Performance Analysis of Energy Detection with Multiple Correlated Antenna Cognitive Radio in Nakagami-m Fading

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***Abstract*—This letter analyzes the performance of energy-detection-based spectrum sensing in a cognitive radio (CR) possessing multiple correlated antennas when the channel from the primary user (PU) to the CR is Nakagami-m faded. The probability of detection of the CR by employing square law combining (SLC) is derived by using the MGF-based approach. Special cases of equally correlated, exponentially correlated and a linear array of 2, 3 and 4 arbitrarily correlated antennas are treated. Numerical and simulation results are used to quantify the detector performance as a function of antenna correlation across the branches, number of antennas, fading severity and the time-bandwidth product.**

***Index Terms*—Cognitive radio, multiple antenna, square law combining, correlation, Nakagami-m, moment generating function.**

I. Introduction

INTELLIGENT cognitive radios (CRs), which are capable of identifying and utilizing gaps in spectrum usage, are necessary to meet the growth in demand for radio spectrum [1], [2]. Thus, the IEEE 802.22 standard on cognitive wireless regional area networks (WRANs) [3] focuses on the exploitation of the unused spectrum without causing any interference to the licensed primary user (PU) of the spectrum. To realize this goal, the CR must be able to detect the presence or absence of the PU signal. Due to ease of implementation, one of the simplest and most widely used detectors is the energy detector, which does not require any a priori knowledge of the PU signal [4], thus rendering the energy detector suitable for wideband applications. However, an inherent problem with the energy detector is the performance degradation at the low signal-to-noise ratio (SNR) region. For example, in a shadowed environment, high noise uncertainty leads to unknown noise variance at the detector, thus causing the hidden terminal problem [5].

To improve the performance in low-SNR conditions, multiple antennas for spectrum detection can be deployed. The need for multiple antennas is also driven by the promise of a high data rate and high efficiency broadband services by standards such as the Long Term Evolution (LTE), WiMax and IMT-Advanced. The notion of using a multiple antenna CR for detecting the spectrum holes has thus attracted much interest. The linear combinations of multiple antenna outputs can be used to improve the detection reliability [6][7]. The detection performance of several linear diversity combiners has thus been analyzed in [7] by using the moment generating function (MGF) of the channel SNR.

However, all these works are contingent on a CR with independent antennas. If a CR is a handheld terminal in cellular and/or adhoc networks, the spatial separation among the antennas may be insufficient to ensure independent fading. In [8], the detection performance of an energy detector with multiple correlated antennas was analyzed for the Rayleigh fading channel by approximating the energy detector statistics as Gaussian by invoking the central limit theorem (CLT). However, due to the high agility requirement of the CRs [5], a large number of signal samples may not be available, and the CLT may not hold. Study [9] includes the exact detection performance analysis by using the probability density function (PDF) approach for a square law combiner (SLC) in correlated Rayleigh faded channels. Since maximal ratio combining (MRC) requires complete knowledge of the channel state information (CSI), a simpler technique is the SLC, which does not require the CSI.

Despite the focus on the use of spatial diversity in CR-based detection, none of the previous works has addressed the issue of energy detection with multiple correlated antennas in a Nakagami-m faded PU-CR environment. Since this distribution encompasses Rayleigh fading ($m=1$) and can also approximate Rician fading [10], the detection performance under this generic fading channel model is of interest to wireless researchers and engineers. In our letter, we thus assume Nakagami-m faded PU-CR channels and a CR with L correlated antennas with SLC reception. The average probability of detection is then derived by using the MGF approach. Specific correlation models like the constant correlation, exponential correlation and linear array of arbitrary correlation are then analyzed.

II. Problem Formulation

For a CR, the detection problem can be formulated as a classical binary hypothesis test of the form $r(t) = n(t)$ under H_0 and $r(t) = hs(t) + n(t)$ under H_1, where $r(t)$ is the received signal at the detector, $s(t)$ is the primary signal, h is the channel coefficient, and $n(t)$ is the additive white Gaussian noise (AWGN) [4]. We assume the CR is equipped with an energy detector, which essentially filters the input signal with a bandpass filter of bandwidth W and squares the filtered signal to be fed to an integrator of period T. The resulting statistic Y is compared against a predefined threshold λ to decide on

