1}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\zeta+\sin^2\theta}\right)^{N_s}\\ &\times\left(1+\frac{\Omega}{m}-\frac{\Omega}{m}\frac{\sin^2\theta}{\zeta+\sin^2\theta}\right)^{-mN_s} d\theta d\lambda \qquad (16)\end{aligned}$$

Applying negative binomial expansion and solving the inner integral by using [9, Eq.(5A.8), Eq. (5A.4b)], then substituting back ζ followed by binomial expansion and finally solving the resulting integral by using [8, eq. (13.2.5)], we obtain the closed-form SER as follows:

$$\begin{aligned}P_s =&\frac{a}{2}\sum_{k=0}^{\infty}\binom{-mN_s}{k}\left(-\frac{\Omega}{m}\right)^k\left(1+\frac{\Omega}{m}\right)^{-mN_s-k}\\ &\left(1-\frac{1}{\Gamma(q)}\sum_{l=0}^{k+N_s-1}\sum_{n=0}^{l}\frac{\binom{2l}{l}\binom{l}{n}\Gamma(q+n+1/2)\sqrt{\eta cb}}{4^l\alpha^q(1+\eta cb)^{q+n+1/2}}\right.\\ &\left.U\left[q+n+\frac{1}{2}, q+n-l+1, \frac{1}{\alpha(1+\eta cb)}\right]\right). \qquad (17)\end{aligned}$$

The above expression is valid for integer $m > 0$. The result for $m = 0$ is straightforward and omitted for brevity.

B. AoF

The AoF is typically used as a measure of the severity of the fading channels. It can be derived via the definition

$$AoF = \frac{\mathbb{E}[\gamma^2] - (\mathbb{E}[\gamma])^2}{(\mathbb{E}[\gamma])^2} \qquad (18)$$

with $\mathbb{E}[\gamma]$ and $\mathbb{E}[\gamma^2]$ given by (10) and (12) respectively.

V. Numerical Results and Conclusion

Validation of our analytical results with Monte Carlo simulations using the Alamouti code is performed here. Fig. 1 shows the average SER of the system for BPSK ($a = b = 1$) and QPSK ($a = 2$ and $b = 0.5$) constellations using different antenna configurations. The analytical plots are obtained using (17) where the infinite series is accurately truncated with 10 terms. The AoF of the system is plotted against β in Fig. 2. A clear match between analytical and simulated results is observed in both figures. Clearly, both the SER and AoF decrease with the increase in N_r, N_d. Reliability is improved due to exploiting spatial diversity from multiple antennas.

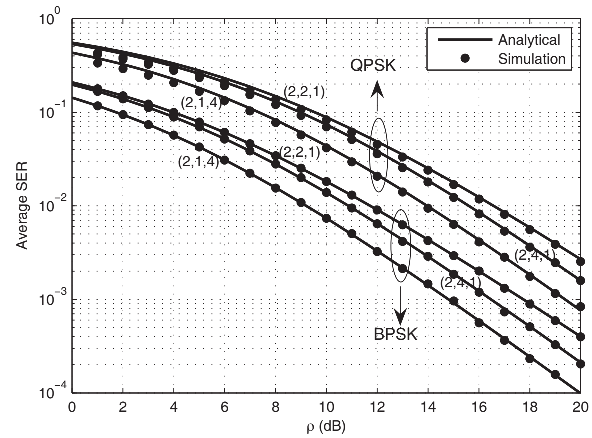

Fig. 1. Average SER of BPSK and QPSK constellations using Alamouti's coding for antenna configuration (N_s, N_r, N_d), $m = 3$, $\Omega = 0.1$ and $\beta = 2\rho$.

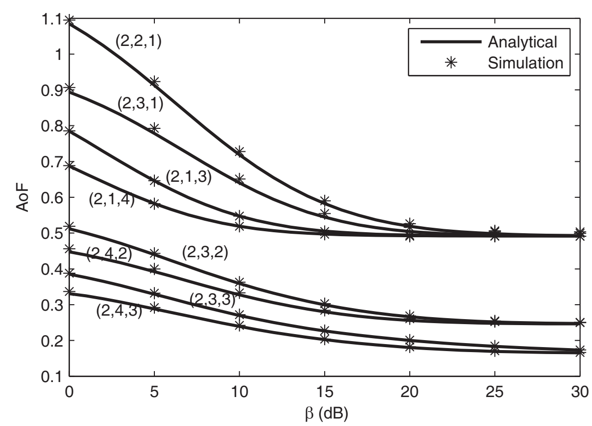

Fig. 2. AoF vs. β using Alamouti's coding for antenna configuration (N_s, N_r, N_d), $m = 2$, $\Omega = 0.22$ and $\rho = 10$dB.

