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# Performance Analysis of MIMO Channel Inversion in Rayleigh Fading

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*Abstract*—The performance of eigenmode transmission over a multiple-input-multiple-output (MIMO) Rayleigh channel under the channel inversion (CI) power allocation scheme is investigated. The moment-generating function of the reciprocal of the received signal-to-noise ratio (SNR) is derived. For the special case when the minimum number of antennas at the transmitter and the receiver is two, the exact closed-form expressions for the probability density function (pdf) and cumulative distribution function (cdf) are also derived. The average symbol error rate (SER) is derived as an application of the results. The extension to Rician and semicorrelated Rayleigh fading scenarios is also outlined.

*Index Terms*—Channel inversion (CI), eigenmode transmission, multiple-input multiple-output (MIMO), Wishart distribution.

### I. INTRODUCTION

**M** ULTIPLE-ANTENNA wireless terminals, which are used along with special signal-processing techniques to achieve diversity and multiplexing benefits, characterize multiple-input-multiple-output (MIMO) wireless technology [3]. MIMO technology exploits the *space dimension*, in addition to the time and frequency dimensions, to deliver data rates and a quality of service unmatched otherwise with comparable spectral resources. MIMO is widely investigated and being deployed (e.g., IEEE 802.11n [4] and Third-Generation Partnership Project (3GPP) Long-Term Evolution (LTE) [5] standards).

A MIMO channel is represented by a *channel matrix*, whose elements are channel gains between transmit–receive antenna pairs. Thus, mathematical tools such as the random matrix theory [6] help the analysis. A MIMO channel can be reduced to a set of noninterfering spatial channels (*virtual channels*) with appropriate transmitter and receiver signal-processing techniques. The ability to independently code, modulate, and power allocate for [7], [8] these virtual channels facilitates the use of single-antenna signal-processing techniques in MIMO. Our

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focus is on applying the channel inversion (CI) power allocation scheme [9] across the virtual channels realized through eigenmode transmission.

In a multichannel communication system with channel state information (CSI) at the transmitter, CI allocates power among the channels such that the total instantaneous transmit power is held constant and the instantaneous received signal-to-noise ratio (SNR) is identical across the channels. Although CI yields worse capacity than optimal water-filling power allocation, it may be suitable for applications with tight delay constraints [10, Ch. 5]. A *temporal* variant of CI can be used in single-carrier single-antenna systems [9]. Its use with MIMO has also been investigated in literature [11]. MIMO CI [12] considered here, by contrast, is *spatial*, i.e., achieved across spatially multiplexed virtual channels.

Hereafter, CI refers to CI power allocation on the virtual channels produced by eigenmode transmission. References [12]–[14] examine the basics of MIMO CI, whereas [15]–[17] investigate certain variants exhibiting improved capacity. Notably, signal processing for CI in certain MIMO configurations does not even require singular value decomposition of the channel matrix.

Another noteworthy fact is the similarity that CI has with zero-forcing (ZF) beamforming [18], [19]; thus, CI might find use in MIMO and multiuser MIMO scenarios [20]. ZF simply inverts the channel at the transmitter or the receiver. Transmit ZF thus causes the instantaneous transmit power to fluctuate unbounded, making its practical realization challenging. CI is immune to this issue. ZF reception employs nonunitary signal processing, which is susceptible to noise enhancement and correlated noise. CI, with only unitary receiver processing (if at all) [12], [14], is free from these disadvantages. However, CI requires nonunitary transmit processing, and the achievable error rate is the same as that of receive ZF but is worse than that of transmit ZF. These distinctions can be observed in terms of error performance (see Fig. 1).

Our main contribution is the development of a mathematical framework for an  $N_r \times N_t$  MIMO system to accurately characterize the per-virtual-channel received SNR  $\Lambda$  under CI. More specifically, we obtain the following:

- 1) for arbitrary  $N_t$  and  $N_r$ : the exact moment-generating function (mgf) of  $\Lambda^{-1}$ ;
- 2) for the case  $\min(N_t, N_r) = 2$ : the exact probability density function (pdf) of  $\Lambda$ , the cumulative density function (cdf) of  $\Lambda$ , and the average symbol error rate (SER) for a class of modulation schemes (presented in [1]).



Fig. 1. Average SER for quadratic phase-shift keying (QPSK) on  $(N_r \times N_t)$  MIMO systems: ZF versus CI. Transmit ZF is used for the 2 × 4 configuration, whereas ZF reception is used with the others. 10<sup>6</sup> channel realizations, with 100 QPSK symbols per virtual channel for each, have been simulated.

Although independent Rayleigh fading is assumed for the main results, the extension of some of the results for Rician fading (presented in [2]) and semicorrelated Rayleigh fading is briefly developed.

This paper is organized as follows: The system model is given in Section II, and the mathematical formulation and the numerical results for independent and identically distributed (i.i.d.) Rayleigh fading follow in Sections III and IV, respectively. Section V extends the analysis for Rician fading and semicorrelated Rayleigh fading. Section VI concludes this paper. Proof of the results is annexed.