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one of the two hypotheses, H_0 and H_1. Under H_0 and H_1, the statistic Y is modeled as a central and non-central chi-square with $2u$ ($u = TW$ is the time bandwidth product) degrees of freedom as $Y \sim \chi^2_{2u}$ and $Y \sim \chi^2_{2u}(2\gamma)$, respectively, where γ is the SNR at the receiver, and 2γ is the non-centrality parameter [4]. Since we use SLC for L diversity branches at the CR, the total SNR will be equal to the sum of the individual branch SNRs (γ_t), thus resulting in a non-centrality parameter of $\sum_{i=1}^{L} 2\gamma = 2\gamma_t$. The probability of detection at the CR then becomes [9]

$$P_{d,SLC} = Q_{Lu}(\sqrt{2\gamma_t}, \sqrt{\lambda}), \tag{1}$$

where $Q_M(a,b) = \int_b^\infty x(\frac{x}{a})^{M-1} exp(-\frac{x^2+a^2}{2}) I_{M-1}(ax)dx$ is the generalized M^{th} order Marcum-Q function [11], and $\Gamma(a,x) = \int_x^\infty t^a e^{-t}$ is the upper incomplete gamma function. The average detection probability over the correlated Nakagami-m channels is thus required. By using the alternative representation of the Marcum-Q function combined with the definition of MGF for SLC, the average detection probability is [7]

$$\overline{P}_d = \frac{e^{-\frac{\lambda}{2}}}{2\pi j} \oint_\triangle M\Big(1 - \frac{1}{z}\Big) \frac{e^{\frac{\lambda}{2}z}}{z^{Lu}(1-z)} dz, \tag{2}$$

where $M(s) = E(e^{-s\gamma})$ is the MGF, with $E(\cdot)$ representing the mathematical expectation. The MGF of SNR with correlated Nakagami-m fading is now required.

III. Average Probability of Detection for L Correlated Antennas

For analytical tractability, we assume the correlated fading channels have an identical fading parameter m (integer). The MGF of SNR for L branches at the CR can then be written as [12]

$$M(s) = \prod_{i=1}^{L} (1 + s\eta_i)^{-m}, \tag{3}$$

where η_i, $\forall i \in \{1, ..., L\}$ is the i^{th} eigenvalue of the $L \times L$ matrix,

$$\mathbf{\Theta_L} = \frac{\gamma_t}{m}\sqrt{\mathbf{C_L}}. \tag{4}$$

The term $\sqrt{\mathbf{C_L}}$ is a notation for the matrix whose elements are equal to the square root of the corresponding elements of the $L \times L$ channel covariance matrix $\mathbf{C_L}$ [13]. Substituting (3) into (2) and following some steps of manipulations, we obtain

$$\overline{P}_d = \frac{e^{-\frac{\lambda}{2}}}{\prod_{i=1}^{L}(1+\eta_i)^m 2\pi j} \oint_\triangle f(z) dz, \tag{5}$$

where $f(z) = \frac{e^{\frac{\lambda}{2}z}}{\prod_{i=1}^{L}(z-\theta_i)^m z^{\beta_1}(1-z)}$ with $\beta_1 = L(u-m)$, and $\theta_i = \frac{\eta_i}{1+\eta_i}$ is the i-th pole of $f(z)$, $\forall i = \{1, ..., L\}$. By applying the residue theorem [14] to (5), the average probability of detection is obtained as follows:

$Case\ 1 : u > m$

$$\overline{P}_d = \frac{e^{-\frac{\lambda}{2}}}{\prod_{i=1}^{L}(1+\eta_i)^m}\left[\sum_{i=1}^{L} Res(f;\theta_i,m) + Res(f;0,\beta_1)\right], \tag{6}$$

$Case\ 2 : u \le m$

$$\overline{P}_d = \frac{e^{-\frac{\lambda}{2}}}{\prod_{i=1}^{L}(1+\eta_i)^m}\left[\sum_{i=1}^{L} Res(f;\theta_i,m)\right], \tag{7}$$

where the residue of $f(z)$ for an l-th order pole at $z = a$ is defined as [14]

$$Res(f;a,l) = \lim_{z\to a} \frac{1}{(l-1)!} \frac{d^{l-1}}{dz^{l-1}}[(z-a)^l f(z)]. \tag{8}$$