In conclusion, an LMS system with a terrestrial relay (fixed gain AF) and without CSI at the source and the relay was considered. The satellite-relay link is Shadowed-Rician fading while the terrestrial link is Rayleigh fading. The MGF, the first two moments of the SNR and the SER were derived. While the independent fading scenario is valid for sufficiently separated antennas, the impact of antenna correlation is omitted due to space limitation and will be addressed in future submissions.

Appendix

A. Proof of Theorem 1

Since $\mathbf{H} = \mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2\mathbf{H}_1 = \mathbf{H}_L + \mathbf{H}_s$ where $\mathbf{H}_L = \mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2\bar{\mathbf{H}}_1$ and $\mathbf{H}_s = \mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2\tilde{\mathbf{H}}_1$, given $\mathbf{H}_L$ and $\mathbf{H}_2$,

$$\begin{aligned}&\mathbb{E}_{\mathbf{H}_s}[(\mathrm{vec}(\mathbf{H}) - \mathrm{vec}(\mathbf{H}_L))(\mathrm{vec}(\mathbf{H}) - \mathrm{vec}(\mathbf{H}_L))^H]\\ &= \mathbb{E}_{\mathbf{H}_s}[\mathrm{vec}(\mathbf{H}_s)\mathrm{vec}(\mathbf{H}_s)^H]\\ &= (\mathbf{I}_{N_s}\otimes\mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2)\mathbb{E}_{\tilde{\mathbf{H}}_1}[\mathrm{vec}(\tilde{\mathbf{H}}_1)\mathrm{vec}(\tilde{\mathbf{H}}_1)^H](\mathbf{I}_{N_s}\otimes\mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2)^H\\ &= \mathbf{I}_{N_s}\otimes\mathbf{H}_2^H K^{-1}\mathbf{H}_2 \qquad (19)\end{aligned}$$

where $\mathrm{vec}(\mathbf{A})$ denotes a vector formed by stacking the columns of matrix $\mathbf{A}$ on top of each other, and $\otimes$

stands for Kronecker product. We used $\mathrm{vec}(\mathbf{H}_s) = (I_{N_s} \otimes \mathbf{K}^{-\frac{1}{2}}\mathbf{H}_2)\mathrm{vec}(\tilde{\mathbf{H}}_1)$ [10, Lemma (2.2.2)] to get the second equality. Third equality follows after substituting $\mathbb{E}_{\tilde{\mathbf{H}}_1}[\mathrm{vec}(\tilde{\mathbf{H}}_1)\mathrm{vec}(\tilde{\mathbf{H}}_1)^H] = (\mathbf{I}_{N_s} \otimes \mathbf{I}_{N_r})$ and applying the properties of Kronecker product. From the above derivation, it follows that given $\mathbf{H}_L$ and $\mathbf{H}_2$,

$$\mathrm{vec}(\mathbf{H})\Big|_{\mathbf{H}_L,\mathbf{H}_2} \sim \mathcal{CN}\left(\mathrm{vec}(\mathbf{H}_L), \mathbf{I}_{N_s} \otimes \mathbf{H}_2^H K^{-1}\mathbf{H}_2\right). \quad (20)$$

$\mathbf{I}_{N_s} \otimes \mathbf{H}_2^H K^{-1}\mathbf{H}_2$ being Hermitian and non-negative definite, using the eigenvalue decomposition, we have,

$$\mathbf{I}_{N_s} \otimes \mathbf{H}_2^H K^{-1}\mathbf{H}_2 = \left[\begin{array}{cc} \mathbf{U} & \mathbf{U}_0 \end{array}\right] \left[\begin{array}{cc} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \mathbf{U}^H \\ \mathbf{U}_0^H \end{array}\right] \quad (21)$$