*Notation:*  $P[\mathcal{A}]$  is the probability of event  $\mathcal{A}$ . The pdf, cdf, complimentary cdf (ccdf), and the mgf of a random variable X are given by  $f_X(x)$ ,  $F_X(x)$ ,  $\bar{F}_X(x)$ , and  $\mathcal{M}_X(s)$ , respectively. The expected value is  $\mathcal{E}_X\{\cdot\}$ .  $\mathcal{I}_{\nu}(\cdot)$  and  $\mathcal{K}_{\nu}(\cdot)$  are the Modified Bessel functions [21, Sec. 9.6] of the first kind and second kind of order  $\nu$ , respectively.  $\mathcal{G}_{p \ q}^{m \ n}(\cdot|_{b_1,\ldots,b_m,b_{m+1},\ldots,b_q}^{a_1,\ldots,a_n,a_{n+1},\ldots,a_p})$  is the Meijer G function [22, Sec. 9.3].  $\mathcal{Q}(\cdot)$  is the Gaussian Q-function [21, eq. (26.2.3)].  $\log_2(\cdot)$  represents the logarithm to base 2, whereas k! is the factorial of k.  $\mathbb{C}^{m \times n}$  is the set of  $m \times n$  complex matrices.

 $\mathbf{B}^{H}$ ,  $\|\mathbf{B}\|_{F}$ , trace( $\mathbf{B}$ ), and eig( $\mathbf{B}$ ) represent the conjugate transpose, Frobenius norm, trace, and eigenvalues of a matrix  $\mathbf{B}$ , respectively. The remainder, in a Maclaurin series of a function of x after the  $x^{n}$  term, is denoted by  $o(x^{n})$ .

# II. SYSTEM MODEL

We consider a MIMO system with  $N_t$  and  $N_r$  transmit and receive antennas. Let  $m = \min(N_t, N_r)$  and  $n = \max(N_t, N_r)$ . The channel matrix is  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ . We also assume perfect transmit CSI and the usual additive i.i.d. complex Gaussian noise at the receiver antennas.

Define  $\mathcal{W}$  as either  $\mathbf{HH}^H$   $(N_t > N_r)$  or  $\mathbf{H}^H \mathbf{H}$   $(N_t \le N_r)$ . The eigenvalues  $\{\lambda_1, \ldots, \lambda_m\}$  of  $\mathcal{W}$  sufficiently characterize the MIMO channel. For instance, they relate to the received SNR along the *m* virtual channels under eigenmode transmission. The total transmit power P is allocated as  $p_i$  for each *i*th virtual channel  $i \in \{1, \ldots, m\}$ , using CI, causing  $\lambda_i p_i = K$  to be held identical for each of them at any given time instant. Thus, we have

$$P = \sum_{i=1}^{m} p_i = K \sum_{i=1}^{m} \lambda_i^{-1}.$$

Let  $\Lambda = K/P$ . Then, we get

$$\Lambda^{-1} = \sum_{i=1}^{m} \lambda_i^{-1} = \operatorname{trace}(\mathcal{W}^{-1}).$$
(1)

Note that (1) holds only under block fading assumptions, which permit averaging out the randomness of additive noise and transmitted data for each channel realization.

#### **III. MATHEMATICAL FORMULATION**

This section analyzes MIMO CI systems. The links between the transmit–receive antenna pairs are assumed to undergo i.i.d. Rayleigh fading. Thus,  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  becomes a complex Gaussian matrix;  $\mathcal{W}$  becomes a rank-*m* complex central Wishart matrix [7], [23] having *n* degrees of freedom; and  $\Lambda^{-1}$ becomes the trace of an inverse-Wishart matrix.

# A. Arbitrary $m \leq n$

The joint pdf of  $\lambda_i, i \in \{1, ..., m\}$ , which are the unordered eigenvalues of  $\mathcal{W}$ , is given [23], [24] by

$$f_{\lambda_1,\dots\lambda_m}(\lambda_1,\dots,\lambda_m) = \frac{e^{-\sum_{i=1}^m \lambda_i}}{m!\mathcal{K}_{m,n}} \prod_{i=1}^m \lambda_i^{n-m} \prod_{1 \le i < j \le m} (\lambda_i - \lambda_j)^2 \quad (2)$$

where  $\mathcal{K}_{m,n} = \prod_{k=1}^{m} (m-k)!(n-k)!$ . The joint pdf of unordered eigenvalues differs only by a factorial term from that of the ordered eigenvalues. All the unordered eigenvalues have the range  $[0, \infty)$ , and therefore, using their joint pdf simplifies further derivations.

The factor  $\prod_{1 \le i < j \le m} (\lambda_i - \lambda_j)^2$  in (2) may be expanded to obtain the more manipulable following form:

$$f_{\lambda_1,\dots\lambda_m}(\lambda_1,\dots,\lambda_m) = \frac{e^{-\sum_{i=1}^m \lambda_i}}{m!\mathcal{K}_{m,n}} \left(\prod_{i=1}^m \lambda_i^{n-m}\right)$$
$$\times \sum_{\substack{k_1,\dots,k_m \in \{0,\dots,2(m-1)\}\\\sum k_i = m(m-1)}} b(k_1,\dots,k_m)\lambda_1^{k_1}\cdots\lambda_m^{k_m}.$$
 (3)

Coefficients  $b(k_1, \ldots, k_m)$ , corresponding to variables  $\lambda_1, \ldots, \lambda_m$  raised to respective powers  $k_1, \ldots, k_m$ , can be obtained by expanding the factor as a multivariate polynomial. The equality  $\sum_{i=1}^{m} k_i = m(m-1)$  is seen to hold for each term of this expansion.

