Partial and Opportunistic Relay Selection with Outdated Channel Estimates

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***Abstract*—This paper investigates the impact of using outdated channel estimates for relay selection and signal amplification on the performance of amplify-and-forward (AF) relays under partial relay selection (PRS) and opportunistic relay selection (ORS). In practice, outdated channel state information (CSI) can occur due to feedback or scheduling delay. Both variable gain (VG) AF and fixed gain AF schemes are considered. Outage probability, the average bit error rate (BER) and simplified high signal-to-noise ratio approximations are derived. The effect of parameters such as the number of relays, the rank of chosen relay, and the correlation between the delayed and current channel state information are analyzed. Outdated CSI for computing relay gains in PRS causes about 2 dB loss. In ORS, a 3% reduction in correlation causes up to an order of magnitude increase in the outage probability.**

***Index Terms*—Relay selection, amplify-and-forward, outdated channel state information, outage probability, error performance.**

I. Introduction

COOPERATIVE communication using relays is capable of efficiently combating wireless impairments and coverage extension [1], [2]. In such systems, performance can be improved by selecting one of the relay terminals [3]–[15]. Relay selection schemes investigated in the literature can be classified as: (1) opportunistic relay selection [3], and (2) partial relay selection [4]. In opportunistic relay selection, a single relay based on the instantaneous global (two hop) channel state information of the network is selected to assist the source [3]. In partial relay selection only local (single-hop) information is used to activate a relay [4].

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The performance of the opportunistic relay selection scheme in dual-hop transmissions has been analyzed in the literature (see [5]–[9]). The outage probability is analyzed in [5], and the error performance of opportunistic relay selection with variable gain (VG) (also known as channel state information (CSI)-assisted) AF relaying under Rayleigh and Rician fading is analyzed in [6] and [7]. The opportunistic relay selection scheme achieves significant performance gains at high signal-to-noise ratio (SNR) at the expense of complexity. In [4], a new reduced complexity partial relay selection scheme using VG AF relays was introduced. Although the performance of this scheme is inferior to that of the opportunistic relay selection scheme, this simpler approach finds wide applicability especially in low complexity ad-hoc and sensor networks because such network nodes may not have resources to implement complex relay selection protocols. Among the works that have covered partial relay selection, in [4], the performance with VG AF relaying has been analyzed. In [11] and [12], the performance of partial relay selection with fixed gain (FG) AF relaying has been studied. Very recently diversity and coding gains of partial relay selection with FG AF relaying over Nakagami-m fading channels have been studied in [13].

So far, only few papers have investigated the impact of outdated CSI on the performance of opportunistic relay selection and partial relay selection systems (e.g. [17]–[20]). In time-varying channels, outdated CSI could be used for relay selection due to feedback delay [15] or scheduling delay [16]. Moreover, outdated CSI may also be used for signal amplification at the relay. Although outdated CSI corresponds to several practical scenarios, it has apparently not been comprehensively investigated in the existing literature.

In this paper, we study the performance of two reduced complexity FG and VG AF partial relay selection systems in the presence of outdated CSI due to feedback/scheduling delay. Systems with FG AF relays retransmit using a constant amplification factor regardless of the amplitude of the first hop; the advantage is that instantaneous channel estimation is not required at the relay. The VG AF protocol studied in this paper differs from [17] as it considers the effect of using outdated CSI for *both* relay selection and amplification gain. During the data transmission phase, the selected relay does not estimate the amplitude of the first hop. Next, we also analyze two VG AF based opportunistic relay selection systems, where selection is based on the bottleneck SNR (i.e., minimum of 'source-relay', 'relay-destination' link SNRs) of the path. The VG AF protocol that uses outdated CSI for both

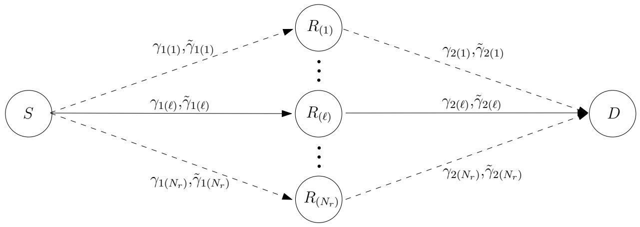

Fig. 1. The cooperative communication system model based on AF relays.

relay selection and amplification gain, and the VG AF protocol presented in [17] are considered. In all four cases of study, we derive new outage probability and average bit error rate (BER) expressions and verify their correctness using Monte Carlo simulations. These expressions reveal new interesting insights into the system design. They are also general because the k-th worst (equivalently choosing the $(N_r - k)$-th best) relay selection with outdated CSI are included. Thus the best relay selection scheme with perfect CSI analyzed in the current literature [4], is a special case of our analysis. Our new results show the relative performance gains between partial relay selection and opportunistic relay selection schemes under a wide variety of conditions when parameters such as the chosen relay rank, the level of CSI accuracy and SNR imbalance are varied.

The rest of the paper is organized as follows. In Section II we introduce the partial relay selection system model with fixed and VG AF relaying schemes, and opportunistic relay selection system model with VG AF relaying. The performance of partial relay selection systems is investigated in Section III. Section IV presents the performance analysis of opportunistic relay selection systems. Numerical and simulation examples are presented in Section V. Finally in Section VI, we conclude with some remarks.

II. System Model

A. Partial Relay Selection

As depicted in Fig. 1, consider a dual-hop AF relay system, with a single source S, a single destination D and N_r relays. The direct link between S and D is absent in this system, which may be a result of high shadowing between S and D. According to partial relay selection, S selects a single relay, $R_{(k)}$, with the k-th worst $S-R$ link, $k = 1, \ldots, N_r$. The analysis of the selection of the kth worst relay is due to two reasons. First, this ensures that analysis is fairly general. Second, while it is always desirable to operate the network with the best relay, sometimes it may not be available to assist the source [14]. In this network we assume that all links are subject to Rayleigh fading, i.e., the channels gains are complex Gaussian random variables (RVs) with zero mean and unit variance. Rayleigh fading is applicable when there is no line-of-sight propagation and there is a rich scattering environment.

When the system employs the FG AF protocol, relays can be relieved from the task of channel estimation as required with VG AF relaying. The system can be implemented using pilots transmitted from the relays [11]. The $S-R$ link CSI estimated from these pilots are used to select a relay at S. However, in some cases the data transmission between S and D may not be scheduled immediately after a relay is selected. Therefore, the FG AF protocol must consider the uplink and downlink imperfect reciprocity conditions [16], e.g. a *scheduling delay*. When the system is implemented with VG AF relays, S can periodically monitor the quality of its connectivity with the relays via transmission of a local *delayed* feedback. Thus, both in VG AF and in FG AF relaying, the CSI used for relay selection may differ substantially from the actual CSI during data transmission.

Let the modulated signal transmitted by S be denoted as x with $E\left(|x|^2\right) = 1$, where $E(\cdot)$ denotes statistical expectation. During the first time slot S communicates with $R_{(k)}$. In the second time slot $R_{(k)}$ transmits its received signal to D. The received signal at $R_{(k)}$ can be written as

$$y_{R_{(k)}} = \sqrt{P_s} h_{S,R_{(k)}} x + n_{R_{(k)}}, \tag{1}$$

where P_s is the transmit power at S, $h_{S,R_{(k)}}$ is the channel between S and $R_{(k)}$ and $n_{R_{(k)}}$ is the additive white Gaussian noise (AWGN) satisfying $E\left(|n_{R_{(k)}}|^2\right) = N_{01}$. The relay multiplies $y_{R_{(k)}}$ by a gain, G and the output is transmitted to D. The received signal at D is given by

$$y_D = h_{R_{(k)},D} G y_{R_{(k)}} + n_D, \tag{2}$$

where $h_{R_{(k)},D}$ is the channel between $R_{(k)}$ and D, and n_D is the AWGN satisfying $E\left(|n_D|^2\right) = N_{02}$.

In order to facilitate the mathematical analysis in the sequel, let $\widetilde{\gamma}_{1(k)} = |h_{S,R_{(k)}}|^2 \eta_1$ and $\widetilde{\gamma}_{2(k)} = |h_{R_{(k)},D}|^2 \eta_2$, where $\eta_1 = \frac{P_s}{N_{01}}$, $\eta_2 = \frac{P_r}{N_{02}}$ and P_r is the transmit power of $R_{(k)}$. Moreover, define $\gamma_{1(k)} = |g_{S,R_{(k)}}|^2 \eta_1$ and $\gamma_{2(k)} = |g_{R_{(k)},D}|^2 \eta_2$ where $g_{S,R_{(k)}}$ and $g_{R_{(k)},D}$ denote the channels at the selection decision instant. Note that the relay selection would be based on $\gamma_{1(k)}$ and $\gamma_{2(k)}$ and the link SNRs experienced by the signal, $\widetilde{\gamma}_{1(k)}$ and $\widetilde{\gamma}_{2(k)}$, are delayed versions of $\gamma_{1(k)}$ and $\gamma_{2(k)}$ respectively.