where $\mathbf{\Sigma} = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_{pN_s}\}$ is a positive definite $pN_s \times pN_s$ diagonal matrix, the diagonal elements of which have p distinct values with each distinct value repeated N_s times such that $\sigma_{(j-1)N_s+1} = \sigma_{(j-1)N_s+2} = \ldots = \sigma_{jN_s} = \frac{\lambda_j}{1+\alpha\lambda_j}$, $j = 1, 2, \ldots, p$ where $\lambda_1 > \lambda_2 > \ldots > \lambda_p > 0$ are the eigenvalues of $\mathbf{H}_2\mathbf{H}_2^H$, $\mathbf{U} \in \mathbb{C}^{N_r N_s \times pN_s}$ and $\mathbf{U}_0 \in \mathbb{C}^{N_r N_s \times (N_r - p)N_s}$ such that $\left[\mathbf{U}\,\mathbf{U}_0\right]\left[\mathbf{U}\,\mathbf{U}_0\right]^H = \mathbf{I}_{N_r N_s}$. If we define $\mathbf{h} = \left[\mathbf{U}\,\mathbf{U}_0\right]^H \mathrm{vec}(\mathbf{H})$, it can be shown that given $\mathbf{H}_L$ and $\mathbf{H}_2$, $\mathbf{h}$ is distributed as

$$\mathbf{h}\Big|_{\mathbf{H}_L,\mathbf{H}_2} \sim \mathcal{CN}\left(\mathbf{h}_L, \left[\begin{array}{cc} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right]\right) \quad (22)$$

where $\mathbf{h}_L = \left[\mathbf{U}\,\mathbf{U}_0\right]^H \mathrm{vec}(\mathbf{H}_L)$. As $\mathbf{\Sigma}$ is diagonal, given $\mathbf{h}_L$ and $\mathbf{\Sigma}$, the elements of $\mathbf{h}$ are independent and distributed as $h_i \sim \mathcal{CN}(h_{L_i}, \sigma_i), i = 1, \ldots, pN_s$ where h_{L_i}s are the elements of $\mathbf{h}_L$. Hence marginal pdf of $|h_i|$ is given by

$$p_{|h_i|}(x_i | h_{L_i}, \sigma_i) = \tfrac{2x_i}{\sigma_i}\exp\left(-\tfrac{x_i^2 + h_{L_i}^2}{\sigma_i}\right) I_0\left(\tfrac{2x_i h_{L_i}}{\sigma_i}\right).$$

Since entries of $\bar{\mathbf{H}}_1$ are independent Nakagami-m RVs such that $\mathbb{E}_{\bar{\mathbf{H}}_1}[\mathrm{vec}(\bar{\mathbf{H}}_1)\mathrm{vec}(\bar{\mathbf{H}}_1)^H] = \Omega(\mathbf{I}_{N_s} \otimes \mathbf{I}_{N_r})$, it can be similarly derived as in (19) that given $\mathbf{H}_2$, $\mathbb{E}_{\mathbf{H}_L}[\mathrm{vec}(\mathbf{H}_L)\mathrm{vec}(\mathbf{H}_L)^H] = \Omega(\mathbf{I}_{N_s} \otimes \mathbf{H}_2^H K^{-1}\mathbf{H}_2)$. Then using (21), it can be shown that given $\mathbf{H}_2$, $\mathbf{h}_L$ is a Nakagami-m random vector with $\mathbb{E}_{\mathbf{h}_L}[\mathbf{h}_L(\mathbf{h}_L)^H] = \Omega\left[\begin{smallmatrix} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{smallmatrix}\right]$. Hence given $\mathbf{\Sigma}$, h_{L_i}s, $i = 1, \ldots, pN_s$ are independent Nakagami-m RVs with marginal pdf

$$p_{h_{L_i}}(y_i | \sigma_i) = \tfrac{2m^m}{\Gamma(m)(\Omega\sigma_i)^m} y_i^{2m-1}\exp\left(-\tfrac{my_i^2}{\Omega\sigma_i}\right).$$