Theorem 1—MGF of  $\Lambda^{-1}$  (for Arbitrary  $m \leq n$ ): Let  $\lambda_1, \ldots, \lambda_m$  be the unordered eigenvalues of an  $m \times m$  complex

central Wishart matrix having n degrees of freedom. The mgf of  $\Lambda^{-1}$  in (1) is given by

$$\mathcal{M}_{1/\Lambda}(s) = \frac{2^m s^{-\frac{mn}{2}}}{m! \mathcal{K}_{m,n}} \sum_{\substack{k_1, \dots, k_m \in \{0, \dots, 2(m-1)\}\\\sum_{k_i} = m(m-1)}} b(k_1, \dots, k_m) \times \prod_{i=1}^m \mathcal{K}_{k_i+n-m+1}(2\sqrt{s}).$$
(4)

Proof: See the Appendix.

#### B. Special Case: m = 2

This scenario occurs in any MIMO channel having two antennas at one end and at least two antennas in the other. The MIMO downlink from a multiantenna base station to twoantenna mobile station (as in  $4 \times 2$  3GPP LTE downlink configuration [5]) is an example. Another is the multiuser MIMO downlink [20] from a multiantenna base station to two singleantenna mobile stations. Owing to antenna spacing constraints, equipping a mobile terminal operating in cellular frequency bands (currently, below 4 GHz) with more antennas, is technically challenging. Therefore, the case m = 2 is realistic.

For the case m = 2, (2) reduces to

$$f_{\lambda_1,\lambda_2}(\lambda_1,\lambda_2) = \frac{1}{2\mathcal{K}_{2,n}} e^{-(\lambda_1+\lambda_2)} (\lambda_1-\lambda_2)^2 \lambda_1^{n-2} \lambda_2^{n-2}$$
(5)

where  $\mathcal{K}_{2,n}$  simplifies to (n-1)!(n-2)!. The distribution of  $\Lambda$  can be found from (1) and (5).

Theorem 2—PDF of  $\Lambda$  (for m = 2): Let  $\lambda_1$  and  $\lambda_2$  be the unordered eigenvalues of a 2 × 2 complex central Wishart matrix having *n* degrees of freedom. The pdf of  $\Lambda$  in (1) is given by

$$f_{\Lambda}(x) = \frac{1}{\mathcal{K}_{2,n}} x^{2(n-1)} e^{-2x} \sum_{k=0}^{2n} \binom{2n}{k} \\ \times \left( (n-k-2x) \mathcal{K}_{k-n}(2x) + 2x \mathcal{K}_{k+1-n}(2x) \right).$$
(6)

*Proof:* See the Appendix.

Corollary 1—CDF of  $\Lambda$  (for m = 2): Let  $\lambda_1$  and  $\lambda_2$  be the unordered eigenvalues of a 2  $\times$  2 complex central Wishart matrix having *n* degrees of freedom. The cdf of  $\Lambda$  in (1) is given by

$$F_{\Lambda}(x) = \frac{2\sqrt{\pi}}{4^{2n}\mathcal{K}_{2,n}} \sum_{k=0}^{2n} \binom{2n}{k}$$

$$\times \left[ 2(n-k)\mathcal{G}_{2\,3}^{2\,1} \left( 4x|_{3n-k-1,n+k-1,0}^{1,2n-0.5} \right) - \mathcal{G}_{2\,3}^{2\,1} \left( 4x|_{3n-k,n+k,0}^{1,2n+0.5} \right) + \mathcal{G}_{2\,3}^{2\,1} \left( 4x|_{3n-k-1,n+k+1,0}^{1,2n+0.5} \right) \right]. \tag{7}$$

*Proof:* See the Appendix.

Although relative uncommonly used in the wireless literature, the Meijer G function is well characterized [22, Sec. 9.3].



Fig. 2. CDF of  $\Lambda$  over  $\{2 \times 2, 3 \times 2, 3 \times 3\}$  MIMO systems using CI; numerical versus simulated ( $\blacksquare$ ).

Moreover, it is directly available in the common computational environments including Mathematica, Maple, and MATLAB. Hence, the results can be easily evaluated at high precision.

#### **IV. NUMERICAL RESULTS**

This section highlights some applications of the characterization of  $\Lambda$  made in Section III. Numerical results on different performance metrics are presented to establish its validity.

# A. Arbitrary $m \leq n$

Further derivations based on the result (4) likely require the use of hypergeometric functions of multiple variables and are not examined here.<sup>1</sup> Having an exact expression for  $\mathcal{M}_{\Lambda^{-1}}(s)$  is more appealing when it comes to numerical evaluation of the performance metrics for it reduces the number of folded integrals that one may have to evaluate. The ccdf of  $\Lambda$ , which relates to the probability of outage, for instance, is given by [25]

$$\bar{F}_{\Lambda}(x) = F_{\Lambda^{-1}}\left(\frac{1}{x}\right) = \frac{2}{\pi} \int_{0}^{\infty} \frac{\Re\left(\mathcal{M}_{\Lambda^{-1}}(j\omega)\right)}{\omega} \sin\left(\frac{\omega}{x}\right) d\omega$$
(8)

whose single integral can be evaluated using a simple quadrature technique. Likewise, evaluating a single integral suffices to obtain the SER [26].