Let $\gamma_{1(1)} \le \gamma_{1(2)} \le \cdots \le \gamma_{1(N_r)}$ be the order statistics obtained by arranging $\gamma_{1(\ell)}$ for $\ell = 1, \ldots, N_r$ in an increasing order of magnitude. For our analysis of partial relay selection, the PDF of $\gamma_{1(k)}$ is required, based on which the selection of the respective relay $R_{(k)}$ is made.

1) Fixed Gain Relaying: Consider a partial relay selection system with FG AF relays [11], [12]. Assuming that $R_{(k)}$ only knows the statistics of the $S-R$ channel, the gain is chosen as:

$$G = \sqrt{\frac{P_r}{P_s E\left(|h_{S,R_{(k)}}|^2\right) + N_{01}}}. \tag{3}$$

In this case, the end-to-end SNR can be written as

$$\gamma_{eq1} = \frac{\widetilde{\gamma}_{1(k)} \widetilde{\gamma}_{2(k)}}{C + \widetilde{\gamma}_{2(k)}}, \tag{4}$$

where $C = \frac{P_r}{G^2 N_{01}}$. From [17, Eq. (9)], we know that the probability density function (pdf) of $\widetilde{\gamma}_{1(k)}$ is given by

$$f_{\widetilde{\gamma}_{1(k)}}(x) = k \binom{N_r}{k} \sum_{m=0}^{k-1} \frac{(-1)^m}{\eta_1} \binom{k-1}{m} \times \frac{1}{(N_r - k + m)(1-\rho) + 1} e^{-\frac{(N_r - k + m + 1)x}{((N_r - k + m)(1-\rho)+1)\eta_1}}, \tag{5}$$

where $0 \leq \rho \leq 1$ is the correlation coefficient between $\widetilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$, and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. Hence, we obtain an expression for C given by

$$\begin{aligned} C &= E\left(\widetilde{\gamma}_{1(k)}+1\right) \\ &= k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{\eta_1((N_r-k+m)(1-\rho)+1)} \\ &\times \int_0^\infty (x+1)e^{-\frac{(N_r-k+m+1)x}{((N_r-k+m)(1-\rho)+1)\eta_1}}dx \\ &= 1+k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)^2}. \end{aligned} \tag{6}$$

2) Variable Gain Relaying: Now consider a partial relay selection system in which each relay only makes one channel measurement based on which the selection of a relay is made at S. The selected relay also uses the same outdated information to amplify the signal, $y_{R_{(k)}}$. Hence the VG factor at the relay can be expressed as

$$G = \sqrt{\frac{P_r}{P_s|g_{S,R_{(k)}}|^2+N_{01}}}. \tag{7}$$

With the choice of the VG factor in (7), this scheme has a reduced implementation complexity compared to the VG scheme presented in [17], since the system in [17] requires the relay to estimate $g_{S,R_{(k)}}$ as well as $h_{S,R_{(k)}}$.

Plugging (7) into (2) and after some manipulations, the end-to-end SNR can be written as

$$\gamma_{eq2} = \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\widetilde{\gamma}_{2(k)}+1}. \tag{8}$$

Note that in order to study the performance metrics of this system further analysis is required. It is seen from (8) that the form of the instantaneous end-to-end SNR (γ_{eq2}) is different from [17, Eq. (3)]. Consequently, there is a need to derive new expressions.

B. Opportunistic Relay Selection

Consider a system in which relay selection is performed based on CSI of all $S-R$ and $R-D$ links. We assume that the instantaneous channel measurements of each $S-R_{(\ell)}$ and $R_{(\ell)}-D$ links are transmitted back to S, and that these channel states may have changed at the time of actual communication. We now describe two schemes for communication:

1) Variable Gain Relaying I: Let $\hat{\gamma}_\ell = \min(\gamma_{1(\ell)}, \gamma_{2(\ell)})$, and let $\hat{\gamma}_k$ be the kth smallest among the $\hat{\gamma}_\ell$s. Then links $S-R_{(k)}$ and $R_{(k)}-D$ will be chosen for communication. The actual SNR experienced during communication on links $S-R_{(k)}$ and $R_{(k)}-D$ would be $\widetilde{\gamma}_{1(k)}$ and $\widetilde{\gamma}_{2(k)}$ respectively. Since we assume VG AF relays, G is given by (7) and the end-to-end SNR (γ_{eq3}) is given by $\gamma_{eq3} = \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\widetilde{\gamma}_{2(k)}+1}$.

2) Variable Gain Relaying II: As in the case of VG relaying I, the relay selection at S is performed using outdated CSI feedback from the relays. However, it is assumed that the selected relay has instantaneous CSI of the $S-R$ link through pilot assisted channel estimation using packets received from S, and will select the amplification gain factor accordingly. We include this analysis in order to quantify the benefit of having perfect CSI at the relays for signal amplification. The VG applied is given by $G = \sqrt{\frac{P_r}{P_s|h_{S,R_{(k)}}|^2+N_{01}}}$ and the resulting end-to-end SNR (γ_{eq4}) can be expressed as $\gamma_{eq4} = \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\widetilde{\gamma}_{1(k)}+\widetilde{\gamma}_{2(k)}+1}$.

III. Partial Relay Selection

In this section, we derive important performance metrics; the outage probability and the average BER for the partial relay selection system with FG and VG relaying.

A. Fixed Gain Relaying

1) Outage Probability: The outage probability defined as the probability that the end-to-end SNR drops below a pre-defined SNR threshold γ_T, is an important quality of service (QoS) measure. For this sytem, the outage probability was found to be,

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &= 1-2k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)} \\ &\times e^{-\frac{(N_r-k+m+1)\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1}}\sqrt{\frac{((N_r-k+m)(1-\rho)+1)C\gamma_T}{(N_r-k+m+1)\eta_1\eta_2}} \\ &\times K_1\left(2\sqrt{\frac{(N_r-k+m+1)C\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1\eta_2}}\right), \end{aligned} \tag{9}$$

where $K_\nu(x)$ is the νth order modified Bessel function of the second kind [23, Sec. (9.6)]. The derivation steps can be found in Appendix I. The outage probability for the special case of $\rho = 1$ and $N_r = k$ is given by [11, Eq. (5)]. Although the above result (9) gives the exact outage probability, a simpler high-SNR approximation is desirable in order to gain further insights, in terms of the diversity order and the coding gain.

The asymptotic outage probability for large η_1 and η_2 with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &\approx \frac{\gamma_T}{\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{(N_r-k+m)(1-\rho)+1} \\ &\times\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)}\right)+\phi\right), \end{aligned} \tag{10}$$

with $\phi = e^{-\Lambda} + \Lambda\left(1-\gamma+\mathrm{Ei}(-\Lambda)-\ln(\Lambda)\right)$, $\Lambda = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\frac{((N_r-k+m)(1-\rho)+1)}{(N_r-k+m+1)^2}$, $\gamma = 0.57721...$ is the Euler–Mascheroni constant and $\mathrm{Ei}(x)$ is the exponential integral function [23, Eq. (5.1.2)]. The proof of (10) is given in Appendix II.

The simple form in (10) helps obtain better insight in to the system. High SNR approximations for the special cases $\rho = 0, 1$ are given below.

$$F_{\gamma_{eq1}}(\gamma_T) \approx \begin{cases} \frac{\gamma_T}{\eta_1}\left(\frac{\Lambda}{\mu}\ln(\eta_1)+1.01004771...\right) & \rho = 0, \\ \sum_{m=0}^{k-1}\frac{k\binom{N_r}{k}\gamma_T(-1)^m\binom{k-1}{m}\ln\left(\frac{\eta_1}{N_r-k+m+1}\right)}{\eta_1} & \rho = 1. \end{cases} \tag{11}$$

2) Average Bit Error Rate: We now proceed to analyze the system's average BER. For many modulation formats used in wireless applications, the average BER can be expressed as

$$P_b = \alpha\, E[Q(\sqrt{\beta\gamma_{eq1}})] = \frac{\alpha}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}} dt, \tag{12}$$

where $\alpha, \beta > 0$ are constants depending on the modulation scheme, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ is the Gaussian Q-function. Eq. (12) can be evaluated with the help of [21, Eq. (4.16.33)] and we arrive at the following expression for the average BER given by

$$P_b = \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta\eta_1}kC}{2\eta_2}\sum_{m=0}^{k-1}\frac{\binom{N_r}{k}(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)} \times \frac{e^{\varsigma_3}\left(K_1(\varsigma_3)-K_0(\varsigma_3)\right)}{\left(\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)}+\beta\eta_1\right)^{\frac{3}{2}}}, \tag{13}$$

where $\varsigma_3 = \frac{C(N_r-k+m+1)}{\eta_2(2(N_r-k+m+1)+\beta\eta_1((N_r-k+m)(1-\rho)+1))}$. The average BER for the special case with $\rho = 1$ and $N_r = k$ is given in [11, Eq. (12)].