Note that the two cases $u > m$ and $u \le m$ must be treated separately as the z^{β_1} term in $f(z)$ appears either as a pole or a zero, respectively. Denoting $T(z) = \frac{e^{\frac{\lambda}{2}z}}{\prod_{i=1, i\neq j}^{L}(z-\theta_i)^m z^{\beta_1}(1-z)}$, we can use the Faà-di Bruno's formula [15] for the nth derivative of a composite function to evaluate the $(m-1)^{th}$ derivative of $e^{\log(T(z))}$. The residues in closed-form may be given by

$$Res(f;\theta_j,m) = \sum_{\mathbf{\Lambda}}\left[\frac{e^{\phi(\theta_j)}\phi^k(\theta_j)}{b_1!b_2!...b_{m-1}!}\left(\frac{\phi'(\theta_j)}{1!}\right)^{b_1}\left(\frac{\phi''(\theta_j)}{2!}\right)^{b_2} ...\left(\frac{\phi^{(m-1)}(\theta_j)}{(m-1)!}\right)^{b_{m-1}}\right], \tag{9}$$

where $\mathbf{\Lambda} = [\{(b_1, ..., b_m) | \sum_{i=1}^{m} ib_i = m, \sum_{i=1}^{n} b_i = k, b_i \ge 0, i = 1, ..., m\}]$ and $\phi(z) = \log(T(z))$ whose nth derivative $\forall n \in \mathbb{Z}^+$ is given by

$$\phi^{(n)}(z) = \frac{\lambda}{2}\delta(n-1) + (-1)^n m(n-1)!\left[\sum_{i=1, i\neq j}^{L}(z-\theta_i)^{-n} + \beta_1 z^{-n} + (1-z)^{-n}\right], \tag{10}$$

with $\delta(x)$ being the Kronecker delta function. The same procedure can be used to obtain the closed-form expression for $Res(f;0,\beta_1)$ as well. Here, we omit the expressions for brevity.

Note that (6) and (7) are given by the residues, which in turn are expressed as higher-order derivatives of $f(z)$. Although the expressions seem tedious, the residues can be computed efficiently by using a platform such as MATHEMATICA for any given set of L, u, m and the channel covariance matrix $\mathbf{C_L}$. In general, for L correlated antennas at the CR, the eigenvalues will be real. Given the matrix $\mathbf{C_L}$, the total received SNR γ_t and the fading index m, the eigenvalues of $\mathbf{\Theta_L}$ can be readily computed. Thus, our approach can be used to analyze a variety of antenna correlation models, some of which are discussed next.

A. *Constant correlation*

This model applies when the antennas are situated close enough in a circular array [16] to result in the same correlation between any two branches. The channel covariance matrix, thus, is $\mathbf{C_L} = [c_{ij}]$, where $c_{ij} = 1$ for $i = j$, and $c_{ij} = \rho$ for $i \neq j$. For this structure, $\mathbf{\Theta_L}$ has only two distinct eigenvalues, $\eta_1 = \gamma_t(1-\sqrt{\rho})/m$ and $\eta_2 = \gamma_t(1+(L-1)\sqrt{\rho})$, where η_1 is repeated $(L-1)$ times. Then, $f(z)$ in (5) reduces to

$g(z) = \frac{e^{\frac{\lambda}{2}z}}{(z-\theta_1)^{\beta_2}(z-\theta_2)^m z^{\beta_1}(1-z)}$, with $\beta_2 = (L-1)m$. Now, we need the residues of $g(z)$ at $z = \theta_1$, $z = \theta_2$ and $z = 0$ (if $u > m$). In this special case, we can evaluate (9) further. Substituting $z = t+\theta_1$ into $g(z)$, performing the Laurent series expansion [14] about $t = 0$, and extracting the coefficient of t^{-1} will give the residue of $g(z)$ at $z = \theta_1$ as

$$Res(f;\theta_1,\beta_2)=\sum_{\mathbf{\Lambda_1}}\left[\left(\frac{\lambda}{2}\right)^{k_1}\binom{-m}{k_2}(\theta_1-\theta_2)^{-m-k_2} \cdot\binom{-\beta_1}{k_3}\theta_1^{-\beta_1-k_3}(1-\theta_1)^{-1-k_4}\cdot e^{\frac{\lambda}{2}\theta_1}\right], \tag{11}$$

where $\mathbf{\Lambda_1} = \{(k_1,k_2,k_3,k_4)|\sum_{i=1}^{4}k_i = \beta_2 - 1, k_i \geq 0\}$. Similarly, the residue at $z = 0$ is

$$Res(f;0,\beta_1) = \sum_{\mathbf{\Lambda_2}}\left(\frac{\lambda}{2}\right)^{k_1}\binom{-\beta_2}{k_2}\binom{-m}{k_3}(-\theta_2)^{-m-k_3}, \tag{12}$$

with $\mathbf{\Lambda_2} = \{k_1,k_2,k_3,k_4|\sum_{i=1}^{4}k_i = \beta_1 - 1, k_i \geq 0\}$. The same steps will give the residue of the mth order pole at $z=\theta_2$. For brevity, we omit the calculations here. The probability of detection can be then expressed as

$$\overline{P}_d = \frac{e^{-\frac{\lambda}{2}}\left[Res(f;\theta_1,\beta_2)+Res(f;\theta_2,m)+Res(f;0,\beta_1)\right]}{(1+\eta_1)^{\beta_2}(1+\eta_2)^m}. \tag{13}$$