Fig. 2 compares the cdf of  $\Lambda$ : numerical versus simulated (10<sup>6</sup>-point semianalytic Monte Carlo). Numerical values have been computed using adaptive quadrature routine quadl in MATLAB after applying the variable transformation  $\omega = (1 + t)^2/(1 - t)^2$  to adjust the range of integration. Since CI holds reciprocity and performs similar to a ZF receiver, the diversity order of CI can be deduced (from [27]) to be  $|N_t - N_r| + 1$ . The slope of cdf curves as  $\lambda \to 0$  agrees with this observation.

<sup>&</sup>lt;sup>1</sup>Native support for special functions of an arbitrary number of variables is not currently available in standard computational environments such as MATLAB and Mathematica. They are nevertheless implementable as cascaded infinite series.



Fig. 3. Ergodic capacity (in bits per second per hertz) for a  $2 \times n$  MIMO system using CI; numerical versus simulated ( $\blacksquare$ ).

#### B. Special Case m = 2

This scenario is more tractable. Three applications of the results are examined next, indirectly verifying (6) and (7).

1) Ergodic Capacity: The Shannon capacity of the MIMO system in concern is given by a random variable  $C = \sum_{k=1}^{2} \log_2(1 + p_i\lambda_i/N_0) = 2\log_2(1 + P\Lambda)$ , where P denotes the transmit SNR (transmit power normalized w.r.t. the noise variance). Its average, i.e., *ergodic capacity*  $\mathcal{E}_C\{C\}$ , can be numerically computed as

$$\mathcal{E}_C\{C\} = \int_0^\infty c f_C(c) dc = 2 \int_0^\infty \log_2(1+Px) f_\Lambda(x) \, dx$$

for any given *P*. Fig. 3 verifies the numerical values thus obtained for  $\mathcal{E}_C\{C\}$  for the cases  $n \in \{2, 3, 5\}$ , against the simulation (10<sup>5</sup>-point semianalytic Monte Carlo) results. As expected, the ergodic capacity logarithmically increases with the transmit SNR (i.e., appears as a straight line at high-transmit SNR, when SNR is given in decibels) and increases (although not linearly) with *n*.

2) Average SER: Since the receiver processing for CI leaves the distribution of additive Gaussian noise unaltered, the average SER  $P_s$  under many modulation schemes [28, Ch. 5] can be given by

$$P_s = \mu \mathcal{E}_{\Lambda} \left\{ \mathcal{Q}(\sqrt{2\nu\Lambda P}) \right\}$$
(9)

where P is the transmit SNR, and  $\mu$ ,  $\nu$  are constants dependent on the modulation scheme. For example,  $\mu = 1$ ,  $\nu = 1$  exactly give the SER for binary phase-shift keying (BPSK), whereas  $\mu = 2(M-1)/M$ ,  $\nu = 3/(M^2 - 1)$  approximate those for other *M*-ary pulse amplitude modulation schemes. The average SER for such systems can be analytically derived using (9) and the distribution of  $\Lambda$ .

Corollary 2—Average SER (for m = 2): Let  $\lambda_1$  and  $\lambda_2$  be the unordered eigenvalues of a 2  $\times$  2 complex central Wishart



Fig. 4. Average SER for BPSK (i.e.,  $\mu = \nu = 1$ ) over a  $2 \times n$  MIMO system using CI; analytical versus simulated ( $\blacksquare$ ). (Dashed lines) Asymptotes to curves.

matrix having n degrees of freedom. The average SER (9) is given by

$$P_{s} = \frac{\mu}{(4^{2n})\mathcal{K}_{2,n}} \sum_{k=0}^{2n} \binom{2n}{k} \times \left[ 2(n-k)\mathcal{G}_{3\,3}^{2\,2} \left(\frac{4}{\nu P}\Big|_{3n-k-1,n+k-1,0}^{0.5,1,2n-0.5}\right) - \mathcal{G}_{3\,3}^{2\,2} \left(\frac{4}{\nu P}\Big|_{3n-k,n+k,0}^{0.5,1,2n+0.5}\right) + \mathcal{G}_{3\,3}^{2\,2} \left(\frac{4}{\nu P}\Big|_{3n-k-1,n+k+1,0}^{0.5,1,2n+0.5}\right) \right].$$
(10)

*Proof:* See the Appendix.

3) High SNR Analysis: The diversity and coding gains of the system can be easily deduced [29, Prop. I] from the least order approximation of x on the pdf of  $\Lambda$ . For the MIMO system in concern, it may be obtained, after some manipulations, using [21, eq. (9.6.9)] on (6).

*Corollary 3:* Let  $\lambda_1$  and  $\lambda_2$  be the unordered eigenvalues of a 2 × 2 complex central Wishart matrix having *n* degrees of freedom. The least order approximation of the pdf of  $\Lambda$  in (1) is given by

$$f_{\Lambda}(x) = \frac{n}{2(n-2)!} x^{n-2} + o(x^{n-2}).$$
(11)

*Proof:* Evidently, only the k = 0 term of the summation in (6) contributes to this approximation. Using the first term of each series expansion [21, eq. (4.2.1)] and [21, eq. (9.6.9)], followed by the selection of the least order term of x, yields the result.