Substituting (10) into (12) and solving the integral, the average BER at high SNR can be written as

$$P_b^\infty \approx \frac{\alpha k}{2\beta\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}}{(N_r-k+m)(1-\rho)+1} \times\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho)+1)\eta_1}{(N_r-k+m+1)}\right)+\phi\right). \tag{14}$$

B. Variable Gain Relaying

1) Outage Probability: Follwing the steps detailed in Appendix III, we can arrive at the following lower bound for the outage probability for the system,

$$F_{\gamma_{eq2}}(\gamma_T) \geq 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}k\rho^p}{(1-\rho)^{p-n-1}\eta_1^{p-n}\eta_2^{n+1}p!} \times\frac{\gamma_T^{p+1}(p+n+1)!}{((N_r-k+m)(1-\rho)+1)^{p+n+2}}e^{-\frac{\gamma_T}{\eta_1(1-\rho)}} \times\mathcal{U}\left(p+n+2, n+2; \frac{\gamma_T}{\eta_2((N_r-k+m)(1-\rho)+1)}\right), \tag{15}$$

where $\mathcal{U}(a,b;z)$ is the confluent hypergeometric function of the second kind [23, Eq. (13.1.3)].

In Section V, extensive simulation results to complement (15) are presented. We stress that the outage probability predicted from (15) and simulations match reasonably well even at low SNRs of $\eta_1 = \eta_2 = 5$ dB. For the special case of $\rho = 1$ and $N_r = k$, the outage probability is given in [4, Eq. (2)]. In order to gather useful insights, we have developed a high SNR approximation for the outage probability. The asymptotic outage probability for large η_1 and η_2 with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by

$$F_{\gamma_{eq2}}(\gamma_T) \approx k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times\left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho)}\right)\left(\frac{\gamma_T}{\eta_1}\right). \tag{16}$$

The proof of (16) is given in Appendix IV with p_1, p_2 and ω, respectively defined.

2) Average Bit Error Rate: Using (12), and [21, Eq. (4.22.16)], we derive the average BER of the partial relay selection system with VG AF relaying as

$$P_b \geq \frac{\alpha}{2} - \frac{\alpha k}{\sqrt{8\pi}}\sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}}{(1-\rho)^{p-n-1}\eta_1^{p-n}\eta_2^{\frac{n}{2}}p!} \times\frac{(p+n+1)!\rho^p\beta^{\frac{n}{2}-p}\Gamma(p-n+\frac{1}{2})\varsigma_2^{\frac{n}{2}+1}}{((N_r-k+m)(1-\rho)+1)^{p+1+\frac{n}{2}}(\varsigma_1+\frac{\varsigma_2}{2})^{p+\frac{3}{2}}} \times\frac{\Gamma(p+\frac{3}{2})}{\Gamma(2p+\frac{5}{2})}{}_2F_1\left(p+\frac{3}{2}, p+n+2, 2p+\frac{5}{2}; \frac{\varsigma_1-\frac{\varsigma_2}{2}}{\varsigma_1+\frac{\varsigma_2}{2}}\right), \tag{17}$$

where $\varsigma_1 = \frac{1}{\beta\eta_1(1-\rho)} - \frac{1}{2\eta_2((N_r-k+m)(1-\rho)+1)\beta} + \frac{1}{2}$, $\varsigma_2 = \frac{1}{\beta\eta_2((N_r-k+m)(1-\rho)+1)}$ and ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function[23, Eq. (15.1.1)]. The average BER for the special case with $\rho = 1$, $N_r = k$ and BPSK modulation is given in [4, Eq. (14)].

Consider the average BER at high SNR. Following a similar approach as in FG AF relaying, the average BER for the VG AF relaying, in the high SNR regime can be written as

$$P_b^\infty \approx \frac{\alpha k\binom{N_r}{k}}{2\beta\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times\left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho)}\right). \tag{18}$$

The average output power at the relay in the case of VG AF relaying would be different from P_r due to selecting the amplification gain factor using outdated CSI. Hence, for a fair comparison of the FG and VG AF schemes, we make an average power normalization, so that the average output power at the relay is equal to unity. In order to do so, a modified amplification gain factor, $G = \sqrt{\frac{P_r/\xi}{P_s|g_{S,R_{(k)}}|^2+N_{01}}}$ is introduced, where

$$\xi = E\left(\frac{\widetilde{\gamma}_{1(k)}+1}{\gamma_{1(k)}+1}\right) = \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1^2}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\int_0^\infty\int_0^\infty\left(\frac{x+1}{y+1}\right) \times I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right)e^{-\left(\frac{x}{(1-\rho)\eta_1}+\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}\right)}dxdy. \tag{19}$$

Using [24, Eq. (9)] and [21, Eq. (4.2.6)], we can evaluate (19) to arrive at a closed-form expression for the power scaling factor given by

$$\xi = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\left(\frac{\rho}{N_r-k+m+1} -(1-\rho)\left(1+\frac{1}{\eta_1}\right)e^{\frac{N_r-k+m+1}{\eta_1}}\mathrm{Ei}\left(-\frac{N_r-k+m+1}{\eta_1}\right)\right). \tag{20}$$

IV. Opportunistic Relay Selection

A. Variable Gain Relaying I

1) Outage probability: To derive the outage probability, the distribution functions of $\widetilde{\gamma}_{1(k)}$ and $\widetilde{\gamma}_{2(k)}$ are required. Using the approach given in Appendix V, the pdf of $\widetilde{\gamma}_{1(k)}$ can be derived as follows:

$$f_{\widetilde{\gamma}_{1(k)}}(x) = \frac{k}{\eta_1}\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m))} \times \left(e^{-\frac{x}{\eta_1}} + \frac{(N_r-k+m)\eta_2 e^{-\frac{(N_r-k+m+1)x}{\rho_1\bar{\eta}+(1-\rho_1)\eta_1(N_r-k+m+1)}}}{\rho_1\bar{\eta}+(1-\rho_1)(N_r-k+m+1)\eta_1}\right). \tag{21}$$

Since the relay selection criteria is symmetric, by interchanging η_1 and ρ_1 with η_2 and ρ_2 respectively, the pdf of $\widetilde{\gamma}_{2(k)}$ can be obtained. An expression for the cdf of $\widetilde{\gamma}_{2(k)}$ can be obtained by integrating the pdf and is given by

$$F_{\widetilde{\gamma}_{2(k)}}(y) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}p_m\sum_{i=1}^{2}q_{m,i}e^{-\frac{r_{m,i}}{\eta_1}y}, \tag{22}$$

where $p_m = \frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_1}{\bar{\eta}}(N_r-k+m))}$, $q_{m,1} = 1$, $q_{m,2} = \frac{(N_r-k+m)\eta_1}{(N_r-k+m+1)\eta_2}$, $r_{m,1} = \frac{\eta_1}{\eta_2}$ and $r_{m,2} = \frac{(N_r-k+m+1)\eta_1}{\rho_2\bar{\eta}+(1-\rho_2)\eta_2(N_r-k+m+1)}$. Observing that the above distribution is a sum of exponentials makes further derivations much convenient. Having derived all the required distributions, we proceed to derive the outage probability, using $\gamma_{eq3} \approx \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\widetilde{\gamma}_{2(k)}}$ as

$$F_{\gamma_{eq3}}(\gamma_T) \approx 1 - \Pr\left(\frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\widetilde{\gamma}_{2(k)}} > \gamma_T\right) = 1 - \int_{\gamma_T}^{\infty}\int_0^{\infty} f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)\left(1 - F_{\widetilde{\gamma}_{2(k)}}\left(\frac{y\gamma_T}{x-\gamma_T}\right)\right)dydx. \tag{23}$$

By modifying the limits of the outer integral, and substituting results from (61) and (22), $F_{\gamma_{eq3}}(\gamma_T)$ can be written as

$$F_{\gamma_{eq3}}(\gamma_T) \approx 1 - \frac{k^2\binom{N_r}{k}^2}{(1-\rho_1)\eta_1^2}\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i} \times \sum_{j=1}^{2}q'_{n,j}\int_0^{\infty}\int_0^{\infty}e^{-\frac{x+\gamma_T}{(1-\rho_1)\eta_1}}e^{-\frac{y}{\eta_1}(r'_{n,j}+\frac{r_{m,i}\gamma_T}{x})} \times I_0\left(\frac{2\sqrt{\rho_1(x+\gamma_T)y}}{(1-\rho_1)\eta_1}\right)dydx. \tag{24}$$