B. Exponential correlation

If the ij-th entry of the covariance matrix $\mathbf{C_L}$ is of the form $\rho_{ij} = \rho^{|i-j|}$, then the signals at the received branches are said to be exponentially correlated [16]. This case is relatively more general than the constant correlation model and is a special case of the linear array of arbitrarily correlated antennas, which is treated in the next section. There are no explicit expressions for the eigenvalues in this case for $L > 4$, but they can be readily computed. Then, (6) and (7) can be used to compute the average probability of detection. This model is particularly employed to show the effect of antenna correlation on the detection performance of the CR.

C. Linear array of arbitrary correlation

If the antenna elements are placed in a linear configuration, then the signals at the branches may be arbitrarily correlated depending upon the incident angle, the spacing between the antennas or the height of the antennas. Therefore, the elements cannot be assumed to have a constant or exponential correlation structure. In such a scenario, the covariance matrix can be modeled to have a Toeplitz structure of the form, $\mathbf{C_L} = [\rho_{|i-j|}]$ with $\rho_0 = 1$ [13].

Explicit number of antennas: If we consider the CR to be equipped with two correlated antennas with a correlation ρ between the branches, the covariance matrix $\mathbf{C_2}$ will be the 2×2 version of $\mathbf{C_L}$. Then the eigenvalues of the corresponding matrix $\mathbf{\Theta_2}$ can be shown to be

$$\eta_1,\eta_2 = \frac{\gamma_t}{m}(1\pm\sqrt{\rho_1}). \tag{14}$$

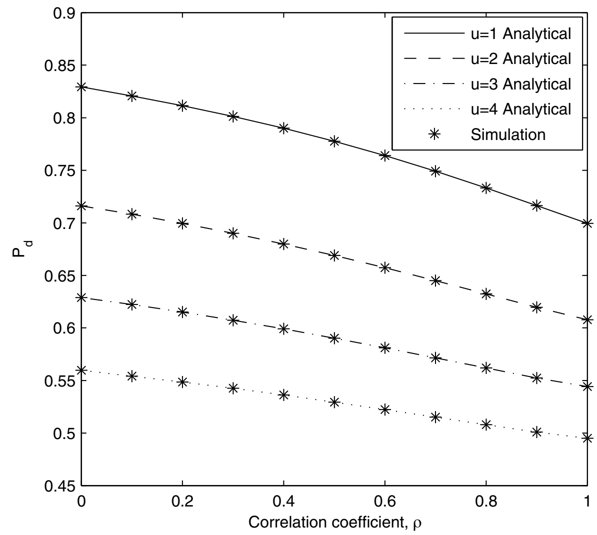

Fig. 1. $\overline{P}_d$ vs ρ for the exponential correlation model with $L = 4$, $Pf = 0.01$, $m = 2$, SNR = 5 dB and $u = \{1,2,3,4\}$: analytical and simulation.

Similarly, if the CR is equipped with three antennas, by using a 3×3 covariance matrix $\mathbf{C_3}$ with correlation coefficients ρ_1 and ρ_2 across the branches, the eigenvalues of $\mathbf{\Theta_3}$ can be shown to be

$$\eta_1 = \frac{\gamma_t}{m}(1-\sqrt{\rho_2});\eta_2,\eta_3 = \frac{\gamma_t}{2m}(2+\sqrt{\rho_2}\pm\sqrt{8\rho_1+\rho_2}). \tag{15}$$

For a four-antenna CR, the eigenvalues of $\mathbf{\Theta_4}$ can be obtained as

$$\begin{aligned}\eta_1,\eta_2 &= \frac{\gamma_t}{2m}(2-\sqrt{\rho_1}-\sqrt{\rho_3}) \\ &\quad\pm\sqrt{5\rho_1-8\sqrt{\rho_1\rho_2}+4\rho_2-2\sqrt{\rho_1\rho_3}+\rho_3}, \\ \eta_3,\eta_4 &= \frac{\gamma_t}{2m}(2+\sqrt{\rho_1}+\sqrt{\rho_3}) \\ &\quad\pm\sqrt{5\rho_1+8\sqrt{\rho_1\rho_2}+4\rho_2-2\sqrt{\rho_1\rho_3}+\rho_3}.\end{aligned} \tag{16}$$

Using (14), (15) and (16) separately along with (6) and (7) for $L = \{2,3,4\}$ number of antennas at the CR, we can obtain the expression for the average probability of detection at the CR for the two cases of $u > m$ and $u \leq m$. We omit the expressions here for brevity. Note that this model reduces to the exponential correlation model for $\rho_i = \rho^i$, $i = \{0,1,...,L-1\}$ for the i-th element of the first row of the covariance matrix $\mathbf{C_L}$.