Fig. 4 shows for the cases  $n \in \{2, 3, 5\}$  how the analytical result (10) for the SER of the BPSK compares with the simulation  $(10^5$ -point semianalytic Monte Carlo) results. It also shows the asymptotes for the curves computed using (11) based on [29]. A reduction of SER with increased n and a diversity order of (n - 1) are observed.

# V. EXTENSION TO RICIAN AND CORRELATED RAYLEIGH CASES

The joint pdf of the unordered eigenvalues of a complex central Wishart distribution resembles those of the noncentral and semicorrelated central Wishart distributions [30]. Therefore, certain results presented in Section III can be generalized for these scenarios. Only pdf results are presented here since the derivation of other results from them (for case m = 2) requires no different approach from the central-Wishart case.

# A. Rician Fading for the Case $\min(N_t, N_r) = 2$

Without further loss of generality, let  $N_t = n \ge 2$  and  $N_r = 2$ . Suppose that the resulting  $2 \times n$  channel matrix **H** is of the form  $\mathbf{H} = a\mathbf{H}_{sp} + b\mathbf{H}_{sc}$ , where  $\mathbf{H}_{sp}$  represents the deterministic specular (line-of-sight) component,  $\mathbf{H}_{sc} \in \mathbb{C}^{2 \times n}$  represents the random scatter component, and  $a^2 + b^2 = 1$ . The specular component is governed by the directional gains of the antennas, presence of dominant multipaths, etc.  $K = a^2 \|\mathbf{H}_{sp}\|_F^2 / b^2 \|\mathbf{H}_{sc}\|_F^2$  is the Rician factor [31].  $\mathbf{\Omega} = (a^2/b^2)\mathbf{H}_{sp}\mathbf{H}_{sp}^{\mathrm{H}}$  is the *noncentrality matrix*.

Let  $\{\lambda_1, \lambda_2\} = \operatorname{eig}(\mathbf{H}\mathbf{H}^H)$  and  $\{\omega_1, \omega_2 | \omega_1 > \omega_2\} = \operatorname{eig}(\mathbf{\Omega})$ . The joint distribution of unordered eigenvalues  $\lambda_1, \lambda_2$  is given by [31, eq. (15)]

$$f_{\lambda_{1},\lambda_{2}}(\lambda_{1},\lambda_{2})$$

$$=\frac{e^{-(\omega_{1}+\omega_{2})}}{2}\frac{|\lambda_{1}-\lambda_{2}|(\lambda_{1}\lambda_{2})^{\frac{n-2}{2}}}{(\omega_{1}-\omega_{2})(\omega_{1}\omega_{2})^{\frac{n-2}{2}}}$$

$$\times e^{-(\lambda_{1}+\lambda_{2})}\left(\mathcal{I}_{n-2}(2\sqrt{\omega_{1}\lambda_{1}})\mathcal{I}_{n-2}(2\sqrt{\omega_{2}\lambda_{2}})\right)$$

$$-\mathcal{I}_{n-2}(2\sqrt{\omega_{2}\lambda_{1}})\mathcal{I}_{n-2}(2\sqrt{\omega_{1}\lambda_{2}})\right). (12)$$

The pdf result corresponding to case  $\omega_1 = \omega_2$  can be obtained through the limiting operation  $\omega_1 \rightarrow \omega_2$  on (12).

Assume perfect transmit CSI and the CI scheme. Given transmit SNR P, the per-channel received SNR  $(b^2/2)P\Lambda$  relates to  $\lambda_1$  and  $\lambda_2$  through (1).

Theorem 3—PDF of  $\Lambda$ : Let  $\lambda_1$  and  $\lambda_2$  be the eigenvalues of a rank-2 complex noncentral Wishart matrix having *n* degrees of freedom and noncentrality matrix  $\Omega$  whose eigenvalues are  $\{\omega_1, \omega_2 | \omega_1 > \omega_2 > 0\}$ . The pdf of  $\Lambda$  in (1) is given by

$$f_{\Lambda}(x) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \frac{g_{i,j}(\omega_1, \omega_2)}{(j+n-2)!(i-j+n-2)!j!(i-j)!} \\ \times \sum_{p=0}^{i+2n-1} {i+2n-1 \choose p} e^{-2x} x^{i+2n-2} \\ \times (\mathcal{K}_{n+j-p}(2x) - \mathcal{K}_{n+j-p-1}(2x))$$
(13)

where

$$g_{i,j}(\omega_1,\omega_2) = \frac{e^{-(\omega_1+\omega_2)}}{(\omega_1-\omega_2)} \left(\omega_1^j \omega_2^{(i-j)} - \omega_1^{(i-j)} \omega_2^j\right).$$
(14)



Fig. 5. PDF of  $\Lambda$  over  $2 \times n$  MIMO systems using CI; analytical versus simulated ( $\blacksquare$ ). Rician fading modeled by a rank-2 noncentrality matrix having eigenvalues [4, 1] assumed.

**Proof:** The identity [21, eq. (9.6.10)] is used to expand each  $\mathcal{I}_n(\cdot)$  as an infinite series. Each term of the resulting cascaded infinite series is of a form similar to (6). Hence, the rest of the proof is similar to that of Theorem 2.

Fig. 5 verifies the analytical pdf results for  $2 \times n$  MIMO systems using CI under Rician fading against the simulated (10<sup>6</sup>-point semianalytic Monte Carlo) results. The noncentrality matrix has eigenvalues  $\omega_1 = 4$  and  $\omega_2 = 1$ .