Simplifying (24) yields

$$F_{\gamma_{eq3}}(\gamma_T) \approx 1 - \frac{k^2\binom{N_r}{k}^2}{(1-\rho_1)\eta_1}\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i} \times \sum_{j=1}^{2}q'_{n,j}\int_0^{\infty}\frac{e^{-\frac{x+\gamma_T}{(1-\rho_1)\eta_1}\left(1-\frac{\rho_1}{(1-\rho_1)(r'_{n,j}+\frac{r_{m,i}\gamma_T}{x})}\right)}}{(r'_{n,j}+\frac{r_{m,i}\gamma_T}{x})}dx, \tag{25}$$

where $p'_n = \frac{\binom{k-1}{n}(-1)^n}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+n)}$, $q'_{n,1} = 1$, $q'_{n,2} = \frac{\eta_2(N_r-k+n)}{\bar{\eta}}$, $r'_{n,1} = \frac{1}{1-\rho_1}$ and $r'_{n,2} = \left(\frac{(N_r-k+n+1)\eta_1}{\bar{\eta}} + \frac{\rho_1}{(1-\rho_1)}\right)$. Since a closed-form solution with standard mathematical functions does not exist for the integral in (25) we proceed to find a high SNR approximation for $F_{\gamma_{eq3}}(\gamma_T)$. Substituting

$$\begin{aligned}&\frac{x+\gamma_T}{(1-\rho_1)\eta_1} - \frac{\rho_1 x(x+\gamma_T)}{(1-\rho_1)^2\eta_1(r'_{n,j}x+r_{m,i}\gamma_T)}\\ &= x\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)\\ &+ \frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)\\ &+ \frac{\gamma_T^2\rho_1 r_{m,i}(r'_{n,j}-r_{m,i})}{r'^2_{n,j}\eta_1(1-\rho_1)^2(\gamma_T r_{m,i}+r'_{n,j}x)},\end{aligned} \tag{26}$$

using the Maclaurin series expansion of $e^{-\frac{\gamma_T^2\rho_1 r_{m,i}(r'_{n,j}-r_{m,i})}{r'^2_{n,j}\eta_1(1-\rho_1)^2(\gamma_T r_{m,i}+r'_{n,j}x)}}$ and ignoring higher order terms of $\frac{\gamma_T}{\eta_1}$ yields the following approximation

$$\begin{aligned}F_{\gamma_{eq3}}(\gamma_T) &\approx 1 - \frac{k^2\binom{N_r}{k}^2}{(1-\rho_1)\eta_1}\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i}\\ &\times \sum_{j=1}^{2}q'_{n,j}e^{-\frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)}\\ &\times \int_0^{\infty}e^{-x\left(\frac{1}{(1-\rho_1)\eta_1}-\frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)}\\ &\times \left(\frac{1}{r'_{n,j}} - \frac{\gamma_T r_{m,i}}{r'_{n,j}(r'_{n,j}x+r_{m,i}\gamma_T)}\right)dx + O\left(\left(\frac{\gamma_T}{\eta_1}\right)^2\right).\end{aligned} \tag{27}$$

Using [25, Eq. (11)], $F_{\gamma_{eq3}}(\gamma_T)$ can be expressed as

$$\begin{aligned}F_{\gamma_{eq3}}(\gamma_T) &\approx 1 - k^2\binom{N_r}{k}^2\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i}\\ &\times \sum_{j=1}^{2}q'_{n,j}e^{-\frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)}\\ &\times \left(\frac{1}{r'_{n,j}\left(1-\frac{\rho_1}{(1-\rho_1)r'_{n,j}}\right)} + \frac{\gamma_T r_{m,i}}{(1-\rho_1)\eta_1 r'^2_{n,j}}\right.\\ &\times \left.\ln\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)\right).\end{aligned} \tag{28}$$

2) Average Bit Error Rate: The average BER can be now derived substituting (28) into (12). Through simplifications after the integration, P_b can be expressed as

$$\begin{aligned}P_b &\approx \frac{\alpha}{2} - \frac{\alpha k^2}{2}\binom{N_r}{k}^2\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i}\\ &\times \sum_{j=1}^{2}q'_{n,j}\sqrt{\frac{\beta}{\beta+\frac{2}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)}}\\ &\times \left(\frac{(1-\rho_1)}{(r'_{n,j}(1-\rho_1)-\rho_1)}\right.\\ &\left. - \frac{r_{m,i}\ln\left(\frac{1}{(1-\rho_1)\eta_1}-\frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)}{r'^2_{n,j}\beta(1-\rho_1)\eta_1+2\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)}+r'^2_{n,j}\right)}\right).\end{aligned} \tag{29}$$

We introduce the modified amplification gain factor ξ_2, for a fair comparison among the systems since the average output power at the relay will be different from P_r due to having outdated CSI. Using a similar approach as earlier, ξ_2 can be expressed as

$$\xi_2 = \sum_{m=0}^{k-1} \frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)} \sum_{i=1}^{2} q'_{m,i} \left(\frac{\rho_1}{s_i} -(1-\rho_1)\left(1+\frac{1}{\eta_1}\right) e^{\frac{s_i}{\eta_1}} \operatorname{Ei}\left(-\frac{s_i}{\eta_1}\right)\right), \tag{30}$$

where $s_1 = 1$ and $s_2 = \frac{(N_r-k+m+1)\eta_1}{\bar{\eta}}$.

B. Variable Gain Relaying II

1) Outage Probability:

$$\begin{aligned} F_{\gamma_{eq4}}(\gamma_T) &= \Pr(\gamma_{eq4} < \gamma_T) \\ &= 1 - \int_0^\infty f_{\widetilde{\gamma}_{1(k)}}(x+\gamma_T)\left(1 - F_{\gamma_{2(k)}}\left(\frac{\gamma_T(x+\gamma_T+1)}{x}\right)\right) dx \\ &= 1 - \frac{2k^2}{\eta_1}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q_{n,1j} \\ &\quad \times e^{-(r_{m,i}+r_{n,1j})\frac{\gamma_T}{\eta_1}} \sqrt{\frac{r_{m,i}(\gamma_T+\gamma_T^2)}{r_{n,1j}}} \\ &\quad \times K_1\left(\frac{2}{\eta_1}\sqrt{r_{m,i} r_{n,1j}(\gamma_T+\gamma_T^2)}\right), \end{aligned} \tag{31}$$

where $p_{1m} = \frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m))}$, $q_{n,11} = 1$, $q_{n,12} = \frac{(N_r-k+n)\eta_2}{\rho_1\bar{\eta}+(1-\rho_1)(N_r-k+n+1)\eta_1}$, $r_{n,11} = 1$ and $r_{n,12} = \frac{(N_r-k+n+1)\eta_1}{\rho_1\bar{\eta}+(1-\rho_1)\eta_1(N_r-k+n+1)}$. A simple high SNR approximation can be obtained using $K_1(x) \approx \frac{1}{x}$ as $x \to 0$ in the above result. Therefore, as η_1, η_2 tends to ∞, $F_{\gamma_{eq4}}(\gamma_T)$ can be expressed as

$$F_{\gamma_{eq4}}(\gamma_T) \approx \frac{\gamma_T k^2}{\eta_1}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \times \sum_{j=1}^{2} q_{n,1j}\left(1+\frac{r_{m,i}}{r_{n,1j}}\right). \tag{32}$$

2) Average Bit Error Rate: The average BER can be now derived substituting (31) in (12). However, as the resulting integral does not have a solution in standard mathematical functions, we use the approximation $\gamma_{eq4} \approx \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\widetilde{\gamma}_{1(k)}+\widetilde{\gamma}_{2(k)}}$. Using [22, Eq. 6.621.3] and further simplifications, we can derive the average BER and is given by

$$\begin{aligned} P_b &\approx \frac{\alpha}{2} - \frac{3\pi\alpha k^2}{\sqrt{2}\beta^2\eta_1^2}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} r_{m,i} \\ &\quad \times \sum_{j=1}^{2} \frac{q_{n,1j}}{\left(\frac{1}{2}+\frac{(\sqrt{r_{m,i}}+\sqrt{r_{n,1j}})^2}{\beta\eta_1}\right)^{\frac{5}{2}}} \\ &\quad \times {}_2F_1\left(\frac{5}{2},\frac{3}{2};2;\frac{\beta\eta_1+2(\sqrt{r_{m,i}}-\sqrt{r_{n,1j}})^2}{\beta\eta_1+2(\sqrt{r_{m,i}}+\sqrt{r_{n,1j}})^2}\right). \end{aligned} \tag{33}$$

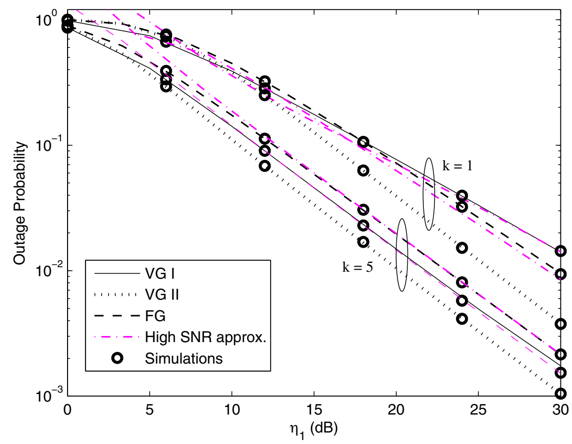

Fig. 2. The outage probability with best and worst relay selection ($N_r = 5, \rho = 0.8, \eta_1 = \eta_2$).