The MGF of $\gamma = \alpha\eta c||\mathbf{H}||_F^2 = \alpha\eta c\,\mathrm{vec}(\mathbf{H})^H\mathrm{vec}(\mathbf{H}) = \alpha\eta c\mathbf{h}^H\mathbf{h} = \alpha\eta c\sum_{i=1}^{pN_s}|h_i|^2$ can be derived as

$$\begin{aligned} M_\gamma(s) &= \mathbb{E}_\gamma[e^{-s\gamma}] \\ &= \mathbb{E}_{\mathbf{H}_2,\mathbf{H}_L}\left[\mathbb{E}_{\mathbf{H}_s}\left[e^{-\alpha\eta cs\sum_{i=1}^{pN_s}|h_i|^2}\Big|\mathbf{H}_L, \mathbf{H}_2\right]\right] \\ &= \mathbb{E}_{\mathbf{\Sigma}}\left[\prod_{i=1}^{pN_s}\mathbb{E}_{h_{L_i}}\left[\int_0^\infty e^{-\alpha\eta cx_i^2} p_{|h_i|}(x_i|h_{L_i}, \sigma_i)dx_i\right]\right] \\ &= \mathbb{E}_{\mathbf{\Sigma}}\left[\prod_{i=1}^{pN_s}\frac{1}{1+\alpha\eta cs\sigma_i}\int_0^\infty e^{-\frac{\alpha\eta csy_i^2}{1+\alpha\eta cs\sigma_i}} p_{h_{L_i}}(y_i|\sigma_i)dy_i\right] \\ &= \mathbb{E}_{\mathbf{\Sigma}}\left[\prod_{i=1}^{pN_s}\frac{(1+\alpha\eta cs\sigma_i)^{m-1}}{(1+(1+\Omega/m)\alpha\eta cs\sigma_i)^m}\right] \\ &= \mathbb{E}_{\mathbf{\Lambda}}\left[\prod_{i=1}^{p}\frac{(1+\alpha\eta cs\frac{\lambda_i}{1+\alpha\lambda_i})^{(m-1)N_s}}{(1+(1+\Omega/m)\alpha\eta cs\frac{\lambda_i}{1+\alpha\lambda_i})^{mN_s}}\right] \end{aligned} \quad (23)$$

where $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$. Integrating (23) with respect to the joint distribution of λ_is, $f(\mathbf{\Lambda}) = \mathcal{K}^{-1}\det(\lambda_i^{j-1})^2\prod_{i=1}^p \lambda_i^{q-p}e^{-\lambda_i}$, $\lambda_1 > \lambda_2 > \ldots > \lambda_p > 0$, we obtain

$$M_\gamma(s) = \mathcal{K}^{-1}\int_{\mathcal{D}}\det(\lambda_i^{j-1})^2\prod_{i=1}^p \xi(i)\,d\lambda_1\ldots d\lambda_p. \quad (24)$$

where $\xi(i) = \left[\frac{(1+\alpha\eta cs\frac{\lambda_i}{1+\alpha\lambda_i})^{(m-1)N_s}}{(1+(1+\frac{\Omega}{m})\alpha\eta cs\frac{\lambda_i}{1+\alpha\lambda_i})^{mN_s}}\lambda_i^{q-p}e^{-\lambda_i}\right]$, and $\mathcal{D} = \{\lambda_1 > \lambda_2 > \ldots > \lambda_p > 0\}$. The integral (24) is solved using [11, eq. (51)] to yield (6).

B. Proof of Theorem 2

Proof: The n-th moment of γ is $(-1)^n$ times the n-th derivative of the MGF of γ evaluated at $s = 0$. Hence by differentiating (6) once, the first moment of γ is obtained. But since MGF (6) is given by a determinant expression, for any $n \times n$ matrix $\mathbf{G}$ with entries that are functions of x, we use

$$\frac{d\big(\det[\mathbf{G}]\big)}{dx} = \sum_{j=1}^n \det[\tilde{\mathbf{G}}(j)] \quad (25)$$

where $\tilde{\mathbf{G}}(j)$ is an $n \times n$ matrix formed by differentiating the j-th column of the matrix $\mathbf{G}$ with all other columns left intact. Using (25), first moment of γ can be expressed as (10) where the (k, l)-th entry of $\mathbf{B}(j)$ is given by

$$\mathbf{B}(j)_{k,l} = \begin{cases} \dfrac{d(\mathbf{A}_{k,l}(s))}{ds}\Big|_{s=0} & l = j \\ \mathbf{A}_{k,l}(s)\Big|_{s=0} & l \neq j. \end{cases} \quad (26)$$

Finally, (11) follows from (26) and (7). ■

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