Equation (13) gets simplified further for the case  $\omega_1 \neq 0$ ,  $\omega_2 = 0$  and the limiting case  $\omega_1 \rightarrow \omega_2$ . Corresponding pdf expressions, along with the cdf and average SER results, have been presented and verified in [2]. The least order approximation of the pdf therein is reproduced here. It shows that the diversity order is the same for both Rayleigh and Rician fading.

Corollary 4: Let  $\lambda_1$  and  $\lambda_2$  be the eigenvalues of a rank-2 noncentral complex Wishart matrix having *n* degrees of freedom and noncentrality matrix  $\Omega$ , whose eigenvalues are  $\{\omega_1, \omega_2 | \omega_1 > \omega_2 > 0\}$ . The first-order approximation of the pdf of  $\Lambda$  in (1) is given by  $f_{\Lambda}(x) = ax^{n-2} + o(x^{n-2})$ , where

$$a = \frac{(n+\omega_1-1)e^{-\omega_2} - (n+\omega_2-1)e^{-\omega_1}}{2(\omega_1-\omega_2)(n-2)!}.$$
 (15)

*Proof:* The proof is similar to that of Corollary 3. Note that the limit of a in (15) as  $\omega_1 \to \omega_2$  and  $\omega_2 \to 0$  is n/2(n-2)!, which corresponds to Rayleigh fading.

# B. Min Semicorrelated Rayleigh Fading for the Case $\min(N_t, N_r) = 2$

Semicorrelated MIMO channel models [30] represent the scenarios where only one of the set of transmit or receive antennas is correlated. Under Rayleigh fading, the joint distributions of corresponding unordered eigenvalues hold certain similarities to that of the complex central Wishart matrix.

Let us extend the analysis of  $\min(N_t, N_r) = 2$  case in Section III for min semicorrelated Rayleigh fading, where correlation exists only at the terminal having a smaller number of antennas. Without loss of generality, let  $N_t = n \ge 2$  and  $N_r = 2$ . The channel matrix becomes  $\mathbf{H} = \boldsymbol{\Sigma}^{1/2} \mathbf{H}_w$ , where  $\mathbf{H}_w \in \mathbb{C}^{N_r \times N_t}$  is complex Gaussian distributed, and  $\boldsymbol{\Sigma}$  is the 2 × 2 receive correlation matrix whose ordered eigenvalues are  $\sigma_1$  and  $\sigma_2$ , such that  $\sigma_2 > \sigma_1$ . The joint distribution of the ordered eigenvalues of  $\mathbf{HH}^H$  is given by [32, eq. (17)]. Thus, we get the joint distribution of the unordered eigenvalues  $\{\lambda_1, \lambda_2\} = \operatorname{eig}(\mathbf{HH}^H)$  as

$$f_{\lambda_1,\lambda_2}(\lambda_1,\lambda_2) = \frac{K}{2} |\lambda_1 - \lambda_2| (\lambda_1 \lambda_2)^{n-2} \\ \times \left( e^{-(\lambda_1/\sigma_1 + \lambda_2/\sigma_2)} - e^{-(\lambda_1/\sigma_2 + \lambda_2/\sigma_1)} \right)$$
(16)

where

$$K = \frac{(\sigma_1 \sigma_2)^{1-n}}{(n-1)!(n-2)!(\sigma_2 - \sigma_1)}.$$
(17)

The result for the case  $\sigma_1 = \sigma_2$  would be given by the limiting operation  $\sigma_2 \rightarrow \sigma_1$ . The distribution of  $\Lambda$  in (1) can be characterized as follows:

Theorem 4—PDF of  $\Lambda$ : Let  $\lambda_1$  and  $\lambda_2$  be the eigenvalues of a rank-2 complex central Wishart matrix having *n* degrees of freedom and a correlation matrix having eigenvalues  $\{\sigma_1, \sigma_2 | \sigma_2 > \sigma_1 > 0\}$ . The pdf of  $\Lambda$  in (1) is given by

$$f_{\lambda}(x) = Kx^{2n-2}e^{-(1/\sigma_1 + 1/\sigma_2)} \left( \mathbb{I}_x(\sigma_1, \sigma_2, 0) - x\mathbb{I}_x(\sigma_1, \sigma_2, 1) - \mathbb{I}_x(\sigma_2, \sigma_1, 0) + x\mathbb{I}_x(\sigma_2, \sigma_1, 1) \right)$$

where

$$\mathbb{I}_{x}(\mu_{1},\mu_{2},a) = \sum_{k=0}^{2n-1} {\binom{2n-1}{k} \left(\frac{\mu_{1}}{\mu_{2}}\right)^{\frac{k-n-a+1}{2}}} \times \frac{\mathcal{K}_{k-n-a+1}\left(\frac{2x}{\sqrt{\mu_{1}\mu_{2}}}\right)}{x^{a}}$$

*Proof:* The proof is omitted, given the similarity to that of Theorem 2.

Fig. 6 verifies the analytical pdf results for  $2 \times n$  MIMO systems using CI against the simulated results. The receive correlation matrix has eigenvalues  $\sigma_1 = 0.1$  and  $\sigma_2 = 0.3$ . Special cases such as  $\sigma_1 = \sigma_2$ , as well as the cdf and average SER results, are mathematically tractable but not attempted here.