Using (32) in (12), as $\eta_1, \eta_2 \to \infty$ a high SNR approximation for the above BER can be obtained and is given by

$$P_b \approx \frac{\alpha k^2}{2\beta\eta_1}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \times \sum_{j=1}^{2} q_{n,1j}\left(1+\frac{r_{m,i}}{r_{n,1j}}\right). \tag{34}$$

V. Numerical and Simulation Results

A. Partial Relay Selection

Figures 2-4 show the performance of the two systems investigated in Section III, and also the system analyzed previously in [17], where the relay benefits from the perfect instantaneous CSI, as a benchmark. The VG AF relaying system investigated in this paper is labeled as "VG I" in the figures, and the FG AF relaying system and the system in [17] are labeled as "FG" and "VG II", respectively.

Figure 2 shows the outage probability against η_1 in dB ($N_r = 5, \mu = 1, \rho = 0.8$). Two cases are presented where the best ($k = 5$) and worst ($k = 1$) relay are chosen. We see that, when the best relay is chosen, VG AF relaying outperforms FG AF relaying. In the case of worst relay selection, at low SNR the VG AF relaying outperforms FG AF relaying, while at high SNR it becomes worse. The simpler high SNR approximations show good proximity to the exact results. Simulation results not included here showed that for higher values of k (e.g. best relay) and ρ, the overlap of high SNR approximation with the exact result happens at SNR in the vicinity of 15 dB. For low values of k and ρ it happens around 40 dB.

Figure 3 shows the influence of the correlation ρ on the outage probability. For best relay selection, i.e. $k = 5$, the performance improves with increasing ρ in all systems, as expected. The VG systems outperform the FG system when current and outdated link SNRs are well correlated ($\rho > 0.3$). In the case of worst relay selection, the FG relaying performs better than the VG I system in $\rho < 0.8$ region and the performance of the FG and VG II systems improve as ρ

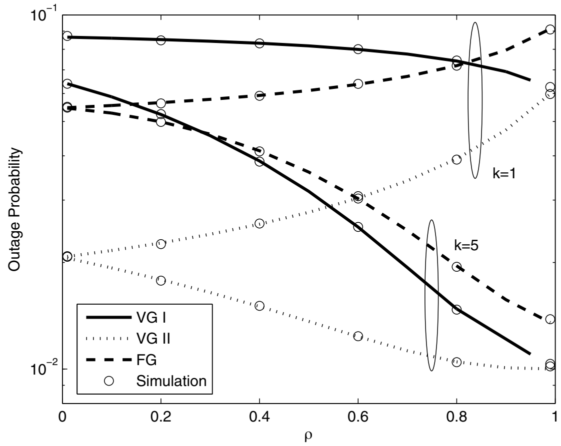

Fig. 3. Outage probability versus ρ, for best ($k = 5$) and worst ($k = 1$) relay selection ($N_r = 5, \gamma_T = 1, \eta_1 = \eta_2 = 20$ dB).

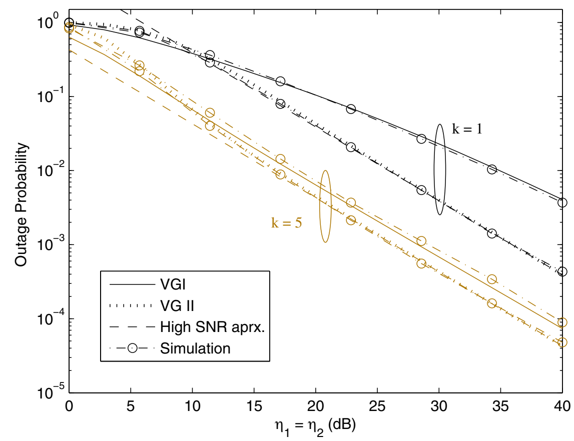

Fig. 5. The outage probability of opportunistic relay selection with best and worst relay selection ($N_r = 5, \gamma_T = 1, \rho_1 = 0.8, \rho_2 = 0.7, \eta_1 = \eta_2$).

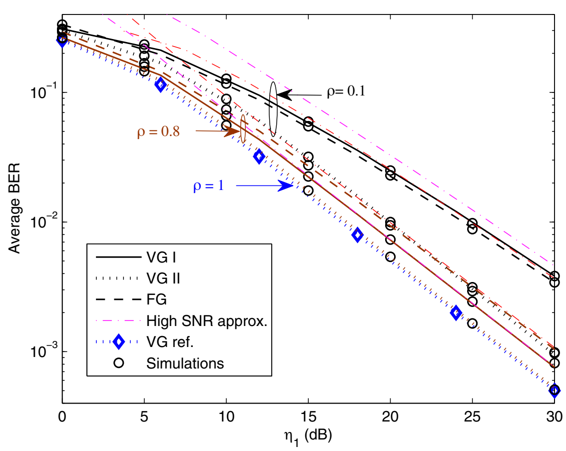

Fig. 4. The average BER under different values of correlation ρ. ($N_r = 5$, $k = 5$, $\eta_1 = \eta_2$).

decreases. With decreasing ρ, the likelihood that the link selected is not the actual worst, increases. The performance of VG I under worst relay selection, in contrast, degrades with decreasing ρ, as the influence of incorrectly selecting the amplification gain factor at the relay becomes more significant. The performance gap between best and worst selection curves in FG and VG II systems vanishes as $\rho \to 0$. If the decision and actual link SNR values are not correlated, the ranking of relays would have no effect on the performance.

Figure 4 presents the average BER with quadrature phase shift keying (QPSK) modulation ($\alpha = \beta = 1$), for two cases where the correlation between the outdated channel estimate and the actual channel is high ($\rho = 0.8$) and low ($\rho = 0.1$). The best relay ($k = 5$) out of all the ($N_r = 5$) relays was chosen and $\eta_1 = \eta_2$. The infinite series in (17) was truncated at 45 terms. In low SNR regions, VG AF relaying outperforms the FG AF counterpart. When the correlation is high ($\rho = 0.8$), VG AF relaying outperforms FG relaying. If the correlation is low ($\rho = 0.1$), at high SNR, the FG system outperform VG I system. We realize that, when the outdated link SNR is less correlated to the current SNR, it is better to use long term statistics of the channel at the relay, when deciding the amplification gain, instead of using the outdated CSI. In all cases, the reference "VG II" system demonstrates better performance. A reference curve for VG AF relaying with $\rho = 1$ is plotted to observe the performance loss due to imperfect information at the relay.

B. Opportunistic Relay Selection

Figures 5-7 show the performance of the two systems investigated in Section IV of this paper. The figures include plots of the high SNR approximation for performance metrics of the system VG I, exact analytic results obtained for the system VG II and its high SNR approximations.

Figure 5 shows the outage probability of the opportunistic relay system for best ($k = 5$) and worst ($k = 1$) relay selection, with $N_r = 5, \gamma_T = 1, \rho_1 = 0.8, \rho_2 = 0.7$ and $\eta_1 = \eta_2$. The VG I system with perfect channel information at the relay performs better than VG II, in both cases. However, in the case of $k = 1$, i.e. the worst relay, the performance loss due to imperfect information at the relay is higher. We further observe that the diversity gain of the systems is one, except for VG II at $k = 1$, in which it is slightly less than one. The performance degradation is enhanced by the power factor correction ξ_2. For low values of k, ξ_2 is higher and increases with η_1. One important thing to note is that, even though there are $N_r(= 5)$ relays, the diversity is not equal to N_r as it would be in the case of having perfect CSI at the source and the relays.

Figure 6 shows the variation of outage probability with the correlation coefficients at $\eta_1 = \eta_2 = 20$ dB, for the best and worst relay selection scenarios. When the best relay is selected ($k = 5$), the outage probabilities of both VG I and II systems decrease as the correlation coefficients ρ_1 and ρ_2 increase. This is expected. It is important to note that as $\rho \to 1$, outage performance changes rapidly. i.e., a small delay may cause the decision link SNRs and actual link SNRs to become different. A significant loss in the performance occurs due to this. If the worst relay was selected, an improvement of outage performance in VG II system is observed as the correlation

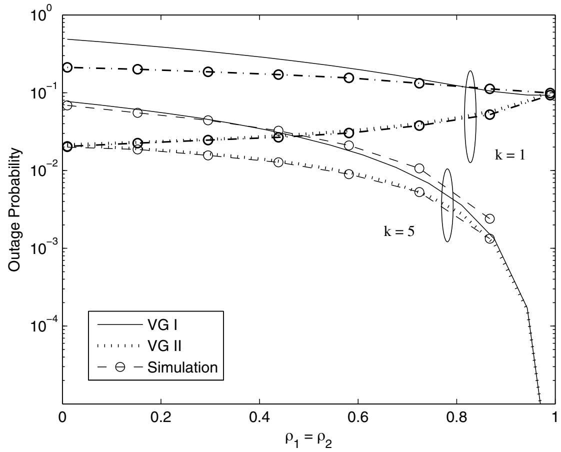

Fig. 6. Outage probability versus ρ_1, for best ($k = 5$) and worst ($k = 1$) of opportunistic relay selection ($N_r = 5, \gamma_T = 1, \eta_1 = \eta_2 = 20$ dB).