IV. Numerical and Simulation Results

The analytical results (in Section III) are evaluated by using MATHEMATICA to obtain theoretical curves for the given parameters of interest. These curves are validated with simulations performed in MATLAB for 10^6 iterations. We study the effect of the degree of antenna correlation, the number of antennas and the time-bandwidth product of the energy detector at the CR. The threshold of detection λ is computed by solving $P_{f,SLC} = \frac{\Gamma(Lu,\lambda/2)}{\Gamma(Lu)}$ [9] for a desired value of the false alarm probability.

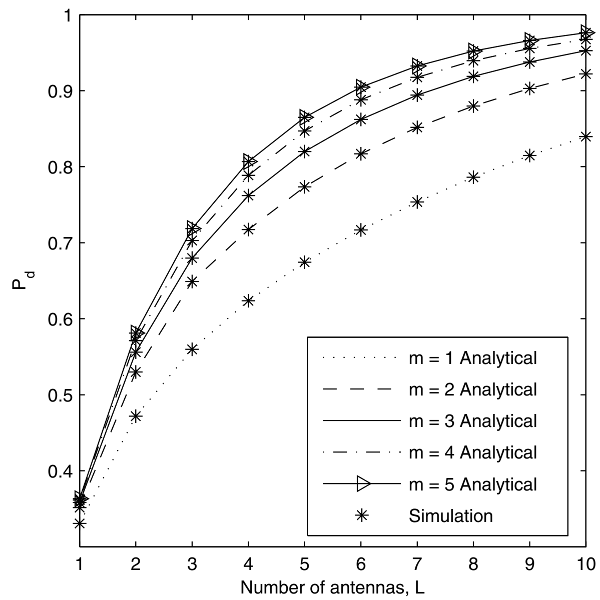

Fig. 2. $\overline{P}_d$ vs L for the linear array with $P_f = 0.01$, $u = 1$, SNR = 5 dB, $m = \{1, 2, 3, 4, 5\}$: analytical and simulation.

To observe the effect of antenna correlation on the CR detection performance, we plot the average probability of detection against the various degrees of correlation across the branches for an exponentially correlated four antenna CR in Fig. 1. As the antenna correlation increases, the system performance gain due to antenna diversity deteriorates [17]. Our results validate this fact for the detection performance of a CR. The probability of detection for the completely correlated scenario ($\rho = 1$) observed for $u = 1$ is about 16% less than that for the no-correlation ($\rho = 0$) case. Also, the detection probability decreases for larger values of the time-bandwidth product u because at higher values of u, the increased incoherence of noise tends to nullify the signal energy at the detector, thus resulting in lower probability of detection [4].

In Fig. 2, the average probability of detection at the CR is plotted against the number of antennas for a linear array with the first row of $\mathbf{C_L}$ being $(1, 0.9, 0.8, \ldots, 0.1)$ (the correlation decreases as the branches become further apart). The detection capability of the CR increases significantly with the increase in the number of antennas even if the antennas are correlated. For example, for $m = 1$, the Rayleigh fading case, we find that the detection probability for a 10-antenna CR is almost double (about 80% more) than that for a 2-antenna CR. Thus, more antennas at the CR boost the detection performance even if high correlations exist across them. The fading index also quantifies the detection performance for several Nakagami-m parameter values. As expected, higher values of the fading index m imply a relatively less degraded received signal and thus lead to a higher probability of detection. Thus, the antenna diversity mitigates the impact of correlation and introduces a significant improvement in the probability of detection.

V. Conclusion

We analyzed the detection performance of a multiple correlated antenna CR by deploying the square law combining technique. The average probability of detection over L correlated Nakagami-m channels from PU to CR was derived. Specific results were developed for constant correlation, exponential correlation and linear array antenna models. The impact of various parameters such as the degree of correlation across the antennas, the time-bandwidth product of the energy detector, the number of antennas and the fading severity index was investigated. Although antenna correlation degrades the detection performance of the CR, the gain due to multiple antenna diversity remains significant.

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