#### VI. CONCLUSION

The performance of MIMO eigenmode transmission under the CI power allocation scheme has been examined. A mathematical framework has been developed to characterize received signal power  $\Lambda$  in CI. The exact mgf of  $\Lambda^{-1}$  (for arbitrary  $N_t$  and  $N_r$ ) and the exact pdf of  $\Lambda$  and the cdf of  $\Lambda$  (for the special case min $(N_t, N_r) = 2$ ) have been derived assuming Rayleigh fading. Some extensions have been derived for Rician fading and semicorrelated Rayleigh fading. Numerical results, including that of the average SER, have been provided and verified through simulation to highlight possible applications of the framework developed.

The results confirm the intuition that CI has the diversity order of the weakest eigenmode, which is  $|N_t - N_r| + 1$  under



Fig. 6. PDF of  $\Lambda$  over  $2 \times n$  MIMO systems using CI; analytical versus simulated ( $\blacksquare$ ). Receive correlation matrix assumed to have [0.1, 0.3] as the eigenvalues.

both Rayleigh and Rician fading. This is because CI allocates power so that the received SNR is the same over all eigenmodes. Note that ZF reception performs similar to CI, without requiring transmit CSI. Hence, CI is not attractive when the receiver has more antennas than the transmitter. When the transmitter has more antennas, just like transmit ZF, CI does not require spatial processing at the receiver. Therefore, in such point-to-point and point-to-multipoint MIMO configurations, CI is an easier to implement alternative to transmit ZF.

#### APPENDIX

This Appendix presents the proofs of Theorems 1 and 2 and Corollaries 1 and 2.

*Proof—Theorem 1: MGF of*  $\Lambda^{-1}$  (*Arbitrary*  $m \in \mathbb{Z}^+$ ): The mgf of  $\Lambda^{-1}$  is given by the *m*-folded integral, i.e.,

$$\mathcal{M}_{1/\Lambda}(s) = \int_{\lambda_1=0}^{\infty} \cdots \int_{\lambda_m=0}^{\infty} e^{-\sum_{i=1}^{m} \frac{1}{\lambda_i} s} \\ \times f_{\lambda_1,\dots,\lambda_m}(\lambda_1,\dots,\lambda_m) \, d\lambda_1 \cdots d\lambda_m.$$
(18)

Substituting (3) into (18) and simplifying with [33, 4.5.1.(9)], we get

$$\mathcal{M}_{1/\Lambda}(s) = \frac{1}{m!\mathcal{K}_{m,n}} \sum_{\substack{k_1,\dots,k_m \in \{0,\dots,2(m-1)\}\\\sum_{k_i}=m(m-1)}}^{m} b(k_1,\dots,k_m)$$

$$\times \prod_{i=1}^{m} \int_{\lambda_1=0}^{\infty} \lambda_i^{k_i+n-m} e^{-\left(\lambda_i + \frac{s}{\lambda_i}\right)} d\lambda_i$$

$$= \frac{1}{m!\mathcal{K}_{m,n}} \sum_{\substack{k_1,\dots,k_m \in \{0,\dots,2(m-1)\}\\\sum_{k_i}=m(m-1)}}^{m} b(k_1,\dots,k_m)$$

$$\times \prod_{i=1}^{m} \left(2s^{-\frac{k_i+nm+1}{2}}\mathcal{K}_{k_i+n-m+1}(2\sqrt{s})\right) \quad (19)$$

and, hence, (4).

definition of the cdf

$$\begin{split} F_{\Lambda}(x) &= \mathcal{P}\left[\frac{\lambda_{1}\lambda_{2}}{\lambda_{1}+\lambda_{2}} \leq x\right] \\ &= \mathcal{P}\left[\lambda_{1}(\lambda_{2}-x) \leq \lambda_{2}x\right] \\ &= \int_{0}^{x} \underbrace{\bar{F}_{\lambda_{1}\mid\lambda_{2}}\left(\frac{\lambda_{2}x}{\lambda_{2}-x}\middle|\lambda_{2}\right)}_{\doteq 1} f_{\lambda_{2}}(\lambda_{2}) d\lambda_{2} \\ &+ \int_{x}^{\infty} F_{\lambda_{1}\mid\lambda_{2}}\left(\frac{\lambda_{2}x}{\lambda_{2}-x}\middle|\lambda_{2}\right) f_{\lambda_{2}}(\lambda_{2}) d\lambda_{2} \\ \bar{F}_{\Lambda}(x) &= \int_{x}^{\infty} \bar{F}_{\lambda_{1}\mid\lambda_{2}}\left(\frac{\lambda_{2}x}{\lambda_{2}-x}\middle|\lambda_{2}\right) f_{\lambda_{2}}(\lambda_{2}) d\lambda_{2}. \end{split}$$

Differentiating it with respect to x, we get

$$f_{\Lambda}(x) = \int_{x}^{\infty} \frac{\lambda_{2}^{2}}{(\lambda_{2} - x)^{2}} f_{\lambda_{1}|\lambda_{2}} \left( \frac{\lambda_{2}x}{\lambda_{2} - x} \middle| \lambda_{2} \right) f_{\lambda_{2}}(\lambda_{2}) d\lambda_{2}$$
$$= \int_{x}^{\infty} \frac{\lambda_{2}^{2}}{(\lambda_{2} - x)^{2}} f_{\lambda_{1},\lambda_{2}} \left( \frac{\lambda_{2}x}{\lambda_{2} - x}, \lambda_{2} \right) d\lambda_{2}$$
$$= \int_{0}^{\infty} \left( \frac{t + x}{t} \right)^{2} f_{\lambda_{1},\lambda_{2}} \left( \frac{x(t + x)}{t}, t + x \right) dt. \quad (20)$$