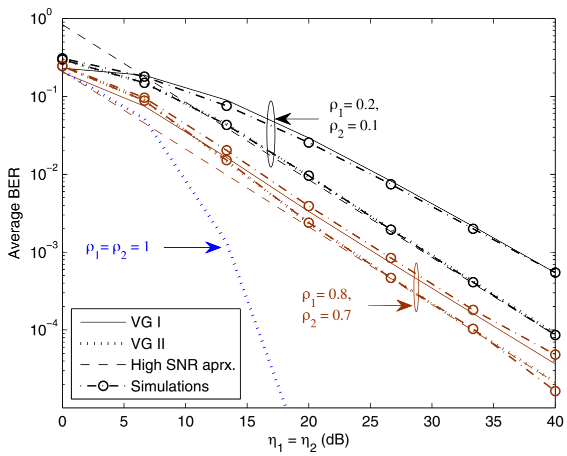

Fig. 7. The average BER for QPSK under different values of correlation ρ_1, ρ_2 for opportunistic relay selection. ($N_r = 5$, $k = 5$).

decreases. This is as expected, because at low correlation there is high likelihood that the worst relay chosen is not actually the worst one. In contrast, the performance degrades in VG I system as the correlation decreases. This is caused by the increase of ξ_2 as $\rho_1 \to 0$. The effect of incorrect selection of gain factor G is apparent. The performance gap between systems VG I and VG II narrows as $\rho_1, \rho_2 \to 1$ for both cases $k = 1$ and $k = 5$. As $\rho_1, \rho_2 \to 0$, in the VG II system, the performance gap between best and worst relay selection scenarios gets reduced. As expected, if the decision and actual link SNRs are not correlated, the ranking of the relays does not make a difference. However for the VG II system, this is not the case due to the effect of incorrect selection of G.

Figure 7 has the plots of average BER versus SNR for the links η_1 and η_2. Here situations of low correlation ($\rho_1 = 0.2$, $\rho_2 = 0.1$) and high correlation($\rho_1 = 0.8$, $\rho_2 = 0.7$) are compared, selecting the best relay ($N_r = k = 5$) with QPSK modulation. As expected, the average BER of the VG II system is lower than that of VG I in both cases, and the performance gap between the systems reduces as ρ_1, ρ_2 increase. A reference curve for the case of $\rho_1 = \rho_2 = 1$ is plotted in the same figure, and from that the major performance degradation due to outdated CSI is clearly visible. With perfect CSI, the diversity gain is equal to the number of relays. However with outdated CSI, it is one, irrespective of the number of relays.

Comparing plots shown in Fig. 3 and Fig. 6, we observe that the outage probability of opportunistic relay selection at 20 dB shows higher variation with changing ρ than partial relay selection. For the case of best relay selection, we notice that at low correlation ($\rho < 0.25$) the performance of partial relay selection slightly outperforms its opportunistic relay selection counterparts. However as correlation value increases, this gap reverses itself. With worst relay selection, partial relay selection systems outperform the respective opportunistic relay selection counterparts. The partial relay selection FG system outperforms opportunistic relay VG I system, but is worse than VG II.

VI. Conclusions

The effect of outdated CSI on partial and opportunistic relay selection with FG AF and VG AF relays was analyzed. The VG AF protocol employs outdated CSI for both relay selection and signal amplification. New analytical expressions and high SNR approximations for the outage probability and the average BER were derived. It was observed that in partial relay selection systems, for low correlation values and with best relay selection, FG relaying performs better than VG AF relaying. However as correlation increases, VG AF relaying outperforms FG AF relaying, while VG AF relaying considered in [17] shows the best performance in all cases. It was further observed that, partial relay selection schemes perform better than opportunistic relay selection counterparts, when the decision CSI and actual CSI has low correlation. However as ρ increases, the opportunistic relay systems shows far superior performance. Performance degradation due to outdated CSI were quantified. In ORS systems, performance degrades by about 10 dB at the error rate of 10^{-3}.

Appendix I : Outage Probability Derivation - Fixed Gain AF Relaying

$$F_{\gamma_{eq1}}(\gamma_T) = \Pr(\gamma_{eq1} < \gamma_T) = \Pr\left(\frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{C + \widetilde{\gamma}_{2(k)}} < \gamma_T\right), \quad (35)$$

where $\Pr(\cdot)$ denotes the probability. Equation (35) can be simplified as

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - \int_{\gamma_T}^{\infty} \Pr\left(\widetilde{\gamma}_{2(k)} > \frac{C\gamma_T}{x - \gamma_T}\right) f_{\widetilde{\gamma}_{1(k)}}(x)dx. \quad (36)$$

Using (5) and the complementary cumulative distribution function (cdf) of $\widetilde{\gamma}_{2(k)}$, we obtain

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_T) &= 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m} \\ &\times \frac{e^{-\frac{(N_r-k+m)\gamma_T}{((N_r-k+m)(1-\rho)+1)\eta_1}}}{(N_r-k+m)(1-\rho)+1} \\ &\times \int_0^{\infty} e^{-\frac{C\gamma_T}{\eta_2 y} - \frac{(N_r-k+m+1)y}{((N_r-k+m)(1-\rho)+1)\eta_1}}dy. \end{aligned} \quad (37)$$

Finally, using [21, Eq. (4.5.25)], the outage probability can be expressed as (9).

APPENDIX II : HIGH SNR APPROXIMATION - FIXED GAIN AF RELAYING

Let $\gamma_u = \min\left\{\widetilde{\gamma}_{1(k)}, \frac{\widetilde{\gamma}_{1(k)}\gamma_2}{C}\right\}$. It is claimed that γ_u is an upper bound for γ_{eq1} in (4)[25].

$$\overline{F}_{\gamma_u}(\gamma_T) = \Pr\{\gamma_u > \gamma_T\} = \Pr\left\{\frac{\gamma_2\widetilde{\gamma}_{1(k)}}{C} > \gamma_T \cap \widetilde{\gamma}_{1(k)} > \gamma_T\right\}$$
$$= \int_{\gamma_T}^{\infty} f_{\widetilde{\gamma}_{1(k)}}(x)\overline{F}_{\gamma_2}(C\gamma_T/x)dx, \tag{38}$$

where $\overline{F}_X(\cdot)$ denotes the complementary cdf of the RV, X. Since the above distributions are known, $\overline{F}_{\gamma_u}(\gamma_T)$ can be expressed as

$$\overline{F}_{\gamma_u}(\gamma_T) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{a}{\eta_1}I, \tag{39}$$

where $I = \int_{\gamma_T}^{\infty} e^{-\left(\frac{C\gamma_T}{\eta_2 x}+\frac{bx}{\eta_1}\right)}dx$, $a = \frac{(-1)^m\binom{k-1}{m}}{(N_r-k+m)(1-\rho)+1}$ and $b = \frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}$. We simplify I as

$$I = \sum_{i=0}^{\infty}\frac{(-1)^i C^i\gamma_T^i}{i!\eta_2^i}\int_{\gamma_T}^{\infty}\frac{e^{-\frac{bx}{\eta_1}}}{x^i}\,dx = \sum_{i=0}^{\infty}\frac{(-1)^i C^i}{i!\eta_2^i}$$
$$\times e^{-\frac{b\gamma_T}{\eta_1}}\int_0^{\infty}\frac{e^{-\frac{bx}{\eta_1}}}{\left(\frac{x}{\gamma_T}+1\right)^i}\,dx. \tag{40}$$

For large η_1 and η_2, I can be approximated using [25, Eq. (11)] as

$$I \approx e^{-\frac{b\gamma_t}{\eta_1}}\left(\sum_{i=2}^{\infty}\frac{(-1)^i C^i\gamma_T}{(i-1)i!\eta_2^i} + \frac{\eta_1}{b} + \frac{C\gamma_T}{\eta_2}\ln\left(\frac{b}{\eta_1}\right)\right). \tag{41}$$

Substituting (41) into (39), using cdf of γ_u, $F_{\gamma_u}(\gamma_T) = 1 - \overline{F}_{\gamma_u}(\gamma_T)$, and after further simplifications, we obtain

$$F_{\gamma_u}(\gamma_T) \approx \frac{\gamma_T}{\eta_1}\sum_{m=0}^{k-1}\frac{k\binom{N_r}{k}(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)}(1 - \frac{C}{\eta_2}$$
$$\times\ln\left(\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1}\right) - \sum_{r=2}^{\infty}\frac{(-1)^r C^r}{\eta_2{}^r r!(r-1)}\Bigg). \tag{42}$$

Simplifying further, we obtain the approximation given in (10).