Substituting (5) into (20) and using the binomial expansion and [22, eqs. (3.471.9, 8.471.1)], we get

$$f_{\Lambda}(x) = \frac{1}{2\mathcal{K}_{2,n}} \int_{0}^{\infty} \left(\frac{t+x}{t}\right)^{2} e^{-\frac{(t+x)^{2}}{t}} \\ \times \left(\frac{(t+x)^{2}(t-x)^{2}}{t^{2}}\right) \left(\frac{x(t+x)^{2}}{t}\right)^{n-2} dt \\ = \frac{1}{2\mathcal{K}_{2,n}} x^{n-2} e^{-2x} \sum_{k=0}^{2n} {2n \choose k} x^{2n-k} \\ \times \int_{0}^{\infty} \frac{(t-x)^{2}}{t^{n+2-k}} e^{-\left(t+\frac{x^{2}}{t}\right)} dt \\ = \frac{1}{\mathcal{K}_{2,n}} x^{2(n-1)} e^{-2x} \sum_{k=0}^{2n} {2n \choose k} \\ \times \left((n-k-2x)\mathcal{K}_{k-n}(2x) + 2x\mathcal{K}_{k+1-n}(2x)\right). \quad (21)$$

*Proof—Corollary 1: CDF of*  $\Lambda$  (*Case:* m = 2): Consider the following integral, which can be simplified into a single Meijer G function [34, p. 419] first by using [22, eq. (9.34.4)]

*Proof—Theorem 2: PDF of*  $\Lambda$  (*Case:* m = 2): From the and then by applying [35, eq. (7.34.21.2.1)] with a substitution  $u \doteq 2t$ :

$$\int_{0}^{x} t^{\mu} \mathcal{K}_{\nu}(t) e^{-t} dt = \sqrt{\pi} \int_{0}^{x} t^{\mu} \mathcal{G}_{1\,2}^{2\,0} \left( 2t |_{-\nu,\nu}^{0.5} \right) dt$$
$$= \frac{\sqrt{\pi}}{2^{\mu+1}} \int_{0}^{2x} u^{\mu} \mathcal{G}_{1\,2}^{2\,0} \left( u |_{-\nu,\nu}^{0.5} \right) du$$
$$= \frac{\sqrt{\pi}}{2^{\mu+1}} \mathcal{G}_{2\,3}^{2\,1} \left( 2x |_{\mu-\nu+1,\mu+\nu+1,0}^{1,\mu+1.5} \right).$$
(22)

Now, let us consider the cdf of  $\Lambda$ , i.e.,

$$F_{\Lambda}(x) = \int_{0}^{x} f_{\Lambda}(t) dt$$
  
=  $\frac{1}{\mathcal{K}_{2,n}} \sum_{k=0}^{2n} {2n \choose k}$   
 $\times \left( (n-k) \int_{0}^{x} t^{2n-2} e^{-2t} \mathcal{K}_{k-n}(2t) dt - 2 \int_{0}^{x} t^{2n-1} e^{-2t} \mathcal{K}_{k-n}(2t) dt + 2 \int_{0}^{x} t^{2n-1} e^{-2t} \mathcal{K}_{k+1-n}(2t) dt \right).$  (23)

Applying the result of (22) in (23), we get (7).

*Proof—Corollary 2: Average SER (Case:* m = 2): Equation (9) can be simplified as follows using integration by parts and the Leibniz's rule for differentiation [36, eq. (32)], [37]:

$$P_{s} = \int_{0}^{\infty} \mu \mathcal{Q}(\sqrt{2\nu xP}) dF_{\Lambda}(x)$$
$$= \mu \int_{0}^{\infty} \frac{e^{-\nu P x}}{\sqrt{2\pi}} \cdot \frac{\sqrt{2\nu P}}{2\sqrt{x}} F_{\Lambda}(x) dx.$$
(24)

Consider the integral

$$\begin{split} \mathbb{I}(q,\alpha,\beta,\gamma) &= \int\limits_{0}^{\infty} x^{-0.5} e^{-qx} \mathcal{G}_{2\,3}^{2\,1} \left( 4x \big|_{\beta,\gamma,0}^{1,\alpha} \right) dx \\ &= \mathcal{L} \left\{ x^{-0.5} \mathcal{G}_{2\,3}^{2\,1} \left( 4x \big|_{\beta,\gamma,0}^{1,\alpha} \right) \right\} \Big|_{s \doteq q} \end{split}$$

where  $q, \alpha, \beta$ , and  $\gamma$  are positive reals, and  $\mathcal{L}\{.\}$  denotes the Laplace transform. It can be solved using [33, eq. (4.23.34)] to get

$$\mathbb{I}(q,\alpha,\beta,\gamma) = q^{-0.5} \mathcal{G}_{3\,3}^{2\,2} \left(\frac{4}{q}\Big|_{\beta,\gamma,0}^{0.5,1,\alpha}\right).$$
(25)

Substituting (7) into (24) and using (25) complete the proof.  $\blacksquare$ 

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