APPENDIX III : OUTAGE PROBABILITY DERIVATION - VARIABLE GAIN AF RELAYING

In order to derive the outage probability of VG AF relaying, it is convenient to obtain a statistical distribution formula for the general form

$$Y = \frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)} + \widetilde{\gamma}_{2(k)} + c}. \tag{43}$$

Note that $c = 1$ gives the exact expression for γ_{eq2} in (8), while $c = 0$ can be substituted to obtain an analytically feasible approximation. The cdf of the RV, Y can be written as

$$F_Y(\gamma_T) = \Pr\left(\frac{\widetilde{\gamma}_{1(k)}\widetilde{\gamma}_{2(k)}}{\gamma_{1(k)} + \widetilde{\gamma}_{2(k)} + c} < \gamma_T\right). \tag{44}$$

After some mathematical manipulations, (44) can be written as follows

$$F_Y(\gamma_T) = 1 - \int_0^{\infty}\int_0^{\infty}\Pr\left(\widetilde{\gamma}_{2(k)} > \frac{\gamma_T(y+c)}{w}\right)$$
$$\times f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(w+\gamma_T, y)dwdy. \tag{45}$$

It is important to note that, $f_{\widetilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) = f_{\widetilde{\gamma}_{(\ell)}|\gamma_{(\ell)}}(x|y)$, where ℓ represents unordered relays. Hence the joint pdf of $\widetilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ can be established from

$$f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{f_{\widetilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y)}{f_{\gamma_{1(\ell)}(y)}} \times f_{\gamma_{1(k)}}(y). \tag{46}$$

Since $\widetilde{\gamma}_{1(\ell)}$ and $\gamma_{1(\ell)}$ are two correlated exponentially distributed RVs, their joint pdf is given by

$$f_{\widetilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y) = \frac{e^{-\frac{x+y}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1^2}I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right), \tag{47}$$

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind. From [17], we know that the pdf $f_{\gamma_{1(k)}}(y)$ is given by $f_{\gamma_{1(k)}}(y) = \frac{N_r!}{(k-1)!(N_r-k)!}[F_{\gamma_{1(\ell)}}(y)]^{k-1}[1 - F_{\gamma_{1(\ell)}}(y)]^{N_r-k}f_{\gamma_{1(\ell)}}(y)$ where $f_{\gamma_{1(\ell)}}(y) = \frac{1}{\eta_1}e^{-\frac{y}{\eta_1}}$ and $F_{\gamma_{1(\ell)}}(y) = 1 - e^{-\frac{y}{\eta_1}}$. Using the above results in (46) and after some simplifications, the joint pdf of $\widetilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$, $f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)$, can be written as

$$f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{k\binom{N_r}{k}e^{-\frac{x}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1^2}I_0\left(\frac{2\sqrt{\rho xy}}{(1-\rho)\eta_1}\right)$$
$$\times\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}}. \tag{48}$$

Substituting (48) and the complementary cdf of $\widetilde{\gamma}_{2(k)}$ into (45), we obtain

$$F_Y(\gamma_T) = 1 - \int_0^{\infty}\int_0^{\infty} e^{-\frac{\gamma_T(y+c)}{\eta_2 w}}\frac{k\binom{N_r}{k}e^{-\frac{(w+\gamma_T)}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1^2}$$
$$\times I_0\left(\frac{2\sqrt{\rho(w+\gamma_T)y}}{(1-\rho)\eta_1}\right)\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}$$
$$\times e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}}dwdy. \tag{49}$$

Using the infinite series expansion $I_0(x) = \sum_{p=0}^{\infty}\frac{x^{2p}}{2^{2p}(p!)^2}$ from [22, Eq. (8.447.1)] in (49) and [21, Eq. (4.5.29)], (49) can be expressed as

$$F_Y(\gamma_T) = 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{k-1}{m}k\binom{N_r}{k}\rho^p e^{-\frac{\gamma_T}{(1-\rho)\eta_1}}}{(1-\rho)^{2p+1}(p!)^{2p}\eta_1^{2p+2}}$$
$$\times 2\binom{p}{n}\gamma_T^{p-n}\int_0^{\infty}y^p e^{-\frac{((N_r-k+m)(1-\rho)+1)y}{(1-\rho)\eta_1}} \tag{50}$$
$$\times\left(\frac{\gamma_T(y+c)(1-\rho)\eta_1}{\eta_2}\right)^{\frac{n+1}{2}}K_{n+1}\left(\sqrt{\frac{4\gamma_T(y+c)}{\eta_1\eta_2(1-\rho)}}\right)dy.$$

The integral in (50) does not appear to have a closed-form solution. Hence, we substitute $c = 0$ and use [21, Eq. (4.16.37)], to obtain a tight lower bound for (50) as in (15)

APPENDIX IV : HIGH SNR APPROXIMATION - VARIABLE GAIN AF RELAYING

Let $\gamma_{u2} = \min\left\{\widetilde{\gamma}_{1(k)}, \frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\gamma_{1(k)}}\right\}$. It can be shown that $\gamma_{u2} \approx \gamma_{eq2}$ given by (8) in high SNR region. Following a similar approach as in Appendix II, we arrive at the following expression.

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \int_{\gamma_T}^{\infty}\int_0^{\infty} f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)e^{-\frac{\gamma_T y}{\eta_2 x}}dydx\\ &= \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1^2}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\int_{\gamma_T}^{\infty}e^{-\frac{x}{(1-\rho)\eta_1}}\\ &\times\int_0^{\infty}e^{-y\left(\frac{\omega}{\eta_1}+\frac{\gamma_T}{\eta_2 x}\right)}I_0\left(\frac{2\sqrt{\rho x y}}{(1-\rho)\eta_1}\right)dy\,dx \end{aligned}\quad (51)$$

where $\omega = \frac{((N_r-k+m)(1-\rho)+1)}{(1-\rho)}$.

Using [21, Eq. (4.16.14)], and substituting $\eta_2 = \mu\eta_1$, and simplifying the result, we get

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\\ &\times\int_{\gamma_T}^{\infty}\frac{\mu x}{\omega\mu x+\gamma_T}e^{\left(-\frac{x}{(1-\rho)\eta_1}\left(1-\frac{\rho\mu x}{(1-\rho)(\omega\mu x+\gamma_T)}\right)\right)}dx.\end{aligned}\quad (52)$$

Using partial fractions, we can write

$$\begin{aligned}\frac{x}{(1-\rho)\eta_1}\left(1-\frac{\rho\mu x}{(1-\rho)(\omega\mu x+\gamma_T)}\right) &= \frac{p_1}{\eta_1}x\\ +\frac{p_2\gamma_T}{\eta_1}+\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)},\end{aligned}\quad (53)$$

where $p_1 = \frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}, p_2 = \frac{\rho}{\mu((N_r-k+m)(1-\rho)+1)^2}$ and $p_3 = \frac{-\gamma_T^2\rho}{\mu((N_r-k+m)(1-\rho)+1)^2}$. Substituting (53) into (52) yields

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{(1-\rho)\eta_1}e^{-\frac{p_2\gamma_T}{\eta_1}}\\ &\times\int_{\gamma_T}^{\infty}\frac{\mu x}{\omega\mu x+\gamma_T}e^{\left(-\frac{p_1 x}{\eta_1}-\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)}\right)}dx.\end{aligned}\quad (54)$$

Using a variable transformation to modify the range of the above integral to $(0,\infty)$, and the Maclaurin series expansion of $\exp\left(\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)}\right)$ yields

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \frac{k\binom{N_r}{k}}{(1-\rho)}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}e^{-\frac{(p_1+p_2)\gamma_T}{\eta_1}}\\ &\times\sum_{i=0}^{\infty}\frac{(-1)^i p_3^i\mu}{i!\eta_1^{i+1}}\int_0^{\infty}\frac{x+\gamma_T}{(\omega\mu x+\gamma_T(1+\omega\mu))^{i+1}}e^{-\frac{p_1}{\eta_1}x}dx.\end{aligned}\quad (55)$$

With further simplifications, using [25, Eq. (11)] and ignoring higher order terms of η_1^{-1}, one can obtain the following high SNR approximation for the complementary cdf. As $\eta_1 \to \infty$,

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &\approx \frac{k\binom{N_r}{k}}{(1-\rho)}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{\omega}e^{-\frac{(p_1+p_2)\gamma_T}{\eta_1}}\\ &\times\left(\frac{1}{p_1}-\frac{\gamma_T\ln(\eta_1/p_1)}{\mu\omega\eta_1}\right).\end{aligned}\quad (56)$$

Using the above result, we obtain the following approximation for the cdf as $\eta_1 \to \infty$

$$\begin{aligned}F_{\gamma_{u2}}(\gamma_T) &\approx \frac{k\binom{N_r}{k}\gamma_T}{\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\\ &\times\left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho)}\right).\end{aligned}\quad (57)$$

APPENDIX V : HOP SNR DISTRIBUTION - OPPORTUNISTIC RELAYING

To derive the distributions of $\widetilde{\gamma}_{2(k)}$ and $\widetilde{\gamma}_{2(k)}$, the distribution functions of $\gamma_{1(k)}$ and $\gamma_{1(k)}$ must be obtained first. The cdf of $\gamma_{1(k)}$ can be written as

$$\begin{aligned}F_{\gamma_{1(k)}}(\gamma_T) &= \Pr\left(\gamma_{1(l)}<\gamma_T\cap\gamma_{1(l)}>\gamma_{2(l)}\cap l=k\right)\\ &+\Pr\left(\gamma_{1(l)}<\gamma_T\cap\gamma_{1(l)}<\gamma_{2(l)}\cap l=k\right)\\ &= \int_0^{\gamma_T}f_{\gamma_{1(l)}}(x)\int_0^{x}f_{\gamma_{2(l)}}(y)N_r\binom{N_r-1}{k-1}\left(F_{\hat{\gamma}_l}(y)\right)^{k-1}\\ &\times\left(1-F_{\hat{\gamma}_l}(y)\right)^{Nr-k}dydx+\int_0^{\gamma_T}f_{\gamma_{1(l)}}(x)\left(F_{\hat{\gamma}_l}(x)\right)^{k-1}\\ &\times N_r\binom{N_r-1}{k-1}\left(1-F_{\hat{\gamma}_l}(x)\right)^{Nr-k}\int_x^{\infty}f_{\gamma_{2(l)}}(y)dydx.\end{aligned}\quad (58)$$

Since $\gamma_{1(i)}$ and $\gamma_{2(i)}$ are exponential RVs, $F_{\hat{\gamma}_i}(x) = 1 - \Pr\left(\min(\gamma_{1(l)},\gamma_{2(l)})>x\right) = 1-e^{-\frac{x}{\bar{\eta}}}$, with $\bar{\eta} = \frac{\eta_1\eta_2}{\eta_1+\eta_2}$. Employing the binomial expansion yields $\left(F_{\hat{\gamma}_l}(x)\right)^{k-1} = \sum_{m=0}^{k-1}\binom{k-1}{m}(-1)^m e^{-\frac{mx}{\bar{\eta}}}$. Using these results along with the exponential pdf in (58), the integrals in (58) can be evaluated. After further simplifications, $F_{\gamma_{1(k)}}(\gamma_T)$ can be expressed as

$$\begin{aligned}F_{\gamma_{1(k)}}(\gamma_T) &= 1-\sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)}\\ &\times\left(e^{-\frac{\gamma_T}{\eta_1}}+\frac{\eta_2(N_r-k+m)}{\eta_1(N_r-k+m+1)}e^{-\frac{\gamma_T}{\bar{\eta}}(N_r-k+m+1)}\right).\end{aligned}\quad (59)$$

The pdf of $\gamma_{1(k)}$ can be obtained by taking the derivative with respect to γ_T of (59) and is given by

$$\begin{aligned}f_{\gamma_{1(k)}}(y) &= \sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)}\\ &\times\left(\frac{1}{\eta_1}e^{-\frac{y}{\eta_1}}+\frac{\eta_2(N_r-k+m)}{\eta_1\bar{\eta}}e^{-\frac{y}{\bar{\eta}}(N_r-k+m+1)}\right).\end{aligned}\quad (60)$$

Using the relationship in (46), the joint pdf of $\widetilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ can be expressed as

$$\begin{aligned}f_{\widetilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) &= \frac{k\binom{N_r}{k}e^{-\frac{x}{(1-\rho_1)\eta_1}}}{(1-\rho_1)\eta_1^2}I_0\left(\frac{2\sqrt{\rho_1 xy}}{(1-\rho_1)\eta_1}\right)\\ &\times\sum_{m=0}^{k-1}\frac{\binom{k-1}{m}(-1)^m}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)}\left(e^{-\frac{y}{(1-\rho_1)\eta_1}}\right.\\ &\left.+\frac{\eta_2(N_r-k+m)}{\bar{\eta}}e^{-y\left(\frac{(N_r-k+m+1)}{\bar{\eta}}+\frac{\rho_1}{(1-\rho_1)\eta_1}\right)}\right).\end{aligned}\quad (61)$$

Integrating the above joint pdf over $\gamma_{1(k)}$, we arrive at the pdf of $\widetilde{\gamma}_{1(k)}$ given in (21).

REFERENCES

[1] M. Uysal (editor), *Cooperative Communications for Improved Wireless Network Transmission: Frameworks for Virtual Antenna Array Applications*. IGI-Global, 2009.

[2] T. A. Tsiftsis, G. K. Karagiannidis, P. T. Mathiopoulos, and S. A. Kotsopoulos, "Nonregenerative dual-hop cooperative links with selection diversity," *EURASIP J. Wireless Commun. Netw.*, vol. 2006, article ID 17862, pp. 1–8.

[3] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 659–672, Mar. 2006.

[4] I. Krikidis, J. S. Thompson, S. McLaughlin, and N. Goertz, "Amplify and forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, pp. 235–237, Apr. 2008.

[5] A. Bletsas, H. Shin, and M. Z. Win, "Outage optimality of opportunistic amplify-and-forward relaying," *IEEE Commun. Lett.*, vol. 11, pp. 261–263, Mar. 2007.

[6] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, pp. 757–759, Nov. 2006.

[7] B. Maham and A. Hjørungnes, "Performance analysis of amplify-and-forward opportunistic relaying in Rician fading," *IEEE Signal Process. Lett.*, vol. 16, pp. 643–646, Aug. 2009.

[8] M. Torabi, W. Ajib, and D. Haccoun, "Performance analysis of amplify-and-forward cooperative networks with relay selection over Rayleigh fading channels," in *Proc. 2009 IEEE VTC – Spring*, pp. 1–5.

[9] S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami-m fading channel," *IEEE Commun. Lett.*, vol. 11, pp. 334–336, Apr. 2007.

[10] B. Barua, H. Q. Ngo, and H. Shin, "On the SEP of cooperative diversity with opportunistic relaying," *IEEE Commun. Lett.*, vol. 12, pp. 727–729, Oct. 2008.

[11] H. A. Suraweera, D. S. Michalopoulos, and G. K. Karagiannidis, "Semi-blind amplify-and-forward with partial relay selection," *Electron. Lett.*, vol. 45, pp. 317–318, Mar. 2009.

[12] D. B. da Costa and S. Aissa, "End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection," *IEEE Trans. Wireless Commun.* vol. 8, pp. 4306–4315, Aug. 2009.

[13] H. Ding, J. Ge, D. B. da Costa, and Z. Jiang, "Diversity and coding gains of fixed-gain amplify-and-forward with partial relay selection in Nakagami-m fading," *IEEE Commun. Lett.*, vol. 14, pp. 734–736, Aug. 2010.

[14] S. S. Ikki and M. H. Ahmed, "On the analysis of the Nth best-relay cooperative-diversity networks," *IEEE Trans. Commun.*, vol. 58, pp. 3062–3069, Nov. 2011.

[15] J. L. Vicario, A. Bel, J. A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 2872–2876, June 2009.

[16] G. Andrieux, J.-F. Diouris, and Y. Wang, "Channel time variation effects on transmit beamforming in the TDD mode of UMTS," in *Proc. IEEE PIMRC 2002*, pp. 1987–1991.

[17] H. A. Suraweera, M. Soysa, C. Tellambura, and H. K. Garg, "Performance analysis of partial relay selection with feedback delay," *IEEE Signal Process. Lett.*, vol. 17, pp. 531–534, June 2010.

[18] D. S. Michalapoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Relay selection with outdated channel estimates," in *Proc. IEEE GLOBECOM 2010*, pp. 1–6.

[19] M. Seyfi, S. Muhaidat, and J. Liang, "Performance analysis of relay selection with feedback delay and channel estimation errors," *IEEE Signal Process. Lett.*, vol. 18, pp. 67–70, Jan. 2011.

[20] M. Soysa, H. A. Suraweera, C. Tellambura, and H. K. Garg, "Amplify-and-forward partial relay selection with feedback delay," in *Proc. IEEE WCNC 2011*, pp. 1304–1309.

[21] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Tables of Integral Transforms*, vol. 1, ser. Bateman Manuscript Project, California Institute of Technology. McGraw-Hill Book Company, Inc., 1954.

[22] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th edition. Academic Press, Inc., 2000.

[23] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. Dover Publications, Inc., 1970.

[24] A. H. Nuttall, "Some integrals involving the Q-function," Naval Underwater Systems Center (NUSC) Technical Report 4297, Apr. 1972.

[25] F. Xu, F. C. M. Lau, and D.-W. Yue, "Diversity order for amplify-and-forward dual-hop systems with fixed-gain relay under Nakagami fading channels," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 92–98, Jan. 2010.

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